CHAPTER VII
NETWORK MODELS

Network diagram is a practical representation of the various events and activities.

Network modeling: Techniques that enable complex projects to be scheduled, taking into account the precedence of each activity.

7.1. PROJECTS

Definition: A project is a set of proposals for the investment of resources in to a clearly identified set of actions that are expected to produce future benefits that exceeds costs.

Examples of a project:

- *New business establishment*
- *Expansion of existing business*
- *Launching new product*
- *Cost reduction project*
- *Total Quality of Management (**TQM**), etc*

Characteristics of a project:

1. *The duration of a project lasts long*
2. *A project is complex in nature involving many interrelated activities and participants*
3. *Delay in completion time may be very costly*
4. *Project activities are sequential*

7.2. NETWORK CONVENTIONS

1. *The project can be sub-divided into a set of predictable independent activities each of which has a clear beginning and ending.*
2. *Each activity can be sequenced as to its predecessors or successors.*
3. *The network is not cyclical*

   i.e: Each activity is executed once and only once during the life of the project.
4. *Activity times may be estimated either as a **single point estimate** (**CPM**) or as a **3- point estimate** (**PERT**)*
   - ***CPM**- Critical Path Method*
   - ***PERT**- Program Evaluation and Review Technique*
5. *The duration of the activities is independent of each other.*

7.3. EVENT ANALYSIS

Events: Events of the network represent protect milestone, such as *the start* or *the completion* of an activity (task) or activities, and occur at a particular instant of time at which some specific part of the project has been or is to be achieved.

Event: An event is a point in time that marks the beginning or ending of activity.

Events are commonly represented by ***circles*** (***nodes***) in the network diagram.

The events can be further classified into the following two categories (types):

i. Merge event

An event which represents the ***joint completion*** of more than one activity is known as ***merge event***.

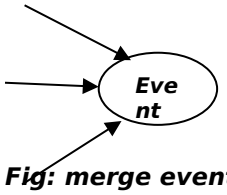


Fig: merge event

ii. Burst event

An event which represents the ***initiation*** (***beginning***) of more than one activity is known as ***burst event***.

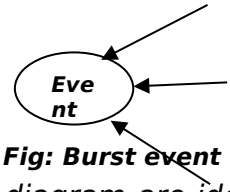


Fig: Burst event

Events in the network diagram are identified by numbers. Each event should be identified by a number higher than that allotted to its immediately preceding event to indicate progress of work. The numbering of events in the network diagram must ***start*** from ***left*** (start of the project) to ***right*** (completion of the project) and ***top*** to the ***bottom***. Care should be taken that there is no duplication in the numbering.

7.4. ACTIVITY ANALYSIS

Activity is a time-consuming job or task that is a key subpart of the total project.

Activities of the net work represent project operations or tasks to be conducted. As such each activity except dummy consumes time and resources and incurs costs. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project.

Activities are identified by the numbers of their starting (tail or initial) *event* and *ending* (head or terminals) *event*. An arrow (***i, j***) extended between two events; the *tail event* ***i*** represents the start of the activity and the *head event* ***j***, represents the completion of the activity.

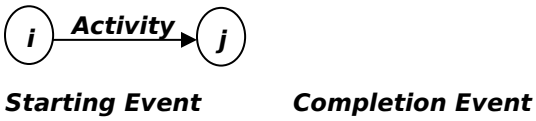


Fig: Activity -Node relationship in network diagram.

Activities can be further classified into the following three categories:

i. Predecessor activity

Predecessor activity is an activity which must be completed before one or more other activities start.

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ii. Successor activity

Successor activity is an activity which started immediately after one or more of other activities are completed.

iii. Dummying activity

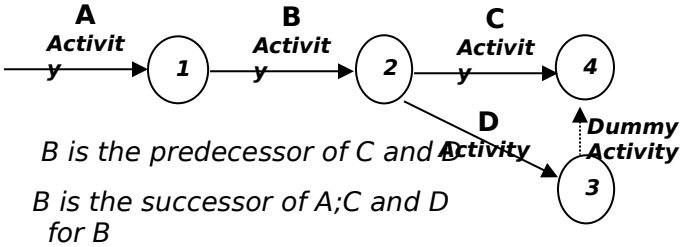
Dummying activity is an activity which does not consume either any resource or time.

A ***dummy activity*** in the network is added only to represent the given precedence relationships among activities of the project and is needed when

a) two or more parallel activities in a project have same head and tail events, or

b) two or more activities have some (but not all) of their immediate predecessor activities in common.

A *dummy activity* is depicted by ***dotted line*** in the network diagram.



7.5. RULES OF NETWORK CONSTRUUCTLON

Following are some of the rules which have to be followed while constructing a network:

1. *In network diagram, arrows represent activities and circles the events. The length of an arrow is of on significance.*
2. *Each activity should be represented by one arrow and must start and end in a circle called event. The tail of an activity represents the start and head the completion of work.*
3. *The event numbered 1 denotes start of the project and is called initial event. All activities emerging (or taking off) from event 1 should not be preceded by any other activity or activities. Event carrying the highest number denotes the completion events. A network should have only one initial event and only one terminal event.*
4. *The general rule of numbering the event is that the number at an activity's head should always be larger than that at its tail. That is, events should be numbered such that for each* ***activity (i, j), i< j****.*
5. *An activity must be uniquely identified by its starting and completion event which implies that*

    a. *an event number should not get repeated or duplicated*

    b. *two activities should not be identified by the same completion event*

    c. *activities must be represented either by their symbols or by the corresponding ordered pair of starting-completion events.*

6. *The logical sequence (or interrelationship) between activities must follow the following rules :*
    a. *An event cannot occur until all the incoming activities into it have been completed.*

    b. *An activity cannot start unless all the preceding activities, on which it depends, have been completed.*

    c. *Though a dummy activity does not consume ether any resource or time, even then it has to follow the rules 6(a) and 6(b).*

7.5. ERRORS AND DUMMIES IN NETWORK

i.Looping and Dangling

Looping (cycling) and dangling are considered as faults in a network. Therefore, these must be avoided.

1. *Looping*

A case of endless loop in a network which is also known as ***looping*** is shown in figure below, where activities ***A***, ***B***, and ***C*** from a cycle.

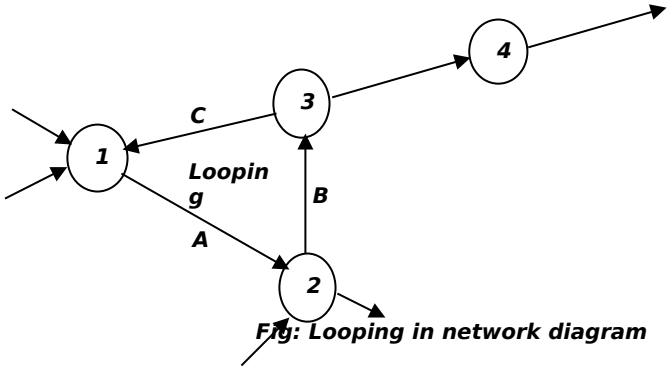


Fig: Looping in network diagram

In this case it is difficult to number three events associated with activity ***A***, ***B***, and ***C*** so as to satisfy ***rule 6*** of constructing the network.

2. Dangling

A case of disconnect activity before the completion of all activities which is also known as ***dangling***.

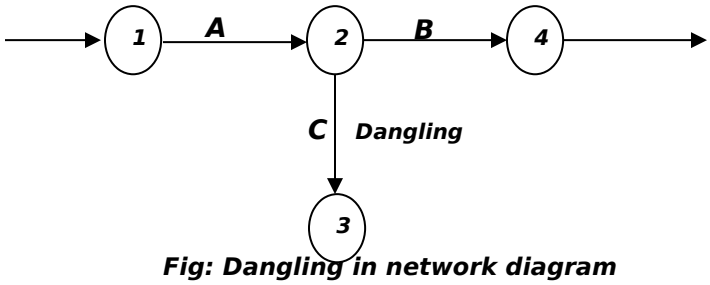


Fig: Dangling in network diagram

In this case, activity ***C*** does not give any result as per the rules of the network. The dangling may be avoided by adopting ***rule 5*** of constructing the network.

i.Dummy or Redundant Activity.

The following is the one of the cases in which the use of dummy activity may help in drawing the network correctly as per the various rules

When two or more parallel activities in a project have the same head and tail events:

i.e: two events are connected with more than one arrow.

In a), activities ***B*** and ***C*** have a common predecessor activity ***A***. At the same time, they have activity ***D*** as a common successor. To get the correct network a dummy activity for the ending event of ***B*** to show that ***D*** may not start before both ***B*** and ***C*** are completed is shown in figure b)

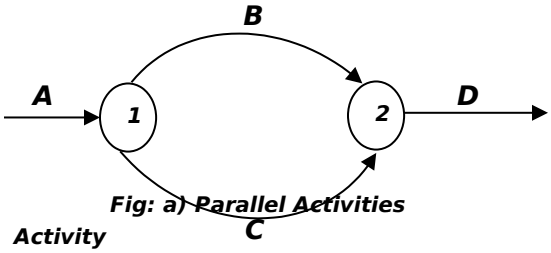
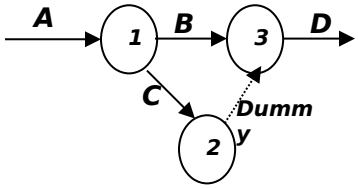


Fig: a) Parallel Activities



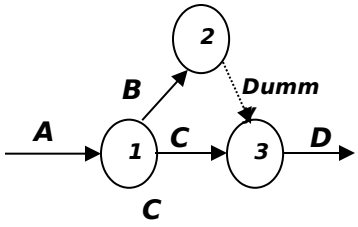


Fig: b) Dummy Activity

Example

1. An assembly is to be made from two parts ***X*** and ***Y***. Both parts must be turned on a lathe and ***Y*** need not be polished. The sequence of activities together with their predecessors is given below:

Activity	*Description*	*Predecessor Activity*
A	Open work order	_
B	Get material for X	A
C	Get material for Y	A
D	Turn X on Lathe	B
E	Turn Y on Lathe	B, C
F	Polish Y	E
G	Assemble X and Y	D, F
H	Pack	G

Draw a network diagram for the project.

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Solution

The network diagram for the project is show below:

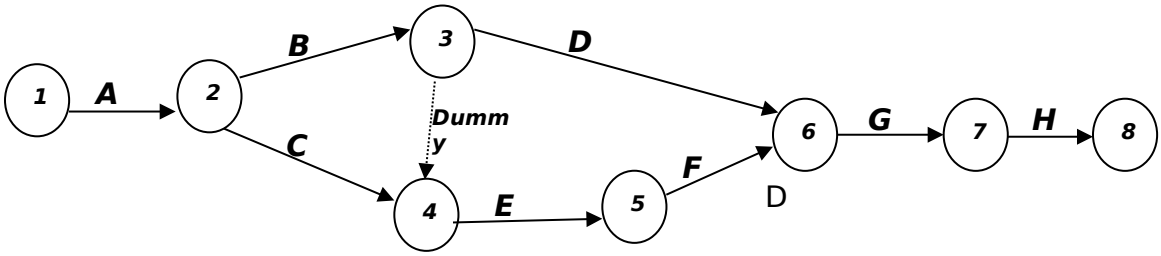


Fig: Network Diagram

2. Listed in the table are the activities and sequencing necessary for a maintenance job on the heat exchangers in a refinery.

Activity	***Description***	***Predecessor Activity***
A	Dismantle pipe connections	-
B	Dismantle header, closure, and floating front	A
C	Remove tube bundle	B
D	Clean bolts	B
E	Clean header and floating head front	B
F	Clean tube bundle	C
G	Clean shell	C
H	Replace tube bundle	F, G
I	Prepare shell pressure test	D, E, H
J	Prepare tube pressure test and reassemble	I

Required:

Draw a network diagram for the project

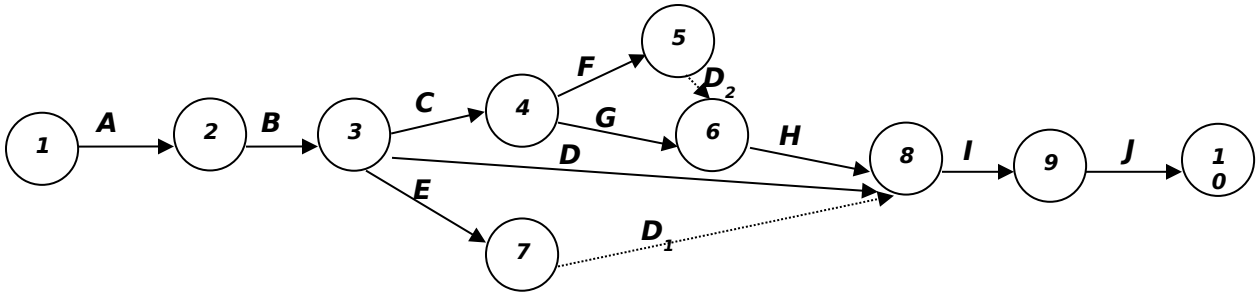


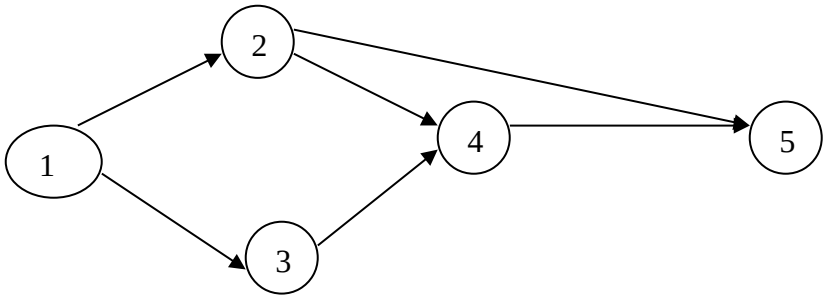
Fig: Network Diagram

3. The following table gives the activities in a construction project:

Activity	Immediate Predecessor
A	-
B	-
C	-
D	A
E	C
F	A
G	D, B, E

Required: Draw an arrow diagram for the project

Solution:

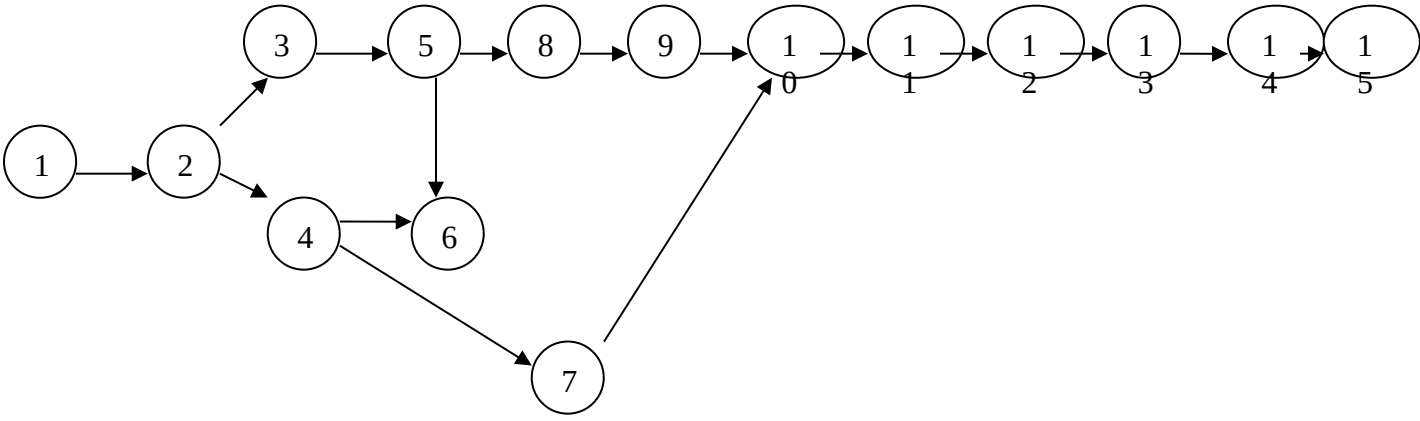


Exercise:

Listed in the table are the activities & sequencing necessary for the completion of a recruitment procedure for management trainees (MT) in a firm.

Activity	Predecessor Activity
A	-
B	A
C	A
D	C
E	B
F	C
G	D, E
H	G
I	H
J	F
K	I, J
L	K
M	L
N	M, N.

Draw a network diagram for the project Ans:



F IG Net work diagram

7.5 CRITICALPATH ANALISIS

The objective of critical path analysis is to estimated the total project duration & to assign starting & finishing times toall activities involved in the project. This helps in checking actual progress against the scheduled duration of the project.

The duration of individual activities may be uniquely determined (in case of CPM) or may involve the three tine estimates (in case of PERT) out of which the expected duration of an activity is computing. Having done this, the following factors should be known to prepare project scheduling

i. Total completion time of the project
ii. Earliest & latest start time of each activity
iii. Float for each activity

i.e. the amount of time by which the completion of an activity can be delayed without delaying the total project completion time.

iv. Critical activities & critical path.

Consider the following notation for the purpose of calculating various times of event & activities

A. Earliest occurrence time of event I Ei It is the earliest time at which an event can occur without affecting the total project time.
B. Latest occurrence time of event i = Li
It is the latest time at which an event can occur without affecting the total project time
C. Earliest start time for activity (i ,j) = ESij
It is the time at which the activity can start without affecting the total project time
D. Latest start time for activity (i, j) = LSij
It is the latest possible time by which an activity must start without affecting the total project time
E. Earliest finish time for activity (i , j)= EFij
It is the earliest possible time at which an activity can finis without affecting the total project time.
F. Latest finish time for activity (i, j) =LF ij
It is the latest time by which an activity must get completed with out delayining the project completion.
G. Duration of activity (i, j) = t ij

Expected time estimate

I n a network diagram, there should only be one initial event & one final event, while other events are numbered consecutively with integers 1,2,.....,n such that i<j , fir any two event i&j connected by an activity which starts at i & finish at j

For calculating the above mentioned times, there are two methods; namely;

1) Forward pass method &
2) Backward pass method

7.5.1 Forward Pass Method
(For Earliest Event Time)

I n this methods calculation begin from the initial event 1, proceed through the network visiting event in an increasing order of event no & end to the final event, say N. At each event we calculate earliest occurrence event time (E) & earliest start & finish time for each activity that begins at that event. When calculations end at the final event N, its earliest occurrence time gives the earliest possible completion time of the entire project.

The method may be summarized as follows:

1) Set the earliest occurrence time of initial event 1 to zero, i.e.

   E1 = 0, i =1

2) Calculate earliest start time for each activity that begins at event I i(=1) This is equal to earliest occurrence time of event, i (tail event). That is

   ESij = Ej, for LL activities (i, j) starting at event i.

3) Calculate the earliest finish time pf each activity that begins at event i. This equal to the earliest start time of the activity plus the duration of the activity. That is,

   EFij = ES ij + tij

   = Ei + tij for all activities (i j) beginning at event

   i

4) Proceed to the next event, say j, j > j

5) Calculate the earliest occurrence time for the event j. This is the maximum of the earliest finish times of all activities ending into that event, i.e.

$$E_j = \max \{EF_{ij}\}$$

$$= \max \{E_i + t_{ij}\}$$ for all immediate predecessor Activities

6) If j = N (final event), then earliest finish time for the project, i.e. the earliest occurrence time EN for the final event is given by

$$EN = \max \{EF_{ij}\}$$, for all terminal activities.

$$= \max \{EN - 1 + t_{ij}\}$$ & stop the method.

7.5.2 Backward Pass Method

(For Latest Allowable Event Time)

In this method calculations begin from final event N, proceed through the network visiting events in the decreasing order of event no & end at the initial event 1. At each event, we calculate the latest occurrence event time (L) for the corresponding event, latest finish & start time for each activity that is terminating at the event, such that the earliest finish time for the project remains the same. The method may be summarized as follows.

1. Set the latest occurrence of last event N equal to its earliest occurrence tome (known from forward pass method), that is

   $$L_N = E_N, \quad j = N$$

2. Calculate latest finish time of each activity which ends at event j. This is equal to latest occurrence time of final event N. That is $LF_{ij} = L_J$, for all activities (i,j) ending at evenj
3. Calculate the latest start times of all activities ending at j. I t is obtained by subtracting the duration of the activity from the latest finish time of the activity, That is,

   $$LF_{ij} = L_j$$

   $$LS_{ij} = LF_{ij} - t_{ij}$$

   $$= L_j - t_{ij}$$, for each activity (i,j) ending at

Event, j

4. Proceed backward to the event in the sequence that decreases j by 1.
5. Calculate the latest occurrence time of event i (i<j). This is the minimum of the latest start time of all activities starting from that event. That is,

   Li = min {LSij}

   = min {Lj – t ij}
6. If j = 1 (initial event), then the latest finish time for project, i.e. latest occurrence time L1 for the initial event is given by.

   Li = min [LSij} for all immediate successor activities

   = min { Lj – t ij} & stop the method

7.5.3. Float (or slack) of an Activity and Event

The ***float*** of an activity is the amount of time by which it is possible to delay its completion time with out affecting the total project completion time.

1. Event Float

The ***float*** (also sometimes called '***slack***') of an event is the difference between its latest time (L_i) and its earliest time (E_i).That is

$$\textbf{Event Float} = L_i$$

It is a measure of how much later than expected a particular event could occur without delaying the completion of the entire project.

2. Activity Float

It is the float (or slack) in the activity time estimates.

Types of activity floats:

a. Total Float

It is the difference between the time available to perform the activity (***measured from the earliest start time to the latest finish time***) and the expected completion time of the activity. That is, for activity (**i,j**), the total float is given by

$$\textbf{Total Float } (TF_{ij}) = (L_j - E_i) - t_{ij}$$

$$= LS_{ij} - ES_{ij}$$

b. Free Float

For calculating the total float, only a particular activity was considered with respect to its tail and head event times or by considering latest start and finish time of an activity with respect to its earliest start and finish time. However, it may be needed to know that how much an activity's scheduling time can be increased or decreased without changing its immediate successor activities.

The time by which the completion of an activity can be delayed beyond its ***earliest finish time*** without affecting the earliest start time of a succeeding activity is called ***free float***.

$$\textbf{Free Float } (FF_{ij}) = (E_j - E_i) - t_{ij}$$
$$= Min\{ ES_{ij}, \text{for all immediate successors of } (i,j) \} - EF_{ij}$$

c. Independent float

Remarks:

1. Since latest event time is always greater than or equal to the earliest event time ***(***i.e $L_i \geq E_i$***)***, $TF_{ij} \geq FF_{ij}$.

This implies the value of free float may range from zero to total float and can never exceed total float value.

2. The calculation of various floats can help the decision-maker in identifying the underutilized resources, flexibility in the total schedule and possibilities of redeployment of resources.

3. Once the float of an activity is disturbed, float of all other activities of the project is changed and should be recalculated.

4. If total float value is

a. Negative (i.e ***L-E<0***),

-***resources are not adequate*** and activity may not finish in time

-Induce extra resources or certain activities (also called critical activities) need crashing to reduce negative float value.

b. Zero (i.e ***L-E=0***)

-***Resources are just sufficient to complete the activity***. Any delay in activity execution will necessarily increase the project cost.

c. Positive (i.e ***L-E>0***)

-Resources are surplus

-Surplus resources can be deployed elsewhere or execution of the activity can be delayed.

7.6 CRITICAL PATH

Path A path is unbroken chain of activities that connects form the start node to the end node (terminal node)

Certain activities in a network diagram of a project are called critical activities because delay in their execution will cause further delay in the project completion time. Thus, all activities having zero total float value are identified as critical activities.

The critical path is the continuous chain of critical activities in a network diagram. It is the longest path starting from first to the last event & is shown by the thick line or double lines in the network diagram.

The length of the critical path is the sum of the individual times of the critical activities living on it & defines the minimum time required to complete the project.

The critical path on a network diagram can be identified as:

i. For all activities (i,j) lying on the critical path the E –values & L-values fortail & head events are equal

That is, Ej=Lj& Ei =Li

ii. On critical path Ej-Ei = Lj-Li=Lij

Finding the critical path is important for directing decision-makers attention & & effort to critical activities where improvement will pay the largest dividend.

Example

1. An established company has decided to add a new product to its line. It will buy the product form manufacturing concern, package is & sell it to a number of distributes selected on a geographical basis. Market research has identified the volume expected & the size of sales force required. The steps

Activity	**Description**	**Time (Week)**
A	Organize sales office	6
B	Hire Salesman	4
C	Train salesman	7
D	Select advertising agency	2
E	Plan advertising campaign	4
F	Conduct	10
G	Design package	2

H	Setup packaging facilities	10
I	Package initial Stocks	6
J	Order Stock from manufacture	13
K	Select distributors	9
L	Sell to distributors	3
M	Ship stocks	5

Required:

a) Indicate the critical path the critical activities & the total project time
b) For each non-critical activity, find the total float

A. the vritical path in the network diagram above has been shown by thick lines by joining all those events where two values Ei& Lj are equal

the critical path of the project is:

1-2-5-6-9-10 &

the critical activities are A,B,C,L&M.

the total project time is 25 weeks

B. For each non-critical activity, the total float is:

Activity (i,j)	Duration (tii)	Earliest time		Last time		Float	
		Start (Ei)	Finish (Ei+tij)	Start (Lj-tij)	Finish (Lj)	Total [(Lj-tij)Ej]	Free (Ej-Ei)-tji
1-3	2	0	2	2	4	2	0
1-4	13	0	13	1	14	1	0
2-6	9	6	15	8	17	2	
2-7	2	6	8	9	11	3	0
3-4	10	2	12	4	14	2	
4-9	6	13	19	14	20	1	
7-8	4	8	13	11	15	3	
8-10	10	12	22	15	25	3	

Remark:

- What does the critical path really mean?

- The critical path shows the shortest time in which a project can be completed. Even though the critical path is the longest path, it represents the shortest time it takes to complete a project.
- There can be more than one critical path on a project. Project managers should closely monitor performance of activities on the critical path to avoid late project completion. If there is more than one critical path, project mangers must keep their ayes on all of them

C. A small maintenance project consists of 5 activities which take place according to the following conditions:

Activity	Immediate predecessors	Estimated Activity Time in Week
A	-	3
B	-	5
C	-	5
D	A	2
E	B	4

Required:

a. Draw the network diagram for the following activities.
b. Find the critical path & the critical activities & the critical path duration
c. Find the total float for each activity

b. Path 1: 1-2-4 length 3+2= 5 weeks
Path 2:1-4 Length =6 weeks
Path 3: 1-3-4 Length = 5=4 9 weeks.

The critical path of the project is 1-3-4

The critical activities are B& E

The critical path duration 9 weeks

Activity (i,j)	Duration (tij)	Earliest start (Ei)	Time Finish (Ei+tij)	Last time		Total Float (Li-tij)-Ej
				Start (Lj-tij)	Finish Lj	
1-2	3	0	3	4	7	4
1-3	5	0	5	0	5	0

1-4 2-4 3-4	6 2 4	0 3 3	6 5 9	3 7 5	9 9 9	3 4 0

Exercise

1. A research & development department is developing a new power supply for a console television set. It has broken the job down into the following form:

Job	Immediate predecessors	Expected tome (days)
A	-	5
B	A	7
C	B	2
D	B	3
E	C	1
F	D	2
G	C	1
H	E,F	3
I	G,H	10

   a. Draw a critical path scheduling arrow diagram, identifying jobs by letters & associating times with each. Indicate the critical path.
   b. What is the minimum time for completion of the project?
   c. Find the total float of each job.

2. Based on the following network diagram a) identify the critical path. What is its length b) Find the total float for each task.

7.7 PERT NETWORKS

Program Evaluation and Review Technique (PERT)- is a network analysis technique used to estimate project duration when there is a high degree of uncertainty about the individual activity duration estimates. PERT applies the critical path method to a weighted average duration estimate.

If the duration of activities in a project is uncertain than activity scheduling calculations are done by using the expected value of the durations. However, such expacted duration estimated may not be given an accurate answer. Thus, rather than estimating directly the expected completion time of an activity, three values are considered in PERT.

The three time estimates are:

1. Optimistic time (to or a):
   - This is the shortest possible time to perform an activity, assuming that everything goes well.
2. Pessimistic time (tp or b):
   This is the maximum (longest) time that is required to perform an activity, under extremely bad conditions. However, such conditions do not include acts of God like earthquakes, flood, etc.
3. Most likely time (tm or m):
   - This is the most realistic time to complete the activity. Statistically, it is the modal value of duration of the activity

   i.e consider the time estimate under ht normal situation

PERT uses probabilistic time estimates duration estimates based on using optimistic most likely & pessimistic estimates of activity durations instead of one specific or discrete duration estimate. Like the CPM, PERT is based on a project network diagram.

To use the PERT method, we have to calculate the expected activity time.

The most likely time (tm) estimate may not coincide with the midpoint (to+tp)/2 & may occur to its left or its right as shown in fig below. Because of these properties, it is justified to assume that the duration of each activity may follow Beta (B) distribution with unmodel point occurring at tm & it's and points at to & tp



In Bata distribution, the mid-point (to+tp)/2is assumed to weight half as much as the most likely point (tm). Thus the expected or mean (te) value of the activity duration can be approximated as the arithmetic mean of (to+tp)/2 &2tm.

Example

1. A project is represented by a network shown be low & has the following data

Task:	A	B	C	D	E	F	G	H	I
Optimistic time	5	18	26	16	15	6	7	7	3
Pessimistic time:	10	22	40	20	25	12	12	9	5
Most Likely time:	8	20	33	18	20	9	10	8	4

Determine the following

a. Expected task time & their variance
b. The earliest & latest expected times to reach each event
c. The critical path

Activity	to	tp	tm	te=(to+4tm+tp)/6	S^2= [(tp-to)/6]^2
1-2	5	10	8	7.8	0.694
1-3	18	22	20	20.0	0.444
1-4	26	40	33	33.0	5.444
2-5	16	20	18	18.0	0.444
2-6	15	25	20	20.0	2.778
3-6	6	12	9	9.0	1.000
4-7	7	12	10	9.8	0.694
5-7	7	9	8	8.0	0.111
6-7	3	5	4	4.0	0.111

Forward Pass Method

Exercise:

A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i)& ending (j) node numbers.

Activity (i,j)	Estimated Duration (in weeks)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

a) Draw the project network
b) Find the expected duration& variance for each activity. What is the expected project length?
c) Calculate the variance & standard deviation of the project length.

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