Addis Ababa University

Department of Mathematics

Math 2042 Applied Mathematics III

Worksheet 1

A. Solve the following differential equations

1. $(1+x^2)\tan^{-1}x\frac{dy}{dx}+y=0$
2. $e^x(y^2+1)dx+(1+e^x)ydy=0$
3. $y-x\frac{dy}{dx}=y^2+\frac{dy}{dx}$
4. $x\sec^2 ydy+(2-\tan y)dx=0$
5. $(1-y)dx+(x^4-x^2)dy=0$
6. $(xy+2x+y+2)dx+(x^2+2x)dy=0$
7. $2r\cos tdr-(r^4+1)dt=0$
8. $ye^{-x}dx+(3y-e^{-x})dy=0$
9. $(3y-7x+3)dx-(5y-3x+1)dy=0$
10. $(2x-2y^2)dx+(12y^2-4xy+1)dy=0$
11. $y'=\frac{2y+x}{2x-y}$
12. $y'=\frac{x-y+4}{x+y-6}$
13. $(-5x^4+3x^2y^2-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy=0$
14. $(\sin x\sec^2 y+\cos(x+y))dy+(\cos x\tan y+\cos(x+y))dx=0$

B. Solve the following differential equations

1. $(xy^2-x^5)dx-x^2ydy=0$
2. $(3xy+4y-1)dx+(x+1)dy=0$
3. $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$
4. $ydx+(x-x^4)dy=0$
5. $y'+1=e^{x-y}$
6. $y'+2y=\cos 3x$
7. $x(1-x^2)y'+(x^2-1)y=x^3$
8. $x^2y'+2xy=\sin^2 x$
9. $(1+x^2)\,dy+2xy\,dx=\cot x\,dx$
10. $(2y-x^3)dx=xdy$

C. Solve the IVP

1. $(y+2)dx+y(x+4)dy=0,\ y(-3)=-1$
2. $(3x+8)(y^2+4)dx-y(x^2+5x+6)dy=0,\ y(1)=2$
3. $(x^2-3x^2y)dx-x^3dy=0,\ y(2)=1$
4. $y'+y\cot x=4x\csc x,\ y(\frac{\pi}{2})=0$

D. Word Problems

1. If 10 % of a radio-active substance disintegrates in 4 days, what is its half-life?
2. Uranium disintegrates at a rate proportional to the amount present at any time. If m_1 and m_2 grams of uranium are present at time t_1 and t_2 respectively, show that the half-life of Uranium is
$$\frac{(t_2-t_1)\ln 2}{\ln(m_1/m_2)}$$
3. A thermometer showing 10^0C is brought into a room whose temperature is 25^0C. After 5 minutes it shows 20^0C. When will the thermometer reach the room temperature 24^0C? (Apply Newton's Law of Cooling)
4. Find a differential equation whose general solution is the class of all upward parabolas with vertex on the x-axis.

5. A culture initially has P_o number of bacteria. At t=1 hour, the number of bacteria is measured to be $\frac{3}{2}$ P_0. If the rate of growth is proportional to the number of bacteria P(t) present at time t, determine the time necessary for the number of bacteria to triple.
6. A tank with a capacity of 600 liters initially contains 200 liters of pure water. A salt solution containing 3 Kg of salt per liter is allowed to run into the tank at a rate of 16 lit/min, and the mixture is then removed at a rate of 12 lit/min. Find the expression for the number of Kilograms of salt in the tank at any time t.

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Work Sheet 2

A. Solve the following differential equations.

1. $y'' + (a+b)y' + aby = 0$
2. $2y'' - y' - y = 0$
3. $6y'' + 7y' - 20y = 0$
4. $y'' - 6y' + 13y = 0$
5. $25y'' - 10y' + y = 0$
6. $y'' + y' - 6y = 0$

B. Given that $y_1 = x$ a solution of each of the following ODEs, determine a solution y_2 such that the Wronskian $W(y_1, y_2) \neq 0$ in each case.

1. $(x-1)y'' - xy' + y = 0$
2. $(1-x^2)y'' - 2xy' + 2y = 0$
3. $x^2y'' + y' - y = 0$
4. $x^2y'' - 2xy' + 2y = 0$

C. Find the general solution of each of the following differential equations.

1. $y'' - 2y' - 8y = 8e^{2x} - 6e^{3x}$
2. $2y'' - y' = 10x^2$
3. $y'' - a^2y = x + e^{-ax}, a > 0$
4. $y'' - 2y' + y = xe^x$
5. $y'' + y' + y = 5 + \cosh x$
6. $y'' - 9y = \sin 3x$
7. $y'' - y = x\cos x$
8. $y'' - 4y' + 4y = x - 1 + e^{2x}$
9. $y'' + 2y' + y = 3e^{-x}\sin x$

D. Using the method of variation of parameters solve

1. $y'' + 3y' + 2y = 4e^x$
2. $y'' - 2y' = x + 1$
3. $y'' + 9y = 2\sec 3x$
4. $y'' + y = \csc^2 x$

E. Solve the following IVP

1. $y'' - 3y' + 2y = 0; y(1) = 0, y'(1) = -1$
2. $y'' + 4y' + 29y = 0; y(0) = 2, y'(0) = 0$
3. $y'' + 4y' + 29y = 0; y(0) = 2, y'(0) = 0$
4. $y'' + y = 3x - 4\sin x; y(0) = 0, y'(0) = -1$
5. $y'' - y = 3x^2e^x;\ y(0) = 1,\ y'(0) = 2$

F. Solve the following Cauchy-Euler equations

1. $4x^2y'' - 4xy' + 3y = 0$
2. $x^2y'' + 4xy' + 2y = 2x^4 + \frac{1}{x}$
3. $x^2y'' - xy' - 3y = x^2\ln x$
4. $(1+x)^2y'' + (1+x)y' + y = 4$
5. $x^2y'' + xy' - y = x^2 + 1; y(1) = 2, y'(1) = -3$

G. Solve the following system of ODEs

a) $\begin{cases} \dot{x} = x - 2y \\ \dot{y} = 2x + y \end{cases}$

b) $\begin{cases} \dot{x} = 3x + 5y \\ \dot{y} = x + 2y \end{cases}$

c) $\begin{cases} \dot{x} = -x + 4y \\ \dot{y} = 3x - 2y \end{cases}$ $x(0) = 3,\ y(0) = 4$

d) $\begin{cases} \dot{x} = x + 4y - t^2 + 6t \\ \dot{y} = x + y - t^2 + t - 1 \end{cases}$ $x(0) = 2,\ y(0) = -1$

H. 1. A 10 kg mass stretches a spring 80 cm in equilibrium. Suppose that a 2 kg mass is attached to the spring, initially displaced 25 cm below equilibrium and given an initial velocity of 2 m/sec. Assuming no resistive force, find the displacement at time t. Find the period, frequency and the amplitude of the motion. (use g =10m/s^2)

2. An electric circuit consists of a resistor of 4 ohms, an inductor of 0.05 henrys, a conductor of 0.008 farads and a source of electromotive force of voltage $E(t) = 90e^{-10t}$ volts. Find the current I and the charge Q at time t, given that at t = 0, Q = 2 coulomb and I =0 ampere.