

Bashewam KG, Primary, G. Secondary and Preparatory School

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2012 E.C 1st Semester 2nd term Physics Handout for Grade 12

Chapter - 6 Magnetic Fields

The **magnetic field** or **B field** is a field that produces a force on a moving charge. This is not the only definition, but it is a good operational definition to start with. The force is given by the vector relation

$$\mathbf{F} = q_0 \mathbf{v} \times \mathbf{B} \quad (33-1)$$

Cross products are discussed in detail in the Introduction, Mathematical Background. In a coordinate system the relation is shown in the accompanying figure.

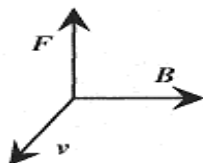


Fig. 33-1

To fix the direction of the force on a positive charge, place the right hand with fingers pointing in the direction of $\mathbf{v}$, and a line normal to the palm of the hand pointing in the direction of $\mathbf{B}$. Curl the fingers naturally into $\mathbf{B}$. The thumb points in the direction of $\mathbf{F}$. For a negative charge the force is in the opposite direction. The force is a right angle to the plane of $\mathbf{v}$ and $\mathbf{B}$. Since this is a vector it can be expressed as a cross product in determinant form,

$$\mathbf{F} = q_0 \mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \quad (33-2)$$

Or in magnitude and direction form, $F = vB \sin \phi$, where ϕ is the angle between $\mathbf{v}$ and $\mathbf{B}$ and $\mathbf{F}$ points in a direction normal to the plane of $\mathbf{v}$ and $\mathbf{B}$ as determined by the right-hand rule. Most problems involve $\mathbf{v}$ and $\mathbf{B}$ at right angles, so the magnitude of the force reduces to the product of q , v , and B with the direction of $\mathbf{F}$ determined by the right-hand rule.

The units of B are (from the force equation 33-1) N.s/C.m or N/A.m, which is called a **Tesla** (T). The **Tesla** is related to the smaller and widely used unit the **gauss** (1 Tesla = 10^4 gauss).

Ex-1) Calculate the magnetic force on electron moves with a speed v through the earth's magnetic field which has a value of 0.40 gauss at a particular location. Take the electron with 10KeV energy and $m_e = 9.1 \times 10^{-31} \text{ kg}$.

Solution: First calculate the velocity of the electrons using an energy relation $1\text{eV} = (1/2)mv^2$

$$10 \times 10^3 \text{ eV} (1.6 \times 10^{-19} \text{ J/eV}) = (1/2)mv^2 = (1/2)9.1 \times 10^{-31} \text{ kg} \cdot v^2$$

$$v = \left[\frac{2 \cdot 1.6 \times 10^{-15} \text{ J}}{9.1 \times 10^{-31} \text{ kg}} \right]^{1/2} = 5.9 \times 10^7 \text{ m/s}$$

Convert gauss to Tesla: $0.40 \text{ gauss} (1\text{T}/10^4 \text{ gauss}) = 4.0 \times 10^{-5} \text{ T}$.

The force on the electron is

$$F = qvB = 1.6 \times 10^{-19} \text{ C} (5.9 \times 10^7 \text{ m/s}) 4.0 \times 10^{-5} \text{ T} = 3.8 \times 10^{-16} \text{ N}$$

Ex-2) Take an electron with velocity (components) $\mathbf{v} = (2.0 \mathbf{i} - 3.0 \mathbf{j}) 10^6 \text{ m/s}$ and a $\mathbf{B}$ field of $\mathbf{B} = 4.0 \text{ T}$. What is the magnetic force on the electron?

Solution: The force vector in component form is

$$\mathbf{F} = 1.6 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.0 & -3.0 & 0 \\ 0 & 0 & 4.0 \end{vmatrix} \times 10^{-16} \text{ N} = 1.6 \left[\begin{vmatrix} -3.0 & 0 \\ 0 & 4.0 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2.0 & 0 \\ 0 & 4.0 \end{vmatrix} \mathbf{j} \right] \times 10^{-16} \text{ N}$$

$$\mathbf{F} = 1.6 [-12 \mathbf{i} - 8.0 \mathbf{j}] \times 10^{-16} \text{ N} = [-1.9 \mathbf{i} - 1.3 \mathbf{j}] \times 10^{-15} \text{ N}$$

The Cyclotron

If a particle with negative charge, mass, and velocity is injected into a $\mathbf{B}$ field in a plane normal to the field the force is at right angles to the velocity and remains at right angles to the velocity throughout the motion causing the charged particle to go in a circle as shown in Fig. 33-4. A positively charged particle would move in a circle in the opposite direction.

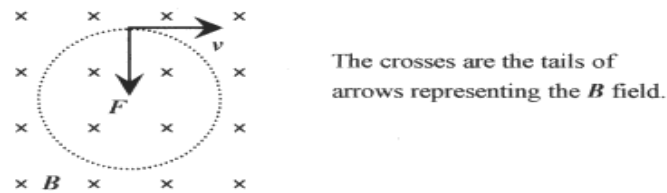


Fig. 33-4

The force is at right angles to the velocity. Apply the $\mathbf{v} \times \mathbf{B}$ law for a negative charge. The magnitude of this force is qvB , and it is always at right angles to the velocity. From mechanics, a motion produced by a force at right angles to the velocity is circular, thus the charged particles move in circular orbits. This qvB force provides the (mechanical force) mv^2/r , so equating these forces

$$qvB = mv^2/r \quad (33-3)$$

There are several relations that follow from this statement, but for the cyclotron the most important is one for the orbit radius

$$r = mv/qB \quad (33-4)$$

And the one for the angular frequency

$$\omega = \frac{v}{r} = \frac{qB}{m} \quad (33-5)$$

Ex-3) An electron of energy 4.0 KeV is circulating in a plane at right angles to a magnetic field of 3.0T. What is the radius of the orbit?

Solution: The geometry is the same as in Fig. 33-4. First, calculate the velocity from the energy

$$4.0 \times 10^3 \text{ eV} (1.6 \times 10^{-19} \text{ J/eV}) = \frac{1}{2} (9.1 \times 10^{-31} \text{ kg}) v^2 \quad \text{or} \quad v = 3.8 \times 10^7 \text{ m/s}$$

Now calculate r

$$r = \frac{mv}{qB} = \frac{9.1 \times 10^{-31} \text{ kg} \cdot 3.8 \times 10^7 \text{ m/s}}{1.6 \times 10^{-19} \text{ C} \cdot 3.0 \text{ T}} = 7.1 \times 10^{-5} \text{ m}$$

Ex-4) An α -particle moves in a circle of radius $5.0 \times 10^{-2} \text{ m}$ in a magnetic field of 2.0 T . Find the speed of the α -particle and the frequency and period of the motion.

Solution: An α -particle is a helium nucleus, which has two protons and two neutrons. The speed of the α -particle is from the defining equation for the cyclotron.

$$v = \frac{q r B}{m} = \frac{2 \cdot 1.6 \times 10^{-19} \text{ C} (5.0 \times 10^{-2} \text{ m}) 2.0 \text{ T}}{4 \cdot 1.7 \times 10^{-27} \text{ kg}} = 4.7 \times 10^6 \text{ m/s}$$

The angular velocity is (analogous to mechanics)

$$\omega = \frac{v}{r} = \frac{q B}{m} = \frac{2 \cdot 1.6 \times 10^{-19} \text{ C} \cdot 2.0 \text{ T}}{4 \cdot 1.7 \times 10^{-27} \text{ kg}} = 9.4 \times 10^7 \text{ 1/s}$$

The frequency, or what is called the cyclotron frequency, is $f = \omega/2\pi = 1.5 \times 10^7 \text{ 1/s}$. The period, or time for one transit of the circle is $T = 1/f = 6.7 \times 10^{-8} \text{ s}$.

Magnetic Force on a Current-Carrying Conductor

If a magnetic field exerts a force on a single charged particle when it moves through a magnetic field, it should be no surprise that magnetic forces are exerted on a current-carrying wire as well. Because the current is a collection of many charged particles in motion, the resultant force on the wire is due to the sum of the individual forces on the charged particles.

Magnetic fields exert forces on moving charges. If these charges are confined to wires then the fields can be thought of as exerting forces on current-carrying wires. The vector equation for the force is

$$\mathbf{F} = i \mathbf{l} \times \mathbf{B} \quad (34-1)$$

Where, l is the length of the wire taken in the direction of the current. In most instances it is not necessary to do a formal vector cross product because the direction of the current and the magnetic field are perpendicular.

Ex-5) A wire carries a current of 22.0 A from west to east. Assume the magnetic field of Earth at this location is horizontal and directed from south to north and it has a magnitude of $0.5 \times 10^{-4} \text{ T}$. Find
(a) the magnitude and direction of the magnetic force on a 36.0-m length of wire, and
(b) gravitational force on the same length of wire if it's made of copper and has a cross-sectional area of $2.5 \times 10^{-6} \text{ m}^2$.

A Current-Carrying Loop in a B Field

The study of torque on a current-carrying loop leads to the definition of the magnetic moment of a circulating current, the understanding of the meter movement (voltmeter and current meter), and electric motors and generators. A loop of wire placed in a magnetic field and oriented as shown in Fig. 34-3 will have no torque or maximum torque, depending on the orientation (rotation) of the loop with respect to the magnetic field.

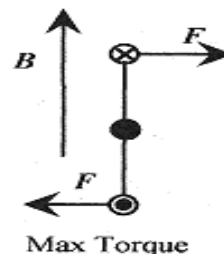
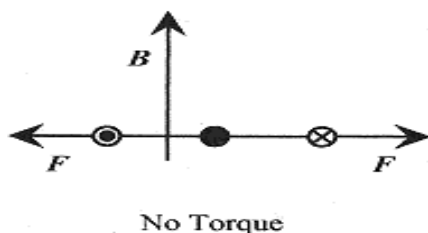
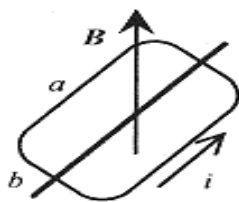


Fig 34-3

As shown in the middle diagram the piece of wire on the right carrying a current, i , represented by the tail of the current arrow, has a force to the right ($\mathbf{I} \times \mathbf{B}$) producing no torque. The piece of wire on the left produces a force to the left also producing no torque. In the diagram on the right the forces are in the same direction but for this orientation of the coil the torque is a maximum. Figure 34-4 shows the torque for angles other than for zero or maximum torque. The torque is the component of this force at right angles to the line connecting the two sides of the coil times $b/2$ (the lever arm) times 2 (two wires).

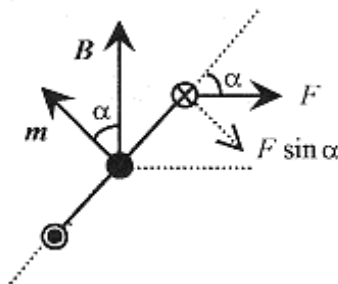


Fig34-4

The appropriate component of the force is $F \sin \alpha$. The vector, $\mathbf{m}$, the **magnetic moment**, is normal to the plane of the coil and points in the direction of the thumb when the fingers are curled naturally in the direction of the current in the coil. The angle between $\mathbf{m}$ and $\mathbf{B}$ is the same as the angle between force and lever arm.

$$\tau = 2(b/2)iaB \sin \alpha = iAB \sin \alpha$$

The product ab is the area, A , of the loop. The $\sin \alpha$ term implies a cross product and if iA is taken as the magnitude of $\mathbf{m}$, the **magnetic moment**, with the direction as defined in Fig. 34-4 then the vector equation for the torque is

$$\tau = \mathbf{m} \times \mathbf{B} \quad \text{or} \quad \tau = mB \sin \alpha \quad \text{with direction defined by } \mathbf{m} \times \mathbf{B} \quad (34-2)$$

Ex-6) Find the maximum torque on a circular coil of 0.050m radius with current of 0.50A and in a magnetic field of 0.0050T.

Solution The maximum torque is $\tau_{\max} = \pi a^2 IB = \pi (0.050\text{m})^2 (0.50\text{A})(0.0050\text{T}) = 2.0 \times 10^{-5} \text{ N} \cdot \text{m}$

Ex-7) The plane of a 10 turn rectangular loop of wire 3.0cm by 4.0cm carrying 2.0A is at an angle of 60° with respect to a magnetic field of 20 T. What is the torque on the loop in this position?

Solution: First calculate the magnetic moment noting that the current is not due to one loop but ten loops (N). $m = NIA = 10 \times 2.0\text{A} \times 12 \times 10^{-4} \text{ m}^2 = 2.4 \times 10^{-2} \text{ A} \cdot \text{m}^2$. Refer to Fig. 34-5 and note that if the angle between the plane of the coil and the magnetic field is 60° , then the angle

$$\tau = mB \sin \alpha = 2.4 \times 10^{-2} \text{ A} \cdot \text{m}^2 \cdot 20\text{T} \cdot \sin 30^\circ = 0.24 \text{ N} \cdot \text{m}$$

between $\mathbf{m}$ and $\mathbf{B}$ is 30° , so the torq

Ampere's Law

Ampere's law describes the magnetic field produced by a current-carrying wire. Two statements give a complete description of the magnetic field around a wire. The **strength** of the field is given by where $\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A}\cdot\text{m}$ is the (magnetic) permeability. The **direction** of the field is (a vector) tangent to a circle about the current-carrying wire normal to the plane of the current. With the thumb of the right hand in the direction of the current the fingers naturally curl in the direction of the **B** fields tangent to concentric circles. The drawing on the left side of Fig. 35-1 is a view of a current coming out of the paper showing the field as a vector tangent to the circle and the drop off of the magnetic field with radius. The diagram on the right side is a three dimensional view of the magnetic field around a current-carrying wire.

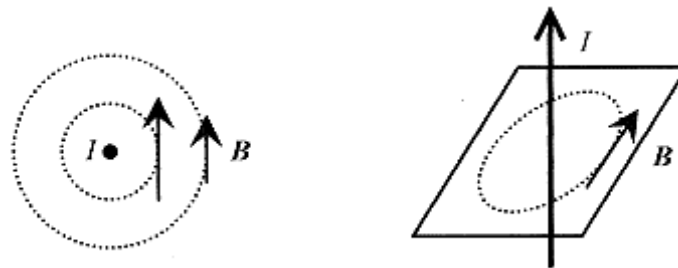


Fig 35-1

Ampere's law is written in the form of an integral

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i \quad (35-2)$$

where i is the current contained within the path of the integral. This integral sign with a circle means that the integral is to be performed over a closed path. In this case the path is a circle in a plane normal to the current (see Fig. 35-1). The circular path is centered about the wire where the magnetic field is a constant. Over this circular path the magnetic field vector and $d\mathbf{l}$, the vector representing an element of length of the circle are parallel so the sum of (integral of) their dot product, $\mathbf{B} \cdot d\mathbf{l}$, is the constant B times the circumference of the circle or $B(2\pi r)$. Remember that the dot product is the product of the projection of one vector onto another.

$$2\pi r B = \mu_0 i \quad \text{or} \quad B = \frac{\mu_0 i}{2\pi r}$$

Ex-8) Calculate the magnetic field 7.0m below a power line carrying 12000A of current.

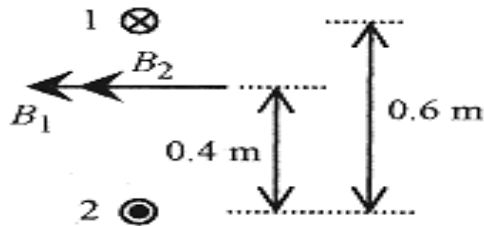
Solution:

$$B = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Wb/A}\cdot\text{m} (12000 \text{ A})}{2\pi \cdot 7.0 \text{ m}} = 3.4 \times 10^{-4} \text{ T}$$

This is comparable to the earth's magnetic field. Assuming the power line is parallel to the surface of the earth

then the magnetic field produced by this power line is also parallel to the surface of the earth.

Ex-9) Two long parallel wires separated by 0.60m carry antiparallel currents (the currents are physically parallel but in opposite directions) of 7.0 A each. What is the resulting field along the line between the wires and 0.40m from the lower wire?



Solution: At the specified position the magnetic field due to wire 1 is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m} (7.0 \text{ A})}{2\pi \cdot 0.20 \text{ m}} = 7.0 \times 10^{-6} \text{ T}$$

and directed as shown (right thumb in direction of current with fingers curled). The magnetic field due to wire 2 is

$$B_2 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \text{ Wb/A} \cdot \text{m} (7.0 \text{ A})}{2\pi \cdot 0.40 \text{ m}} = 3.5 \times 10^{-6} \text{ T}$$

and directed as shown. The total magnetic field is $10.5 \times 10^{-6} \text{ T}$.

The Force on Current Carrying Wires

Parallel wires carrying currents I_1 and I_2 produce a force on one other. Take the currents as parallel as shown in

Fig. 35-3. The current in wire 1 produces a magnetic field at wire 2 of magnitude $B_1 = \mu_0 I_1 / 2\pi d$. This field is directed down as shown and produces a force on wire 2 of magnitude $F_2 = B_1 I_2 \ell$. These forces are usually written in terms of force per unit length and this is most convenient in this problem. The force per unit length on wire 2 produced by the magnetic field associated with I_1 is

$$\frac{F_2}{\ell} = B_1 I_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$$

The force on wire 2 due to the magnetic field produced by wire 1 is toward wire 1. Go through the $\mathbf{v} \times \mathbf{B}$ operation until this is clear in your mind. If you encounter a test problem concerning forces on wires inevitably one part of the question will concern the direction and you don't want to get something this simple wrong because you didn't practice the $\mathbf{v} \times \mathbf{B}$ operation

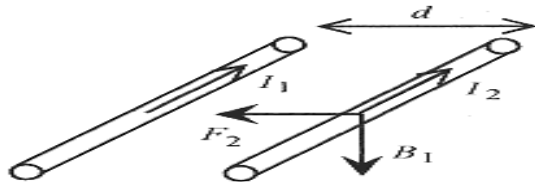


Fig. 35-3

This is the experimental setup that is used to define the ampere. A current produces a magnetic field that produces a force on another current. Using Ampere's law and this setup current can be related directly to mechanical force.

The Solenoid

A popular device that is based on the application of Ampere's law is the **solenoid**, a tightly wound coil producing a nearly uniform magnetic field along the axis of the coil. The coil is shown in Fig. 35-9.

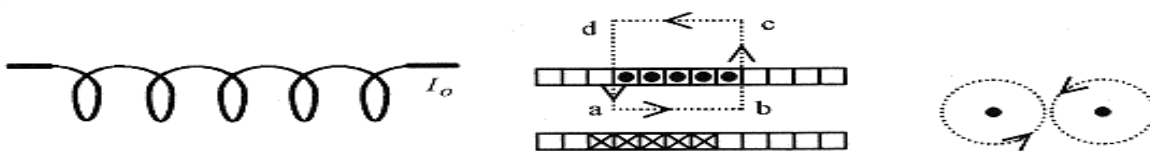


Fig. 35-9

For tightly wound coils the windings take on the form of a sheet of current, and a cross section looks as shown in the middle drawing of the figure. The magnetic field around each coil of wire is such that the magnetic field is concentrated and nearly uniform inside the coil. The integral around the path $a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$ is conveniently done in four parts.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \int_a^b B dl + \int_b^c B dl + \int_c^d B dl + \int_d^a B dl$$

The drawing on the right of Fig. 35-9 shows the magnetic fields of adjacent wires. Between the wires, the fields add to zero so the integrals along the paths $b \rightarrow c$ and $d \rightarrow a$ are zero. The path from $c \rightarrow d$ can be moved sufficiently far away so that the integral along this path approaches zero. The only non-zero integral is along the center line of the solenoid, the $a \rightarrow b$ path. Therefore, if the path length is taken as L then the Ampere's law becomes

$$BL = \mu_0 I_0 N L \quad \text{or} \quad B = \mu_0 I_0 N \quad (35-3)$$

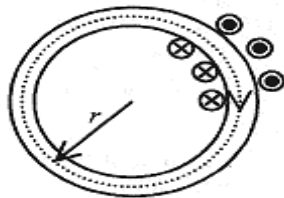
where N is the number of turns per length, so $I_0 N L$ is the total current within the path.

Ex-10) Calculate the magnetic field inside a solenoid of 300 turns per centimeter carrying a current of 0.050A. **Solution :**

$$B = \mu_0 I_0 N = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.050 \text{ A})(300/10^{-2} \text{ m}) = 1.9 \times 10^{-3} \text{ T}$$

The Toroid

A **toroid** is a solenoid wrapped so as to form a circle. A section through a toroid is shown in Fig. 35-10.



The application of Ampere's law is very straight forward. The path for the integral is along the center line of the toroid where the magnetic field is constant, so

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \text{Reduces to} \quad B(2\pi r) = \mu_0 n I_0 \quad \text{Where } n \text{ is the total number of turns in the toroid.}$$

But $n/2\pi r$ is the turns per unit length. Call this N , as before, so the field in a toroid is

$$B = \mu_0 N I_0 \quad (35-4)$$

Ex-11) Calculate the magnetic field in a toroid of 600 turns, current of 0.76A and center line radius 0.50m.

Solution :

$$B = \mu_0 N I_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \left(\frac{600}{2\pi \cdot 0.50 \text{ m}} \right) 0.76 \text{ A} = 1.8 \times 10^{-4} \text{ T}$$

Choose the correct answer from the given alternatives

- 1) A wire of length 0.50 m carries a current of 0.10 A in the positive x-direction, parallel to the ground. If the wire has a weight of 1.0×10^{-2} N, what is the minimum magnitude magnetic field that exerts a magnetic force on the wire equal to the wire's weight?
(a) 0.20 T (b) 0.30 T (c) 0.40 T (d) 0.50 T
- 2) A rectangular coil of wire consisting of ten loops, each with length 0.20 m and width 0.30 m, lies in the xy plane. If the coil carries a current of 2.0 A, what is the torque exerted by a magnetic field of magnitude 0.01T directed at an angle of 30.0° with respect to the positive z-axis?
(a) 24m N (b) 12m N (c) 6mN (d) 4mN
- 3) A long wire carries a current of 1 A. What is the magnitude of the magnetic field 2 m away from the wire?
(a) $0.1 \mu\text{T}$ (b) $0.5 \mu\text{T}$ (c) $10 \mu\text{T}$ (d) $50 \mu\text{T}$
- 4) A proton enters a constant magnetic field of magnitude 0.050 T and traverses a semicircle of radius 1.0 mm before leaving the field. What is the proton's speed?
(a) 1.6×10^3 m/s (b) 2.3×10^3 m/s (c) 2.8×10^4 m/s (d) 4.8×10^3 m/s
- 5) Estimate the magnitude of the magnetic force per unit length between a pair of parallel wires separated by 2 m if they each carry a current of 3 A.
(a) 1×10^{-1} N/m (b) 1×10^{-3} N/m (c) 1×10^{-4} N/m (d) 1×10^{-6} N/m
- 6) What is the magnitude of the magnetic field at the core of a 120-turn solenoid of length 0.50 m carrying a current of 2.0 A?
(a) 2.4×10^{-4} T (b) 4.8×10^{-5} T (c) 1.2×10^{-6} T (d) 6.0×10^{-4} T
- 7) Solenoid A has length L and N turns, solenoid B has length $2L$ and N turns, and solenoid C has length $L/2$ and $2N$ turns. If each solenoid carries the same current, rank by the strength of the magnetic field in the center of each solenoid from largest to smallest.
(a) A, B, C (b) A, C, B (c) B, C, A (d) C, A, B
- 8) A spatially uniform magnetic field cannot exert a magnetic force on a particle in which of the following circumstances?
(a) The particle is charged.
(b) The particle moves perpendicular to the field.
(c) The particle moves parallel to the field.
(d) The magnitude of the magnetic field changes with time.
- 9) Two long, parallel wires each carry the same current I in the same direction. What will be the direction of the total magnetic field at the point P midway between the wires?
(a) zero
(b) directed into the page
(c) directed out of the page
(d) directed to the left
- 10) A magnetic field of magnitude 5.0×10^{-4} T is to be produced at a distance of 5cm from a long straight wire. What current is required to produce this field?
(a) 125A (b) 25A (c) 10A (d) 5A

Answers: 1) a 2) c 3) a 4) d 5) d 6) d 7) d 8) c
9) a 10) a

Chapter - 7

Electromagnetic induction and AC-circuits

7.1- Magnetic flux (ϕ_m)

Magnetic flux is defined in the same way as electric flux and is proportional to both the strength of the magnetic field passing through the plane of a loop of wire and the area of the loop.

The magnetic flux ϕ_m through a loop of wire with area A is defined by

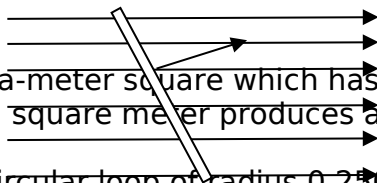
$$\Phi = BA \quad (37-1) \quad \text{or} \quad \phi_m = BA \cos \theta$$

where $BA \cos \theta$ is the component of a uniform magnetic field B perpendicular to the plane of the loop, and θ is the angle between B and the normal (perpendicular) to the plane of the loop.

*SI unit: Weber (Wb)

A

A = area vector



B = uniform external magnetic field

The unit of flux is Tesla-meter square which has a special name, Weber. One Weber's worth of flux passing through a square meter produces a field strength of one Tesla.

Ex-1) A conducting circular loop of radius 0.250 m is placed in the xy-plane in a uniform magnetic field of 0.360 T that points in the positive z-direction, the same direction as the normal to the plane. **(a)** Calculate the magnetic flux through the loop.

(b) Suppose the loop is rotated clockwise around the x-axis, so the normal direction now points at a 45.0° angle with respect to the z-axis.

Solution : $r=0.25\text{m}$, $B=0.36\text{ T}$, $A= \pi(0.25\text{m})^2 =0.2\text{m}^2$

(a) But, $\theta =0^\circ$

$$\Phi = (0.36\text{T})(0.2\text{m}^2) = 0.072\text{ Wb}$$

(b) For $\theta =45^\circ$, $\Phi = BA \cos \theta = (0.36\text{T})(0.2\text{m}^2) \cos(45^\circ) = 0.05\text{ Wb}$

Faraday's law of induction and Lenz's

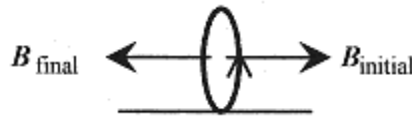
• Faraday's law

If a circuit contains N tightly wound loops and the magnetic flux through each loop changes by the amount $\Delta\phi_m$ during the interval Δt , the average emf induced in the circuit during time is

$$\mathcal{E} = N \frac{\Delta\phi_m}{\Delta t}$$

- **Lenz's law** states that the current from the induced emf $\mathcal{E}$ creates a magnetic field with flux opposing the *change* in magnetic flux through a circuit.

Ex-2) Place a 20 turn coil of radius 0.05m inside a solenoid where the magnetic field is changed from $40 \times 10^{-3}\text{T}$ in one direction to $40 \times 10^{-3}\text{T}$ in the other direction in $60 \times 10^{-3}\text{s}$. Find the induced **emf**.



Solution: Inside the coil the flux is $\Phi_m = BA = (40 \times 10^{-3} \text{ T})\pi(0.05\text{m})^2 = 3.1 \times 10^{-4} \text{ Wb}$. The change in flux is $\Delta\Phi_m = 6.2 \times 10^{-4} \text{ Wb}$, and this occurs over $60 \times 10^{-3} \text{ s}$ in the coil of 20 turns so the induced **emf** is

$$emf = -N \frac{\Delta\Phi}{\Delta t} = -20 \frac{6.2 \times 10^{-4} \text{ Wb}}{60 \times 10^{-3} \text{ s}} = -0.21 \text{ V}$$

Ex-3) A circular loop 0.040m in radius is perpendicular to a magnetic field of 0.50T. The loop is stretched, across a diameter, so that its area goes to zero in 0.20s. What is the induced voltage and direction of the current through the resistor?

Solution: The flux through the loop is $\Phi_m = BA = 0.50\text{T} \cdot \pi(0.04\text{m})^2 = 2.5 \times 10^{-3} \text{ Wb}$. The **emf** is

$$emf = -\frac{\Delta\Phi}{\Delta t} = -\frac{2.5 \times 10^{-3} \text{ Wb}}{0.20\text{s}} = -0.012 \text{ V}$$

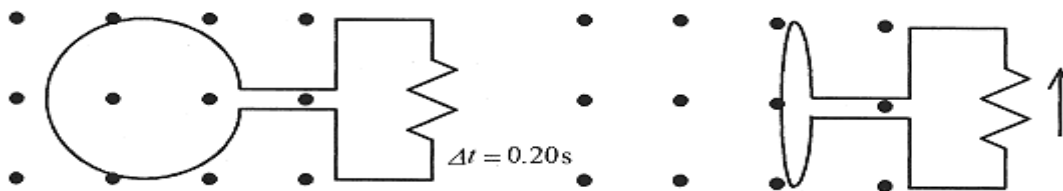


Fig. 37-4

The current is such as to produce a magnetic field out of the page (or in opposition to the collapsing magnetic field). The current is counterclockwise in the stretched loop and up on the resistor.

Ex-4) Take a 50 turn rectangular coil of dimensions 0.10m by 0.20m and rotate it from a position perpendicular to a field of 0.50T to parallel to the field in 0.10s and calculate the induced **emf**

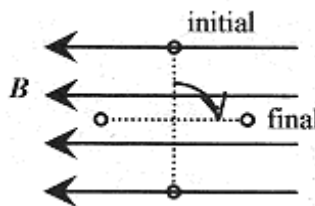


Fig. 37-5

Solution: The flux $\Phi_m = BA$. In addition, $\Phi_m = BA$ since the flux goes from maximum to zero.

$$\Delta\Phi = BA = 0.50\text{T} \cdot 2.0 \times 10^{-2} \text{ m}^2 = 1.0 \times 10^{-2} \text{ T} \cdot \text{m}^2$$

$$emf = -N \frac{\Delta\Phi}{\Delta t} = -50 \frac{1.0 \times 10^{-2} \text{ T} \cdot \text{m}^2}{0.10\text{s}} = -5.0 \text{ V}$$

The coil is 50 turns, so the induced **emf** is

Motional Electromotive force

Consider a rectangular loop of wire being pulled through a magnetic field. The field is strength **B**, width of the loop **L**, length of the loop in the field **x**, and velocity with which the loop is being pulled through the field **v**.

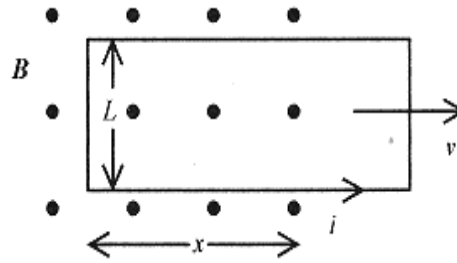


Fig. 37-7

The total flux within the loop is $\Phi = BA = BLx$

The induced **emf** is

$$emf = -\frac{\Delta\Phi}{\Delta t} = -BL \frac{\Delta x}{\Delta t} = -BLv$$

$$emf = -\frac{d\Phi}{dt} = -BL \frac{dx}{dt} = -BLv$$

As the loop is being pulled through the field there is a decrease in the number of lines of magnetic flux pointing out of the page. Therefore the induced current will be in a direction so as to create more flux out of the page. This requires a counterclockwise current around the loop. Interestingly this current produces forces on the sides of the loop. Apply the right-hand rule to the forces and note that the force on the bottom section of the wire is down, while the force on the top section of the wire is up. These two forces are equal and opposite and so add to zero. The force on the vertical section of the wire is to the left in opposition to the force producing the velocity. The force on the wire due to the induced current is BiL . The work performed by this force is (in analog with mechanics) force times distance or $F \cdot x = BiLx$.

The power (again in analog with mechanics) is force times velocity or $P = F \cdot v = BiLv$.

Ex-5) Pull a loop of width 0.20m, length 0.80m, and resistance 200Ω through a magnetic field of 0.40T at 0.20m/s. First find the induced **emf** and current. Then find the force necessary to pull the loop, the work performed, and the power, the rate of doing the work.

Solution: The induced **emf** is $emf = -BLv = -0.40T \cdot 0.20m \cdot 0.20m/s = -1.6 \times 10^{-2} V$.

The induced current (in the direction shown on Fig. 37-7) is

$$i = \frac{emf}{R} = \frac{1.6 \times 10^{-2} V}{200\Omega} = 8.0 \times 10^{-5} A$$

The force necessary to pull the wire out of the field is

$$F = BiL = 0.40T \cdot 8.0 \times 10^{-5} A \cdot 0.20m = 6.4 \times 10^{-6} N$$

The work performed in completely removing the loop is this force times the length of the loop.

$$W = F \cdot x = 6.4 \times 10^{-6} N \cdot 0.80m = 5.1 \times 10^{-6} J$$

The power delivered to the loop is

$$P = F \cdot v = 6.4 \times 10^{-6} N \cdot 0.20m/s = 1.3 \times 10^{-6} W.$$

Inductance

If two coils are placed nearby, the current in one sets up a magnetic field or flux through the other. If the current in the one is changing, then there is a changing magnetic field and flux in the other producing an induced **emf** and current. In equation-like statements:

- * Current in one produces flux in the other.
- * Changing current in one produces an **emf** in the other.

Self Inductance

Further, a changing current in an isolated coil produces an **emf** in itself. This is called a self-induced **emf**. The direction of the self-induced **emf** is so as to produce a current that opposes the current that set it up. Calculation of the self induced **emf** follows from a consideration of the geometry of the coil. Consider a close packed coil with a flux, Φ , passing through each of N turns of the coil. The number of turns times the flux is called the flux linkages, $N\Phi$. The self-induced **emf** is proportional to the time rate of change of these flux linkages. In the language of calculus, the self induced **emf** is proportional to d/dt of $N\Phi$, so

$$emf = -\frac{\Delta(N\Phi)}{\Delta t} \quad \text{or} \quad emf = -\frac{d(N\Phi)}{dt} \quad (38-1)$$

The quantity $N\Phi$, the flux linkages, is proportional to the current, so make $N\Phi = Li$ where L is a constant that depends on how much flux is linked to the coil. Rewriting equation 38-1 with the defined constant L

$$emf = -L \frac{di}{dt} \quad (38-2)$$

The unit of inductance is the Henry which, from equation 38-2, must have units of V.s/A. The self inductance, or inductance as it is often called, can be calculated for a solenoid. Consider a solenoid of length ℓ having N turns or n turns per unit length. Inside the solenoid the flux is BA , so the number of flux linkages is $N\Phi = (n\ell)BA$. For a solenoid the magnetic field is $\mu_0 ni$, so

$$N\Phi = (n\ell)BA = \mu_0 n^2 \ell i A$$

The quantity $N\Phi$ is equal to the constant L times i so

$$L = \mu_0 n^2 \ell A \quad (38-3)$$

The inductance of a solenoid is entirely geometry dependent. Adding a magnetic material to enhance the flux linkages only makes the inductance also dependent on material.

Ex-6) Calculate the inductance of a solenoid of radius 0.0040m, length 0.015m, and 1.0×10^4 turns per meter.

Solution:

$$L = \mu_0 n^2 \ell A = (4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A})(1.0 \times 10^8 \text{ 1/m}^2)(0.015\text{m})\pi(0.0040\text{m})^2 = 9.5 \times 10^{-5} \text{ H}$$

Ex-7) For a coil of 300 turns and inductance $12 \times 10^{-3} \text{ H}$, find the flux through the coil when the current is $5.0 \times 10^{-3} \text{ A}$.

Solution: Use the relationship between flux linkages and inductance $N\Phi = Li$.

$$\Phi = \frac{Li}{N} = \frac{12 \times 10^{-3} \text{ H} \cdot 5.0 \times 10^{-3} \text{ A}}{300} = 2.0 \times 10^{-7} \text{ Wb}$$

*** Mutual Inductance**

If there are coils within, or adjacent to, other coils, then we can define mutual inductance, the

linking of one coil with another via the flux. If the flux is changing, then the current in the one coil is linked to the current in the other coil.

The simplest case to consider is one solenoid-like coil inside another solenoid-like coil. The voltage induced in the second coil is proportional, via the flux linkage, to the changing current in the first coil. The mathematical statement is

$$\text{emf}_2 = M \frac{di_1}{dt}$$

The reverse is also true. The voltage induced in the first coil is proportional to the changing current in the second coil.

$$\text{emf}_1 = M \frac{di_2}{dt}$$

The constants are the same. The geometry is the same. These "geometric coupling constants" are again labeled L , the inductance.

Ex-8) Two coils wound together as a single package have an inductance of 6.0×10^{-3} H. Find the voltage induced in the second coil when the current is changing in the first at the instantaneous rate of 20×10^{-3} A/s.

Solution:

$$\text{emf} = L(di/dt) = 6.0 \times 10^{-3} \text{ H} \cdot 20 \times 10^{-3} \text{ A/s} = 1.2 \times 10^{-4} \text{ V}$$

Power and Energy Storage

If the **emf** generated in an inductor is $L(di/dt)$ and the power in an electric circuit is voltage times current, then the general expression for the power in an inductor is

$$P = Li \frac{di}{dt} \quad (38-4)$$

The energy stored in a conductor is the integral over time of the power (remember power is work over time), or in differential form $dU = Lidi$, so the total energy stored in a coil (the energy is actually stored in the magnetic field) when the current goes from zero to some value i is the integral of this expression

$$U = \int_0^i Lidi = \frac{Li^2}{2} \quad (38-5)$$

The energy density in a solenoid is the total energy stored in the solenoid divided by the volume.

$$u_B = \frac{U_B}{A\ell} = \frac{Li^2}{2A\ell} \quad \text{Using } L = \mu_0 n^2 \ell A, \quad u_B = \frac{\mu_0 n^2 i^2}{2}$$

Or using $B = \mu_0 ni$.

$$u_B = \frac{B^2}{2\mu_0} \quad (38-6)$$

Ex-9) (a) If an inductor carrying a 1.70-A current stores an energy of 0.30 mJ, what is its inductance?

(b) How much energy does the same inductor store if it carries a 3.0A current?

Solution

$$(a) U = L \frac{i^2}{2}, \Rightarrow L = 2 \frac{U}{i^2} \Rightarrow L = \frac{2 \times 0.3 \text{ mJ}}{1.7^2} = 2.08 \times 10^{-4} \text{ H} = 0.208 \text{ mH}$$

$$(b) \text{ If } I = 3.0 \text{ A, the stored energy will be } PE_L = \frac{1}{2} Li^2 = \frac{1}{2} (0.208 \text{ mH}) (3 \text{ A})^2 = 0.936 \text{ mJ}$$

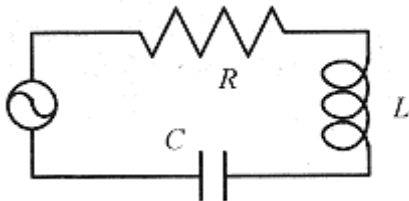
Alternating current circuits

An AC circuit consists of combinations of circuit elements and an AC generator or an AC source, which provides the alternating current. We have seen that the output of an AC generator is sinusoidal and varies with time according to $V(t) = V_{\max} \sin(2\pi ft)$

This chapter is devoted to the study of R's, L's and C's, individually and in combination, in response to sinusoidal voltages.

Alternating Currents

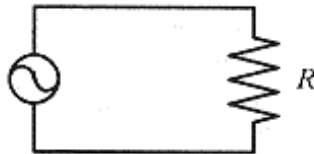
Faraday's Law, describe how a loop rotated in a magnetic field produces a sinusoidal voltage and current. This is the basis of alternating current generators. An external agent, such as falling water or steam, is used to rotate the loop of wire in a magnetic field thus generating a sinusoidal, or alternating, voltage and current. This alternating current, ac for short, has two basic advantages over direct current. First, it is easy to increase the voltage for transmission over long distances and later decrease the voltage for distribution to individual users or for specific applications. (See the "Transformers" section in your text book .) Second, an alternating current in a loop placed in a magnetic field rotates, providing rotational power for all sorts of machines. See the "A Current-Carrying Loop in a B Field" section in the previous topics, Magnetic Forces. The basic circuit for the study of alternating current in R-L-C circuits as shown below.



The circuit is driven by the ac source.

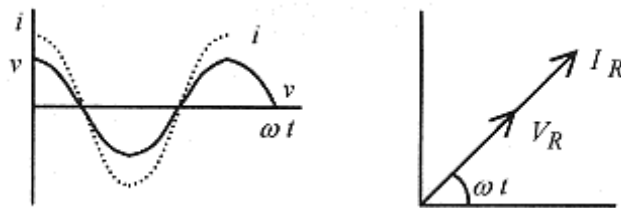
The Resistive Circuit.

The simplest way to start the study of ac circuits is with a resistor and ac source as shown in the Fig. below. The time varying voltage is $v = V_R \cos \omega t$.



The current tracks (is in phase with) the voltage and is $I(t) = (V_R/R) \cos \omega t = I_R \cos \omega t$.

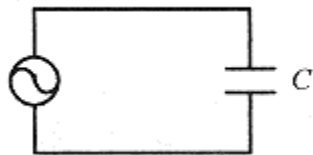
The fig below shows the voltage and current as a function of time and (on the right) what is known as a **phasor diagram**, an important analysis tool in the study of R-L-C circuits.



The individual vector-like arrows are called phasors. The diagram is started by drawing a phasor I_R at some arbitrary angle ωt . The phasor rotates counterclockwise with the length proportional to I_R , and the projection on the horizontal axis proportional to the instantaneous current. An increase in ωt toward the right on the graph corresponds to an increase in ωt in the counterclockwise direction on the phasor diagram. When an entire cycle of a sine wave is complete the phasor will have rotated through 360° . Next the phasor representing the voltage is drawn at the same arbitrary angle ωt . In a resistor the instantaneous voltage and current are in phase. The length is proportional to V_R , and the projection along the horizontal axis is proportional to the instantaneous voltage across R .

The Capacitive Circuit

The Figure below shows a capacitive circuit driven by an ac source.



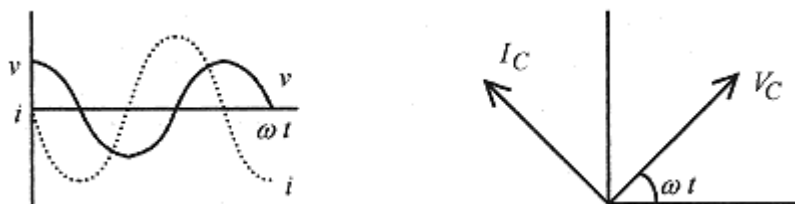
The time varying voltage is $v = V_C \cos \omega t$. The phase relation between this v and i_C is different than for a resistor.

When an alternating voltage is applied to a capacitor the current alternates (flows in one direction, then in the opposite direction) but does not track with (is not in phase with) the voltage across the capacitor. When the voltage reaches a maximum the capacitor is fully charged and the current is zero! When the voltage reaches a maximum in the other direction the capacitor is again fully, but oppositely, charged and the current is again zero. The current then must be a maximum when the (alternating) voltage is passing through zero. The charge is in phase with the voltage

$$q = Cv = CV_C \cos \omega t.$$

The current is $i_C = (dq/dt) = -\omega CV_C \sin \omega t$

The voltage and current are plotted as a function of time in the Fig. below along with the phasor diagram.



The phasor diagram is drawn starting with V_C at an arbitrary angle ωt . (Starting with I_C produces the same result.) The hard part in drawing the I_C phasor is to figure out how to orient it with respect to V_C . The easiest way to do this is to look at the graph of v and i versus time and ask the question, "Which quantity leads the other and by how much?" By looking at adjacent peaks note that i reaches its maximum 90° before v . Therefore we say " i lead v by 90° in a capacitive circuit." The I_C phasor is 90° ahead (rotated 90° further counterclockwise) of V_C .

The maximum current is $I_C = \omega CV_C$ or $V_C = I_C (1/\omega C) = I_C X_C$.

The $1/\omega C$ term plays the role of resistance and is called capacitive reactance X_C .

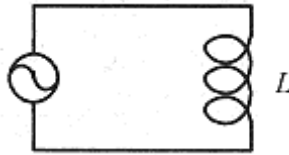
Ex-1) A $20\mu\text{F}$ capacitor is connected to a variable frequency ac source with maximum voltage 30V. What is the capacitive reactance at 60 Hz, 600 Hz, and 60 kHz?

Solution:

$$X_C|_{60} = \frac{1}{\omega C} = \frac{10^6 \text{ s}}{2\pi \cdot 60 \cdot 20\text{F}} = 130\Omega \quad X_C|_{600} = 13\Omega \quad X_C|_{60\text{k}} = 0.13\Omega$$

The Inductive Circuit

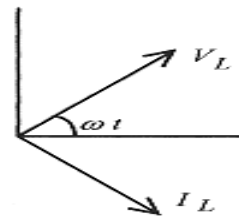
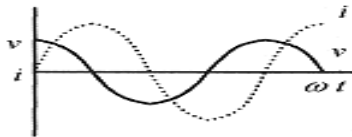
The Figure below shows an inductor driven by an ac source. The time varying voltage is $v = V_L \cos \omega t$. Again the phase relationship between v and i_L is different from either the resistor or capacitor. The maximum voltage across an inductor is proportional to the rate of change of current. Therefore the maximum voltage corresponds not to maximum current but to maximum rate of change of current. A quick look at a sine curve indicates that the maximum rate of change (slope) is when the curve crosses the axis, so we expect the current to be 90° out of phase with the voltage



The Kirchhoff-type voltage statement for this circuit is $V \cos \omega t = L(di/dt)$

This statement is easily integrated $\int V \cos \omega t dt = \int L di$ to $i = (V/\omega L) \sin \omega t$.

The voltage and current are plotted as a function of time in the Fig. below along with the phasor diagram



The phasor diagram is drawn by starting with V_L . Now look at the adjacent peaks in the graph of v and i versus ωt and note that v reaches its peak earlier in time than i_L . Therefore we say " v leads i by 90° in an inductive circuit." Notice how, as the phasors rotate at this fixed 90° difference, the voltage phasor traces out the cosine function on the horizontal axis and the current phasor traces out the sine function.

The maximum current is $I_L = V_L/\omega L = V_L/X_L$ or $V_L = I_L X_L$.

The ωL term plays the role of resistance and is called inductive reactance X_L .

Ex-2) A 120mH inductor is connected to a variable frequency ac source with maximum voltage 10V. What is the inductive reactance at 100 Hz, and 1.0 MHz?

Solution:

$$X_L|_{100} = \omega L = 2\pi \cdot 100\text{Hz} \cdot 0.12\text{H} = 75\Omega$$

$$X_L|_{1.0\text{M}} = 2\pi \cdot 1.0 \times 10^6\text{Hz} \cdot 0.12\text{H} = 7.5 \times 10^5\Omega$$

The basic relation $v = L(di/dt)$ shows that $[H = V \cdot s/A]$ making X_L have the units of Ω .

Ex-3) At what frequency do a 65mH inductor and a 20 μ F capacitor have the same reactance?

Solution: When $X_L = X_C$, or $\omega L = \frac{1}{\omega C}$, The frequency then is

$$\omega = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f = \frac{1}{2\pi\sqrt{LC}} \quad (41-1)$$

This is the condition for resonance or oscillation in the L-C circuit!

Ex-4) An ac source of 100Hz and maximum voltage of 20V is connected to a 70mH inductor. What is the maximum current? When the current is maximum, what is the voltage of the source?

Solution: First calculate the inductive reactance

$$X_L = \omega L = 2\pi \cdot 100\text{Hz} \cdot 0.070\text{H} = 44\Omega$$

$$I_L|_{\text{max}} = V_L / X_L = 20\text{V} / 44\Omega = 0.45\text{A}$$

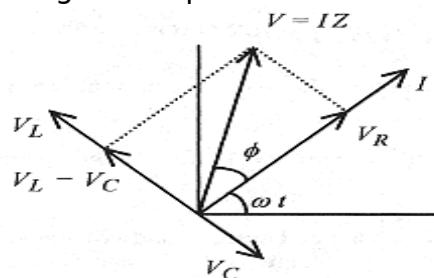
The maximum current is

Look at the graphs in Fig. 41-7 and note that when the current is a maximum, the voltage is zero

$$f = \frac{1}{2\pi\sqrt{65 \times 10^{-3}\text{H} \cdot 20 \times 10^{-6}\text{F}}} = 140\text{Hz}$$

The R-L-C Circuit

The phasor diagrams are most helpful in understanding R-L-C circuits. There are two important points to keep in mind in the analysis of these circuits. First, the sum of the instantaneous voltages must equal the source voltage $V \cos \omega t = V_R + V_L + V_C$. Second, since there is only one current path, the current is everywhere the same. Voltages on the various components have different phase relationships, but the current is the same everywhere in the circuit. The phasor diagram for a typical R-L-C circuit is shown in the Fig. below. Do not try to take this in all at once. Follow along the steps in the construction of the diagram.



Place the I phasor at the arbitrary angle ωt

Place the V_R phasor over I . The voltage and current in the resistor are in phase.

Add $V_L = I X_L$ leading V_R by 90° .

Add $V_C = I X_C$ lagging V_R by 90° .

On an axis perpendicular to V_R and I , V_L and V_C , add in a vector manner to produce $V_L - V_C$. In this example $V_L > V_C$.

If the load is resistive, voltage is in phase with current. If the load is entirely inductive or entirely capacitive the voltage is 90° out of phase with current. In this situation, with all elements present, the voltage is the vector-like sum of V_R and $V_L - V_C$.

$$V^2 = V_R^2 + (V_L - V_C)^2 = I^2 R^2 + (I X_L - I X_C)^2 = I^2 [R^2 + (X_L - X_C)^2] \quad (41-2)$$

In equation form

$$\text{Or} \quad V = I \sqrt{R^2 + (X_L - X_C)^2}$$

This suggests another resistance-like expression

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (41-3)$$

This is called impedance. We now have the numeric relations between voltage, current, and the values of R, L, and C. The phase relation between V and I is seen from the phasor diagram as

$$\tan \phi = \frac{V_L - V_C}{V_R} \quad (41-4)$$

To obtain a better picture of what is going on here imagine measuring the ac voltages of the source, resistor, capacitor, and inductor, and the current in the circuit. The voltages across the resistor, capacitor, and inductor do not add up to the source voltage! They are not in phase! These voltages will satisfy equation 41-2. The source voltage divided by the impedance, equation 41-3, will equal the current. Finally the phase angle between the source voltage and current comes from equation 41-4.

Ex-5) An R-L-C circuit with $R = 200\Omega$, $L = 0.40\text{H}$, and $C = 3.0\mu\text{F}$ is driven by an ac source of 20 V maximum and frequency 100 Hz. Find the reactances, impedance, and maximum current and voltages across each of the components.

Solution: R-L-C circuit problems can be confusing. The key to successfully solving them is to follow a logical path through the problem. The current is everywhere the same and is determined by the source voltage and the impedance. The impedance is determined by the resistance and the reactances, and the reactances are frequency dependent.

$$X_L = \omega L = 2\pi \cdot 100\text{Hz} \cdot 0.40\text{H} = 251\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{10^6}{2\pi \cdot 100\text{Hz} \cdot 3.0\text{F}} = 530\Omega$$

$$(X_L - X_C)^2 = (251\Omega - 530\Omega)^2 = 77800\Omega^2$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200)^2 + 77800\Omega} = 343\Omega$$

$$I = V/Z = 20\text{V}/343\Omega = 0.058\text{A}$$

$$V_R = 0.058\text{A} \cdot 200\Omega = 11.6\text{V}$$

$$V_L = 0.058\text{A} \cdot 251\Omega = 14.6\text{V}$$

$$V_C = 0.058\text{A} \cdot 530\Omega = 30.7\text{V}$$

Ex-6) An R-L-C circuit consists of a 300Ω resistor, 0.15H inductor, and $4.5\ \mu\text{F}$ capacitor driven by an ac source of 800 rad/s . Find the phase angle of the source voltage with respect to the current.

Solution: Notice that the maximum voltage is not given in the problem, yet a phase diagram is to be drawn. The problem is written this way to emphasize that it is not necessary to know the maximum source voltage to find the phase angle. The phase diagrams so far have used voltage. The voltage across each component is the resistance or reactance times a constant, the current. Thus the diagram could as well be drawn with resistance and reactances as well as voltage. Calculate the reactances:

$$X_L = \omega L = 800\text{rad/s} \cdot 0.15\text{H} = 120\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(800\text{rad/s})4.5 \times 10^{-6}\text{F}} = 278\Omega$$

Draw R along I proportional to $300\ \Omega$. Draw X_L leading R by 90° proportional to $120\ \Omega$. Draw X_C lagging R by 90° proportional to $278\ \Omega$. Draw $X_L - X_C = -158\ \Omega$. Complete the rectangle formed by R and $X_L - X_C$ and draw the diagonal, which is proportional to Z . The phase angle is

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{-158}{300} \quad \text{or} \quad \phi = -28^\circ$$

The load is resistive and capacitive, and the phase angle is -28° with the source voltage lagging the current

Ex-7) A series RLC AC circuit has resistance $R = 2.5 \times 10^2\ \Omega$, inductance $L = 0.60\ \text{H}$, capacitance $C = 3.50\ \text{mF}$, frequency $f = 60.0\ \text{Hz}$, and maximum voltage $V_{\text{max}} = 1.50 \times 10^2\ \text{V}$. Find

- | | |
|-----------------------------------|---|
| (a) the impedance of the circuit, | (b) the maximum current in the circuit, |
| (c) the phase angle, and | (d) the maximum voltages across the |
| elements. | |

Solution:

(a) Find the impedance of the circuit. First, calculate the inductive and capacitive reactances:

$$X_L = 2\pi fL = 2\pi(60\text{Hz})(0.60\text{H}) = 226\Omega$$

$$X_C = 1/2\pi fC = 1/2\pi(3.5\text{mF}) = 758\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{250^2 + (226^2 - 758^2)} = 588\Omega$$

$$(b) I_{\max} = \frac{V_{\max}}{Z} = 0.255\text{A}$$

$$(c) \tan(\phi) = \frac{X_L}{R} \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 64.8^\circ$$

$$(d) V_{\max,R} = I_{\max}R = (0.255\text{A})(250\Omega) = 63.8\text{V}$$

$$V_{\max,C} = I_{\max}X_C = (0.255\text{A})(250\Omega) = 193\text{V}$$

$$V_{\max,L} = I_{\max}X_L = (0.255\text{A})(250\Omega) = 57.6\text{V}$$

Power in ac Circuits

No power losses are associated with pure capacitors and pure inductors in an AC circuit. A pure capacitor, by definition, has no resistance or inductance, whereas a pure inductor has no resistance or capacitance. (These definitions are idealizations: in a real capacitor, for example, inductive effects could become important at high frequencies.) We begin by analyzing the power dissipated in an AC circuit that contains only a generator and a capacitor. When the current increases in one direction in an AC circuit, charge accumulates on the capacitor and a voltage drop appears across it. When the voltage reaches its maximum value, the energy stored in the capacitor is

$$- PE_C = \frac{1}{2}CV_{\max}^2$$

This energy storage is only momentary, however: When the current reverses direction, the charge leaves the capacitor plates and returns to the voltage source. During one-half of each cycle the capacitor is being charged, and during the other half the charge is being returned to the voltage source. Therefore, the average power supplied by the source is zero. In other words, **no power losses occur in a capacitor in an AC circuit.**

Similarly, the source must do work against the back emf of an inductor that is carrying a current. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by

$$- PE_L = \frac{1}{2}LI_{\max}^2$$

When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit. The average power delivered to a resistor in an RLC circuit is

$$- P_{\text{av}} = I_{\text{rms}}^2 R$$

-The average power delivered by the generator is converted to internal energy in the resistor. No power loss occurs in an ideal capacitor or inductor.

* An alternate equation for the average power loss in an AC circuit can be found by substituting (from Ohm's law)

$$R = \frac{V_{R,\text{rms}}}{I_{\text{rms}}} \text{ in to equation } P_{\text{av}} = I_{\text{rms}} V_{R,\text{rms}}$$

$$- \text{But, } V_{R,\text{rms}} = V_{\text{rms}} \cos \phi$$

-Hence, the average power delivered by a generator in an AC circuit is $P_{\text{av}} = I_{\text{rms}} V_{R,\text{rms}}$

$$P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos \phi \quad \text{where the quantity } \cos \phi \text{ is called the } \mathbf{power\ factor}.$$