

CHAPTER 3

Time Value of Money

- **Future value**
- **Present value**

Time Value of Money

- **Time value of money is a critical consideration in financial and investment decisions.**
- **Future value determination [compounding] are needed to determine future sums of money**
- **Discounting [present value] is used to evaluate future cash flow**

Time Value of Money

- **A dollar in hand today is worth more than a dollar to be received tomorrow** because of the interest it could earn from putting it in a savings account or placing it in an investment account.
- **Compounding interest means that interest earns interest.**

Time Value of Money

- The key concept that underlies this is the time value of money: that a birr today is worth more than a birr received a year from now.
- This is because of the fact that the value of one birr after a year will grow to one birr and an interest earned on it for it can be invested, or given as loans during the year.
- **Interest is the price paid for the use of money overtime.**

Time Value of Money

- **FV= Future Value , the amount of money at the end of year n**
- **P= Principal amount**
- **i= annual interest rate**
- **n= number of years**
- **$FV1 = P (1+i)$, $FV2 = FV1 (1+i)$ FVn
 $= FV_{n-1} (1+i) = PV(1+i)^n$**

Example

1. You deposit Br. 100 in a bank at 10% interest what would be amount after 3 years.

- $FV \text{ at } 1^{\text{st}} \text{ yr} = PV + (PV) (\text{Rate of interest})$
- $= 100 + 100 (.10)$
- $= 100 + 10$
- $= \underline{110}$
- $FV \text{ at } 2^{\text{nd}} \text{ yr} = 110 + (110) (.10)$
- $= 110 + 11$
- $= \underline{121}$
- $FV \text{ at } 3^{\text{rd}} \text{ yr} = 121 + (121) (.10)$
- $= 121 + 12.10$
- $= \underline{133.10}$

For example

- **Mr X placed \$1,000 in a savings account earning 8 percent interest compounded annually.**
- **How much money will he have in the account at the end of 4 years?**

For example

$$\begin{aligned} \blacksquare FV &= PV(1+i)^n \\ &= \$1000 (1+.08)^4 \\ &= \$ 1000(1.3605) \\ &= \$ 1,360.5 \end{aligned}$$

Annuity

Ordinary annuity

- **An annuity is a series of equal payments or receipts occurring over a specified number of periods.**
- **In an *ordinary annuity*, *payments or receipts occur at the end of* each period.**
- **Figure 3.3 shows the cash flow sequence for an ordinary annuity on a time line.**

ORDINARY ANNUITY FORMULAS

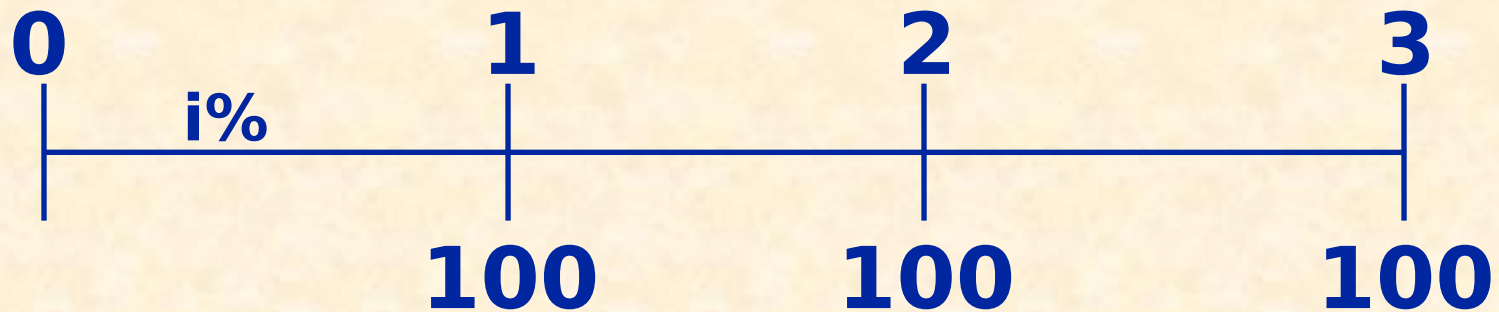
$$FVA_n = (PMT) \left[\frac{(1+i)^n - 1}{i} \right]$$

$$PVA_n = (PMT) \left[\frac{1 - (1+i)^{-n}}{i} \right], \text{ or } PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

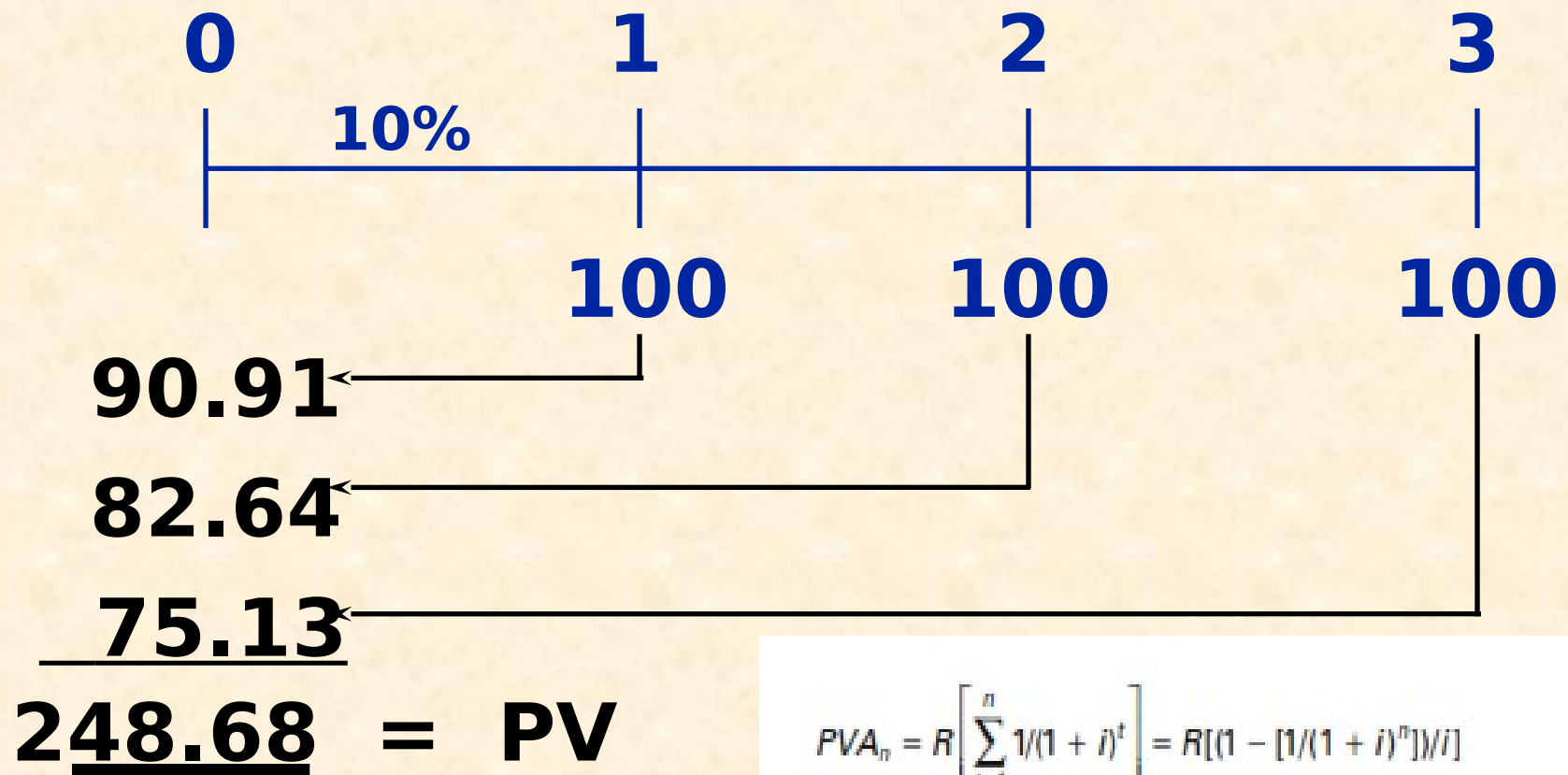
Future Value of an Annuity

- **The future value of an annuity is a compound annuity which involves depositing or investing an equal sum of money at the end of each year for a certain number of years and allowing it to grow.**

**Time line for an ordinary annuity
of \$100 for 3 years.**



What's the PV of this ordinary annuity?



$$PVA_n = R \left[\sum_{t=1}^n \frac{1}{(1+i)^t} \right] = R \left[\frac{1 - [1/(1+i)^n]}{i} \right]$$

Present value-discounting

- **Present value is the present worth of future sums of money.**
- **The process of calculating present values, or discounting, is actually the **opposite** of finding the compounded future value.**

Example-finding single payment FV

1.If you deposit Br. 55,650 in a Bank at 12% interest for a period of ten years what will be the future value?

- $FV = 55,650 (CVF_{10} .12)$
- $= 55,650 (3.106)$
- $= \underline{\underline{172,848.90}}$

Example- FINDING FV OF ANNUITY

You deposit a sum of Br. 5,000 at the end of each year for four years at 6%. How much would this annuity accumulate at the end of the fourth year?

- $FV = A (CVAF_4, .06)$
- $= 5,000(4.375)$

Illustration-finding interest

- assume that ^{rate} you have invested 15,000 Birr today at a bank where it can grow to the future value of 17,000 Birr within three years from now into the future.
- What is the interest rate that the bank should pay for your account in order to fulfill your desire?

Illustration-finding interest rate

- $FV_3 = PV (1+i)^3$
- $17900 = 15,000 (1+i)^3$
- $(1+i)^3 = FVIF_{i,3} = 1.193$
- The future value interest factor in the interest (future value of single payment table) corresponding to the unknown interest rate (i) and a period of 3 years ($n = 3$) is 1.193.
- Hence, look up the three year ($n=3$) row and read horizontally until you find the table value (future value interest factor) that is equal or the closest to the computed value of 1.193.
- There is no table value that is exactly equal to 1.193.
- The table value of 1.191 is found to be the closest value to 1.193 and it corresponds to 6 percent.
- Thus, the interest that the bank actually has to pay to your account is slightly greater than 6 percent.

Illustration-finding interest rate

Portion of future value table for single payment,

Period (n)	4%	5%	6%
1	1.0400	1.0500	1.0600
2	1.0816	1.1025	1.1236
3	1.1249	1.1576	1.1910*
4	1.1699	1.2155	1.2625
5	1.2167	1.2763	1.3382
6	1.2653	1.3401	1.4185

Example-finding number of years

- Assume, for example, a deposit of 1000 Birr is made in an interest bearing account that pays 10 percent compounded yearly. Your goal as a depositor is to collect 1,500 Birr after an unknown number of years.
- How many years should you wait for the desired amount to be realized?

Example-finding number of years

By substituting the values into the future value of single payment equation, you get:

$$FV_n = 1000 (1+i)^n$$

$$1,500 = 1000 (1+0.1)^n$$

$$1500 = 1000 (1.1)^n$$

$$(1.1)^n = \frac{1500}{1000}$$

$$1.1^n = 1.5, \text{ by using logarithm}$$

$$n = \frac{\log 1.5}{\log 1.1} = \frac{0.176}{0.041} = 4.29 \text{ years}$$

Illustration-finding the number of payments

- how many annual deposits of 1000 Birr each must be made into an account that pays 6 percent interest compounded yearly in order to accumulate 5,500 Birr immediately after the last deposit?

Illustration-finding the number of payments

$$FVA_n = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

$$5500 = 1000 (FVIFA_{i,n}) \text{ because } \frac{(1+i)^n - 1}{i} \text{ is equal to } FVIFA_{i,n}$$

$$FVIFA_{6\%, n} = \frac{5500}{1000} = 5.5$$

Illustration-finding the number of payments

A Portion of the Future Value of an Annuity Table

Periods (n)	6%	7%	8%	9%	10%
1	1.000	1.000	1.000	1.000	1.000
2	2.060	2.070	2.080	2.090	2.100
3	3.184	3.215	3.246	3.278	3.310
4	4.375*	4.440	4.506	4.573	4.641
5	5.637 ⁺	5.751	5.866	5.985	6.105

Example – loan amortization

- assume that a 50,000 Birr loan is to be repaid in yearly equal installments of 14,000 Birr. The loan carries a 6 percent annual interest (discount) rate.
- How many payments are required to fully repay this loan?

Example – loan amortization

$$\begin{aligned}PVA_{i,n} &= (PMT) (PVIFA_{i,n}) \\50,000 &= (14,000) (PVIFA_{6\%, n}) \\PVIFA_{6\%, n} &= \frac{50,000}{14,000} = 3.571\end{aligned}$$

Example – loan amortization

- Then look at the 6 percent column in the present value of an annuity table, and read the column down until you arrive at a table value that is equal, or exceed the computed value of 3.571.
- The desired table value lies between 3.465 which corresponds to four payments ($n=4$) and 4.212 which corresponds to five payments ($n=5$).

Example – loan amortization

Period	Un paid Principal	Interest on Principal	Total amount unpaid	Yearly payment	Interest payment	Principal payment	New balance
1	50,000	3,000	53,000	14,000	3,000	11,000	39,000
2	39,000	2,340	41,340	14,000	2,340	11,660	27,3340
3	27,340	1,640	28,980	14,000	1,640	12,360	14,980
4	14,980	899	15,879	14,000	899	13,101	1,879
5	1,879	113	1,992	14,000	113	1,879	0