# DECISION THEORY



# Basic Concepts in Decision Theory

- Operational research is all about decision rules, evaluating alternative decisions, optimizing decisions, predicting the outcome of decisions, helping to cope with uncertainty and risk, and sorting out the complexity of the situations in which decisions are frequently made so that management can quickly exercise judgment on what is the best course of action in the circumstances

# Cont….

Meaning of Decision Theory (Decision Analysis):

- Techniques used to find optimal solutions in situations where a decision maker is faced with several alternatives (Actions) and an uncertain or risk-filled futures (Events or States of Nature).

- The techniques can utilize knowledge of the possible actions that the decision maker can take, probabilities that the events will occur, and the outcome (payoff) associated with each combination of action and Event.

# Steps in Decision theory Approach

- **Step 1:** List all the variable alternatives- strategies or courses of actions like; expand the current plant, construct a new plant or subcontract production for extra demand
- **step 2:** identify the expected future events – they are not under the control of the decision maker, termed as State of Natures. They are mutually exclusive and exhaustive set of outcomes, which describe the possible behavior of the environment in which the decision is made. E.g.
  - high demand
  - moderate demand
  - low demand
  - no demand as State of Natures

# Cont…

- **Step 3:** Construct a payoff table: the decision maker now constructs a payoff table (table representing profit, cost, benefit , etc) for each possible combination of alternative course of actions and state of natures.
- **step 4:** Select optimum decision criteria – the decision maker will choose criterion which results in largest payoff

# Decision Making Environments

Decisions can be made under three situations:

1. Decision Making under certainty: decision maker **knows which event** (state of nature) will occur. Only one state of nature exists. linear programming, transportation, assignment models use such type of situations.
2. Decision Making under risk: decision maker has only **probabilistic estimates** of events that might occur. There is sufficient information to assign probabilities to the different state of natures
3. Decision Making under uncertainty: - decision maker **does not have information** even to estimate the probabilities for the happening of different state of natures

# Example

Assume that a general manager of ABC Company is concerned about building a new plant. Demand for products of the new plant may be high, medium or low.

The manager wants to decide whether to build a large- (L), just right- (JR) (medium) or small- (S) sized plant.

The following payoff table summarises payoffs (millions of Birr profit under different alternative action versus state of nature combinations.

| A | S | | |
|---|---|---|---|
| | *H* | *M* | *W* |
| *L* | 15 | 3 | -6 |
| *JR* | 9 | 4 | -2 |
| *S* | 3 | 2 | 1 |

Pa

# 4.2. Decision Making under certainty

- The main characteristics of this decision making is that the decision maker know, with certainty, which event will occur.
- He need only consider one event and the method of solution is obvious. (He can look at the outcome for each alternative, and choose the best)
- In our example, if we certainly know the demand is say medium, the best alternative is to construct mid size (just right) plan

| A | S | | |
|---|---|---|---|
| | H | M | W |
| L | 15 | 3 | -6 |
| JR | 9 | 4 | -2 |
| S | 3 | 2 | 1 |

## 4.3. Decision Making Under Uncertainty

- The decision situation where there is no way in which the decision maker can assess the probabilities of the various states of nature
- In such situations, the decision maker has no idea at all as to which of the possible states of nature would occur nor has a reason to believe why a given state is more, or less likely to occur as another
- Decision Making under Complete Uncertainty criterion are:
  1) Maximax (Optimistic Criterion)
  2) Maximin (Pessimistic, Wald Criterion)
  3) Minimax regret: Savage Minimax Regret Criterion

## 1) Maximax/Minimin

This optimist criterion attempts to describe the decision-making behavior of people who are perfectly optimistic in their expectations.

- An optimistic decision maker is attracted by large rewards and is willing to risk high losses in order to obtain them.
- **Steps in Maximax**

1. For each action alternative (matrix row) determine the maximum payoff possible.
2. From these maxima, select the maximum payoff. The action alternative leading to this payoff is the chosen decision.

- It is possible to model the optimist profile with the MAXIMAX decision rule (when the payoffs are positive-flow rewards) such as profits or revenue.
- When payoffs are given as negative-flow rewards, (such as costs) the optimist decision rule is MINIMIN. Note that negative-flow rewards are expressed with positive numbers.)

| A | S | | |
|---|---|---|---|
| | H | M | W |
| L | 15 | 3 | -6 |
| JR | 9 | 4 | -2 |
| S | 3 | 2 | 1 |

| A | S | | | | | |
|---|---|---|---|---|---|---|
| | H | M | W | Row Max | Maximax | Decision |
| L | 15 | 3 | -6 | 15 | 15 | L |
| JR | 9 | 4 | -2 | 9 | | |
| S | 3 | 2 | 1 | 3 | | |

**The decision is shown by circling the maximax value, thus indicating the row of the chosen action alternative:**

| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | Maximax |
| *L* | 15 | 3 | -6 | 15 ◀ denotes decision is *L* |
| *JR* | 9 | 4 | -2 | 9 |
| *S* | 3 | 2 | 1 | 3 |

# 2) Maximin/Minimax decision (Wald Criterion)

- It is possible to model the pessimist profile with the MAXIMIN decision rule (when the payoffs are positive-flow rewards, such as profits or income.)
- When payoffs are given as negative-flow rewards, the pessimist decision rule is MINIMAX.

**Rule**

1. For each action alternative (matrix row) determine the minimum payoff possible. This represents the worst possible outcome if that decision alternative were chosen.
2. From these minima, select the maximum payoff. The action alternative leading to this payoff is the chosen decision

| A | S | | | | | |
|---|---|---|---|---|---|---|
| | *H* | *M* | *W* | | | |
| | | | | Row Min | Maximin | Decision |
| *L* | 15 | 3 | -6 | -6 | | |
| *JR* | 9 | 4 | -2 | -2 | | |
| *S* | 3 | 2 | 1 | 1 | 1 | *S* |

**Using our matrix defined previously:**

| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | |
| | | | | Maximin |
| *L* | 15 | 3 | -6 | -6 |
| *JR* | 9 | 4 | -2 | -2 |
| *S* | 3 | 2 | 1 | 1 |

◀ denotes decision is *S*

### 3) Savage Minimax Regret Criterion

- The Minimax Regret criterion focuses on avoiding the worst possible consequences that could result when making a decision.
- **Regret** is defined as the opportunity loss to the decision maker if action alternative *Ai* is chosen and state of nature *Sj* happens to occur.
- **Opportunity loss** is the payoff difference between **the best** possible outcome under *Sj* and the actual outcome resulting from choosing *Ai* given that *Sj* occurs.

## Savage Minimax Regret rule:

- For each event find the best possible outcome (the best entry in each column)
- Find the regret for every entry in the column (the difference b/n the best and the entry)
- Put the regrets found in step 2 in to regret matrix. There is at least one zero in each column and regret are always positive
- For each alternative/row/ find the highest regret
- Choose the alternative with the lowest value of these highest regrets

*Note that in the case of cost payoff; opportunity loss table is determined by considering the d/c b/b the least cost and entry in each column and then after MAXIMIN rule will be followed*

**Payoff Matrix**

| A | S | | |
|---|---|---|---|
| | *H* | *M* | *W* |
| *L* | 15 | 3 | -6 |
| *JR* | 9 | 4 | -2 |
| *S* | 3 | 2 | 1 |
| * | 15 | 4 | 1 |

◄ best payoff for each $S_j$

where the star denotes "best payoff under state of nature *Sj* ".

- The OL matrix can now be obtained by subtracting each entry *Rij* from its column's best payoff. The minimax rule is then applied to the OL (regret) matrix:

| A | S | | |
|---|---|---|---|
| | H | M | W |
| L | 15-15=0 | 4-3=1 | 1- (-6)=7 |
| JR | 15-9= 6 | 4-4=0 | 1- (-2)= 3 |
| S | 15-3 =12 | 4-2=2 | 1 - 1= 0 |

## Opportunity Loss Matrix

| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | Minimax Regret |
| *L* | 0 | 1 | 7 | 7 |
| *JR* | 6 | 0 | 3 | 6 |
| *S* | 12 | 2 | 0 | 12 |

◀ decision is *JR*

## 4) Equality likely/Laplace Criterion

- It finds the alternative with the highest average outcomes
- The Laplace criterion is the first to make explicit use of probability assessments regarding the likelihood of occurrence of the states of nature
- The principle of insufficient reason – since the probability associated with the events are unknown, there is no enough information to conclude that these probabilities will be different

## Laplace decision rule

- First calculate the average outcome for every alternatives, which is the sum of all outcomes divided by the number of events. Then, then pick the best alternative  of the averages.

- Let's assume that all the three demand states (*H*, *M*, *W* ) to be equally probable. Then *and only then* is the use of the Laplace criterion acceptable.

| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | $p_j$'s |
| | 1/3 | 1/3 | 1/3 | **Laplace** |
| *L* | 15 | 3 | -6 | 4 ◀ decision is *L* |
| *JR* | 9 | 4 | -2 | 3.67 |
| *S* | 3 | 2 | 1 | 2 |

# 5. The Hurwicz Criterion

- The Hurwicz criterion attempts to find a middle ground between the extremes posed by the optimist and pessimist criteria.
- Instead of assuming total optimism or pessimism, Hurwicz incorporates a measure of both by assigning a certain percentage weight to optimism and the balance to pessimism.
- A weighted average can be computed for every action alternative with an alpha-weight $\alpha$, called the coefficient of realism. Note that $0 \leq \alpha \leq 1$

# Cont...

- An α = 1 denotes absolute optimism (Maximax) while an α = 0 indicates absolute pessimism (Maximin). The α is selected subjectively by the decision maker.
- Selecting a value for α simultaneously produces a *coefficient of pessimism* 1 - α , which reflects the decision maker's aversion to risk.
- A Hurwicz weighted average H can now be computed for every action alternative *Ai* in Alternatives

H (*Ai* ) = α (row maximum) + ( 1 - α ) (row minimum)
for positive-flow payoffs (profits, revenues)

H (*Ai* ) = α (row minimum) + ( 1 - α ) (row maximum)
for negative-flow payoffs (costs, losses)

**Hurwicz decision rule:**

1. Select a coefficient of optimism value α .
2. For every action alternative compute its Hurwicz weighted average H.
3. Choose the action alternative with the best H as the chosen decision.

("Best" means Max {H} for positive-flow payoffs, and Min {H} for negative-flow payoffs.)

- Let's assume that the managers have agreed to assess their level of optimism at 60%. Thus α = 0.6. Using the decision matrix defined previously:

| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | Hurwicz |
| *L* | 15 | 3 | -6 | 6.6 ◀ decision is *L* |
| *JR* | 9 | 4 | -2 | 4.6 |
| *S* | 3 | 2 | 1 | 2.2 |

## Summary of Methods for Decision Making under Complete Uncertainty

| Method | Description of the Decision-Making Criteria |
|---|---|
| Maximax | Choose the largest of the largest payoffs from the payoff table (optimistic) |
| Maximin | Choose the largest of the smallest payoffs from the payoff table (pessimistic) |
| Minimax regret | Choose the minimum of the maximum losses from the opportunity loss table (loss oriented) |
| Hurwicz | Weighted-average approach where the largest payoff for an alternative is weighted with the coefficient of optimism ($\alpha$) and the smallest payoff is weighted with $1 - \alpha$ |
| Equal likelihood | Weighted-average approach where each decision alternative is weighted equally and the largest average is chosen |

# Decision Making Under Risk

- In this situation the decision maker has to face several states of nature. But he has some knowledge or experience which will enable him to assign probabilities to the occurrence of each state of nature. The objective is to optimize the expected profit or to minimize the opportunity loss.
- **Criterion of decision making under Risk are**:

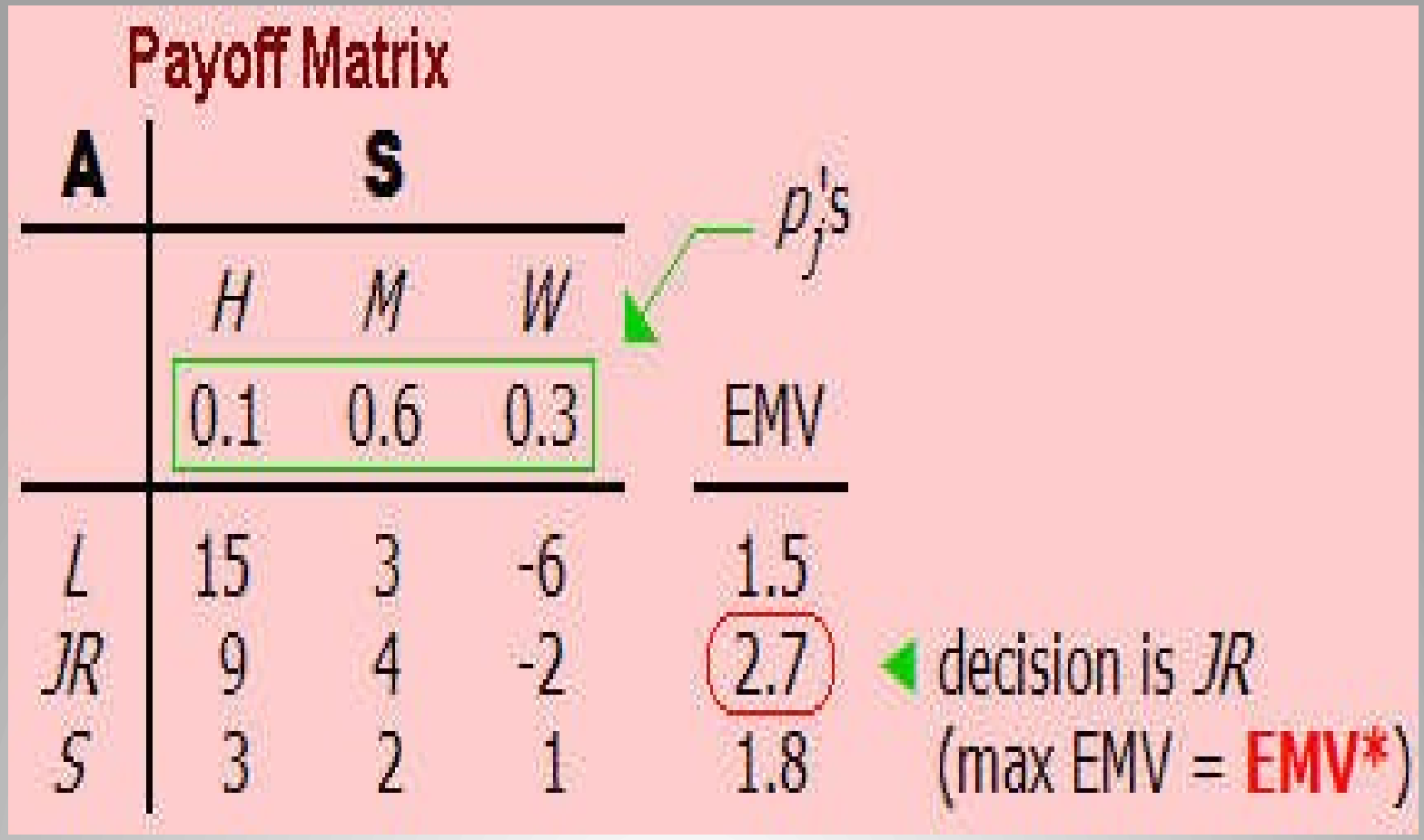
1) **Expected Monetary Value (EMV)**
2) **Expected Opportunity Loss (EOL)**
3) **Expected Value of Perfect Information**

# Decision Making Under Risk

- The difference b/n decision making under uncertainty and risk is the presence of probabilities for the occurrence of the various states of nature under decision making under risk.
- The probabilities may be:
  - Subjective estimates from managers
  - Subjective estimates from experts in the field
  - Historical frequencies
- If they are reasonably correct, they provide the decision maker with additional information that can dramatically improve the decision making process.
- The sum of the probabilities for all states of nature must be 1

## 1. Criteria of Expected Monetary Value (EMV)

- EMV approach provides the decision maker with a value which represents an *average payoff* for each alternative.
- The best alternative is, then, the one that has the highest expected monetary value.
- The average or expected payoff of each alternative is weighted average; the state of nature probabilities are used to weight the respective payoffs.


Payoff Matrix
A
S
$p_j$'s
H
M
W
0.1
0.6
0.3
EMV
L
15
3
-6
1.5
JR
9
4
-2
2.7
S
3
2
1
1.8
decision is JR
(max EMV = EMV*)

# 2. Criteria of Expected Opportunity Loss

- Using EOL is an alternate method for incorporating probabilities in to a decision making process.
- The approach is nearly the same to the EMV approach, except that a table of opportunity losses is used rather than a table of payoff.
- The opportunity losses for each alternative are weighted by the probabilities of their respective states of nature, and the alternative with the smallest expected loss is selected as the best.

## Opportunity Loss Matrix

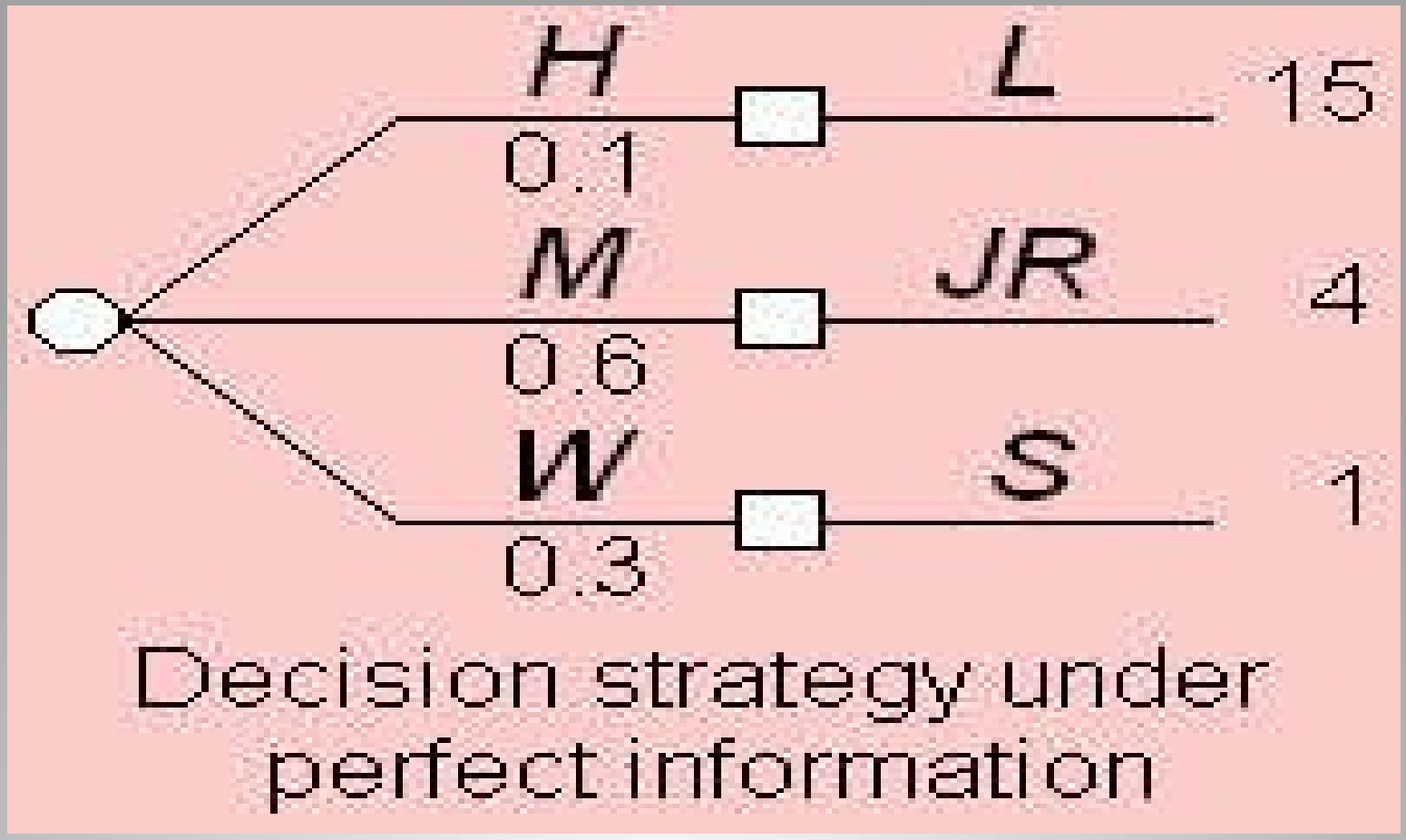
| A | S | | | |
|---|---|---|---|---|
| | *H* | *M* | *W* | |
| | 0.1 | 0.6 | 0.3 | EOL |
| *L* | 0 | 1 | 7 | 2.7 |
| *JR* | 6 | 0 | 3 | 1.5 |
| *S* | 12 | 2 | 0 | 2.4 |

◀ decision is *JR*
(min EOL = **EOL***)

## 3. Expected Value of Perfect Information

- Sometimes the decision maker might go for to delay a decision until it is evident which state of nature is going to materialize (be certain).
- Such delays will involve a cost of some sort.
- Hence, the question is whether the cost of waiting outweighs the potential benefits that could be realized.
- If it would be worth the cost to refine or eliminate probabilities of states of natures.
- The EVPI is thus, the measure of the difference between the certain payoff that could be realized under a condition of certainty and expected payoff under condition involving risk.

- The probabilities of the original state of nature can be used to weight the payoff, one of which will occur under certainty.
- This is called the expected payoff under certainty (EPC).




Decision strategy under perfect information

- EPC= 0.1(15) + .6(4) + .3(1) = 4.2
- The difference between this figure and the expected payoff under risk (EMV) is the expected value of perfect information.

$$EVPI=EPC-EMV$$

$$EVPI= 4.2 - 2.7 =1.5$$

- Perfect information would increase ABC Company's expected payoff by $1.5 million, so that is what the perfect information is worth.

- The expected opportunity loss due to imperfect information is equal to the expected payoff that could be achieved by having perfect information.
- Therefore EVPI is always equal to EOL

$$EVPI=EOL$$

1) Burger Prince Restaurant is contemplating opening a new restaurant on Main Street. It has three different models, each with a different seating capacity. Burger Prince estimates that the average number of customers per hour will be 80, 100, or 120. The payoff table (costs) for the three models is on the next slide.

**Average Number of Customers Per Hour**

| | $s_1 = 80$ | $s_2 = 100$ | $s_3 = 120$ |
|---|---|---|---|
| Model A | $10,000 | $15,000 | $14,000 |
| Model B | $ 8,000 | $18,000 | $12,000 |
| Model C | $ 6,000 | $16,000 | $21,000 |

Required: Which model of restaurant should be opened? Determine the best model under each of the following decision criterion:

a) Optimist criterion
b) Pessimist criterion
c) Hurwicz criterion
d) Laplace criterion
e) EMV, assuming that S1,S2, and S3 have a probability of 0.2,0.7,0.1; respectively

# Decision Trees

- It is a graphical representation of the decision processes indicating decision alternatives, state of nature, probabilities attached to the states of nature and conditional benefits and losses
- It consists of a network of nodes and branches
- Two types of nodes are used: decision node represented by a square and state of nature (chance or event) node represented by a circle
- Alternative courses of action (strategies) originate from the decision node as main braches (decision branches)
- The respective payoffs and the probabilities associated with alternative courses and the chance events are shown alongside these

# Decision Tree Approach

- Decision makers sometimes use tree like diagram to obtain a visual portrays of decision alternatives
- It is an alternative approach to payoff tables.
- A tree is composed of squares, circles, & lines.
  - The squares indicate decision points.
  - Circles represent chance events
  - Lines emanate from the square represent alternatives
  - Lines emanate from a circle represent state of nature.

# Steps in Decision tree analysis

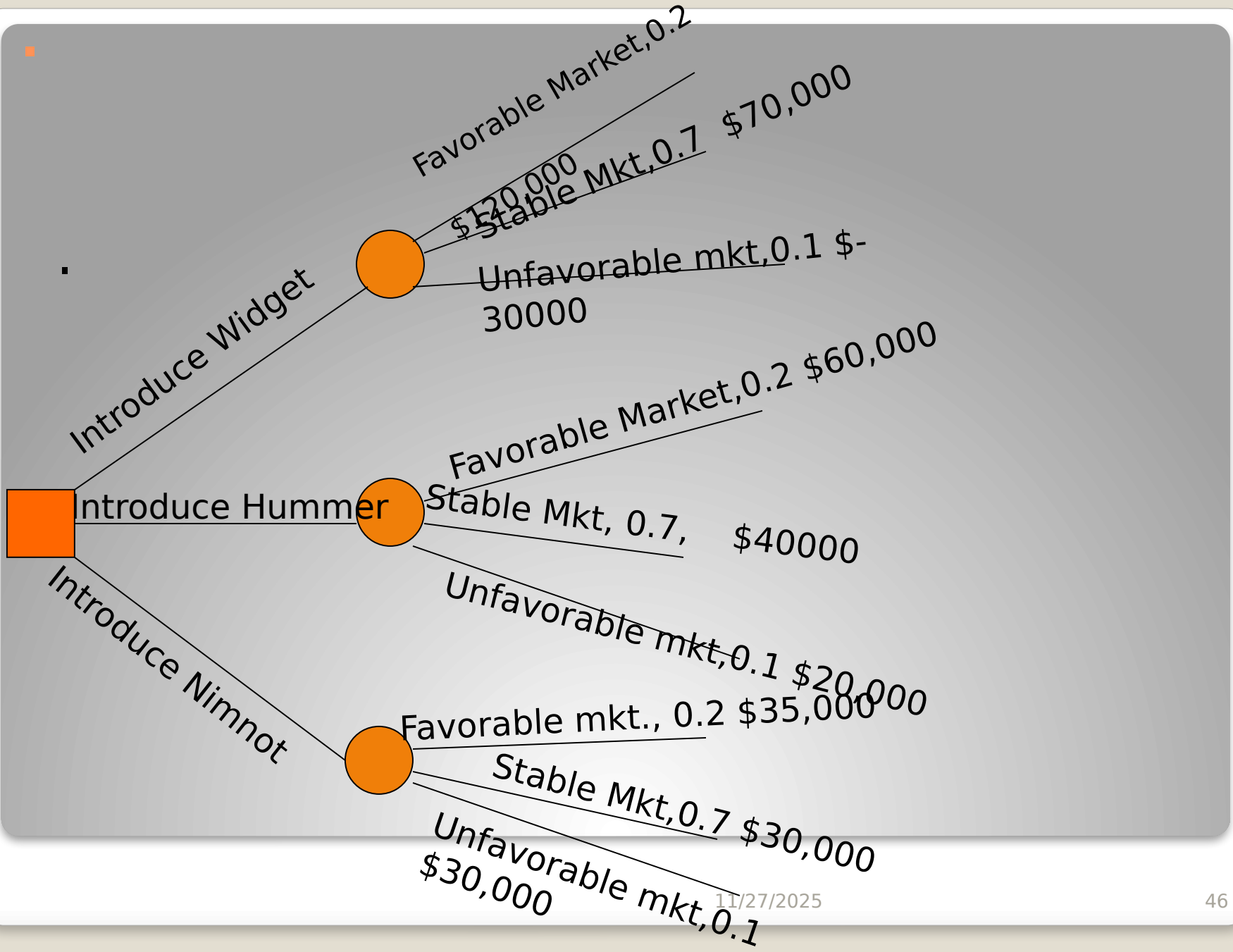
- Identify the decision points and the alternative courses of action at each decision point systematically
- At each decision point determine the probability and the payoff associated with each course of action
- Commencing from the extreme right end, compute the expected payoffs (EMV) for each course of action
- Choose the course of action that yields the best payoff for each of the decisions
- Proceed backwards to the next stage of decision points
- Repeat above steps till the first decision point is

**Example 1:** The Miramar Company is going to introduce one of the three new products: a widget, a hummer or a nimnot. The market conditions (favorable, stabe or unfavorable) will determine the profit or loss the company realizes. The estimated payoffs if widget is introduced are $120,000,$70,000 and $-30,000 respectively for favorable, stable and unfavorable market. The figures for hummer are $60,000,$40,000 and $20,000; and for that of nimnot are $35,000,$30,000 and $30,000 respectively for favorable, stable and unfavorable market conditions.

The probabilities for favorable, stable and unfavorable market conditions are 0.2,0.7 and 0.1 respectively.

**Required:** Use decision tree and locate the optimal strategy for the company.

# …decision tree examples


Favorable Market,0.2
$120,000
Stable Mkt,0.7 $70,000
Unfavorable mkt,0.1 $-
30000
Introduce Widget
Favorable Market,0.2 $60,000
Introduce Hummer
Stable Mkt, 0.7, $40000
Unfavorable mkt,0.1 $20,000
Introduce Nimnot
Favorable mkt., 0.2 $35,000
Stable Mkt,0.7 $30,000
Unfavorable mkt,0.1
$30,000