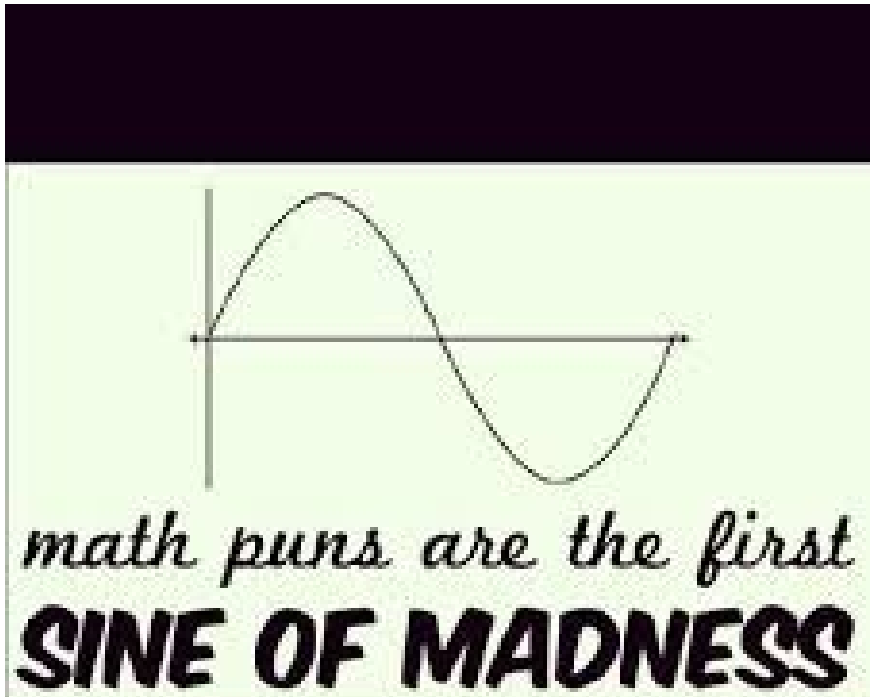
RATIONAL FUNCTIONS AND THEIR GRAPHS

@remedialtutorial2015



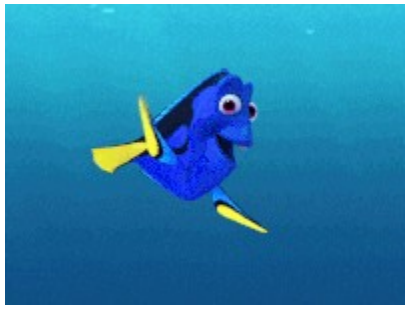
SUMMARY OF HOW TO FIND ASYMPTOTES

Vertical Asymptotes are the values that are NOT in the domain. To find them, set the denominator = 0 and solve.

"WHAT VALUES CAN I NOT PUT IN THE DENOMINATOR????"

To determine horizontal or oblique asymptotes, compare the degrees of the numerator and denominator.

1. **If the degree of the top < the bottom, horizontal asymptote along the *x* axis (*y* = 0)**
2. **If the degree of the top = bottom, horizontal asymptote at *y* = leading coefficient of top over leading coefficient of bottom**
3. **If the degree of the top > the bottom, oblique asymptote found by long division.**



Finding Asymptotes

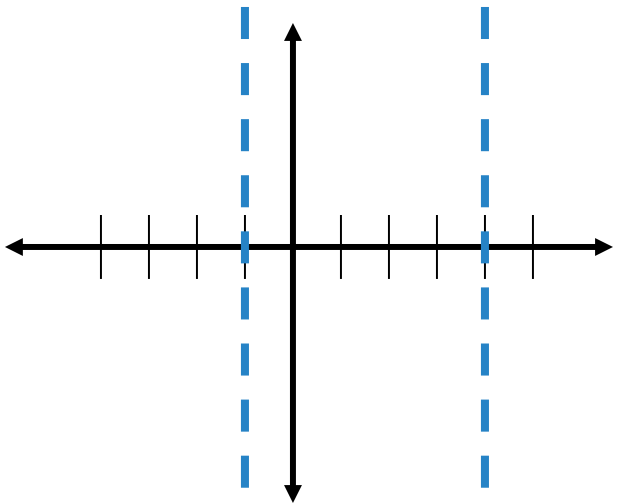
VERTICAL ASYMPTOTES

There will be a vertical asymptote at any "illegal" *x* value, so anywhere that would make the denominator = 0

$$f(x) = \frac{x^2 + 2x + 5}{(x-4)(x+1)}$$

Let's set the bottom = 0 and factor and solve to find where the vertical asymptote(s) should be.

So there are vertical asymptotes at *x* = 4 and *x* = -1.



HORIZONTAL ASYMPTOTES

We compare the degrees of the polynomial in the numerator and the polynomial in the denominator to tell us about horizontal asymptotes.

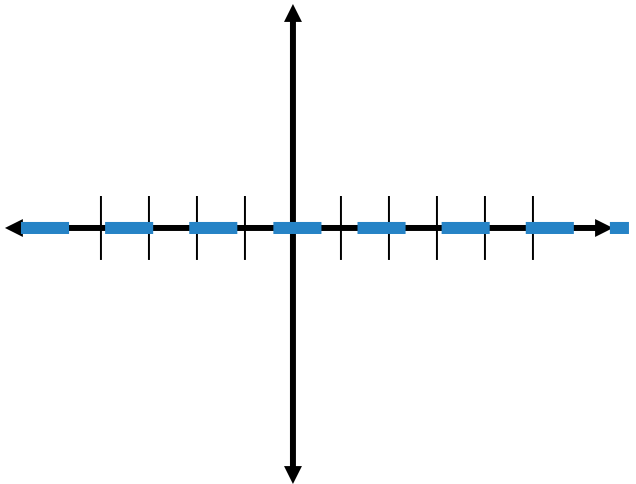
degree of top = 1

$$f(x) = \frac{2x+5}{x^2-3x+4}$$

degree of bottom = 2

1 < 2

If the degree of the numerator is less than the degree of the denominator, the *x* axis is a horizontal asymptote. This is along the line *y* = 0.



HORIZONTAL ASYMPTOTES

The leading coefficient is the number in front of the highest powered *x* term.

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at:

$$y = \frac{\text{leading coefficient of top}}{\text{leading coefficient of bottom}}$$

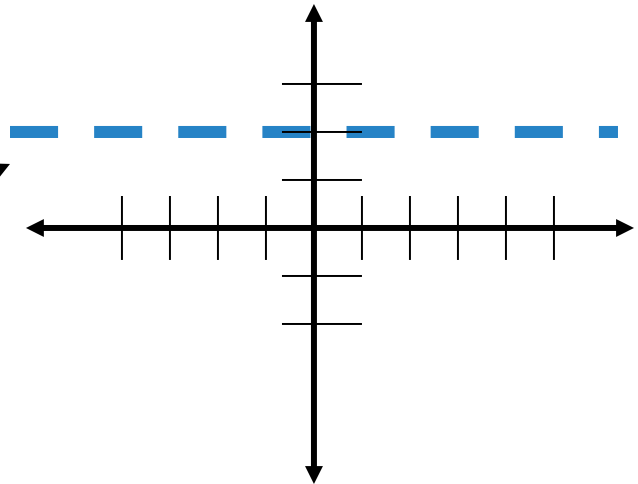
degree of top = 2

$$f(x) = \frac{2x^2 + 4x + 5}{1x^2 - 3x + 4}$$

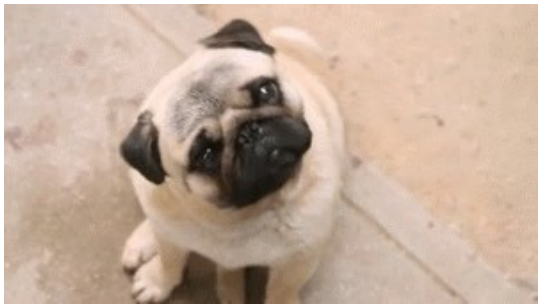
degree of bottom = 2

horizontal asymptote at:

$$y = \frac{2}{1} = 2$$



OBLIQUE ASYMPTOTES - Slanted



If the degree of the numerator is greater than the degree of the denominator, then there is not a horizontal asymptote, but an oblique one. The equation is found by doing long division and the quotient is the equation of the oblique asymptote ignoring the remainder.

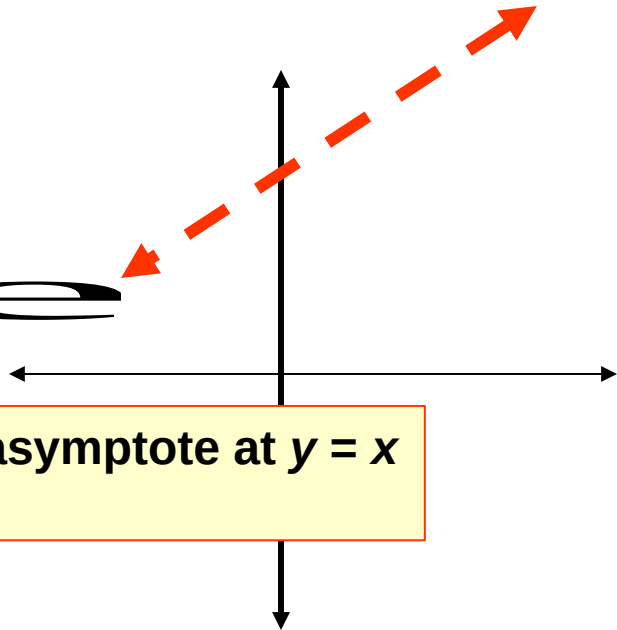
degree of top = 3

$$f(x) = \frac{x^3 - 2x^2 - 3x + 5}{x^2 - 3x + 4}$$

degree of bottom = 2

$$x^2 - 3x + 4 \;\overline{)\; x^3 - 2x^2 - 3x + 5}$$

$$x + 1 \text{ remainder } 1$$



Oblique asymptote at $y = x + 5$

STRATEGY FOR GRAPHING A RATIONAL FUNCTION

1. Graph your asymptotes
2. Plot points to the left and right of each asymptote to see the curve

SKETCH THE GRAPH OF

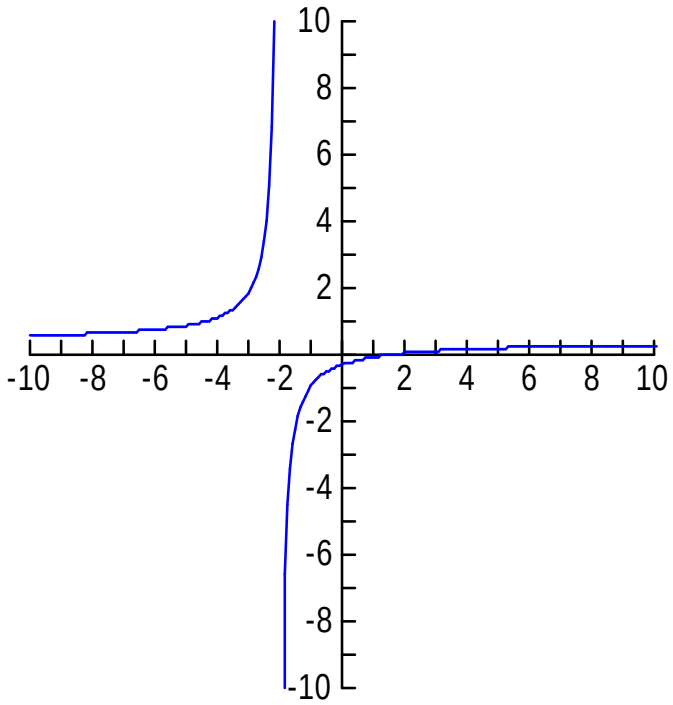
$$f(x) = \frac{2x-3}{5x+1}$$

$$f(x) = \frac{2x - 3}{5x + 1}$$

The vertical asymptote is x = -2

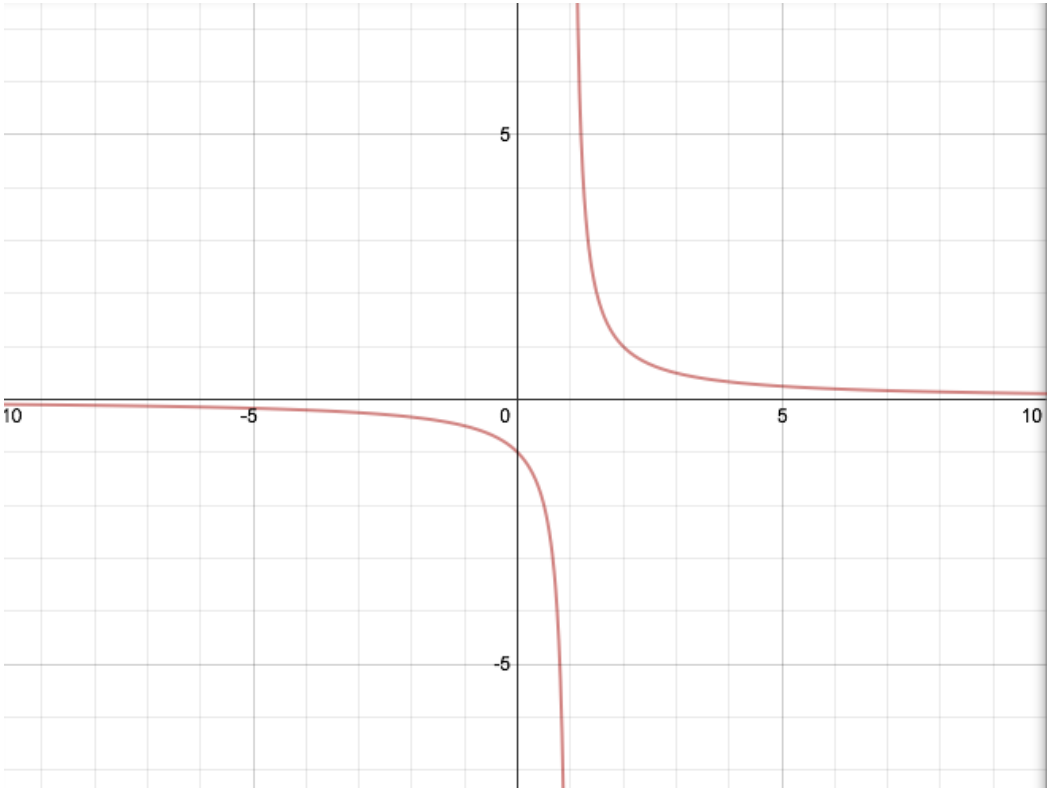
The horizontal asymptote is y = 2/5

$$f(x) = \frac{2x-3}{5x+1}$$



SKETCH THE GRAPH OF:

$$g(x) = \frac{1}{x-1}$$



Vertical asymptotes at??

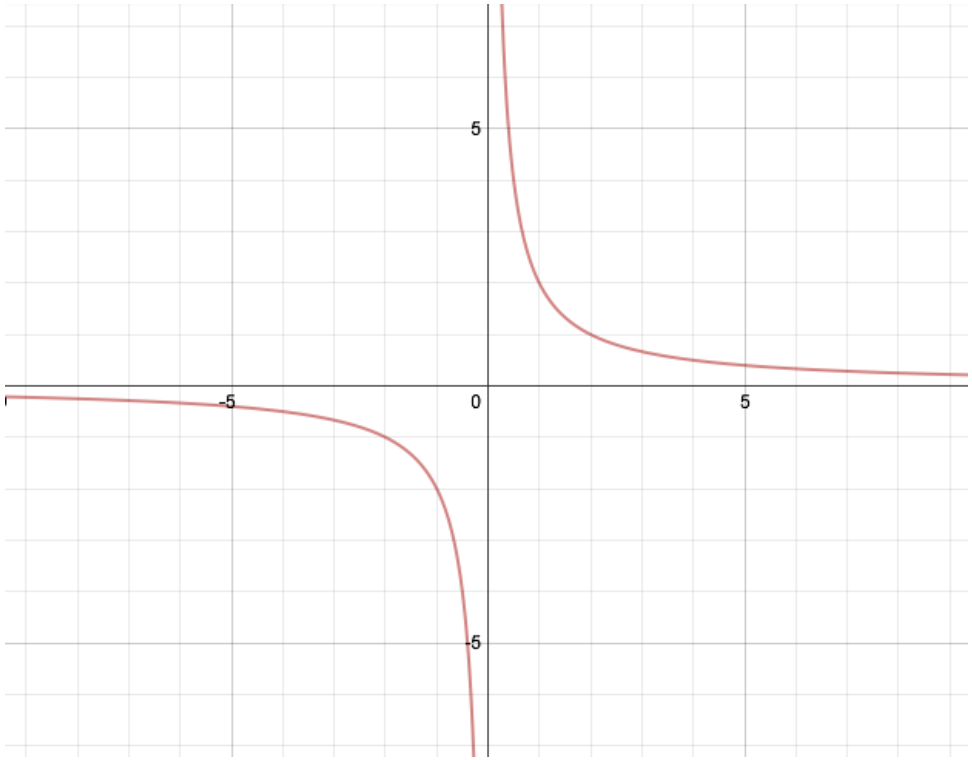
x = 1

Horizontal asymptote at??

y = 0

SKETCH THE GRAPH OF:

$$f(x) = \frac{2}{x}$$



Vertical asymptotes at??

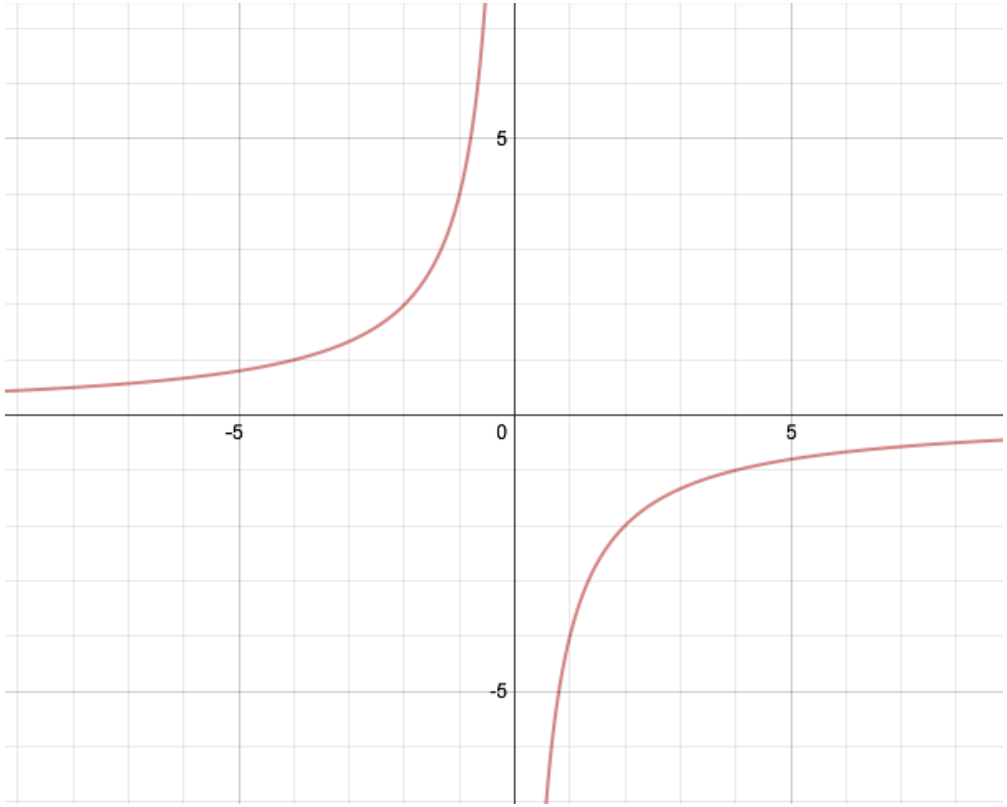
x = 0

Horizontal asymptote at??

y = 0

SKETCH THE GRAPH OF:

$$h(x) = \frac{-4}{x}$$



Vertical asymptotes at??

x = 0

Horizontal asymptote at??

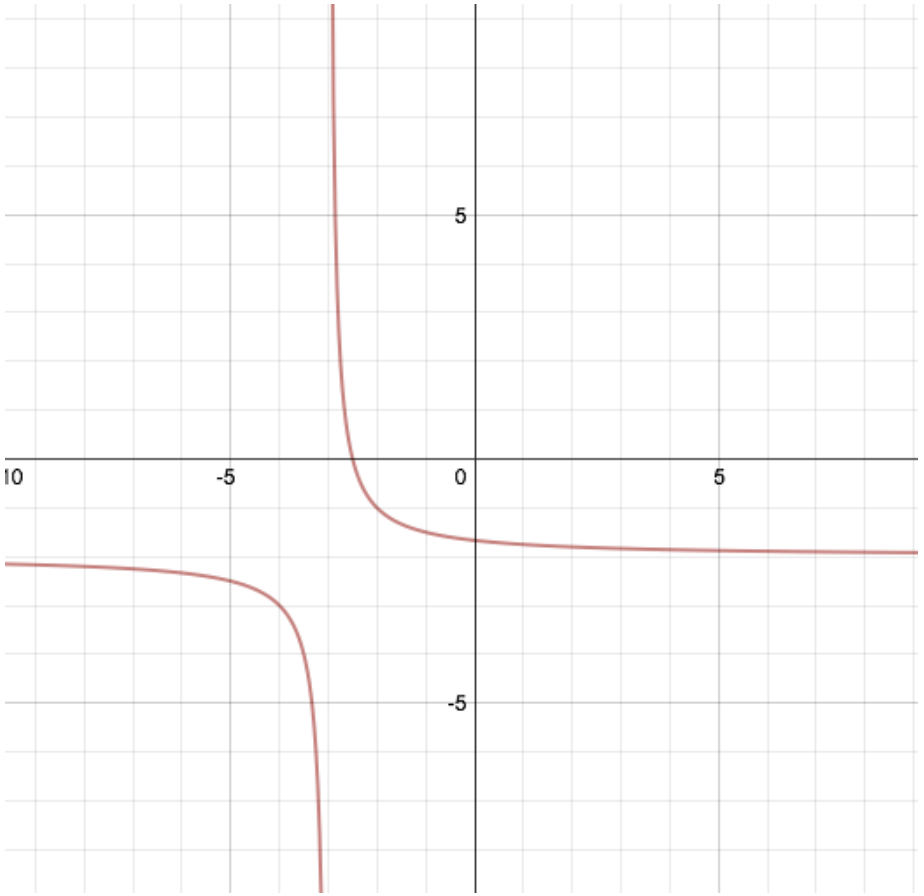
y = 0

SKETCH THE GRAPH OF:

$$y = \frac{1}{x+3} - 2$$

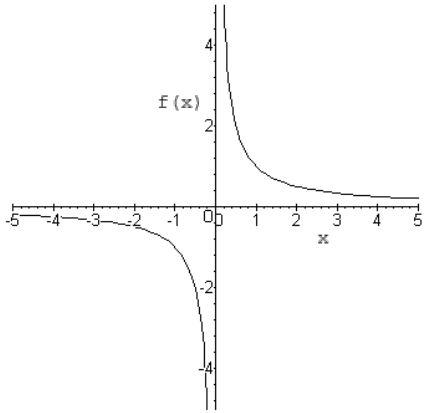
Vertical asymptotes at?? x = 1

Horizontal asymptote at?? y = 0



Hopefully you remember,
y = 1/x graph and it's asymptotes:

Vertical asymptote: x = 0
Horizontal asymptote: y = 0



OR…

We have the function:

$$y = \frac{1}{x+3} - 2$$

But what if we simplified this and combined like terms:

$$y = \frac{1}{x+3} - \frac{2(x+3)}{x+3}$$

$$y = \frac{1 - 2x - 6}{x+3}$$

$$y = \frac{-2x - 5}{x+3}$$

Now looking at this:
Vertical Asymptotes??

x = -3

Horizontal asymptotes??

y = -2

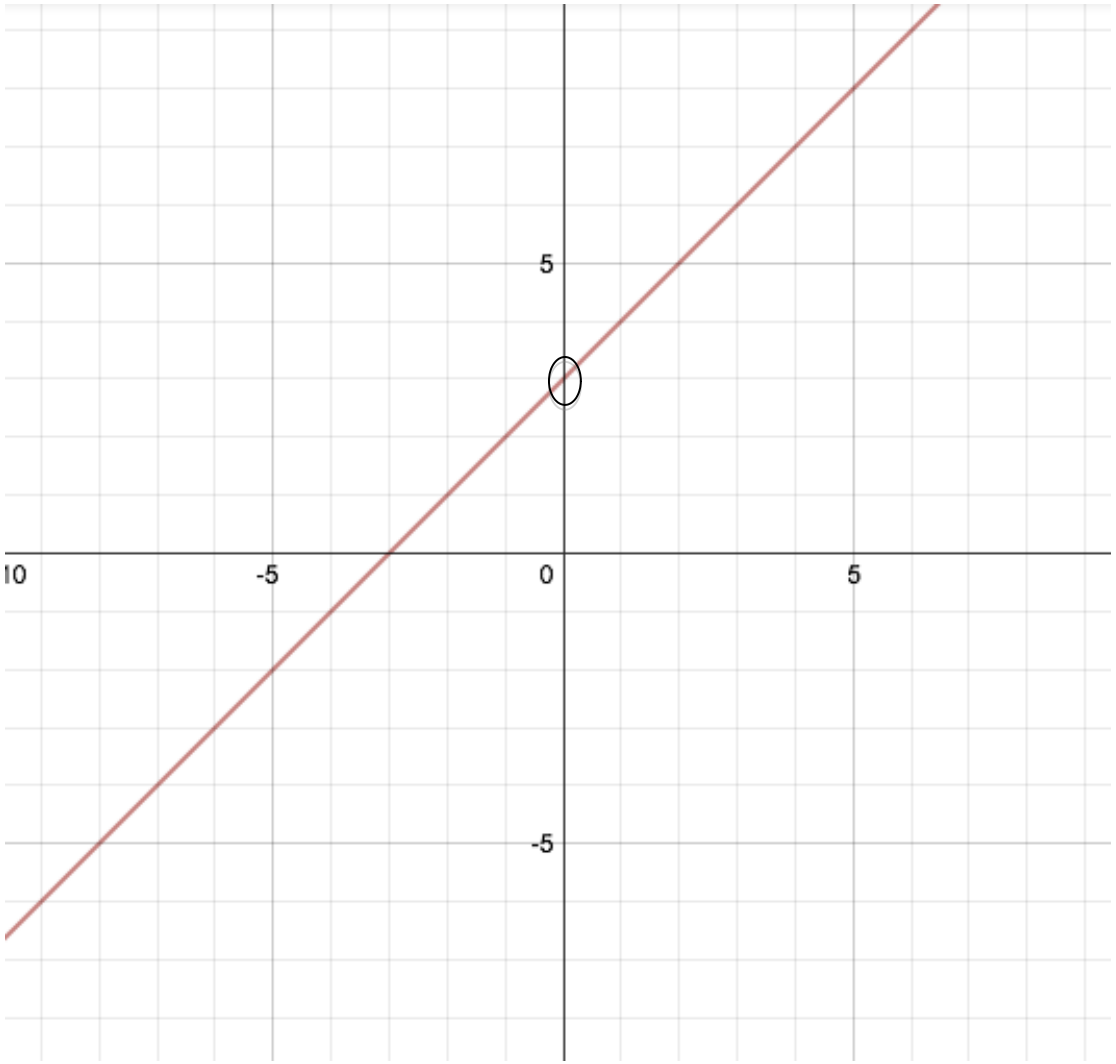
SKETCH THE GRAPH OF:

$$h(x) = \frac{x^2+3x}{x}$$

$$h(x) = \frac{x(x+3)}{x}$$

Hole at??

x = 0



FIND THE ASYMPTOTES OF EACH FUNCTION:

$$y = \frac{x^2 + 3x - 4}{x}$$

$$y = \frac{x^2}{x} + \frac{3x}{x} - \frac{4}{x}$$

$$y = x + 3 - \frac{4}{x}$$

Vertical Asymptote:

x = 0

Slant Asymptote:

y = x + 3

$$y = \frac{x^2 + 3x - 28}{x^3 - 11x + 28}$$

$$y = \frac{(x+7)(x-4)}{(x-7)(x-4)}$$

Hole at x = 4

Vertical Asymptote:

x = 0 and x = 7

Horizontal Asymptote:

y = 0

WHAT MAKES A FUNCTION CONTINUOUS?

Continuous functions are predictable…

1) No breaks in the graph

A limit must exist at every x-value or the graph will break.

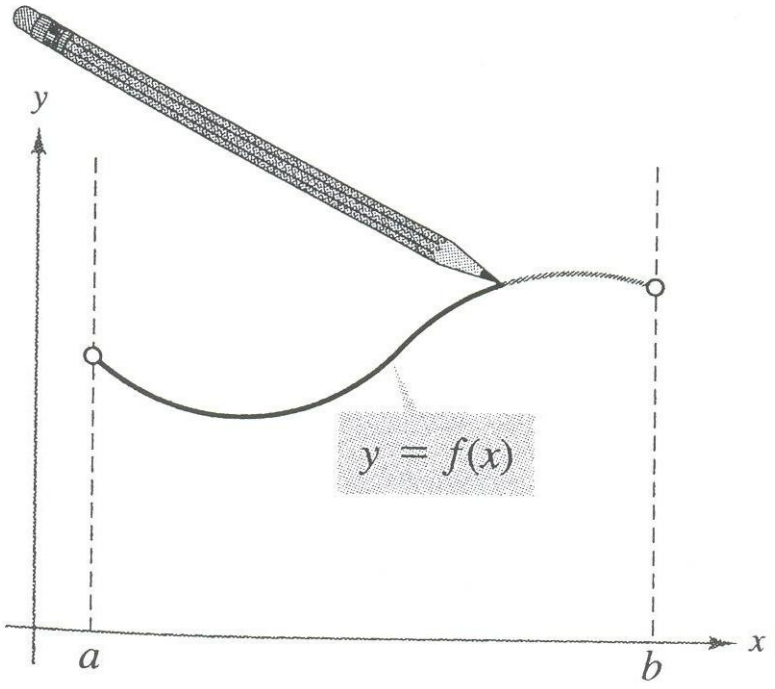
2) No holes or jumps

The function cannot have undefined points or vertical asymptotes.

CONTINUITY

Key Point:

Continuous functions can be drawn with a single, unbroken pencil stroke.



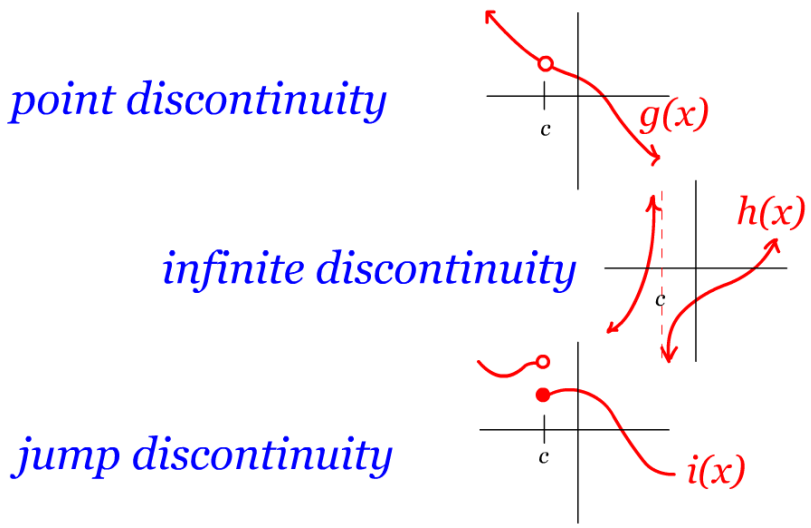
CONTINUITY OF POLYNOMIAL AND RATIONAL FUNCTIONS

A polynomial function is continuous at every real number.

A rational function is continuous at every real number in its domain.

DISCONTINUITY

Discontinuity: a point at which a function is not continuous



DISCONTINUITY

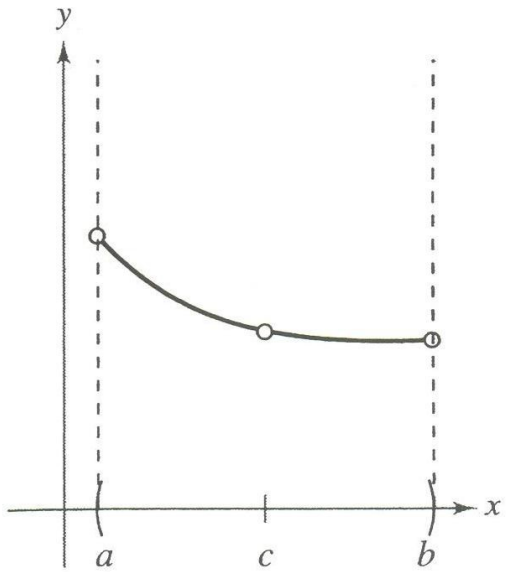
Two Types of Discontinuities

1) Removable (hole in the graph)

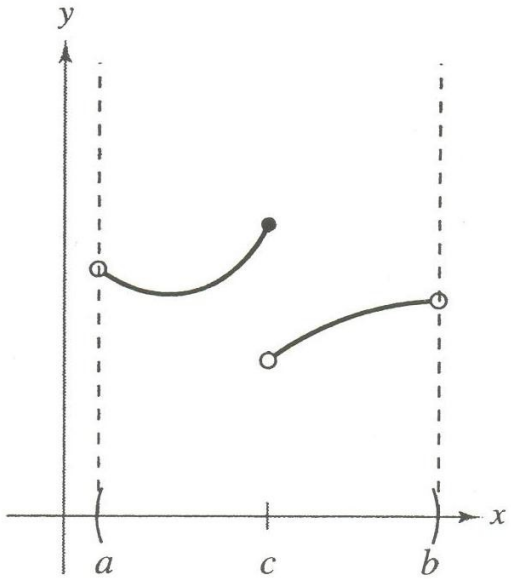
2) Non-removable (break or vertical asymptote)

A discontinuity is called removable if a function can be made continuous by defining (or redefining) a point.

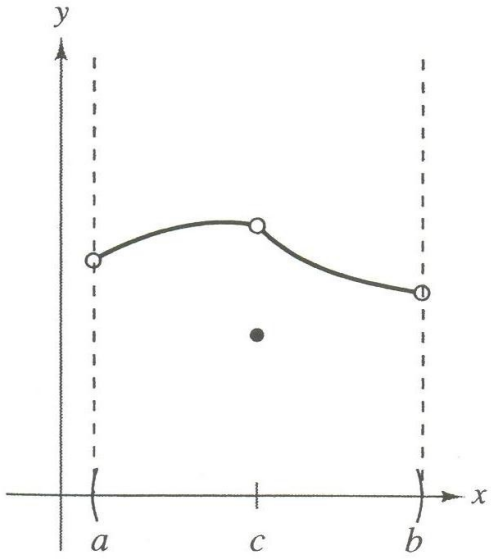
TWO TYPES OF DISCONTINUITIES



(a) Removable discontinuity



(b) Nonremovable discontinuity



(c) Removable discontinuity

DISCONTINUITY

Find the intervals on which these function are continuous.

$$f(x) = \frac{x+2}{x^2-3x-10}$$

$$= \frac{x+2}{(x+2)(x-5)}$$

$$= \frac{1}{(x-5)}$$

Point of discontinuity:

$x+2=0$

$x=-2$

Removable discontinuity

Vertical Asymptote:

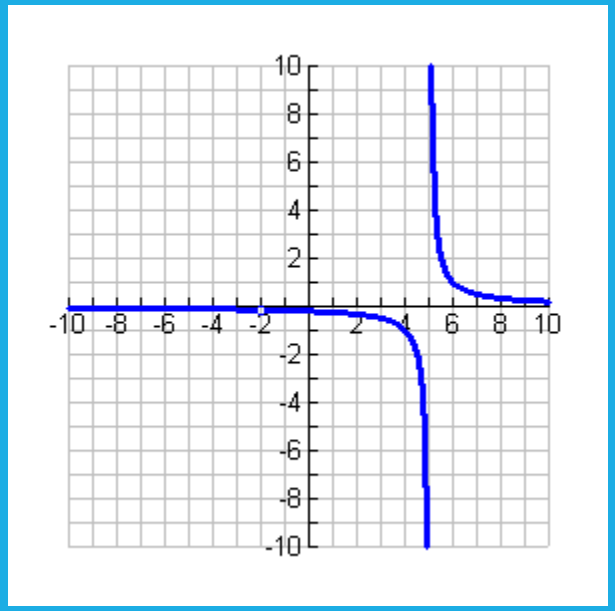
$x-5=0$

$x=5$

Non-removable discontinuity

DISCONTINUITY

$$f(x) = \frac{x+2}{x^2-3x+1}$$



Continuous on:

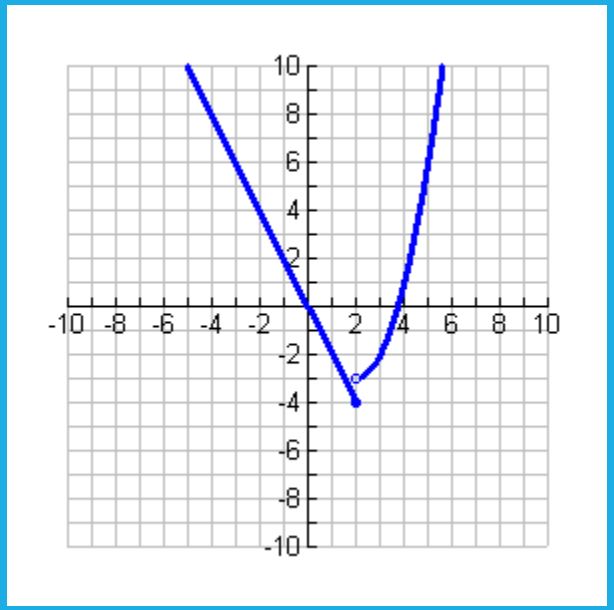
DISCONTINUITY

$$f(x)=\begin{cases}-2x+2 & x<2\\ x^2-4x+1 & x\geq 2\end{cases}$$

$$\lim_{x\to 2^-}(-2x+2)=$$

$$\lim_{x\to 2^+}(x^2-4x+1)=$$

$$f(2)=$$



Continuous on:

DISCONTINUITY

Determine the value(s) of x at which the function is discontinuous. Describe the discontinuity as removable or non-removable.

(A) $f(x) = \dfrac{x^2 - 1}{x^2 - 5x - 6}$

(B) $f(x) = \dfrac{x^2 + 10x + 9}{x^2 - 81}$

(C) $f(x) = \dfrac{x^2 - 4x - 5}{x^2 - 25}$

(D) $f(x) = \dfrac{x^2 - 4}{x^2 - 2x - 8}$

DISCONTINUITY

(A) $f(x) = \dfrac{x^2 - 1}{x^2 - 5x - 6}$

$$= \frac{(x-1)(x+1)}{(x-6)(x+1)}$$

$x = 1$ Removable discontinuity

$x = 6$ Non-removable discontinuity

DISCONTINUITY

(B) $f(x)=\dfrac{x^2+10x+9}{x^2-81}$

$=\dfrac{(x+9)(x+1)}{(x+9)(x-9)}$

$x=-9$ Removable discontinuity

$x=9$ Non-removable discontinuity

DISCONTINUITY

(C) $f(x)=\dfrac{x^2-4x-5}{x^2-25}$

$=\dfrac{(x-5)(x+1)}{(x-5)(x+5)}$

$x=5$ Removable discontinuity

$x=-5$ Non-removable discontinuity

DISCONTINUITY

(D) $f(x)=\dfrac{x^2-4}{x^2-2x-8}$

$$=\frac{(x-2)(x+2)}{(x-4)(x+2)}$$

$x=-2$ Removable discontinuity

$x=4$ Non-removable discontinuity

CONCLUSION

Continuous functions have no breaks, no holes, and no jumps.

If you can evaluate any limit on the function using only the substitution method, then the function is continuous.