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35. Which one of the following is not true?

A. $p \Rightarrow q \equiv \neg p \vee q$ C. $p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

B. $\neg(p \vee q) \equiv \neg p \wedge \neg q$ D. $p \Rightarrow q \equiv q \Rightarrow p$

36. Which one of the following is a proposition?

A. 123 is a prime number C. $x^2 - 4 = 0$

B. What is your name? D. May God bless you!

37. Which of the following is logically equivalent to $\neg [\neg p \Rightarrow q]$

A. $p \vee \neg q$ B. $\neg p \wedge q$ C. $\neg p \wedge q$ D. $\neg p \wedge \neg$

38. Inverse of the statement pattern $(p \vee q) \Rightarrow (p \wedge q)$ is

A. $(p \wedge q) \Rightarrow (p \vee q)$ C. $(\neg p \vee \neg q) \Rightarrow (\neg p \wedge \neg q)$

B. $\neg(p \vee q) \Rightarrow (p \wedge q)$ D. $(\neg p \wedge \neg q) \Rightarrow (\neg p \vee \neg q)$

39. A conditional statement is false only when the hypothesis is

A. True and the conclusion is false C. True and the conclusion is true

B. False and the conclusion is false D. False and the conclusion is true

40. Let p: It rains and q: Crops are good. Which of the following symbol represent the statement: doesn't rain, the crops will not be good"

A. $p \Rightarrow \neg q$ B. $\neg p \Rightarrow \neg q$ C. $\neg p \Leftrightarrow q$ D. $\neg p \Leftrightarrow \neg q$

41. The number of subsets of the set $A = \{t : t \text{ is even integer and } -1 \leq t \leq 3\}$ is

A. 4 B. 2 C. 32 D. 8

42. If for sets A, B and C, $A \cup C = \{1,2,3,6\}$ and $B \cup C = \{1,3,6,7,8\}$, then $(A \cap B) \cup C$ is equal

A. $\{1,8,7,6,3\}$ B. $\{1,3,6\}$ C. $\{7,8\}$ D. $\{1,2,3,6\}$

43. Show that the compound proposition $(p \wedge q) \Leftrightarrow (p \Rightarrow \neg q)$ is contradiction.

44. Show that the compound propositions $(p \wedge q) \Leftrightarrow p$ and $p \Rightarrow q$ are equivalent.

45. Without using truth table, prove that $[(p \vee q) \wedge \neg p] \Rightarrow q$ is a tautology.

46. Using formal proof (rules of inferences), show that the following argument is valid.

$(p \wedge t) \Rightarrow (r \vee s), q \Rightarrow (u \wedge t), u \Rightarrow p, s, q \vdash r$

47. How many distinct subsets do {3, 7, and 9} has? List them.

48. In a class 100 students, 35 like science and 45 like mathematics. 10 like both. How many like e them? And how many like neither?

49. Among a group of students 50 played cricket, 50 played hockey and 40 played volley ball. 1 both cricket and hockey, 20 played both hockey and volley ball, 15 played both cricket and and 10 played all the three. If every student played at least one game. Find the number of stude how many played only cricket, only hockey and only volley ball?

16. Find the truth value of each of the following statements where p, q, r *and* s are assigned the truth values T, F, F, and T respectively.

a) $(s \Rightarrow p) \Leftrightarrow (\neg p \wedge s)$ c) $(\neg r \wedge s) \Rightarrow (q \Rightarrow \neg p \vee s)$

b) $(p \vee \neg q) \vee r \Leftrightarrow (\neg s \wedge r)$ d) $(p \Rightarrow \neg q) \wedge s \Leftrightarrow \neg r \vee p$

17. Find the truth value of the following statement under the given conditions.

a) $(q \Rightarrow p) \Rightarrow r$ given that r is True.

b) $\neg q \Leftrightarrow p$ given that $p \Leftrightarrow q$ is False.

c) $\neg p \wedge \neg q \Leftrightarrow \neg(p \vee q)$ given that $p \vee q$ is False.

18. Show that whether the following formulas are a tautology or Contradiction or not.

a) $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ c) $(p \wedge q) \Leftrightarrow (p \Rightarrow \neg q)$

b) $(p \Rightarrow q) \Rightarrow (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

19. Show whether the following are equivalent proposition or not.

a) $P: p \vee q$ *and* $Q: \neg p \Rightarrow q$ c) $R: p \Rightarrow (q \wedge \neg q)$ *and* $S: \neg p$

b) $T: p \wedge q \Rightarrow p$ *and* $U: (p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

20. Show the validity of the following arguments using truth table

a) $p, r, q \wedge p \Rightarrow \neg r \vdash \neg q$ b) $p \Rightarrow q, \neg(q \vee r) \vdash \neg p$

21. Let p and q are two propositions. Then which of the following logical expressions are Equivalent?

i. $p \vee \neg q$ iii. $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$

ii. $\neg(\neg p \wedge q)$ iv. $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

a) Only i & ii b) Only i, ii & iii c) Only i, ii & iv d) All of the above

22. For propositions p and q, $(p \wedge q) \wedge (\neg p \vee \neg q)$ is a contradiction.

23. The conditional statement $p \Rightarrow q$ and its contra-positive are logical equivalent.

24. If the truth values of the statements p, q, r *and* s are assigned the values T, F, F *and* T respectively, then the truth value of $(s \Rightarrow p) \Leftrightarrow (\neg p \wedge s) =$ _____

25. If $(p \vee \neg p) \Rightarrow \neg q$ is True, then the truth value of q is _____

26. The truth value for negation of $(\exists x)(\forall y)(x < y \wedge y = x + 1)$ is _____

27. If $(p \vee \neg p) \wedge \neg q$ is true, then the truth value of q is _____

28. The negation of $(\exists x \in \mathbb{R})(x^2 + 1 = 0)$ is _____

29. Let $A = \{x \in \mathbb{R}: x^2 - 3x + 2 = 0\}$. Then $P(A) =$ _____

30. Let $\neg p \Rightarrow q$ is false. Then the truth value of $p \vee \neg q =$ _____

31. If $A = \{1, 2, \{4, 5\}\}$ and $B = \{3, 4, 5\}$, then $A \cap B =$ _____

32. Given that r is true, the true value of the statement $(q \Rightarrow p) \Rightarrow r$ is: _____

33. The number of subsets of the set $A = \{x: x \text{ is an even integer and } -1 \leq x \leq 3\}$ is equal to _____

34. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then $(A \times B) \cup (A \times C) =$ _____

Work Sheet

7. Use the truth table method to show that the following argument forms are valid or not.

a) $p \Longrightarrow \neg p, q, r \Longrightarrow q \vdash r$

b) $\neg q \vee \neg r, (\neg r \Longrightarrow p) \Longrightarrow q \vdash p \Longrightarrow q$

c) $p \Longrightarrow q, \neg p \Longrightarrow r, r \Longrightarrow s \vdash \neg q \Longrightarrow s$

8. Give a formal proof for the validity of the following arguments.

a) $p \Longrightarrow q, \neg r \Longrightarrow \neg q \vdash \neg r \Longrightarrow \neg p$

b) $p \Longrightarrow q, r \Longrightarrow \neg q \vdash p \Longrightarrow \neg r$

c) $p \Longrightarrow q, \neg r \Longrightarrow (s \Longrightarrow t), r \vee (p \vee s), \neg r \vdash q \vee t$

d) $(p \vee \neg q) \vee r, \neg p \vee (q \vee \neg p) \vdash q \Longrightarrow r$

9. List all the elements of each of the following and determine their power set.

a) $A = \{\{1,2\}, 3, \{0,\{1\}\}\}$

b) $B = \{\{a\}, (b,c), x, \{\{y,\{z\}\}\}\}$

10. Determine which of the following statements are true and which are false.

a) $\emptyset \in \{\emptyset\}$

b) $\{\emptyset\} \subseteq \emptyset$

c) $\{\emptyset\} \subseteq \{\emptyset\}$

d) $\emptyset \cap \{\emptyset\} = \{\emptyset\}$

e) $\{\emptyset, \{\emptyset, \{\emptyset\}\}\} - \{\{\emptyset\}\} = \emptyset$

f) $\{\emptyset, \{\emptyset\}\} - \{\emptyset\} = \{\emptyset\}$

11. For any two sets A and B, find the truth value of each of the following.

a) $\emptyset \in P(\emptyset)$

b) $P(\emptyset) = \{\emptyset\}$

c) $\emptyset \subseteq P(A)$

d) $A \subset P(A)$

e) $\emptyset \subset P(A)$

f) $A \in P(A)$

g) $A \Delta \emptyset = A$

h) $A \Delta A = A$

i) $A - B = A \Delta (A \cap B)$

12. Let $A \cup B = \{a, b, c, d, x, y, z\}$ and $A \cap B = \{a, x\}$.

a) If $B - A = \{b, c\}$, then $A - B =$ _____

b) If $B - A = \emptyset$, then $A =$ _____ and $B =$ _____

c) If $A = \{a; b, c, x\}$, then $B - A =$ _____

d) $A \Delta A =$ _____

13. In a class of 51 students, 42 students study Mathematics, 39 students study Physics and 6 students study neither. Calculate the number of students who study both Mathematics and Physics.

14. Test Validity of the following propositions.

a) If 6 is a composite number, then 12 is a composite number. If 12 is a composite number. If 12 is a composite number, then there exists a prime number greater than 12. If there exists a prime number greater than 12, then there exists a composite number greater than 12. If 2 divide 6, then 6 is a composite number. The number 12 is composite. Therefore, 6 is composite number.

b) If 8 is even, then 2 does not divide 7. Either 5 is not prime or 2 divides 7. But 5 is prime. Therefore, 8 is odd.

c) Either I shall go home or stay and have a drink. I shall not go home. Therefore, I shall stay and have a drink.

15. Let the universe be the set natural numbers and let

$P(x,y): y = x + 5$, $Q(x,y): x = y$, $H(x,y): x > y$, $G(x,y,z): x + y = z$

Determine the truth –value of each of the following statements.

a) $(\forall x)(\forall y)P(x,y)$

b) $(\exists x)(\exists y)Q(x,y)$

c) $(\exists x)(\forall y)(\exists Z)G(x,y,Z)$

d) $(\forall x)(\exists y)[\neg P(x,y) \Rightarrow Q(x,y)$

e) $(\forall x)(\exists y)[H(x,y) \wedge \neg Q(x,y)]$

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Mathematics for Natural Science (Math 101N) Work Sheet I

1. Which of the following sentences are propositions? For those that are indicate the truth value (if possible).
   a) I wish the area of every circle was equal to π.
   b) A rectangle has four equal sides.
   c) Today is my Birthday.
   d) I am determined to get a grade of A in Math 1011.
   e) $x^2 < 0$, for every real number x.
   f) $x^2 < 0$.
   g) $a + (b \cdot c) = (a + b) \cdot (a + c)$, for all real numbers a, b and c.
   h) The number of stars in the universe is finite.
   i) Emperor Menilik II ate Injera with chicken soup the night after the battle of Adewa.
2. Determine the truth value of:
   a) All Africans are black men only if all black men are Africans.
   b) All prime numbers are odd if 2 is prime.
   c) All squares are rectangles provided that all quadrilaterals are rectangles.
3. Suppose that the truth value of $p \Leftrightarrow q$ is F, what can be said about the truth value of:
   a) $(p \vee q) \Rightarrow (p \wedge q)$ c) $(p \Rightarrow q) \wedge (q \Rightarrow p)$
   b) $(p \Rightarrow q) \vee (q \Rightarrow p)$ d) $\neg p \Leftrightarrow q$
4. For statements p, q and r, use the truth table or a formal method to show that each of the following pair of statements is logically equivalent.
   a) $p \Rightarrow q$ and $(p \wedge q) \Leftrightarrow p$ c) $p \Rightarrow (q \vee r)$ and $\neg r \Rightarrow (p \Rightarrow q)$
   b) $(p \vee q) \Rightarrow r$ and $(p \Rightarrow q) \wedge (q \Rightarrow p)$ d) $p \Rightarrow (q \vee r)$ and $\neg q \Rightarrow (\neg p \vee r)$
5. Let $p \equiv$ He is rich. Let $q \equiv$ He is happy. Then write the following in symbol form.
   a) He is poor but happy.
   b) He is neither rich nor happy.
   c) He is unhappy when he is poor.
   d) He is happy only if he is rich.
   e) He is poor or else he is both rich and unhappy.
   f) He is rich if he is happy.
6. Let the universal set, $U = \mathbb{R}$. Assign a truth value to each one of them.
   a) $(\forall x)(x^2 + 2x \geq 1)$
   b) $(\forall x)(\sqrt{x^2} = x)$
   c) $(\forall x)(\exists y)(x + y = x)$
   d) $(\forall x)(\exists y)(x \cdot y = 1)$
   e) $(\exists x)(x^2 + 2x < 1)$
   f) $(\exists x)(\forall y)(x + y = x)$
   g) $(\exists y)(\forall x)(x \cdot y = y)$
   h) $(\exists x)(\forall y)(x \leq y^2)$
   i) $(\forall x)(\forall y)[(x < y) \vee (x > y)]$
   j) $(\forall x)(\forall y)[x^2 - y^2 = (x + y)(x - y)]$