

# ME13A: CHAPTER SIX



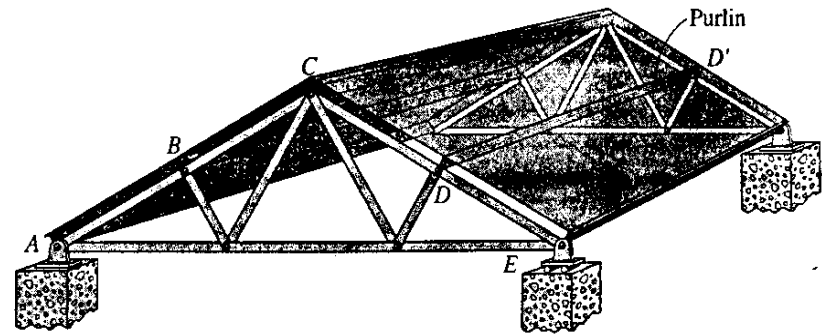
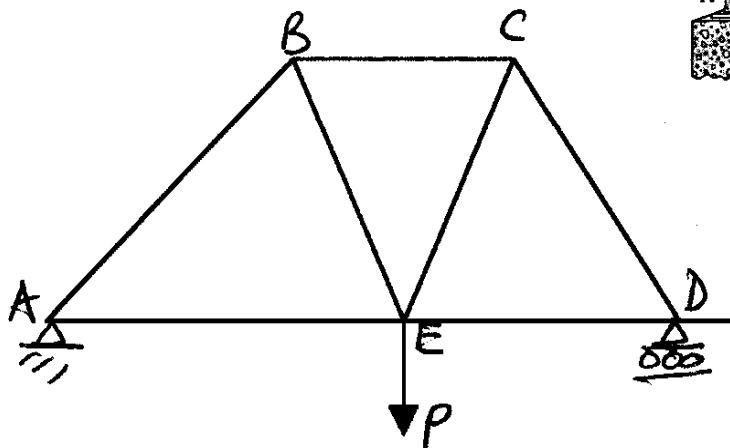
## ANALYSIS OF STRUCTURES

# STRUCTURE DEFINED

- A structure is a rigid body made up of several connected parts or members designed to withstand some externally applied forces.
- The analysis of structures is based on the principle that if a structure is in equilibrium, then each of its members is also in equilibrium.
- By applying the equations of equilibrium to the various parts of simple truss, frame or machine, the forces acting on the connections can be determined.

## 6.2 TRUSSES

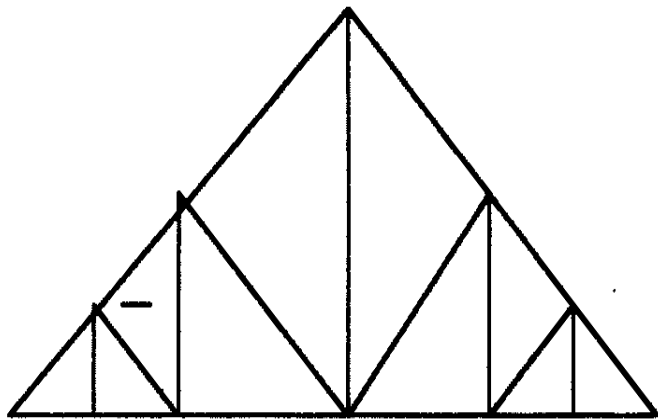
- A truss is a structure made up of straight members which are connected at the joints, and having the joints at the ends of the members. Trusses are used to support roofs, bridges and other structures.



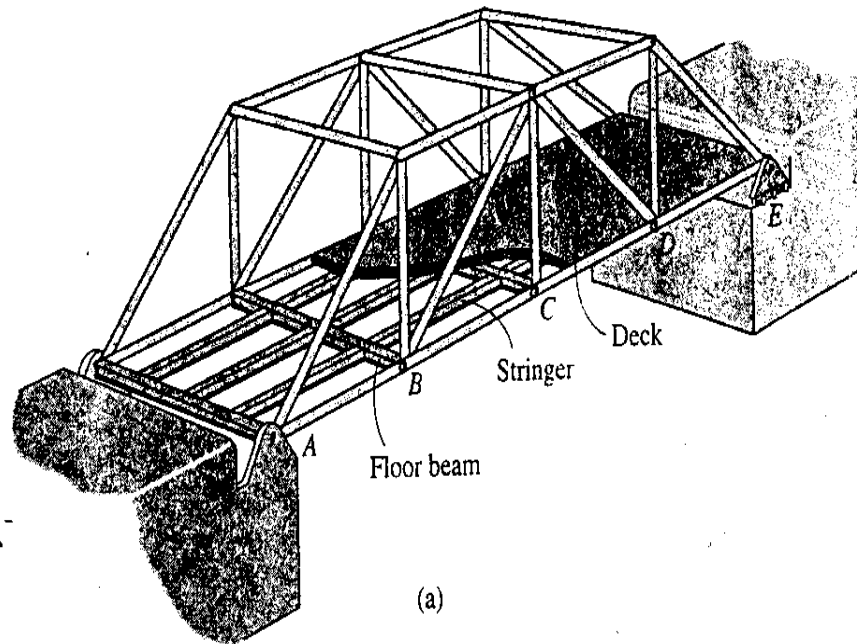
## 6.2.1 Types of Trusses

- (a) **Simple Trusses:** A simple truss is one which is generated from a basic triangle. To any two ends of a member, two additional members are attached and connected at a single new joint.

t a single new joint.



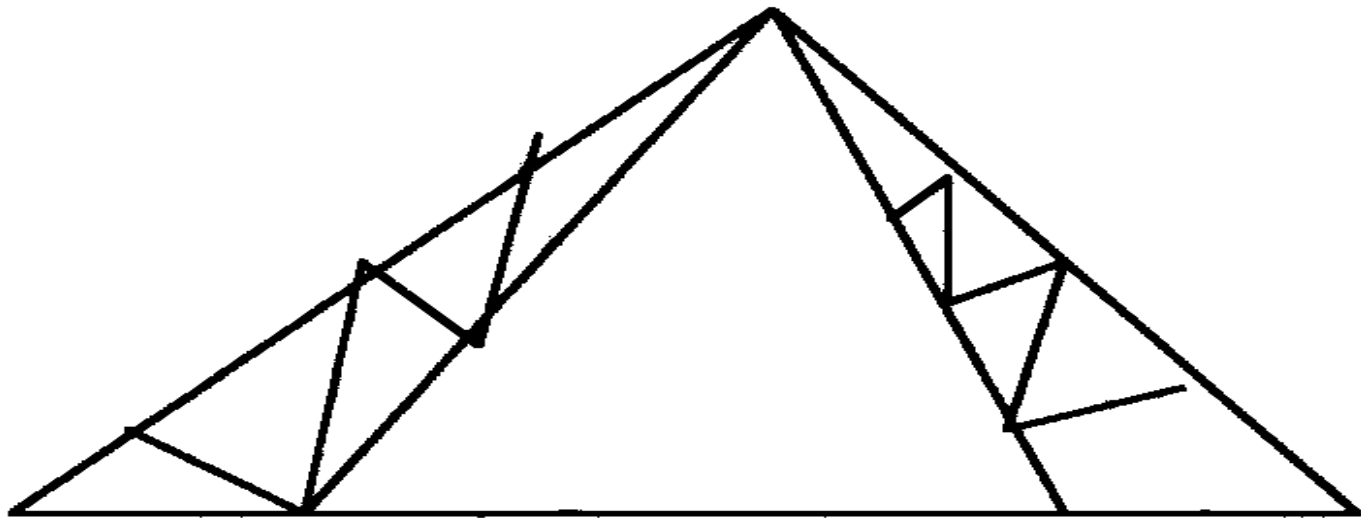
Howe's Roof Truss



(a)

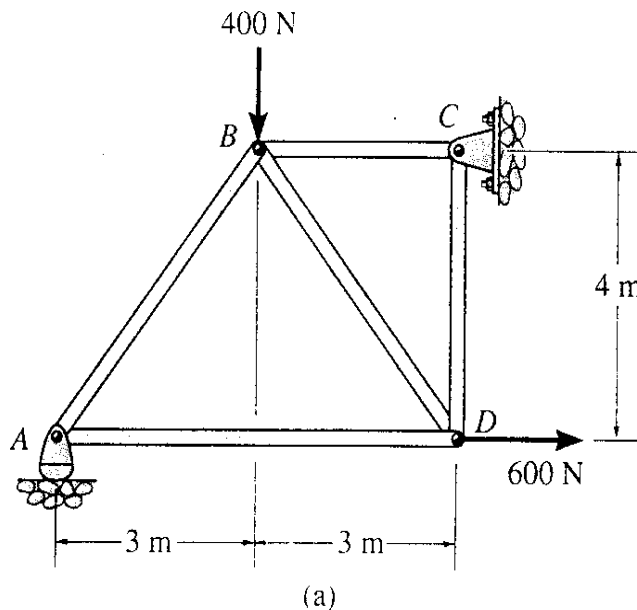
# Types of Trusses Contd.

- (a) Non-Simple Truss-Fink's Roof Truss

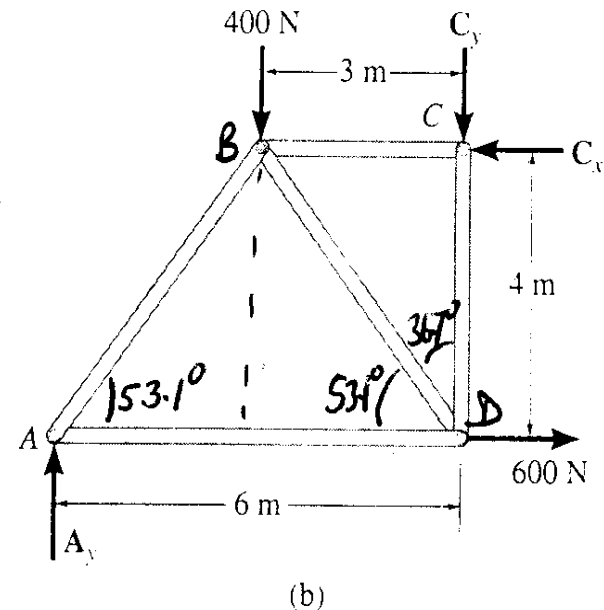


# 6.1.1 Analysis of Trusses- Method of Joints

- **Example:** Determine the force in each member of the truss shown. Indicate whether the members are in tension or compression.

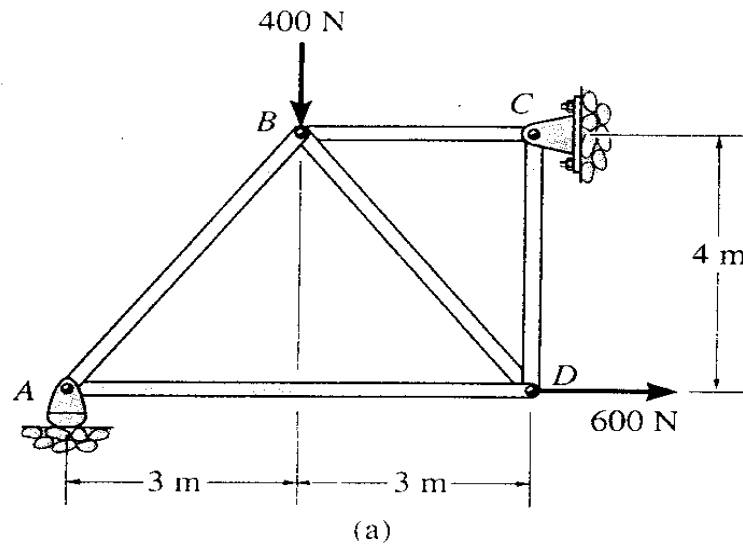


Space diagram

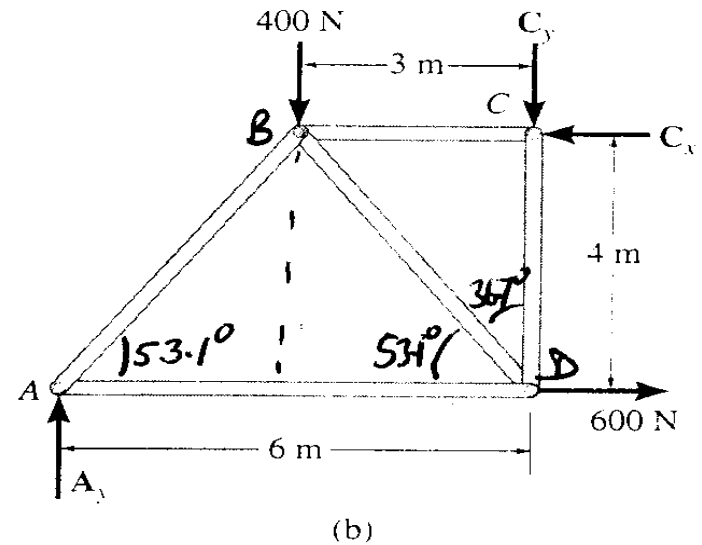


Free-body Diagram.

# External Forces Determination



Space diagram



Free-body Diagram.

## External Forces Determination

$$\sum F_x = 0 \text{ i.e. } 600 \text{ N} - C_x = 0, \quad C_x = 600 \text{ N}$$

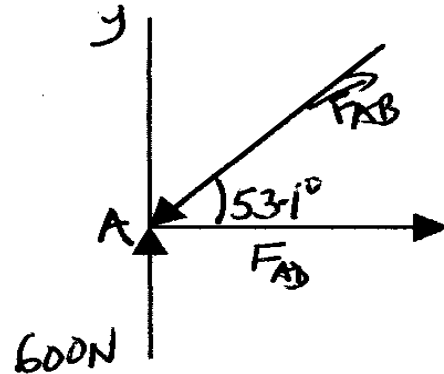
$$\sum M_c = 0 \text{ i.e. } -A_y (6 \text{ m}) + 400 \text{ N} (3 \text{ m}) + 600 \text{ N} (4 \text{ m}) = 0$$

$$6 A_y = 1200 + 2400 = 3600 \text{ N}, \quad A_y = 600 \text{ N}$$

$$\sum F_y = 0 \text{ i.e. } 600 \text{ N} - 400 \text{ N} - C_y = 0, \quad C_y = 200 \text{ N}$$

Choose a joint where there are no more than two unknowns. Start from A or C.

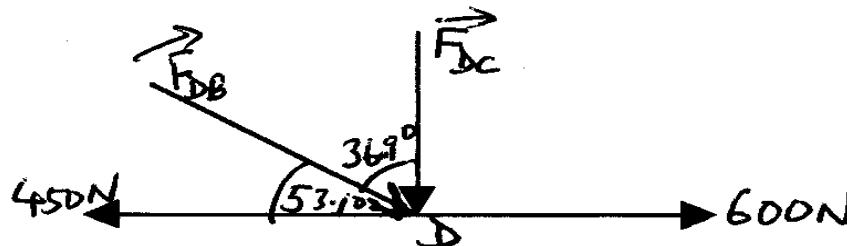
### Joint A



$$\sum F_y = 0 \quad \text{i.e.} \quad 600 \text{ N} = F_{AB} \sin 53.1^\circ; \quad F_{AB} = 750 \text{ N (C)}$$

$$\sum F_x = 0 \quad \text{i.e.} \quad F_{AD} = F_{AB} \cos 53.1^\circ = 750 \cos 53.1^\circ = 450 \text{ N (T)}$$

### Joint D



$$\sum F_x = 0 \quad \text{i.e.} \quad -450 + F_{DB} \cos 53.1^\circ + 600 = 0$$

$$0.6 F_{DB} = -150 \text{ N i.e.} \quad F_{DB} = -250 \text{ N}$$

The negative sign indicates that  $F_{DB}$  acts in the opposite sense to that shown:

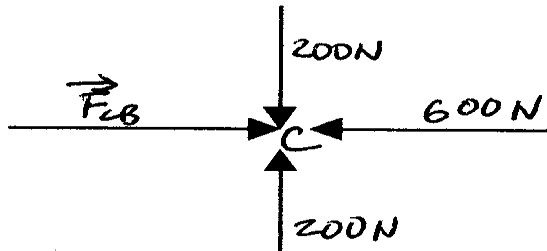
$$\text{Hence } F_{DB} = 250 \text{ N (T)}$$

**Note:** We could have known that  $F_{DB}$  will move upwards before analyzing the joint.

To determine  $F_{DC}$ , we can either correct the sense of  $F_{DB}$  and then apply  $\sum F_y = 0$  or apply this equation and retain the negative sign for  $F_{DB}$ .

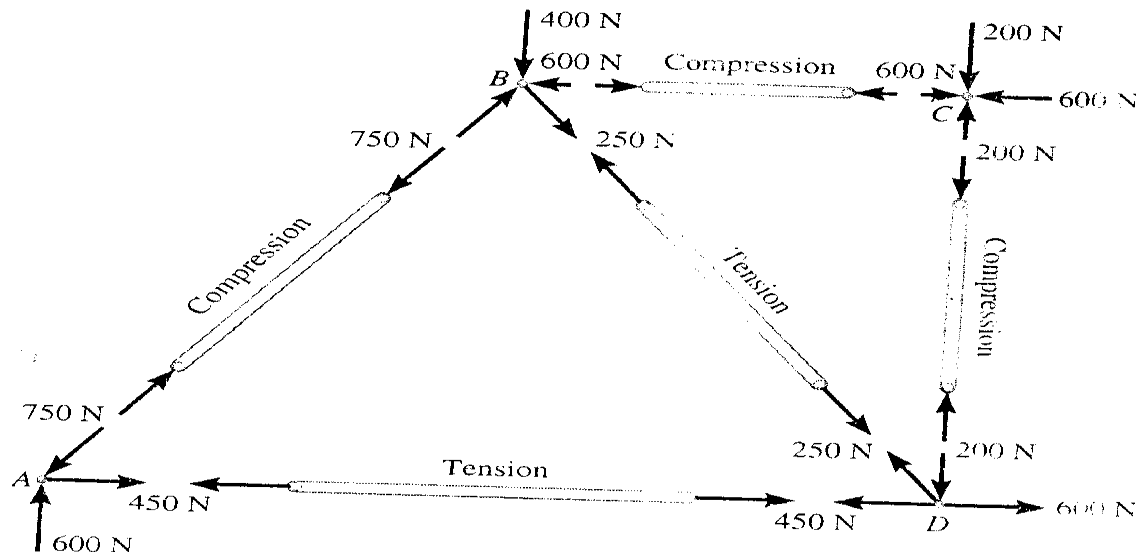
$$\sum F_y = 0 \text{ i.e. } -F_{DC} - (-250) \cos 36.9^\circ = 0; \quad F_{DC} = 200 \text{ N (C)}$$

**Joint C**



$$\sum F_x = 0; F_{CB} - 600 \text{ N} = 0 \text{ i.e. } F_{CB} = 600 \text{ N (C)}$$

$$\sum F_y = 0; 200 - 200 = 0 \text{ (Check).}$$

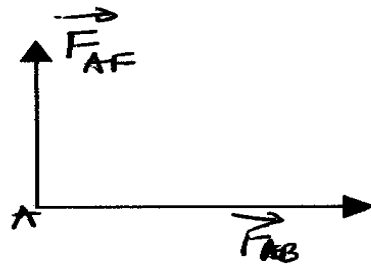


## 2.3 Zero Force Members

- These members are used to increase the stability of the truss during construction and to provide support if the applied loading is changed. There are two conditions:
- (i) If only two members form a truss joint and no external load or support reaction is applied to the joint, the members must be zero force members.

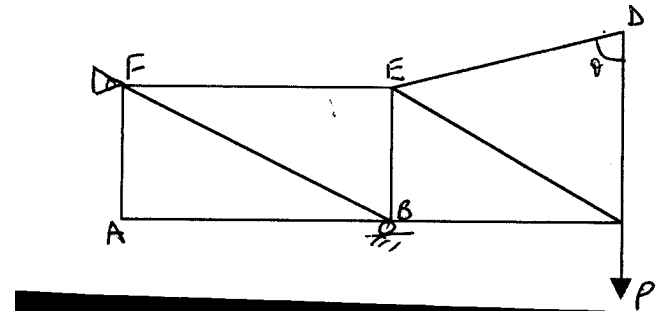
# Zero Force Members Contd.

Consider Joint A

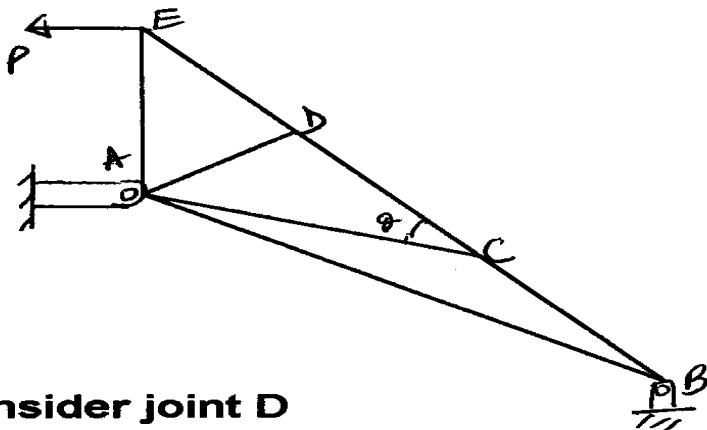


$$\sum F_x = 0, F_{AB} = 0$$

$$\sum F_y = 0, F_{AF} = 0$$



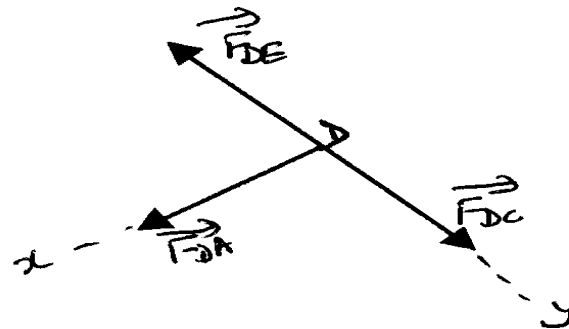
(ii) If three members form a truss joint for which two of the members are collinear, the third member is a zero force member provided no external force or support reaction is applied to the joint.



Consider joint D

$$\sum F_x = 0 \text{ i.e. } F_{DA} = 0$$

$$\sum F_y = 0 \text{ i.e. } F_{DE} = F_{DC}$$





# Analysis of Trusses - Method of Sections

- If there is no need to solve for all the forces in the members, and all the external forces, then the method of joints would be laborious. Method of sections can be used.
- Steps
- (i) Determine the external forces analytically
- (ii) Draw a line which splits the free body diagram into two halves such that the line crosses the members whose forces are required.
- The line should not cross more than three members whose forces are unknown.

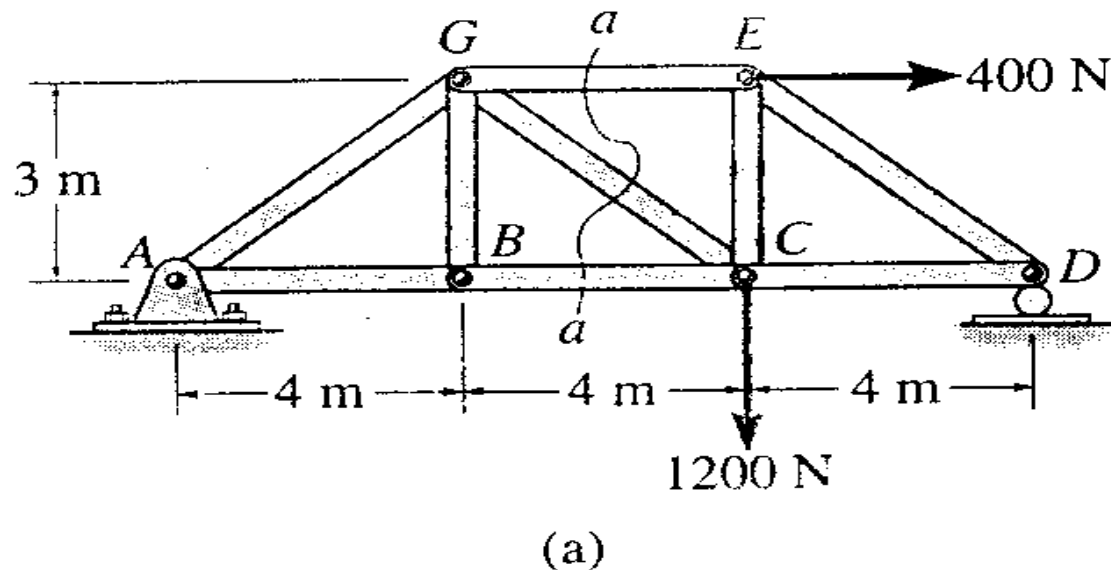
# Steps in the Method of Sections

## Contd.

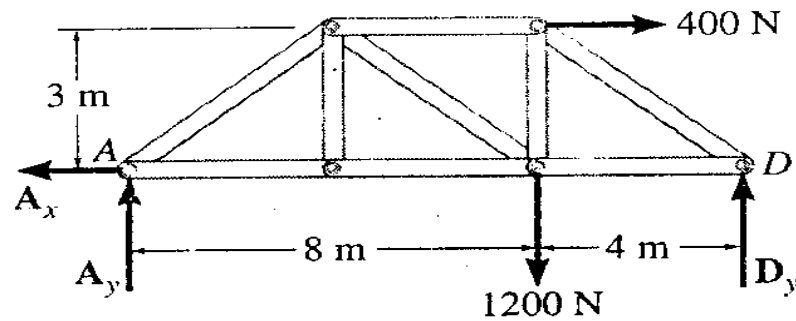
- (iii) Choose one of the halves and draw the free body diagram. Use arbitrary directions for the forces in the members. The solution will give the actual direction.
- (iv) Assuming the external forces have been found, then since the sections chosen must be in equilibrium, the three equations of equilibrium for a 2-dimension rigid body are sufficient to determine the maximum three unknowns.

# Example

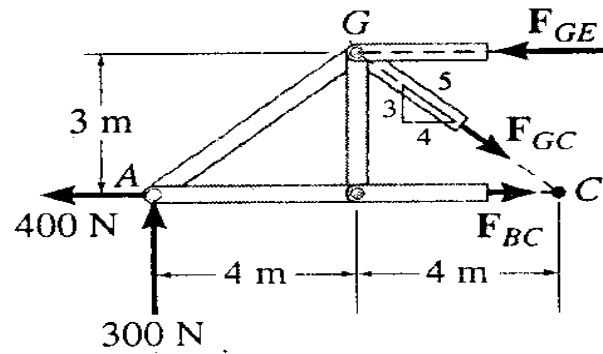
- Determine the force in members GE, GC and BC of the truss shown in the Figure. Indicate whether the members are in tension or compression



(a)



(b)



(c)

## External Forces Determination

$$\sum F_x = 0 \quad \text{i.e.} \quad A_x = 400 \text{ N}$$

$$\sum M_A = 0 ; \quad \text{i.e.} \quad -1200 \times 8 - 400 \times 3 + 12 D_y = 0; \quad D_y = 900 \text{ N}$$

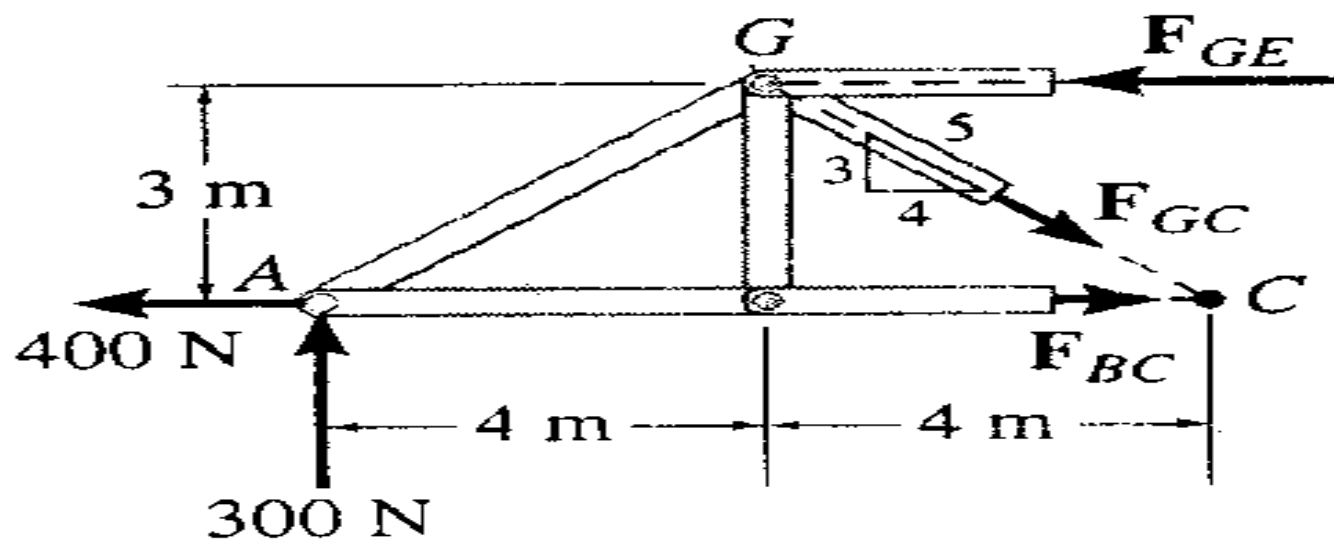
$$\sum F_y = 0 \quad \text{i.e.} \quad A_y - 1200 + 900 = 0; \quad A_y = 300 \text{ N}$$

**Considering the LHS of Cut section**

$$\sum M_G = 0 \text{ i.e. } -300 \times 4 - 400 \times 3 + 3 F_{BC} = 0; F_{BC} = 800 \text{ N (T)}$$

$$\sum M_C = 0 \text{ i.e. } -300 \times 8 + 3 F_{GE} = 0; F_{GE} = 800 \text{ N (C)}$$

$$\sum F_y = 0 \text{ i.e. } 300 = F_{GC} \sin 36.9^\circ; F_{GC} = 500 \text{ N (T)}$$



(c)



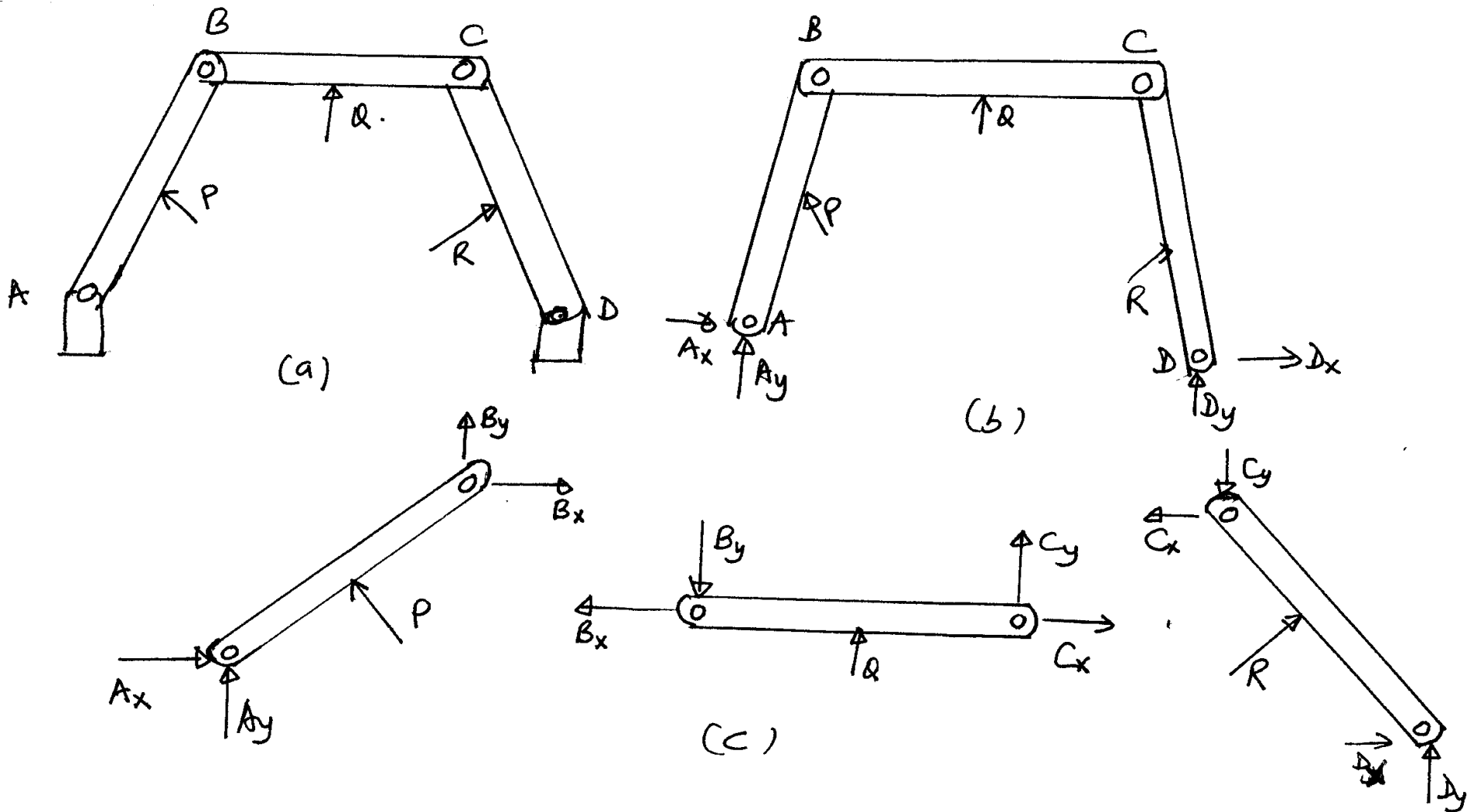
## 6.1 Frames and Machines

- Frames and machines are two common types of structures which are often composed of pin-connecting multi-force members i.e. subjected to three or more forces.
- Frames are stationary and are used to support loads while machines contain moving parts and are designed to transmit and alter the effect of forces.

## 6.1.1 Types of Frames:

- Frames are divided into two:
- (a) **Rigid Frames** where the shape does not change
- (b) **Non-rigid frame**: Where the removal or alteration of the supports of a frame causes the shape to change e.g. diagram below shows a four-link mechanism as an example of a non-rigid frame.

# Non-Rigid Frame



The ground itself forms the 4<sup>th</sup> link.

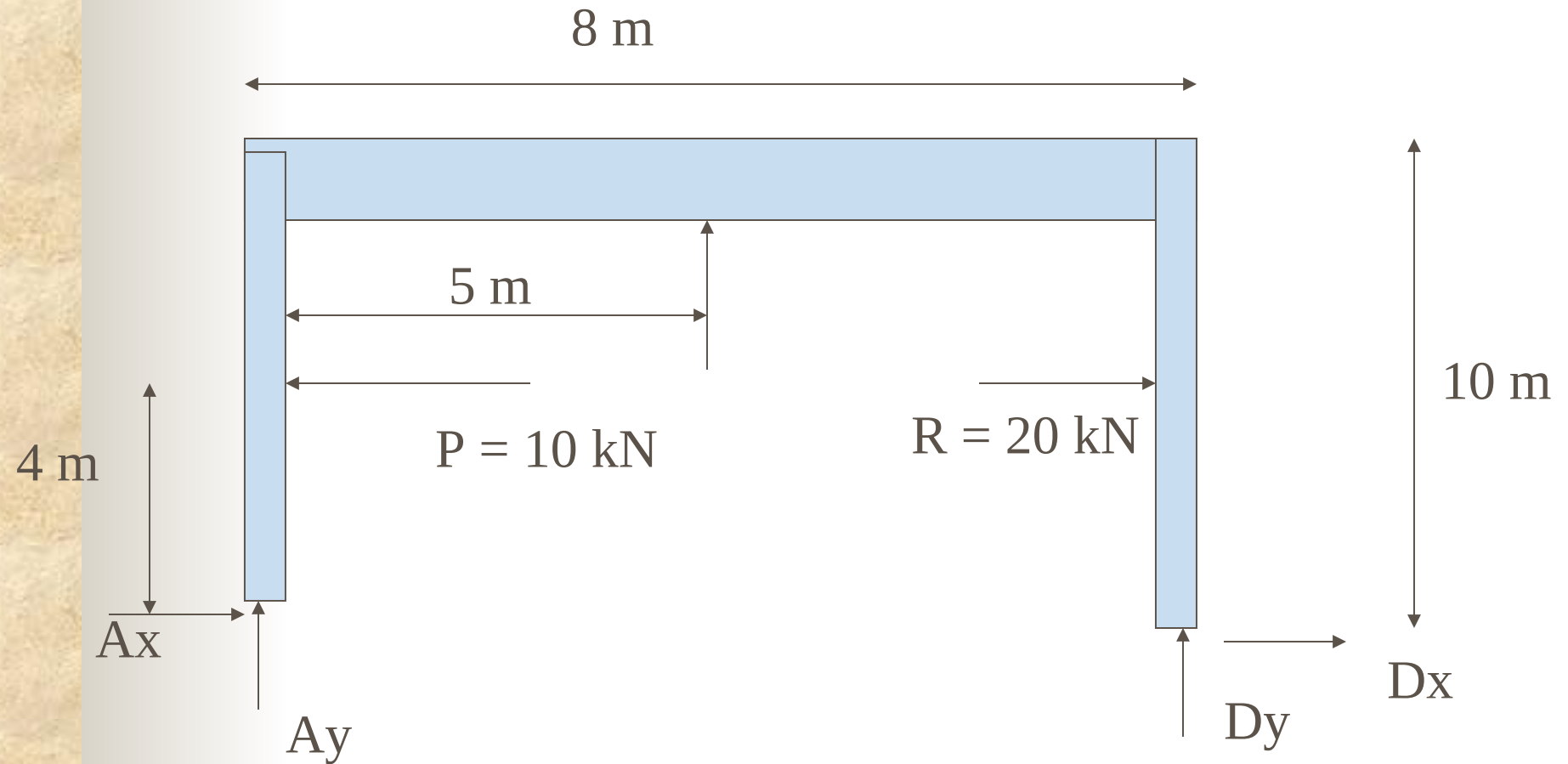
# Non-Rigid Frames

- Non-rigid frames are analyzed in the same way but not all the reaction forces can be obtained from the equilibrium of the entire non-rigid frame.
- See diagram (b) above. There are four unknowns and three equations of equilibrium.

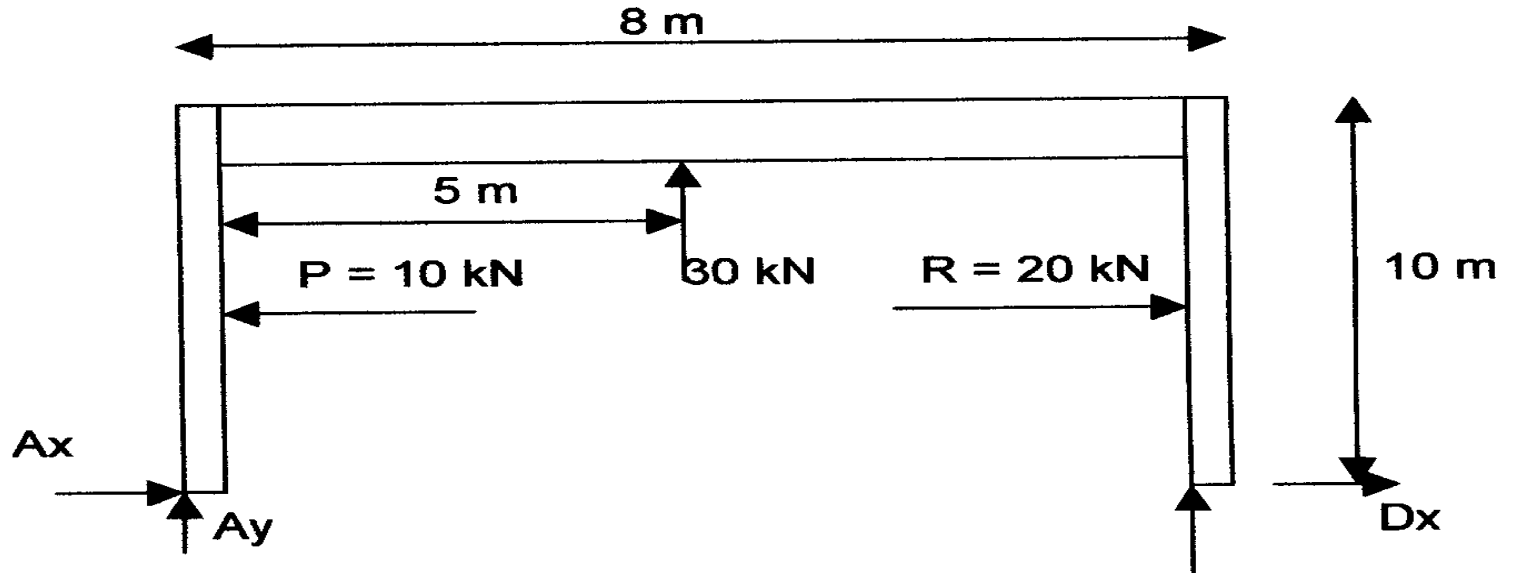
# Non-Rigid Frames Contd.

- From (c), the free body diagrams of the members show 8 unknown forces, the four reaction forces  $A_x$ ,  $A_y$ ,  $D_x$ ,  $D_y$  and four internal forces  $B_x$ ,  $B_y$ ,  $C_x$  and  $C_y$ . Since there are eight independent equilibrium equations, the structure is statically determinate.

# Example



## Example



$$\sum M_A = 0: 30 \times 5 \text{ m} + 10 \times 4 - 20 \times 4 + 8 D_y = 0 \quad D_y$$

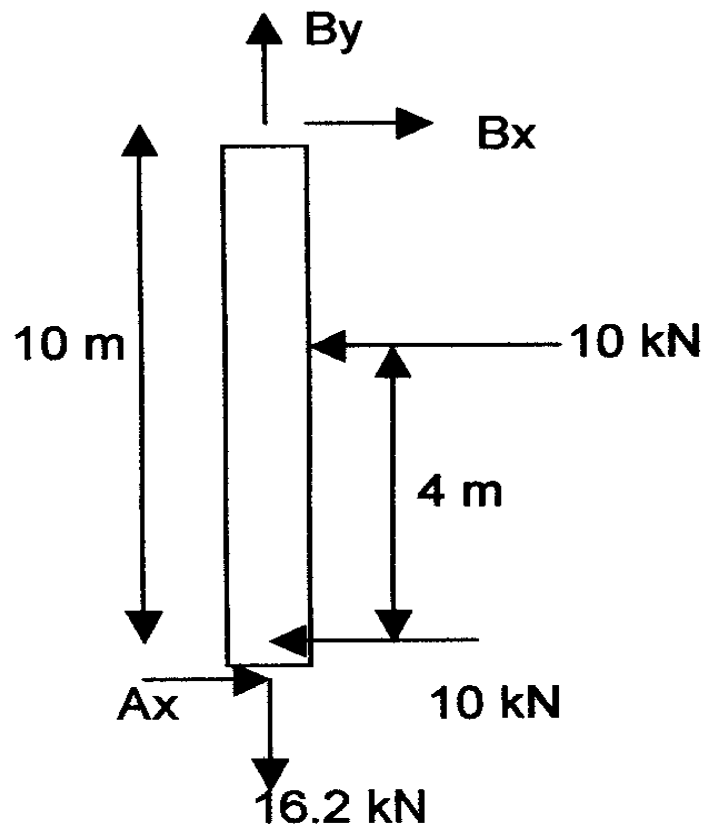
$$D_y = -13.8 \text{ N} = 13.8 \text{ kN} \downarrow$$

$$\sum F_y = 0: A_y - 13.8 + 30 = 0$$

$$A_y = -16.2 \text{ kN} = 16.2 \text{ kN} \downarrow$$

$$\sum F_x = 0: D_x + A_x - 10 + 20 = 0$$

$$\text{i.e. } D_x + A_x = -10 \quad (1)$$



$$\sum M_B = 0: -10 \times 6 + 10 A_x = 0$$

$$A_x = 6 \text{ kN} \rightarrow$$

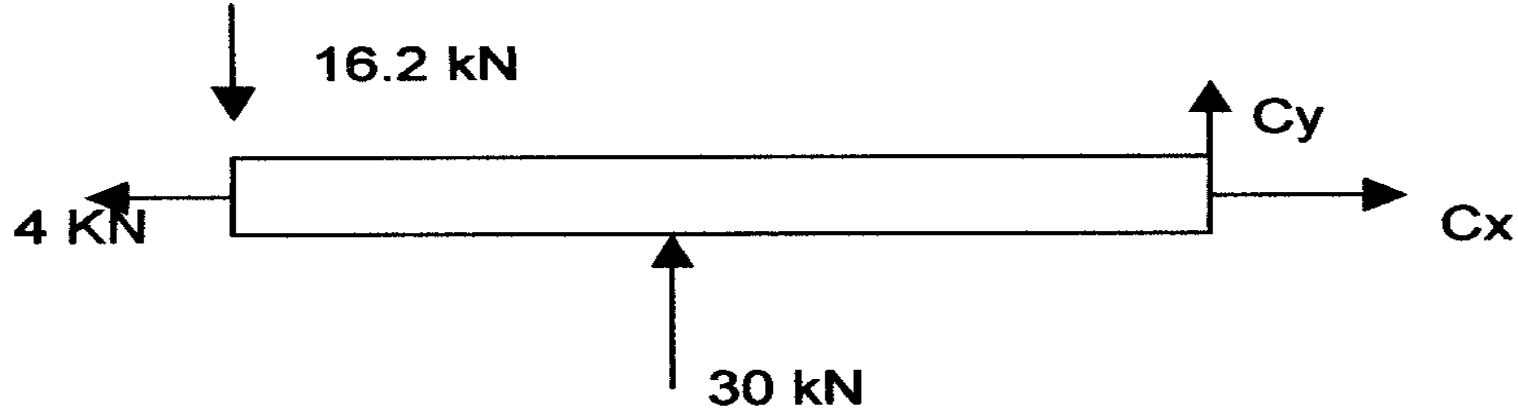
$$\begin{aligned} \text{From Eqn (1): } D_x &= -10 - 6 = -16 \text{ kN} \\ &= 16 \text{ kN} \leftarrow \end{aligned}$$

$$\sum F_y = 0: B_y - 16.2 = 0$$

$$B_y = 16.2 \text{ kN} \uparrow$$

$$\sum F_x = 0: A_x + B_x - 10 = 0$$

$$\text{i.e. } B_x = 4 \text{ kN} \rightarrow$$



$$\sum F_y = 0 : C_y - 16.2 + 30 = 0; C_y = -13.8 = 13.8 \text{ kN} \downarrow$$

$$\sum F_x = 0 : C_x = 4 \text{ kN} \longrightarrow$$

### Summary

$$A_x = 6 \text{ kN} \longrightarrow$$

$$A_y = 16.2 \text{ kN} \downarrow$$

$$D_x = 16 \text{ kN} \longleftarrow$$

$$D_y = 13.8 \text{ kN} \downarrow$$

$$B_x = 4 \text{ kN} \longrightarrow$$

$$B_y = 16.2 \text{ kN} \uparrow$$

$$C_x = 4 \text{ kN} \longrightarrow$$

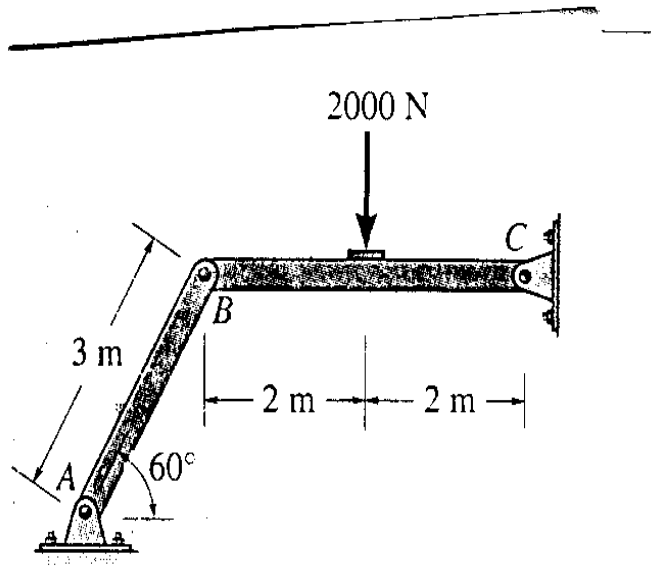
$$C_y = 13.8 \text{ kN} \downarrow$$

# Rigid Frames

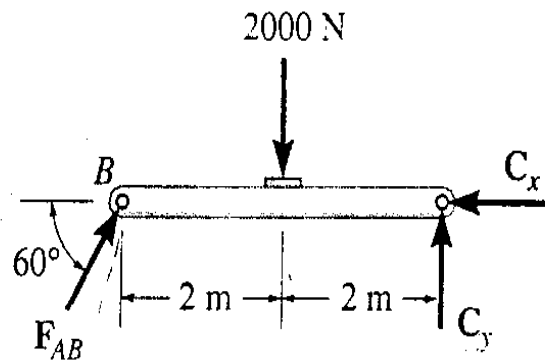
- Rigid frames are analyzed by first drawing the free-body diagram of the entire structure so as to determine the reaction forces.
- A free-body diagram of each member is then drawn and equilibrium equations are used to determine the internal forces. Consider the two-force members first before the multi-force ones.

# Example

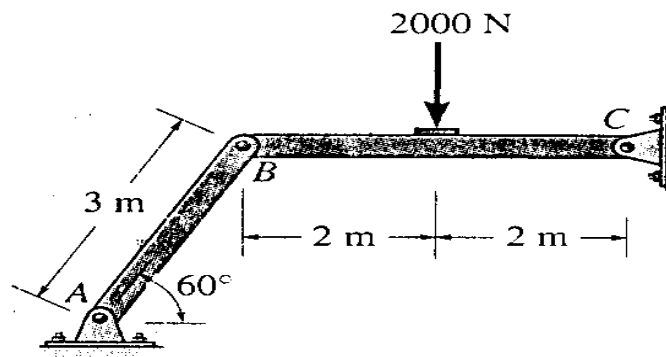
**Example:** Determine the horizontal and vertical components of force which the pin at C exerts on member CB of the frame.



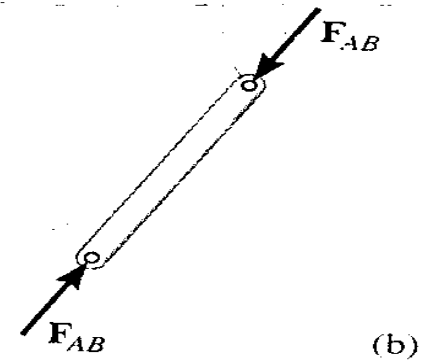
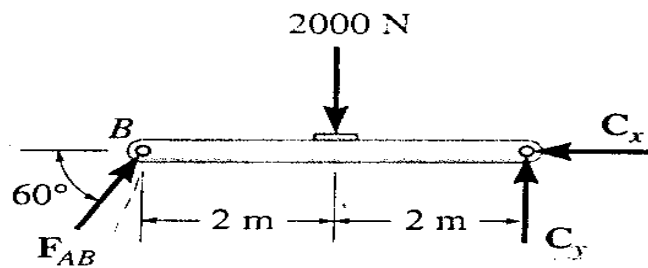
(a)



(b)



(a)



(b)

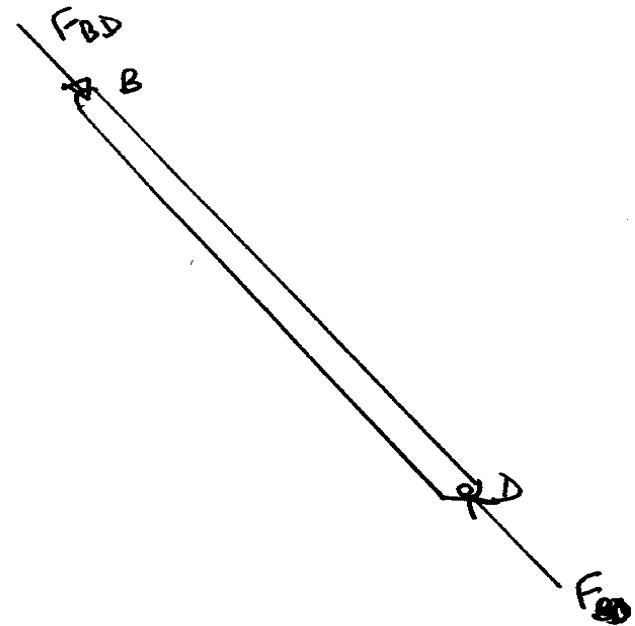
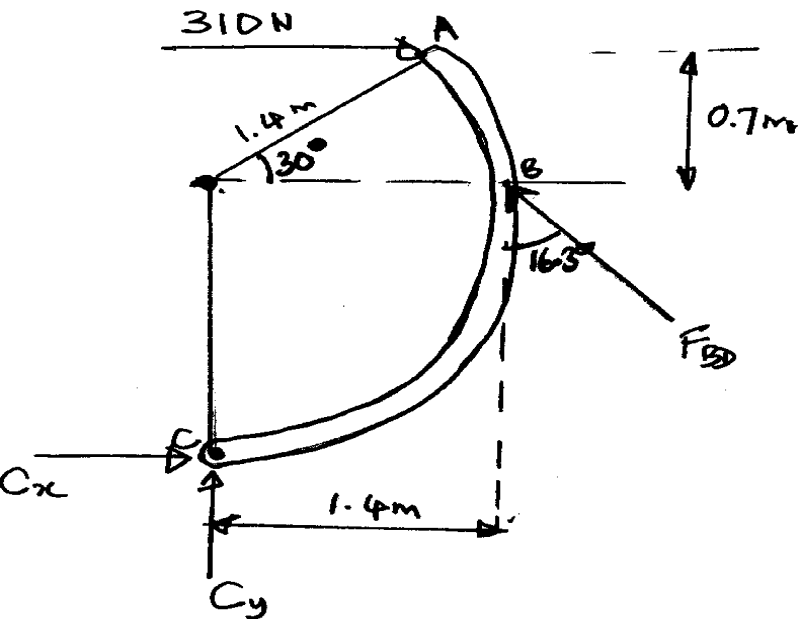
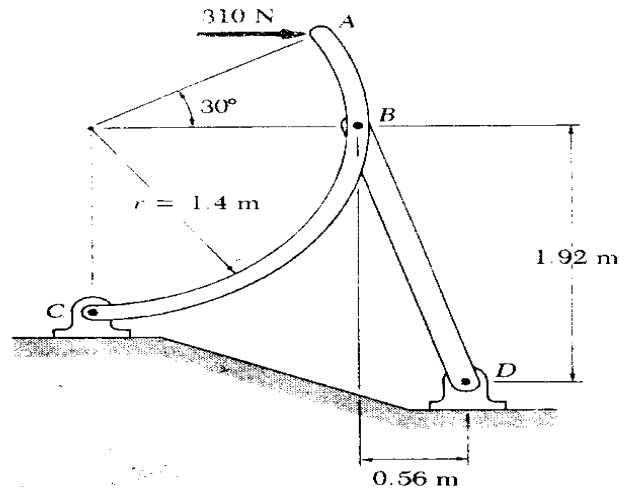
**Reaksi di Batas-batas: Setelektrodigasi**

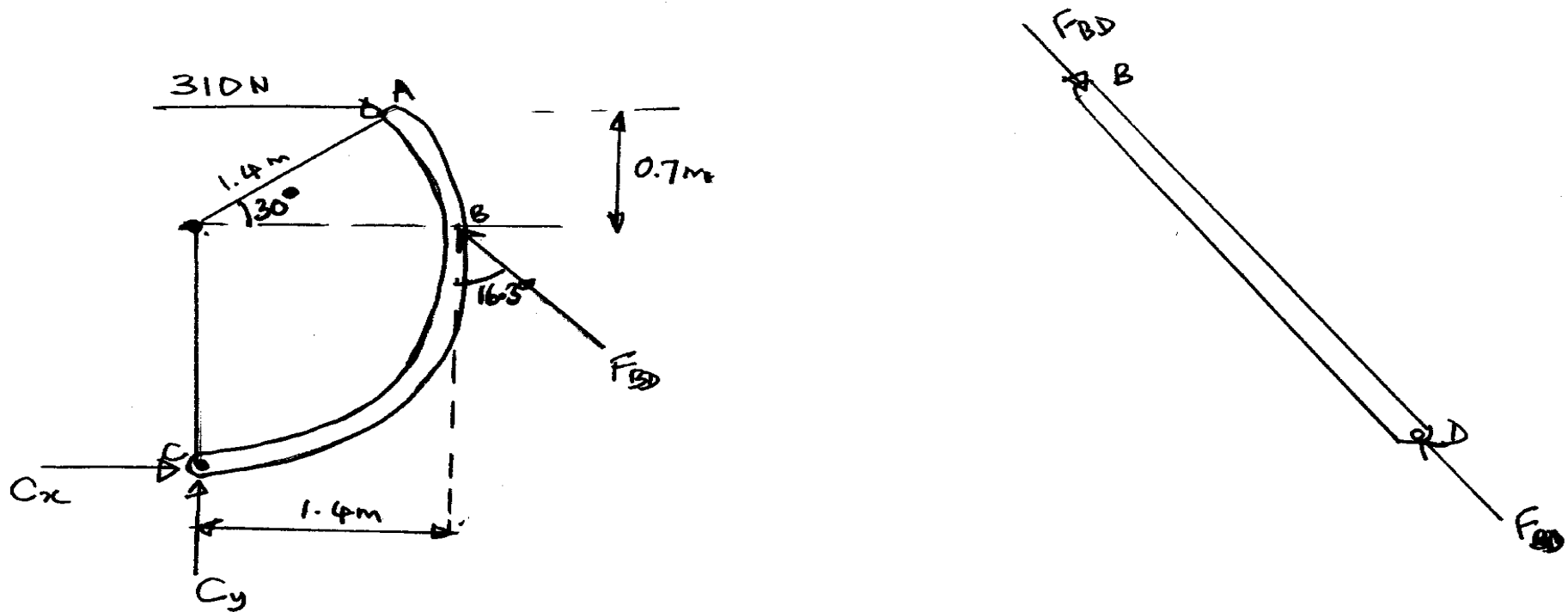
$$\sum M_A = 0: 2000 \times 2\text{ m} - F_{AB} \sin 60^\circ = 0 \quad F_{AB} = 1147 \text{ N}$$

$$\sum F_x = 0: 1147 \cos 60^\circ - C_x = 0 \text{ ie } C_x = 574 \text{ N}$$

$$\sum F_y = 0: 1147 \sin 60^\circ - 2000 + C_y = 0 \quad C_y = 100 \text{ N}$$

**Example. Problem 6.51.** Determine the force in member BD and the components of the reaction at C.





Consider LHS of structure:

$$\sum M_c = 0 \text{ i.e. } F_{BD} \sin 16.3^\circ \times 1.4 + F_{BD} \cos 16.3^\circ \times 1.4 = 310 \times 2.1$$

$$0.393 F_{BD} + 1.344 F_{BD} = 651 ; F_{BD} \approx 375 \text{ N}$$

$$\sum F_y = 0 \text{ i.e. } C_y + 375 \cos 16.3^\circ = 0 , C_y = - 360 \text{ N} = 360 \text{ N} \downarrow$$

$$\sum F_x = 0 \text{ i.e. } C_x - 375 \sin 16.3^\circ + 310 = 0$$

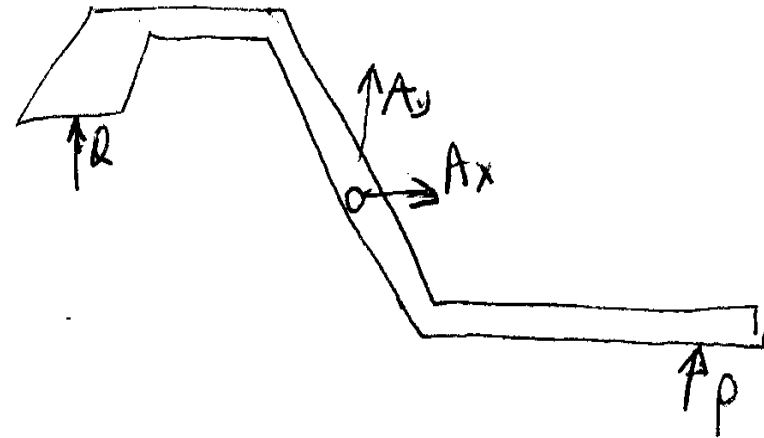
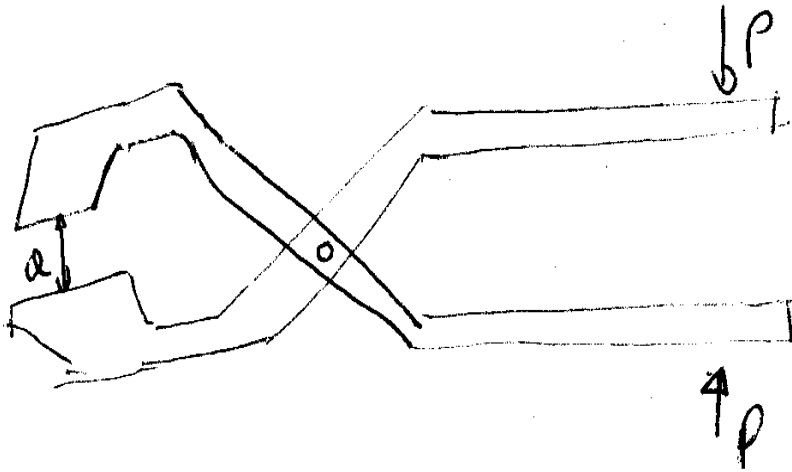
$$C_x = 105.25 - 310 = - 204.7 = - 205 \text{ N} = 205 \text{ N} \leftarrow$$

$$\underline{\text{Force in BD} = 375 \text{ N}; \quad C_y = 360 \text{ N} \downarrow ; \quad C_x = 205 \text{ N} \leftarrow}$$

## 6.3.2 Machines

- A machine has moving parts and is usually not considered a rigid structure. Machines are designed to transmit loads rather than support them
- e.g the pair of tongs below has a force  $P$  applied to each tong that transmits the gripping force  $Q$ .

# Machine Contd.



This gripping force,  $Q$  as well as reactions at the pin can be obtained as a function of the applied force,  $P$  by drawing the free-body diagram of a part of the machine. Equilibrium equations can be used to obtain  $Q$  as well as the reactions.

**Example: Problem 6.104:** A 3600 kg steel ingot is lifted by a pair of tongs as shown.

Determine the forces exerted at C and E on tong BCE.

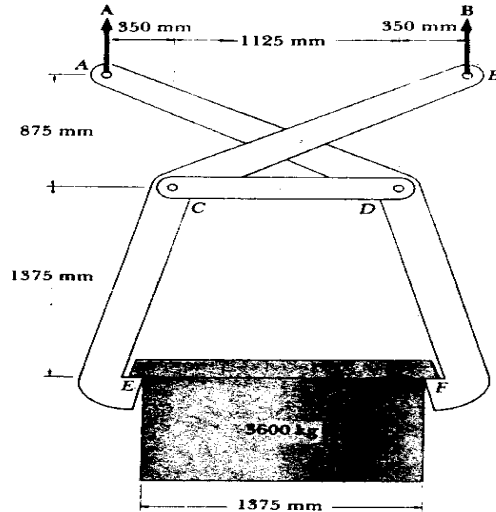
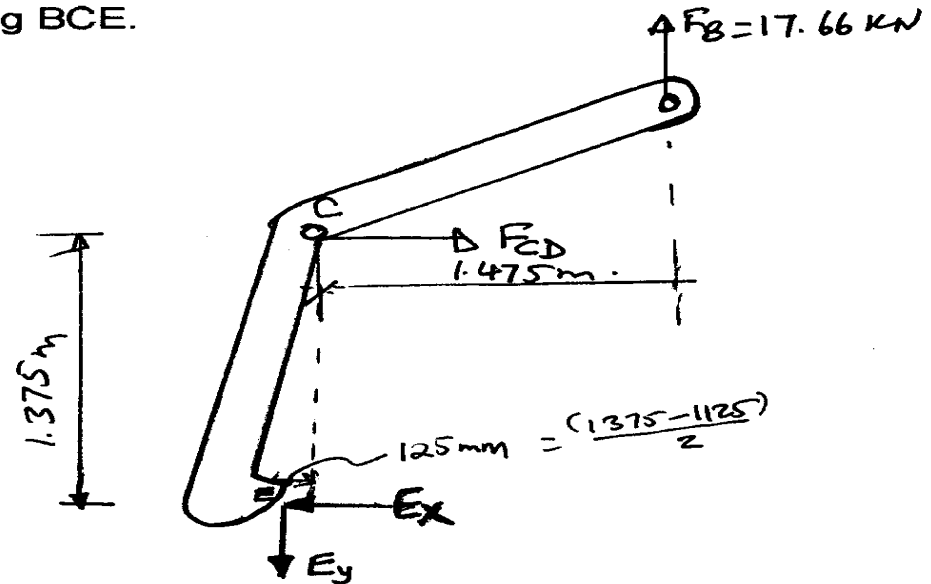


Fig. P6.104



**Solution:** For the whole machine,  $F_A + F_B = (3600 \times 9.81)/1000 = 35.3 \text{ kN}$

Because of symmetry of loading:  $F_B = 35.3/2 = 17.66 \text{ N}$ .

For the isolated member (diagram RHS above):  $\sum F_y = 0$  i.e.  $E_y = 17.66 \text{ kN} \downarrow$

$\sum M_c = 0$  i.e.  $17.66 \times 1.475 - E_x \times 1.375 + 17.66 \times 0.125 = 0$

$$26.05 - 1.375 E_x + 2.21 = 0; E_x = 20.55 \text{ kN} \leftarrow$$

$\sum F_x = 0$  i.e.  $F_{CD} = E_x = 20.55 \text{ kN} \rightarrow$

Reaction at E =  $(E_x^2 + E_y^2)^{0.5} = (20.55^2 + 17.66^2)^{0.5} = 27.1 \text{ kN}$

$$\theta = \tan^{-1} \frac{17.65}{20.55} = 40.7^\circ \quad \text{i.e.} \quad E = 27.1 \text{ kN} \searrow 40.7$$

