ASSIGNMENT

MTH622

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QUESTION:1

In a two-dimensional, incompressible flow of the fluid velocity field is given by $V=(x-4y)\hat{i}-(4x+y)\hat{j}+0\hat{k}$ **.show that the flow is potential(irrational).**

Answer:

$$V=(x-4y)\hat{i}-(4x+y)\hat{j}+0\hat{k}$$

$$\nabla=\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}$$

$$Curl(\vec{V})=\Delta\times\vec{V}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x-4y & 4x+y & 0\end{vmatrix}$$

$$\Delta\times\vec{V}=\left(\frac{\partial(0)}{\partial y}-\frac{\partial(4x+y)}{\partial z}\right)\hat{i}-\left(\frac{\partial(0)}{\partial x}-\frac{\partial(4x-y)}{\partial z}\right)\hat{j}+\left(\frac{\partial(4x+y)}{\partial x}-\frac{\partial(4x-y)}{\partial y}\right)\hat{k}$$

$$\Delta\times\vec{V}=(0)\hat{i}-(0)\hat{j}+(4-4)\hat{k}$$

$$\Delta\times V=0\hat{i}-0\hat{j}+0\hat{k}$$

$$Since the Curl(V)=0, So the flow is irrational$$

QUESTION:2

A particle moves from a point t=0, to t= 2π **under the action of a force** $F=x\hat{i}+y\hat{j}-z\hat{k}$ **along a helical curve** $x=t\cos t\ ,\ \ y=t\sin t\ \ ,\ \ z=t$**. Find the total work done by the particle moving along the curve.**

Answer:

Since,

$\vec{F} = x\hat{i} + y\hat{j} - z\hat{k}$

substituting values of

$x = t\cos t \quad , \quad y = t\sin t \quad , \quad z = t$

$\vec{F} = (t\cos t)\hat{i} + (t\sin t)\hat{j} - t\hat{k}$

$\vec{F} = t\cos t\hat{i} + t\sin t\hat{j} - t\hat{k}$

Also,

$\vec{dr} = dx\hat{i} + dy\hat{j} - dz\hat{k}$

$\frac{dr}{dt} = \left(\frac{d}{dt}t\cos t\right)\hat{i} + \left(\frac{d}{dt}t\sin t\right)\hat{j} + \left(\frac{d}{dt}(t)\right)\hat{k}$

$\vec{dr} = (-t\sin t + \cos t)\hat{i} + (t\cos t + \sin t)\hat{j} + (1)\hat{k})dt$

$\vec{dr} = (\cos t - t\sin t)\hat{i} + (\sin t + t\cos t)\hat{j} + \hat{k})dt$

Now we can calculate the work done by a force $\vec{F}$ along the curve $(x = t\cos t \quad , y = t\sin t \quad , z = t)$

Total work done $= \int_c \vec{F}\,d\vec{r}$

$= \int_0^{2\pi} t\cos t\hat{i} + t\sin t\hat{j} - t\hat{k}[(\cos t - t\sin t)\hat{i} + (\sin t + t\cos t)\hat{j} + \hat{k})]dt$

$= \int_0^{2\pi} (t\cos^2 t - t\sin^2 t\cos t + t\sin^2 t + t^2\sin t\cos t - t)\,dt$

$= \int_0^{2\pi} (t\cos^2 t + t\sin^2 t - t)dt$

$= \int_0^{2\pi} t(\cos^2 t + \sin^2 t) - t)dt$

$= \int_0^{2\pi} t(1) - t)dt \qquad \because \cos^2 t + \sin^2 t = 1$

Here, the limits does not exist in this condition

$= \int 0dt$

Therefore, the total work done by the particle moving along the given helical curve is zero.