

Chapter 7

Hypothesis testing

7.1 Basic principles in design and evaluation of hypothesis

- **A hypothesis** is an idea about something which is given by the researcher /person/ as an option for the given issue or
- it is an assertion regarding the value of a population parameter based on the sample information.
- Statistical hypothesis testing is used for different fields of study and for economics in particular.

We have two types of statistical hypothesis:

i. Null hypothesis (H_0):

- It is the hypothesis to be tested.
- It is an assertion that a population parameter assumes a fixed (beginning) value.
- Usually it is a statement of inert (no effect).
- It always includes the equality sign
- It is denoted by H_0 .
- It is the hypothesis of equality or the hypothesis of no difference.

ii. **Alternative hypothesis (H_1):**

- It is the alternative available when the null hypothesis has to be rejected.
- It is the complement of the null hypothesis.
- It always includes the inequality sign (It is the hypothesis of difference).
- It is denoted by H_1 .
- The Researcher cannot study every item or individual in the population.
- Thus, there is a possibility of 2 types of error.
 - i. Type I Error
 - ii. Type II Error

i. Type I error

- ✓ The rejecting of H_0 , when it is actually true.
- ✓ It is the probability of rejecting H_0 , while it is actually true is equal to “ α ”
- ✓ It is α is called level of significance.

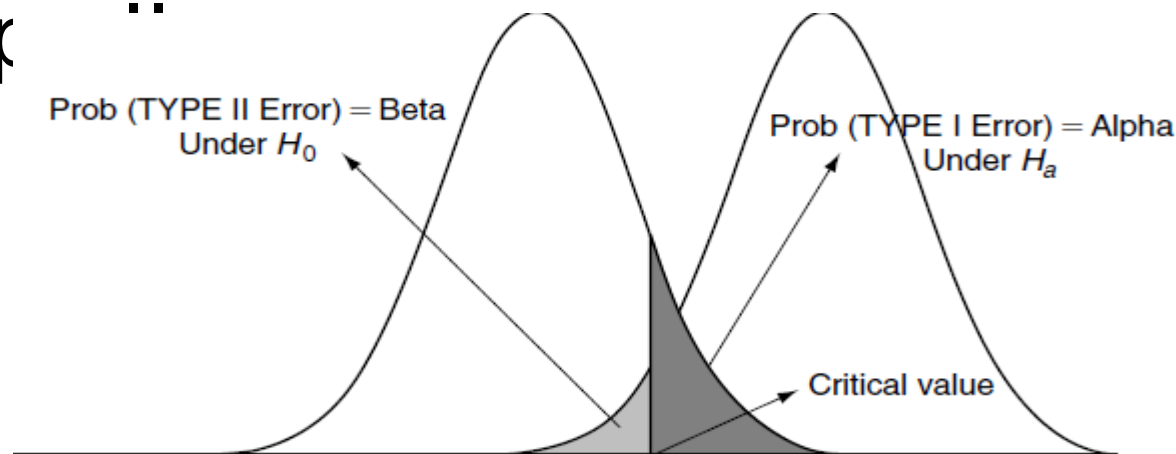
ii. Type II error

- ✓ Not rejecting H_0 , while it is false.
- ✓ The probability of type II error is equal to β .

Level of significance (α): It is the probability of type I error, which is the probability that depicts the level of confidence for the researchers conclusion.

It is usually understood as the allowable error.

- Note: **Type I error**(α) and **Type II error**(β) have inverse relationship, then can not be minimized at the same time.
- In practice we set(α) at some value & design a test minimize β .
- This is because type I error is often considered to be more serious, and therefore more important to avoid than Type II



<u>Decision</u>	<u>Accept H_0</u>	<u>Reject H_0</u>
<u>If H_0 is true and</u>	<u>Correct decision</u>	<u>Type I error(α)</u>
<u>If H_0 is false and</u>	<u>Type II error(β)</u>	<u>Correct decision</u>

Types of testing:

- **one - tailed test:** A test of hypothesis, when the alternative hypothesis (H_1) is given by the inequality sign, because it is given in one direction

Example: - if $H_0: \mu = \mu_0$

$$H_1: \mu > \mu_0 \text{ or } H_1: \mu < \mu_0$$

- **Two - tailed tests:** If the alternative hypothesis is given by the statement or symbol ($\neq$)

- **Example:** if $H_0: \mu = \mu_0$

$$H_1: \mu \neq \mu_0, \text{ since } H_1: \text{ has a}$$

compound statement this type of testing is called two tailed test.

Hypothesis testing:

It is a procedure for examining the validity of the statistical hypothesis.

Critical value:

It is the boundary/ demarcation point between the acceptance and the rejection area.

Test statistic:

It is a value which is computed from a sample that is used to make a decision whether the null hypothesis should be accepted or rejected.

Hypothesis testing procedures (steps)

In hypothesis testing following general steps:

Step 1: Determining(state) the null and alternative hypothesis

Suppose the assumed or hypothesized value of mean is denoted by μ_0 , then formulate two side or one side hypothesis, such as **two sided test, one sided test to the**

1. $H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$
2. $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$
3. $H_0 : \mu = \mu_0$ vs $H_1 : \mu < \mu_0$

Step 2: Choosing the level of significance.

It is referring to the probability of making a type I error (α)

Step 3: Determining the test statistic

Case 1: Population variance (σ^2) is known

✓ The relevant test statistic is $Z_{cal} =$

Case 2: Population variance (σ^2) is unknown
& when the sample size is large ($n \geq 30$),

✓ The relevant test statistic is $Z_{cal} =$

Case 3: when sampling is from normally distribution population with **unknown variance and the sample size is small ($n < 30$)**

✓ The relevant test statistic is $t_{cal} =$ **with $n-1$ d.f**

Step 4: Define a decision rule (the rejection or critical regions) based on the level of significance chosen in step 2. And the sampling distribution from step 3

Hypothesis	Reject H_0 if	Accept H_0 if
$H_0: \mu = \mu_0$ Vs $H_1: \mu \neq \mu_0$	$ Z_{cal} > Z_{\alpha/2}$	$ Z_{cal} \leq Z_{\alpha/2}$
$H_0: \mu = \mu_0$ Vs $H_1: \mu > \mu_0$	$Z_{cal} > Z_{\alpha}$	$Z_{cal} \leq Z_{\alpha}$
$H_0: \mu = \mu_0$ Vs $H_1: \mu < \mu_0$	$Z_{cal} < -Z_{\alpha}$	$Z_{cal} \geq -Z_{\alpha}$

Note: $t_{cal} =$ with $n-1$ d.f

Hypothesis	Reject H_0 if	Accept H_0 if
$H_0: \mu = \mu_0$ Vs $H_1: \mu \neq \mu_0$	$t_{cal} \geq t_{\alpha/2}$	$t_{cal} < t_{\alpha/2}$
$H_0: \mu = \mu_0$ Vs $H_1: \mu > \mu_0$	$t_{cal} \geq t_{\alpha}$	$t_{cal} < t_{\alpha}$
$H_0: \mu = \mu_0$ Vs $H_1: \mu < \mu_0$	$t_{cal} \leq -t_{\alpha}$	$t_{cal} > -t_{\alpha}$

Step 5: performing the statistical test.

Calculate the test statistic based on the available information from the sample.

Step 6: Decision: compare the test statistic with the decision rules

Step 7: **Conclusion:** draw the statistical conclusion.

7.2 Tests about the mean

Example 1: The ADLI(Agricultural Development Led Industrialization) strategy will continue its policy if it brings an impact on house hold earnings. So that it will be resumed, if the mean annual house hold income reaches **at least 24,000 Birr**. If the government is checking the condition frequently and took a random sample of 100 house holds to be 23, 600 Birr with a standard deviation of 4000 Birr. At the 5% level of significant, would you conclude that ADLI should

Solution: Given, $\mu_o = 24,000$, $\bar{x} = 23,600$, $S = 4000$, $n = 100$

Step 1: **Determining(state) the hypothesis**

$$\mathbf{H_o: \mu = 24,000 \quad Vs \quad H_1: \mu > 24,000}$$

Step 2: choosing(select) the level of significance $\alpha = 0.05$

Step 3: Determining the test statistic

Z-statistics is appropriate because the sample size is large

The relevant test statistic is $Z_{cal} =$

Step 4: identify the critical region

$$Z_{0.05} = 1.64, \quad Z_{0.05} = 1.64$$

Step 5: Computation $Z_{cal} = -1$

Step 6: Decision rule: Z_{cal} less than 1.64 ,

Accept then H_o

Step 7: There is no enough evidence to reject H_o at $\alpha = 0.05$

Therefore, the strategy may continue as a policy instrument.

Example 2: The production of garment in Bahir Dar textile factory is normally distributed with mean 400 and standard deviation 32, assuming that a random sample of 100 parts is taken from the whole production in a particular period of time yielded a mean 406. Test the mean using the level of significance = 0.05.

Solution: Given $\mu_0=400$, $n=100$, $\bar{x}=406$, $\sigma=32$

Step 1: Determining(state) the hypothesis

$H_0:\mu=400$ Vs $H_1:\mu\neq 400$

Step 2: choosing(select) the level of significance
 $\alpha=0.05$

Step 3: Select the appropriate test statistics

Z- distribution appropriate because the variance known and sample size large (n

Step 4: identify the critical region

$Z_{0.05/2} = Z_{0.025} = 1.96$, the acceptance region $(-1.96, 1.96)$

Step 5: Computation, $Z_{cal} = 1.875$

Step 6: Decision rule

Accept H_0 , since H_0 is acceptance region

Step 7: The null hypothesis is not rejected. No significant evidence from the sample at $\alpha = 0.05$, to conclude that the mean different from 400.

The P-value for the test is : $P[|Z| \geq 1.88] = 2[0.5 - P(0 \leq Z \leq 1.88)] = 2[0.5 - 0.4699] = 2(0.0301) = 0.0602$

Hence, since $p\text{-value} = 0.0602 > \alpha$, accept H_0

Tests involving two populations' means (large samples)

- According to the socio-economic phenomenon the researcher may be interested in comparing two populations with respect to a given characteristics (in particular two population means).

For example,

- ✓ mean productivity of labor of a company under different conditions,
- ✓ mean performances of two groups of farmers,
- ✓ mean performance of two groups of students in this course,...

- Here the step hypothesis – testing procedure is the same one used for one sample tests.
- And the type of testing have the same as in the previous, i.e., posses one-tailed test and two-tailed test.
- However, the formula for the test statistic (Z_{cal}) is slightly different, and The difference between two normal distribution is also a normal distribution.

Tests' concerning two populations means (large samples)

The test distribution, where the sample size (n) is large, and the population standard deviations are given (known) or if population standard deviations are

$$\text{Use, } Z_{\text{cal}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \text{ for } n - \text{is large and } \sigma_1^2 \text{ \& } \sigma_2^2 \text{ are known}$$

$$\text{Use } Z_{\text{cal}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}, \text{ for } n - \text{is large, but } \sigma_1^2 \text{ \& } \sigma_2^2 \text{ are unknown (not given)}$$

if these samples have the variances , for the two same normal populations and assuming that the two populations have **equal variances**, then we can find the common standard deviation the so called **pooled standard deviation**.

- Hence, $S_p =$
- Then use student (t – distribution) for test statistic.
- Then $t_{cal} =$ is t-distribution with $n_1 + n_2 - 2$ degrees of freedom and for $n < 30$ (small)

Example 3: A farmer is interested to know the efficiency of teff production. It is hypothesized that there is no difference between the mean product of teff in << red soil>> and << Black soil>>. A random sample of 35 hectares of red soil land and 45 hectares of black soil land were tested.

- The mean product of teff in red soil was 3.8 tons with variance 0.64 tone.
- The mean product of teff in black soil was 3.5 tons with variance of 0.25 tones. At the 0.05 level of significance, determine for the farmer if there is a significant difference in the outputs of the two groups of lands.

Solution:

Given: $\bar{x}_1 = 3.8$, $s_1^2 = 0.64$, $n_1 = 35$

$\bar{x}_2 = 3.5$, $s_2^2 = 0.25$, $n_2 = 45$, $\alpha = 0.05$

and let μ_1 and μ_2 are the mean output production of teff in the red soil and black soil respectively.

Step 1: state the hypothesis

H_0 := H_1 :

Step 2: select the level of significance $\alpha = 0.05$

Step 3: select the appropriate test statistics

Z- distribution is the appropriate test statistics because the population variance known and sample size large (n_1 and n_2 are large)

Step 4: identify the critical region

= , Acceptance region (-1.96, 1.96)

Step 5: computation: $z_{cal} = 1.94288$

Step 6: conclusion: **$|z_{cal}| < z_{0.025}$ Accepts H_0 ,
(1.94288 < 1.96)**

Step 7: There is no enough evidence to reject H_0 at $\alpha = 0.05$ level of significance

Therefore there is no significant difference between the mean products (output) of the two groups of lands.

Example 4: Suppose the teacher is interested to compare the performance of students in two groups, assuming that the 1st group has 5 students while the 2nd group has 6 students. And the sample means and variances are 4 and 5; 8.5 and 4.4 for the 1st and 2nd group respectively. Test whether the two groups have different population means at level of significance 0.05.

Solution:

Given: $n_1 = 5, \bar{x}_1 = 8.5$; $n_2 = 6, \bar{x}_2 = 4.4$ and $\alpha = 0.05$.

Step 1: state the null and alternative hypothesis

$H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$

Step 2: Select the level of significance $\alpha = 0.05$

Step 3: Select the appropriate test statistics
t – distribution $n_1 < 30$ and $n_2 < 30$ &
population variance unknown

Step 4: identify the critical region
,Acceptance region $(-, 2.262)$

Step 5:Computation: Hence, $S_p =$
 $S_p = 2.49$, then $t_{cal} = -0.66$

Step 6: $|t_{cal}| < \alpha/2$, **Accept H_0**

Step 7:Hence, there is no significant
evidence from the sample to say that the
two populations have the **same**
mean. ???????

Test about the variances

- There are lot of reasons why this type of test is important to test hypotheses concerning the variance of populations.
- There distinguishes between direct and indirect applications.
- **The direct applications** will have to perform tests about the variability of his product;
- **Indirect applications** are concerned, tests about variances are usually prerequisites for tests regarding to other parameters.

For example in the case of two samples t-test requires the two population variances be equal, and hence etc.

The test regarding whether a given population variance () equals to a specified value may take one of the following form:

I. Two-tailed test:

$$H_0: \sigma^2 = \sigma_0^2$$

$$H_1: \sigma^2 \neq \sigma_0^2$$

II. One-tailed test

a. $H_0: \sigma^2 = \sigma_0^2$

$$H_1: \sigma^2 < \sigma_0^2$$

b. $H_0: \sigma^2 = \sigma_0^2$

$$H_1: \sigma^2 > \sigma_0^2$$

Test statistic:

Chi – square distribution is employed for the test-statistic.

then $X_{cal}^2 = \frac{(n-1)S^2}{\sigma_0^2}$, where $S^2 = \frac{\sum (xi - \bar{x})^2}{n-1}$

Test rule

For chi-square distribution and for a fixed α at $n - 1$ degrees of freedom, we have:

i) $H_0: \sigma^2 = \sigma_0^2$

$$H_1: \sigma^2 \neq \sigma_0^2$$

Reject H_0 : if $X_{cal}^2 > X_{\frac{\alpha}{2}}^2$ or $X_{cal}^2 \leq X_{\frac{1-\alpha}{2}}^2$

ii. $H_0: \sigma^2 = \sigma_0^2$

$$H_1: \sigma^2 < \sigma_0^2$$

Reject H_0 : if $X_{cal}^2 \leq X_{1-\alpha}^2$

iii. $H_0: \sigma^2 = \sigma_0^2$

$$H_1: \sigma^2 > \sigma_0^2$$

Reject H_0 : if $X_{cal}^2 \geq X_{\alpha}^2$

Testing the equality of two populations variances

Given independent random samples of size n_1 & n_2 from two normal populations with the variances and the test for equality of the two variances may take one of the following form:

- i. Two tailed test: $H_0: V_s = V_t$
- ii. one tailed test left: $H_0: V_s \geq V_t$
- iii. one tailed test right: $H_0: V_s \leq V_t$

Test statistic: F – Distribution is employed for the test, statistic then $F_{cal} = \frac{s_1^2}{s_2^2}$, where s_1^2 and s_2^2 are the sample variances drawn from sample one of population one and sample two of population two.

- **Test rule:**

Two tailed test: $H_0: V_s \square$

Reject H_0 , if $(-1, -1)$ or
 $(-1, -1)$

one tailed test left: $H_0: V_s$

Reject H_0 , if $(-1, -1)$

one tailed test right: $H_0: V_s$

Reject H_0 , if $(-1, -1)$

Example: A sample of $n = 19$ drawn a normally distributed population depicts a variance of $S^2 = 2.5$ Test the hypothesis that the population variance $= 3.5$ against the alternative 3.5 at 5% level of significance

Solution:

Step 1: state the null & alternative hypothesis

$= 3.5$ Vs 3.5

Step 2: select the level of significance $\alpha = 0.05$

Step 3: Select the appropriate test statistics
-is the appropriate test statistics

Step 4: Specify the critical region

Step 5: Computation

$= 12.857$

Step 6: decision rule:

Step 7: Conclusion

Which means that there is no significant evidence from the sample to conclude that alternative is different from 3.5

In comparing the variability of the test of two kinds of beers, an experiment gave the following results: $n_1 = 13, s_1^2 = 21$ and $n_2 = 16, s_2^2 = 5$. Assuming that these two independent samples evolved from two normal populations, test $H_0: \sigma_1^2 = \sigma_2^2$ Vs $\sigma_1^2 \neq \sigma_2^2$ at 5% level of significance.

Solution:

Step 1: State the null and alternative hypothesis

$\sigma_1^2 = \sigma_2^2$ Vs $\sigma_1^2 \neq \sigma_2^2$

Step 2: Select the level of significance $\alpha = 0.05$

Step 3: Select the appropriate test statistics

F-distribution is the appropriate test statistics

Step 4: specify the critical region

$$\cancel{2.48} = 2.9633$$

Step 5: Computation

$$F_{cal} = 4.2$$

Step 6: Decision rule

If $(-1, -1)$ reject H_0 . Such as $(4.2 > 2.48)$
reject H_0

Step 7: Conclusion

Since $F_{cal} > F_{tab}$, the H_0 : rejected and conclude that the variability of the test of the two kinds of Beers is not the same (i.e., they are different one with another).

Reading Assignment

- Sampling Distribution of the sample proportion
- Confidence interval for population proportion
- Hypothesis testing for population proportion

THANK
YOU

