



College of Natural and Social Sciences (Department of Mathematics)

Mathematics for Natural Science (Math 1011)

Questions on the Set of Real and Complex Number Systems

1. Find an odd natural number x such that $\text{LCM}(x, 40) = 1400$.
2. There are between 50 and 60 number of eggs in a basket. When Loza counts by 3's, there are 2 eggs left over. When she counts by 5's, there are 4 left over. How many eggs are there in the basket?
3. The GCF of two numbers is 3 and their LCM is 180. If one of the numbers is 45, then find the second number.
4. Using Mathematical Induction, prove the following:
 - a. $6^n - 1$ is divisible by 5, for $n \geq 0$.
 - b. $2^n \leq (n+1)!$, for $n \geq 0$.
 - c. $x^n + y^n$ is divisible by $x + y$ for odd natural numbers $n \geq 1$.
 - d. $2 + 4 + 6 + \dots + 2n = n(n+1)$
 - e. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
 - f. $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$
 - g. $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$
5. Express each of the following rational numbers as decimal:
 - a) $\frac{4}{9}$
 - b) $\frac{3}{25}$
 - c) $\frac{11}{7}$
 - d) $-5 \frac{2}{3}$
 - e) $\frac{2}{77}$
6. Write each of the following as decimal and then as a fraction:
 - a) three tenths
 - b) four thousands
7. Write each of the following in meters as a fraction and then as a decimal
 - a) 4mm
 - b) 6cm and 4mm
 - c) 56cm and 4mm



8. Classify each of the following as terminating or non-terminating periodic

a) $\frac{5}{13}$ b) $\frac{7}{10}$ c) $\frac{69}{64}$ d) $\frac{11}{60}$ e) $\frac{5}{12}$

9. Convert the following decimals to fractions:

a) $3.2\bar{5}$ b) $0.3\bar{14}$ c) $0.\overline{275}$

10.

Determine

whether the following are rational or irrational:

a) $2.7\bar{5}$ b) $0.272727\ldots$ c) $\sqrt{8} - \frac{1}{\sqrt{2}}$

11.

Which

of the following statements are true and which of them are false?

- a. The sum of any two rational numbers is rational
- b. The sum of any two irrational numbers is irrational
- c. The product of any two rational numbers is rational
- d. The product of any two irrational numbers is irrational.

12. Find two rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

13. Verify that

a) $(\sqrt{2}-i) - i(1-\sqrt{2}i) = -2i$ b) $(2, -3)(-2, 1) = (-1, 8)$
 c) $(3, 1)(3, -1)\left(\frac{1}{5}, \frac{1}{10}\right) = (2, 1)$ d) $(2+3i)^2 - (3i-6) = 1+9i$

14. Show that

a) $\operatorname{Re}(iz) = -\operatorname{Im}(z)$ b) $\operatorname{Im}(iz) = \operatorname{Re}(z)$ c) $(z+1)^2 = z^2 + 2z + 1$

15. Do the following operations and simplify your answer.

a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ b) $\frac{5i}{(1-i)(2-i)(3-i)}$ c) $(1-i)^3$

16. Locate the complex numbers $z_1 + z_2$ and $z_1 - z_2$, as vectors where



$$\begin{aligned} a) \ z_1 &= 2i, \ z_2 = \frac{2}{3} - i & b) \ z_1 &= (-\sqrt{3}, 1), \ z_2 = (\sqrt{3}, 0) \\ c) \ z_1 &= (-3, 1), \ z_2 = (1, 4) & d) \ z_1 &= a + ib, \ z_2 = a - ib \end{aligned}$$

17. Sketch the following set of points determined by the condition given below:

$$a) \ |z - 1 + i| = 1 \quad b) \ |z + i| \leq 3 \quad c) \ |z - 4i| \geq 4$$

18. Using properties of conjugate and modulus, show that

$$a) \ \overline{\overline{z} + 3i} = z - 3i \quad b) \ \overline{iz} = -i \overline{z} \quad c) \ \overline{(2+i)^2} = 3 - 4i$$

19. Show that $(-1+i)^7 = 8(-1-i)$.

20. Using mathematical induction, show that (when $n = 2, 3, \dots$)

$$a) \ \overline{z_1 + z_2 + \dots + z_n} = \overline{z_1} + \overline{z_2} + \dots + \overline{z_n} \quad b) \ \overline{z_1 z_2 \dots z_n} = \overline{z_1} \overline{z_2} \dots \overline{z_n}$$

21. Show that the equation $|z - z_0| = r$ which is a circle of radius r centered at z_0 can be written as $|z|^2 - 2\operatorname{Re}(z \overline{z_0}) + |z_0|^2 = r^2$.

22. Find

the argument of the following complex numbers: $a) \frac{3i}{-1-i} \quad b) (\sqrt{3}-i)^6$

23. Show

$$\text{that } a) \ |e^{i\theta}| = 1 \quad b) \ \overline{e^{i\theta}} = e^{-i\theta}$$

24. Using

mathematical induction, show that

$$e^{i\theta_1} \cdot e^{i\theta_2} \cdot \dots \cdot e^{i\theta_n} = e^{i(\theta_1 + \theta_2 + \dots + \theta_n)}, \quad n = 2, 3, \dots$$

25. Show

$$\text{that } a) \ \cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta \quad b) \ \sin 3\theta = 3\cos^2 \theta \sin \theta - \sin^3 \theta$$

26. Show

$$\text{that } 1 + z + z^2 + \dots + z^n = \frac{1 - z^{n+1}}{1 - z}, \quad \text{for } z \neq 1.$$



27. Find
the square roots of $z = 9i$
28. Find
the cube roots of $z = -8i$
29. Solve
the following equations:
 $a) z^{3/2} = 8i$ $b) z^2 + 4i = 0$ $c) z^2 - 4i = 0$