Chapter Four

The Theory of Production and Cost

4.1 Theory of production in the short run

4.1.1 Definition of production

- Raw materials yield less satisfaction to the consumer by themselves.
- In order to get better utility from raw materials, they must be transformed into outputs.
- However, transforming raw materials into outputs requires inputs such as land, labour, capital and entrepreneurial ability.
- Production is the process of transforming inputs into outputs.
- The end products of the production process are outputs which could be tangible (goods) or intangible (services).

4.1.2 Production function

- Production function is a technical relationship between inputs and outputs.
- It shows the maximum output that can be produced with fixed amount of inputs and the existing technology.

$$Q = f(X, X, X, \ldots, X)$$

- where, Q is output and X1, X2, X3,…, Xn are different types of inputs.
- ❖ Inputs are commonly classified as fixed inputs or variable inputs.
- **Fixed inputs** are those inputs whose quantity cannot readily be changed when market conditions indicate that an immediate adjustment in output is required.

Variable inputs are those inputs whose quantity can be altered almost instantaneously in response to desired changes in output.

- That is, their quantities can easily be diminished when the market demand for the product decreases and vice versa.
- The best example of variable input is unskilled labour.

Short run refers to a period of time in which the quantity of at least one input is fixed.

- Consider a firm that uses two inputs: capital (fixed input) and labour (variable input).
- Given the assumptions of short run production, the firm can increase output only by increasing the amount of labour it uses.
- Hence, its production function can be given by: $Q = f(L)$

- Where, Q is output and L is the quantity of labour.
- Thus, output can change only when the amount of labour changes

Total, average, and marginal product

Total product (TP): it is the total amount of output that can be produced by efficiently utilizing specific combinations of the variable input and fixed input.

- Increasing the variable input (while some other inputs are fixed) can increase the total product only up to a certain point.
- Initially, as we combine more and more units of the variable input with the fixed input, output continues to increase, but eventually if we employ more and more unit of the variable input beyond the carrying capacity of the fixed input, output tends to decline.

In general, the TP function in the short-run: initially increases at an increasing rate, then increases at a decreasing rate, reaches a maximum point and eventually falls as the quantity of the variable input rises.

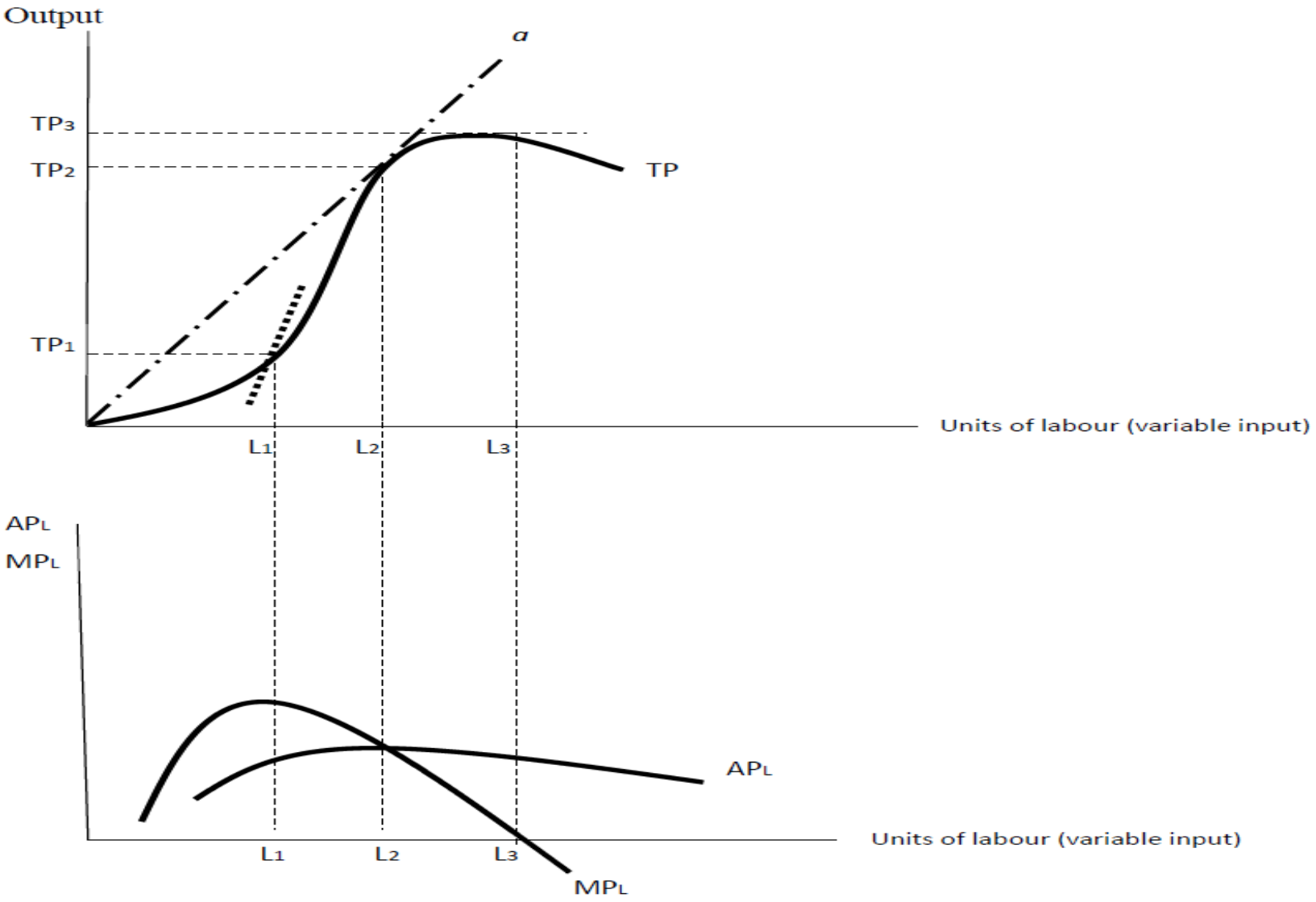


Figure 4.1: Total product, average product and marginal product curves

The relationship between MP_L and AP_L can be stated as follows.

- When AP_L is increasing, $MP_L > AP_L$.
- When AP_L is at its maximum, $MP_L = AP_L$.
- When AP_L is decreasing, $MP_L < AP_L$.

Example: Suppose that the short-run production function of certain cut-flower firm is given by: $Q = 4KL - 0.6K^2 - 0.1L^2$ where Q is quantity of cut-flower produced, L is labour input and K is fixed capital input (K=5).

a) Determine the average product of labour (AP_L) function.

b) At what level of labour does the total output of cut-flower reach the maximum?

c) What will be the maximum achievable amount of cut-flower production?

Solution:

a) $$AP_L = \frac{Q}{L} = \frac{4KL - 0.6K^2 - 0.1L^2}{L} = 4K - \frac{0.6K^2}{L} - 0.1L = 20 - \frac{15}{L} - 0.1L = \underline{\underline{\frac{20L - 15 - 0.1L^2}{L}}}$$

b) When total product (Q) is maximum, MP will be zero.

$$MP_L = \frac{\partial Q}{\partial L} = \frac{\partial(4KL - 0.6K^2 - 0.1L^2)}{\partial L} = 4K - 0.2L = 0$$

$$\Rightarrow 20 - 0.2L = 0 \;\Rightarrow\; L = \frac{20}{0.2} = \underline{\underline{100}}$$

Hence, total output will be the maximum when 100 workers are employed.

c) Substituting the optimal values of labor (L=100) and capital (K=5) into the original production function (Q):

$$\underline{Q_{max}} = 4KL - 0.6K^2 - 0.1L^2 = 4 * 5 * 100 - 0.6 * 5^2 - 0.1 * 100^2 = \underline{\underline{985}}$$

4.1.4 The law of variable proportions

- The law of variable proportions states that as successive units of a variable input(say, labour) are added to a fixed input (say, capital or land), beyond some point the extra, or marginal, product that can be attributed to each additional unit of the variable resource will decline.
- **For example**, if additional workers are hired to work with a constant amount of capital equipment, output will eventually rise by smaller and smaller amounts as more workers are hired.

4.1.5 Stages of production

Stage I

This stage of production covers the range of variable input levels over which the average product (APL) continues to increase

- It goes from the origin to the point where the APL is maximum, which is the equality of MPL and APL.
- This stage is not an efficient region of production though the MP of variable input is positive.
- The reason is that the variable input (the number of workers) is too small to efficiently run the fixed input so that the fixed input is under-utilized (not efficiently utilized).

Stage II

- It ranges from the point where APL is at its maximum (MPL=APL) to the point where MPL is zero
- Here, as the labour input increases by one unit, output still increases but at a decreasing rate.
- Due to this, the second stage of production is termed as the stage of diminishing marginal returns.

- Additional inputs are contributing positively to the total product and MP of successive units of variable input is declining.
- Hence, the efficient region of production is where the marginal product of the variable input is declining but positive.
- As a result, the marginal product diminishes.
- This stage is the efficient region of production.

Stage III

- In this stage, an increase in the variable input is accompanied by decline in the total product.
- Thus, the total product curve slopes downwards, and the marginal product of labour becomes negative.

- This stage is also known as the stage of negative marginal returns to the variable input.
- The cause of negative marginal returns is the fact that the volume of the variable inputs is quite excessive relative to the fixed input; the fixed input is over-utilized.
- Obviously, a rational firm should not operate in stage III because additional units of variable input are contributing negatively to the total product (MP of the variable input is negative).

4.2 Theory of costs in the short run

4.2.1 Definition and types of costs

- To produce goods and services, firms need factors of production or simply inputs.

- To acquire these inputs, they have to buy them from resource suppliers.
- *Cost is, therefore, the monetary value of inputs used in the production of an item.*
- To the economist, **economic profit** is **total revenue** less **economic costs** (explicit and implicit costs).
- **Accounting cost** is the monetary ***value of all purchased inputs used in production***; it ***ignores the cost of non-purchased*** (self-owned) inputs.
- Explicit costs are out of pocket expenses for the purchased inputs.
- **Accounting profit = Total revenue – Accounting cost**

  = Total revenue – Explicit cost

- Economic cost is given by the sum of implicit cost and explicit cost.
- **Economic profit = Total revenue – Economic cost**

  (Explicit cost + Implicit cost)
- Economic profit will give the real profit of the firm since all costs are taken into account.
- Accounting profit of a firm will be greater than economic profit by the amount of implicit cost.
- If all inputs are purchased from the market, accounting and economic profit will be the same.
- However, if implicit costs exist, then accounting profit will be larger than economic profit.

4.2.2 Total, average and marginal costs in the short run

- A cost function shows the total cost of producing a given level of output.
- C = f (Q), where C is the total cost of production and Q is the level of output.
- In the short run, total cost (TC) can be broken down in to two – total fixed cost (TFC) and total variable cost (TVC).

Fixed costs we mean costs which do not vary with the level of output.

- The firm can avoid fixed costs only if he/she stops operation (shuts down the business).
- The fixed costs may include salaries of administrative staff, expenses for building depreciation and repairs, expenses for land maintenance and the rent of building used for production.

Variable costs, on the other hand, include all costs which directly vary with the level of output.

- These costs may include the cost of raw materials, the cost of direct labour and the running expenses of fuel, water, electricity, etc.

$$TC = TFC + TVC$$

Total fixed cost (TFC): Total fixed cost is denoted by a straight line parallel to the output axis. This is because such costs do not vary with the level of output.

- Total variable cost (TVC): The total variable cost of a firm has an inverse S-shape.
- The shape indicates the law of variable proportions in production.

- At the initial stage of production with a given plant, as more of the variable factor is employed, its productivity increases.
- Hence, the TVC increases at a decreasing rate.
- This continues until the optimal combination of the fixed and variable factor is reached.
- Beyond this point, as increased quantities of the variable factor are combined with the fixed factor, the productivity of the variable factor declines, and the TVC increases at an increasing rate.

Total Cost (TC): The total cost curve is obtained by vertically adding TFC and TVC at each level of output.

- The shape of the TC curve follows the shape of the TVC curve, i.e. the TC has also an inverse S-shape.
- It should be noted that when the level of output is zero, TVC is also zero which implies TC = TFC.

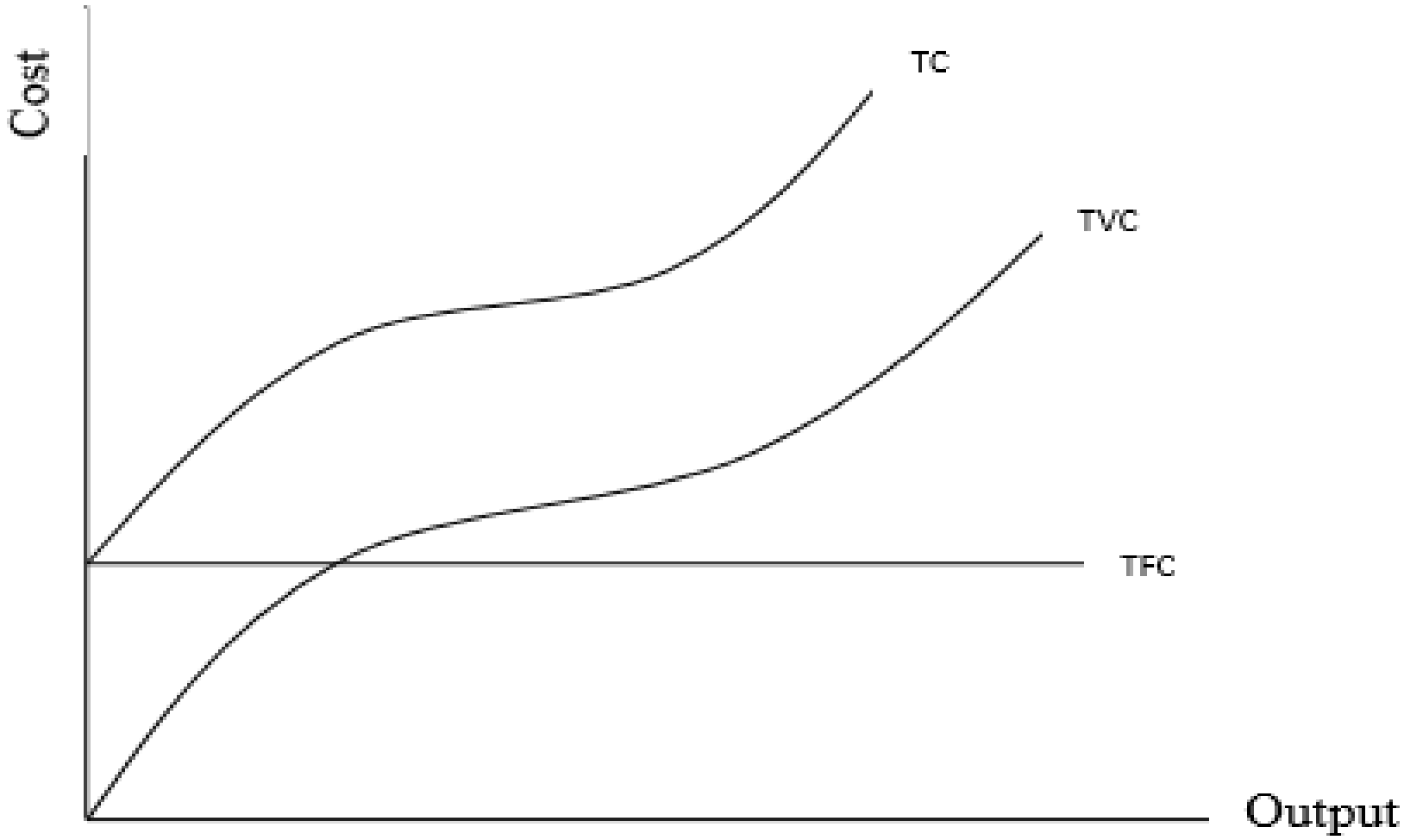


Figure 4.2: ***Short run TC, TFC and TVC curves***

a) **Average fixed cost (AFC)** - Average fixed cost is total fixed cost per unit of output. It is calculated by dividing TFC by the corresponding level of output. The curve declines continuously and approaches both axes asymptotically.

$$AFC = \frac{TFC}{Q}$$

b) **Average variable cost (AVC)** - Average variable cost is total variable cost per unit of output. It is obtained by dividing total variable cost by the level of output.

$$AVC = \frac{TVC}{Q}$$

The short run AVC falls initially, reaches its minimum, and then starts to increase. Hence, the AVC curve has U-shape and the reason behind is the law of variable proportions.

c) **Average total cost (ATC) or simply Average cost (AC)** - Average total cost is the total cost per unit of output. It is calculated by dividing the total cost by the level of output.

$$AC = \frac{TC}{Q}$$

Equivalently, $AC = \frac{TVC + TFC}{Q} = \frac{TVC}{Q} + \frac{TFC}{Q} = AVC + AFC$

Thus, AC can also be given by the vertical sum of AVC and AFC.

Marginal Cost (MC)

Marginal cost is defined as the additional cost that a firm incurs to produce one extra unit of output. In other words, it is the change in total cost which results from a unit change in output. Graphically, MC is the slope of TC function.

$$MC = \frac{dTC}{dQ}$$

In fact, MC is also a change in TVC with respect to a unit change in the level of output.

$$MC = \frac{dTFC + dTVC}{dQ} = \frac{dTVC}{dQ}, \quad \text{since} \quad \frac{dTFC}{dQ} = 0$$

- Given inverse S-shaped TC and TVC curves, MC initially decreases, reaches its minimum and then starts to rise.
- AVC, AC and MC curves are all U-shaped due to the law o

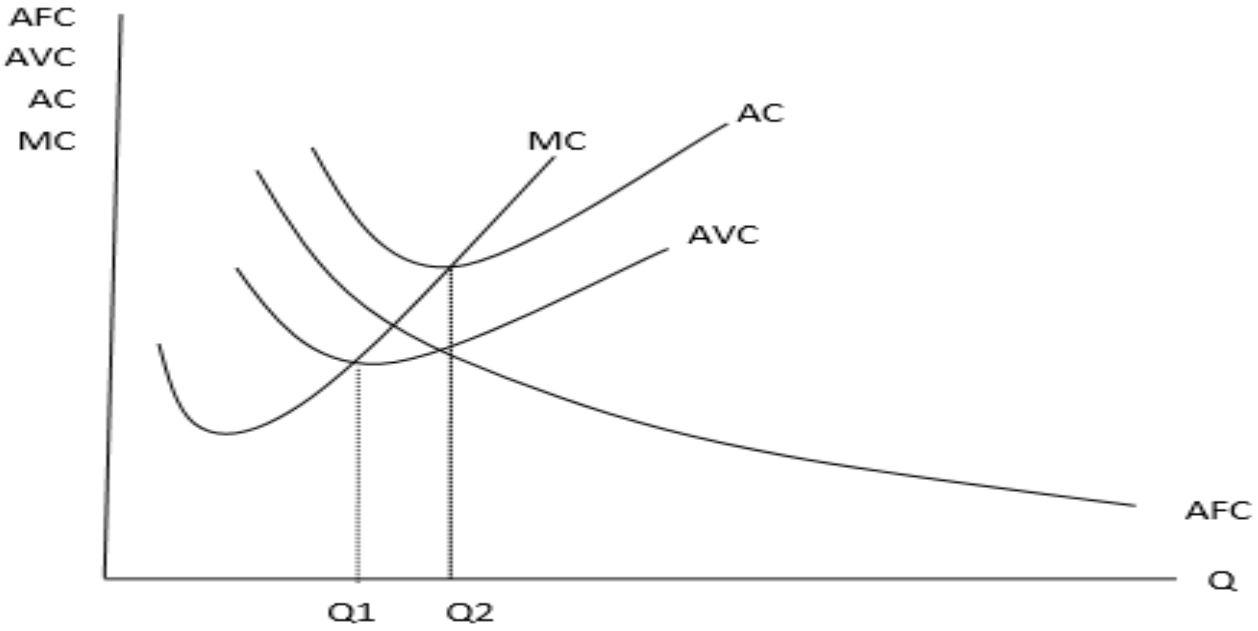


Figure 4.3: ***Short run AFC, AVC, AC and MC Curves***

- In the above figure, the AVC curve reaches its minimum point at Q1 level of output and AC reaches its minimum point at Q2 level of output.
- The vertical distance between AC and AVC, that is, AFC decreases continuously as output increases.
- It can also be noted that the MC curve passes

Example: Suppose the short run cost function of a firm is given by: $TC=2Q^3-2Q^2+Q+10$.

a) Find the expression of TFC & TVC

b) Derive the expressions of AFC, AVC, AC and MC

c) Find the levels of output that minimize MC and AVC and then find the minimum values of MC and AVC

4.2.3 The relationship between short run production and cost curves

- Suppose a firm in the short run uses labour as a variable input and capital as a fixed input.
- Let the price of labour be given by w, which is constant.
- Given these conditions, we can derive the relation between /C and APL.

i) Marginal Cost and Marginal Product of Labour

$$MC = \frac{\Delta TVC}{\Delta Q}, \text{ where } TVC = w.L$$

$$MC = \frac{\Delta(w.L)}{\Delta Q} = w.\frac{\Delta L}{\Delta Q}, \text{ but } \frac{\Delta L}{\Delta Q} = \frac{1}{MP_L}$$

$$\text{Therefore, } MC = \frac{w}{MP_L}$$

- The above expression shows that MC and MPL are inversely related.
- When initially MPL increases, MC decreases; when MPL is at its maximum, MC must be at a minimum and when finally MPL declines, MC increases.

ii) Average Variable Cost and Average Product of Labour

$$AVC = \frac{TVC}{Q}, \text{ where, } TVC = w.L$$

$$AVC = \frac{w.L}{Q} = w.\frac{L}{Q}, \text{ but } \frac{L}{Q} = \frac{1}{AP_L}$$

$$\text{Therefore, } AVC = \frac{w}{AP_L}$$

- This expression also shows inverse relation between AVC and APL. When APL increases, AVC decreases; when APL is at a maximum, AVC is at a minimum and when finally APL declines, AVC increases.

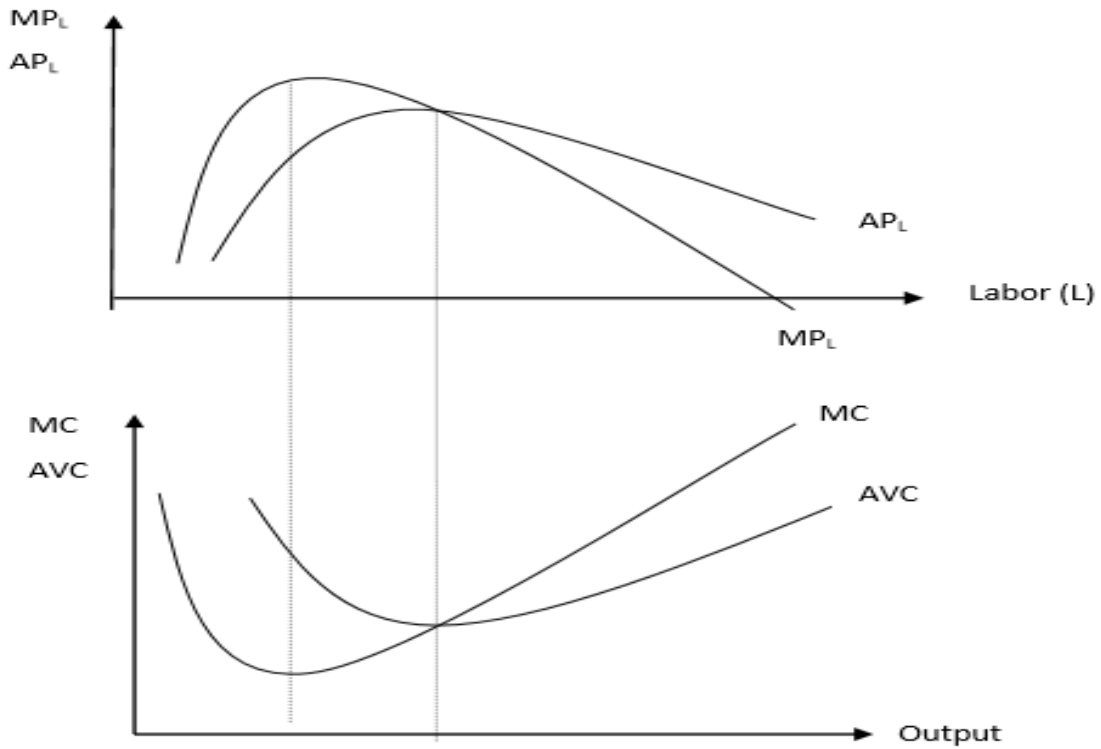


Figure 4.4: relationship between short run production and cost curves

Review questions

1. What is the main difference between fixed inputs and variable inputs?
2. Explain the law of variable proportions.
3. Which stage of short run production is efficient? Why?
4. Show the relationship between short-run MC and MPL both mathematically and graphically.

The End!

Thank U!