

## **PART TWO: STOCHASTIC MODELS**

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### **UNIT 8: DECISION ANALYSIS**

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#### **8.0 AIMS AND OBJECTIVES**

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After studying this unit the student should be able to:

- explain components of decision making
- understand the decision making processes with out probabilities
- describe the different decision making criterion with probabilities
- define the decision analysis with additional information

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#### **8.1 INTRODUCTION**

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In the previous units dealing with linear programming, models were formulated and solved in order to aid the manager in making a decision. The solutions to the models were represented by values for decision variables. However, these linear programming models were all formulated under the assumption that certainty existed. In other words, it was assumed that all of the model coefficients, constraint values, and solution values were known with certainty and did not vary.

In actual practice, however, many decision making situations occur under conditions of uncertainty. For example, the demand for a product may be

not 100 units next week but 50 or 200 units, depending on the state of the market (which is uncertain). Several decision making techniques are available to aid the decision maker in dealing with this type of decision situation in which there is uncertainty.

Decision situations can be categorized into two classes: Situations where probabilities cannot be assigned to future occurrences and situations where probabilities can be assigned. In this chapter we will discuss each of these classes of decision situations separately and demonstrate the decision making criterion most commonly associated with each.

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## **8.2 COMPONENTS OF DECISION MAKING**

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A decision making situation includes several components-the decisions themselves and the actual events that may occur in the future, known as states of nature. At the time the decision is made, the decision maker is uncertain which states of nature will occur in the future and has no control over them.

Suppose a distribution company is considering purchasing a computer in order to increase the number of orders it can process and thus increase its business. If economic conditions remain good, the company will realize a large increase in profit; however, if the economy takes a downturn the company will lose money. In this decision situation the possible decisions are to purchase the computer and to not purchase the computer. The states of nature are good economic conditions and bad economic conditions. The state of nature that occurs will determine the outcome of the decision and it is obvious that the decision maker has no control over which state will occur.

As another example, consider a concessions vendor who must decide whether to stock coffee for the concession stands at a football game in November. If the weather is cold, most of the coffee will be sold, but if

the weather is warm, very little coffee will be sold. The decision is to order or not order coffee, and the states of nature are warm and cold weather.

To facilitate the analysis of these types of decision situations so that the best decisions result, they are organized into payoff tables. In general, a payoff table is a means of organizing and illustrating the payoffs from the different decisions, given the various states of nature in a decision problem. A payoff table is constructed as shown in table 8.1.

Each decision, 1 or 2 in table 8.1 will result in an outcome, or payoff for the particular state of nature that will occur in the future. Payoffs are typically expressed in terms of profit, revenues, or cost (although they can be expressed in terms of a variety of quantities). For example, if decision 1 is to purchase a computer and state of nature “a” is good economic conditions; payoff 1a could be \$100,000 in profit.

| Decision | State of nature |           |
|----------|-----------------|-----------|
|          | a               | b         |
| 1        | Payoff 1a       | Payoff 2a |
| 2        | Payoff 2a       | Payoff 2b |

*Table 8.1 Payoff table*

It is often possible to assign probabilities to the states of nature that aid the decision maker in selecting the decision that has the best outcome. However, in some cases the decision maker is not able to assign probabilities, and it is this type of decision making situation that we will address first.

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### **8.3 DECISION MAKING WITHOUT PROBABILITIES**

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The following example will illustrate the development of a payoff table without probabilities. An investor is going to purchase one of three types of real estate. The investor must decide among an apartment building, an office building, and a warehouse. The future states of nature that will determine how much profit the investor will make are good economic conditions and poor economic conditions. The profits that will result from each decision in the event of each state of nature are shown in the table 8.2.

*Table 8.2 Payoff table for the Real Estate Investments*

| Decision<br>(Purchase) | States of Nature           |                             |
|------------------------|----------------------------|-----------------------------|
|                        | Good economic<br>condition | Poor economic<br>conditions |
| Apartment building     | \$ 50,000                  | \$ 30,000                   |
| Office building        | 100,000                    | -40,000                     |
| Warehouse              | 30,000                     | 10,000                      |

## **Decision Making Criteria**

Once the decision situation has been organized into a payoff table, several criteria are available for making the actual decision. These decision criteria, which will be presented in this section, include ***maximax, maximin, minimax regret, Hurwicz, and equal likelihood***. On occasion these criteria will result in the same decision; however, often they will yield different decisions. The decision maker must select the criterion or combination of criteria that best suits his or her needs.

### **1. The Maximax criterion**

With the maximax criterion, the decision maker selects the decision that will result in the maximum of the maximum payoffs. (In fact, this is how this criterion derives its name—a maximum of maximum.) The maximax criterion is very *optimistic*. The decision maker assumes that the most favorable state of nature for each decision alternative will occur. Thus,

for example, using this criterion, the investor would optimistically assume that good economic conditions will prevail in the future.

The maximax criterion is applied in Table 8.3. The decision maker first selects the maximum payoff for each decision. Notice that all three maximum payoffs occur under good economic condition. Of the three maximum payoffs -\$50,000, \$100,000, and \$30,000- the maximum is \$100,000; thus, the corresponding decision is to purchase the office building.

Table 8.3 Payoff table illustrating a maximax decision

| Decision<br>(Purchase) | States of Nature        |                          |
|------------------------|-------------------------|--------------------------|
|                        | Good economic condition | Poor economic conditions |
| Apartment building     | \$ 50,000               | \$ 30,000                |
| Office building        | 100,000                 | -40,000                  |
| warehouse              | 30,000                  | 10,000                   |

### ***Maximum payoff***

Although the decision to purchase an office building will result in the largest payoff of \$100,000, such a decision completely ignores the possibility of a potential loss of \$40,000. The decision maker who uses the maximax criterion assumes a very optimistic future with respect to the states of nature.

Before the next criterion is presented, it should be pointed out that the maximax decision rule as presented above deals with profit. However if the payoff table consisted of costs, the opposite selection would be indicated: the minimum of the minimum costs, or a minimin criterion. For the subsequent decision criteria we encounter, the same logic in the case of costs can be used.

## 2. The maximin criterion

In contrast to the maximax criterion, which is very optimistic, the maximin criterion is *pessimistic*. With the maximin criterion, the decision maker selects the decision that will reflect the maximum of the minimum payoffs. For each decision alternative, the decision maker assumes that the minimum payoff will occur. Of these minimum payoffs, the maximum is selected. The maximin criterion for our investment example is demonstrated in Table 8.4.

Table 8.4 Payoff table illustrating a maximin decision

| Decision<br>(Purchase) | States of Nature           |                             |
|------------------------|----------------------------|-----------------------------|
|                        | Good economic<br>condition | Poor economic<br>conditions |
| Apartment building     | \$ 50,000                  | \$ 30,000                   |
| Office building        | 100,000                    | -40,000                     |
| warehouse              | 30,000                     | 10,000                      |

### ***Maximum payoff***

The minimum payoffs for our example are \$30,000, -\$40,000, and \$10,000. The maximum of these three payoffs is \$30,000; thus the decision arrived at using the maximin criterion would be to purchase the apartment building. This decision is relatively *conservative*, since the alternatives considered included only the worst outcomes that could occur. The decision to purchase the office building as determined by the maximax criterion included the possibility of a large loss (-\$40,000). The worst that can occur from the decision to purchase the apartment building, however, is a gain of \$30,000. On the other hand, the largest

possible gain from purchasing the apartment building is much less than that of purchasing an office building (i.e., \$50,000 vs. \$100,000).

If Table 11.4 contained costs instead of profits as the payoffs, the conservative approach would be to select the maximum cost for each decision. Then the decision that resulted in the minimum of these costs would be selected.

### ***3. The Minimax regret criterion***

In our example, suppose the investor decided to purchase the warehouse, only to discover that economic conditions in the future were better than expected. Naturally, the investor would be disappointed that the office building had not been purchased, since it would have resulted in the largest payoff (\$100,000) under good economic conditions. In fact, the investor would regret the decision to purchase the warehouse and the degree of regret would be \$70,000 the difference between the investor's choice and the best choice.

This brief example demonstrates the principle underlying the decision criterion known as minimax regret or the regret criterion. With this decision criterion, the decision maker attempts to avoid regret by selection the decision alternative that minimizes the maximum regret.

To use the minimax regret criterion, a decision maker first selects the maximum payoff under each state of nature. For our example, the maximum payoff under good economic conditions is \$100,000, and the maximum payoff under poor economic conditions is \$30,000. All other payoffs under the respective states of nature are subtracted from these amounts, as shown below.

**Good Economic Conditions**

**Poor Economic**

**Conditions**

$$\begin{array}{ll}
 \$100,000 - 50,000 = \$50,000 & \$30,000 - 30,000 = \\
 \$0 & \\
 \$100,000 - 100,000 = \$0 & \$30,000 - (-40,000) = \\
 \$70,000 & \\
 \$100,000 - 30,000 = \$70,000 & \$30,000 - 10,000 = \\
 \$20,000 & 
 \end{array}$$

These values represent the regret that would be experienced by the decision maker if a decision were made that resulted in less than the maximum payoff. The values are summarized in a modified version of the payoff table known as a regret table, shown in Table 8.5 (Such a table is sometimes referred to as an opportunity loss table, in which case the term opportunity loss is synonymous with regret.)

**Table 8.5 the regret table**

| Decision<br>(Purchase) | States of Nature           |                             |
|------------------------|----------------------------|-----------------------------|
|                        | Good economic<br>condition | Poor economic<br>conditions |
| Apartment building     | \$ 50,000                  | \$ 0                        |
| Office building        | 0                          | 70,000                      |
| warehouse              | 70,000                     | 20,000                      |

In order to make the decision according to the minimax regret criterion, the maximum regret for each decision must be determined. The decision corresponding to the minimum of these regret values is then selected. This process is illustrated in Table 8.6.

Table 8.6 Regret Table Illustrating the Minimax Regret Decision

| Decision<br>(Purchase) | States of Nature           |                             |
|------------------------|----------------------------|-----------------------------|
|                        | Good economic<br>condition | Poor economic<br>conditions |
|                        |                            |                             |

|                    |           |        |
|--------------------|-----------|--------|
| Apartment building | \$ 50,000 | \$ 0   |
| Office building    | 0         | 70,000 |
| warehouse          | 70,000    | 20,000 |

### ***The minimum regret***

#### ***value***

According to the minimax regret criterion, the decision should be to purchase the apartment building rather than the office building or the warehouse. This particular decision is based on the philosophy that the investor will experience the least amount of regret by purchasing the apartment building. In other words, if the investor purchased either the office building or the warehouse, \$70,000 worth of regret could result; however, the purchase of the apartment building will result in, at most, \$50,000 in regret.

#### ***4. The Hurwicz criteria***

The Hurwicz criterion strikes a compromise between the maximax and maximin criteria. The principle underlying this decision criterion is that the decision maker is neither totally optimistic (as the maximax criterion assumes) nor totally pessimistic (as the maximin criterion assumes). With the Hurwicz criterion, the decision payoffs are weighted by a coefficient of ***optimism***, a measure of the decision maker's optimism. The coefficient of optimism, which we will define as  $\alpha$ , is between zero and one (i.e.,  $0 \leq \alpha \leq 1$ ). If  $\alpha = 1.0$ , then the decision maker is said to be completely optimistic; if  $\alpha = 0$ , then the decision maker is completely

pessimistic. (Given this definition, if  $\alpha$  is the coefficient of optimism,  $1-\alpha$  is the coefficient of pessimism.)

The Hurwicz criterion requires that, for each decision alternative, the maximum payoff be multiplied by  $Q$  and the minimum payoff be multiplied by  $1-\alpha$ . For our investment example, if  $Q$  equals 0.4 (i.e., the investor is slightly pessimistic),  $1-\alpha = 0.6$ , and the following values will result.

| <b><i>Decision</i></b> | <b><i>Values</i></b>                       |
|------------------------|--------------------------------------------|
| Apartment building     | $\$50,000 (0.4) + 30,000(0.6) = \$38,000$  |
| Office building        | $\$100,000 (0.4) - 40,000(0.6) = \$16,000$ |
| Warehouse              | $\$30,000 (0.4) + 10,000(0.6) = \$18,000$  |

The Hurwicz criterion specifies selection of the decision alternative corresponding to the maximum weighted value, which is \$38,000 for this example. Thus, the decision would be to purchase the apartment building.

It should be pointed out that when  $\alpha = 1.0$ , it is the maximax criterion. A limitation of the Hurwicz criterion is the fact that  $\alpha$  must be determined by the decision maker. It can be quite difficult for a decision maker to accurately determine his or her degree of optimism. Regardless of how the decision maker determines  $\alpha$ , it is still a completely *subjective* measure of the decision maker's degree of optimism. Therefore the Hurwicz criterion is a completely subjective decision making criterion.

### ***5. The Equal likelihood criterion***

When the maximax criterion is applied to a decision situation, the decision maker implicitly assumes that the most favorable state of nature for each decision will occur. Alternatively, when the maximin criterion is applied, the least favorable states of nature are assumed. The equal likelihood (or La Place) criterion weights each state of nature equally, thus assuming that the states of nature are equally likely to occur.

Since there are two states of nature in our example, we assign a weight of 0.5 to each one. Next, we multiply these weights by each payoff for each decision.

| <b><i>Decision</i></b> | <b><i>Values</i></b>                       |
|------------------------|--------------------------------------------|
| Apartment building     | $\$50,000 (0.5) + 30,000(0.5) = \$40,000$  |
| Office building        | $\$100,000 (0.5) - 40,000(0.5) = \$30,000$ |
| Warehouse              | $\$30,000 (0.5) + 10,000(0.5) = \$20,000$  |

As with the Hurwicz criterion, we select the decision that has the maximum of these weighted values. Since \$40,000 is the highest weighted value, the investor's decision would be to purchase the apartment building.

In applying the equal likelihood criterion, we are assuming a 50% chance, or 0.50 probabilities, that either state of nature will occur. Using this same basic logic, it is possible to weight the states of nature differently (i.e., unequally) in many decision problems. In other words, different probabilities can be assigned to each state of nature, indicating that one state is more likely than another to occur. The application of different probabilities to the states of nature is the principle behind the decision criteria to be presented in the section on expected value.

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## **8.4 SUMMARY OF CRITERIA RESULTS**

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The decision indicated by the decision criteria examined so far can be summarized as follows.

| <b><i>Criterion</i></b> | <b><i>Decision (Purchase)</i></b> |
|-------------------------|-----------------------------------|
| Maximax                 | Office building                   |
| Maximin                 | Apartment building                |
| Minimax regret          | Apartment building                |
| Hurwicz                 | Apartment Building                |
| Equal likelihood        | Apartment building                |

The decision to purchase the apartment building was designated most often by the various decision criteria. Notice that the decision to purchase the warehouse was never indicated by any criterion. This is because the payoffs for an apartment building, under either set of future economic condition are always better than the payoffs for a warehouse. Thus, given any situation with these two alternatives (and any other choice, such as purchasing the office building), the decision to purchase an apartment will always be made over the decision to purchase a warehouse. In fact, the warehouse decision alternative could have been eliminated from consideration under each of our criteria. The alternative of purchasing a warehouse is said to be dominated by the alternative of purchasing an apartment building. In general, dominated decision alternatives can be removed from the payoff table and not considered when the various decision making criteria are applied. This reduces the complexity of the decision analysis somewhat. However, in our discussions throughout this unit of the application of decision criteria, we will leave the dominated alternative in the payoff table for demonstration purposes.

The use of several decision criteria often results in a mix of decisions; with no one decision being selected more than the others. The criterion or collection of criteria used and the resulting decision depend on the characteristics and philosophy of the decision maker. For example, the extremely optimistic decision maker might eschew the majority of the results above and make the decision to purchase the office building because the maximax criterion most closely reflects his or her personal decision making philosophy.

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## **8.5 DECISION MAKING WITH PROBABILITIES**

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The decision making criteria just presented were based on the assumption that no information regarding the likelihood of the states of nature was available. Thus, no probabilities of occurrence were assigned to the states of nature, except in the case of the equally likely and assigning a weight of 0.5 to each state of nature in our example; we were implicitly assigning a probability of 0.5 to the occurrence of each state of nature.

It is often possible for the decision maker to know enough about the future states of nature to assign probabilities to their occurrence. Given that probabilities can be assigned, several decision criteria are available to aid the decision maker. We will consider two of these criteria: expected value and expected opportunity loss (although several others, including the maximum likelihood criterion, are available).

**1. Expected value**

In order to apply the concept of expected value as a decision making criterion, the decision maker must first estimate the probability of occurrence of each state of nature. Once these estimates have been made, the expected value for each decision alternative can be computed. Recall that expected value is computed by multiplying each outcome (of a decision) by the probability of its occurrence and then summing these products.

Using our real estate investment example, let us suppose that, based on several economic forecasts, the investor is able to estimate a 0.60 probability that good economic conditions will prevail and a 0.40 probability that poor economic conditions will prevail. This new information is shown in table 8.7.

Table 8.7 payoff table with probabilities for states of nature

|  |                  |
|--|------------------|
|  | States of Nature |
|--|------------------|

| Decision<br>(Purchase) | Good economic<br>Conditions<br>0.60 | Poor economic<br>Conditions<br>0.40 |
|------------------------|-------------------------------------|-------------------------------------|
| Apartment building     | \$ 50,000                           | \$ 30,000                           |
| Office building        | 100,000                             | -40,000                             |
| warehouse              | 30,000                              | 10,000                              |

The expected value for each decision is computed as follows.

$$EV(\text{apartment}) = \$50,000(0.6) + \$30,000(0.4) = \$42,000$$

$$EV(\text{office}) = \$100,000(0.6) + -\$40,000(0.4) = \$44,000$$

$$EV(\text{warehouse}) = \$30,000(0.6) + \$10,000(0.4) = \$22,000$$

The best decision is the one with the greatest expected value, since the greatest expected value is \$44,000; the best decision is to purchase the office building. This does not mean that \$44,000 will result if the investor purchases an office building; rather, it is assumed that one of the payoff values will result (either \$100,000 or -\$40,000). The expected value means that if this decision situation occurred a large number of times, an average payoff of \$44,000 would result. Alternatively, if the payoffs were in terms of costs, the best decision would be the one with the lowest expected value.

## ***2. The expected opportunity loss***

A decision criterion closely related to expected value is expected opportunity loss. To use this criterion we multiply the probabilities by the regret (i.e. opportunity loss) for each decision outcome, rather than multiplying the decision outcomes by the probabilities of their occurrence, as we did for expected monetary value.

The concept of regret was introduced in our discussion of the minimax regret criterion. The regret values for each decision outcome in our example were shown in table 8.6. These values are repeated in table 8.8,

with the addition of the probabilities of occurrence for each state of nature.

Table 8.8 Regret (Opportunity loss) table with probabilities of states of nature

| Decision<br>(Purchase) | States of Nature                    |                                     |
|------------------------|-------------------------------------|-------------------------------------|
|                        | Good economic<br>Conditions<br>0.60 | Poor economic<br>Conditions<br>0.40 |
| Apartment building     | \$ 50,000                           | \$ 0                                |
| Office building        | 0                                   | 70,000                              |
| warehouse              | 70,000                              | 20,000                              |

The expected opportunity loss for each decision is computed as follows.

$$\text{EOL (apartment)} = \$50,000(0.60) + 0(0.40) = \$30,000$$

$$\text{EOL (office)} = \$0(0.60) + 70,000(0.40) = \$28,000$$

$$\text{EOL (warehouse)} = \$70,000(0.60) + 20,000(0.40) = \$50,000$$

As with the minimax regret criterion, the best decision results from minimizing the regret, or, in this case, minimizing the expected regret or opportunity loss. Since \$28,000 is the minimum expected regret, the decision is to purchase the office building.

Notice that the decisions recommended by the expected value and expected opportunity loss criteria were the same- to purchase the office building. This is not a coincidence, as these two methods always result in the same decision. Thus, it is repetitious to apply both methods to a decision situation when one of the two will suffice.

In addition, note that the decisions from the expected value and expected opportunity loss criteria are totally dependent on the probability estimates determined by the decision maker. Thus, if inaccurate probabilities are used, erroneous decisions will result. It is there fore

important that the decision maker be as accurate as possible in determining the probabilities of each state of nature.

### ***Expected value of perfect information***

It is often possible to purchase additional information regarding future events and thus make a better decision. For example, a real estate investor could hire an economic forecaster to perform an analysis of the economy in order to more accurately determine which economic condition will occur in the future. However, the investor (or and decision maker) would be foolish to pay more for this information than he or she stands to gain in extra profit from having the information. That is, the information has some maximum value that represents the limit of what the decision maker would be willing to spend. This value of information can be computed as an expected value-hence its name, the expected value of perfect information (also referred to as EVPI).

In order to compute the expected value of perfect information, we first look at the decisions under each state of nature. If we could obtain information that assured us which state of nature was going to occur (i.e., perfect information), we could select the best decision for that state of nature. For example, in our real estate investment example, if we know for sure that good economic conditions will prevail, then we will decide to purchase the office building. Similarly, if we know for sure that, poor economic conditions will occur, then we will decide to purchase the apartment building. These Hypothetical “perfect” decisions are summarized in table 8.9.

The probabilities of each state of nature(i.e. 0.60 and 0.40) tell us that good economic conditions will prevail 60% of the time and poor economic conditions will prevail 40% of the time( if this decision situation is repeated many times). In other words, even though perfect information enables the investor to make the right decision, each state of nature will

occur only a certain portion of the time. Thus, each of the decision outcomes obtained using perfect information must be weighted by its respective probability.

$$100,000(0.60) + 30,000(0.40) = \$72,000$$

Table 8.9 payoff table with decisions, given perfect information

| Decision<br>(Purchase) | States of Nature                    |                                     |
|------------------------|-------------------------------------|-------------------------------------|
|                        | Good economic<br>Conditions<br>0.60 | Poor economic<br>Conditions<br>0.40 |
| Apartment building     | \$ 50,000                           | \$ 30,000                           |
| Office building        | 100,000                             | -40,000                             |
| warehouse              | 30,000                              | 10,000                              |

The amount \$72,000 is the expected value of the decision, given perfect information, not the expected value of perfect information. The expected value of perfect information is the maximum amount that would be paid to gain information that would result in a decision better than the one made without perfect information. Recall that the expected value decision without perfect information was to purchase an office building, and the expected value was computed as:

$$EV(\text{office}) = \$100,000(0.60) - 40,000(0.40) = \$44,000$$

The expected value of perfect information, \$28,000 is the maximum amount that the investor would pay to purchase perfect information from some other source, such as an economic forecaster. Of course, perfect information is rare and usually unobtainable. Typically, the decision maker would be willing to pay some amount less than \$28,000, depending on how accurate (i.e., close to perfection) the decision maker believes the information is.

It is interesting to notice that the expected value of perfect information, \$28,000 for our example, is the same as the expected opportunity loss for the decision selected using this latter criterion:

$$\text{EOL (office)} = \$0(0.60) + 70,000(0.40) = \$28,000$$

This will always be the case, and logically so, since regret reflects the difference between the best decision under a state of nature and the decision actually made. This is actually the same thing determined by the expected value of perfect information.

### ***Decision trees***

Another useful technique for analyzing a decision situation is a decision tree. A decision tree is a graphical diagram consisting of nodes and branches. However, rather than determining the probability of each branch(i.e. , outcome) as in a probability tree, in a decision tree the user computes the expected value of each outcome and makes a decision based on the expected values. The primary benefit of a decision tree is that it provides an illustration (or picture) of the decision making process. This makes it easier to correctly compute the necessary expected values and to understand the process of making the decision.

Our example of the real estate investor will be used to demonstrate the fundamentals of decision tree analysis. The various decisions, probabilities and outcomes of this example, initially presented in table 8.7 are repeated in table 8.10. The decision tree for this example is shown in figure 8.1.

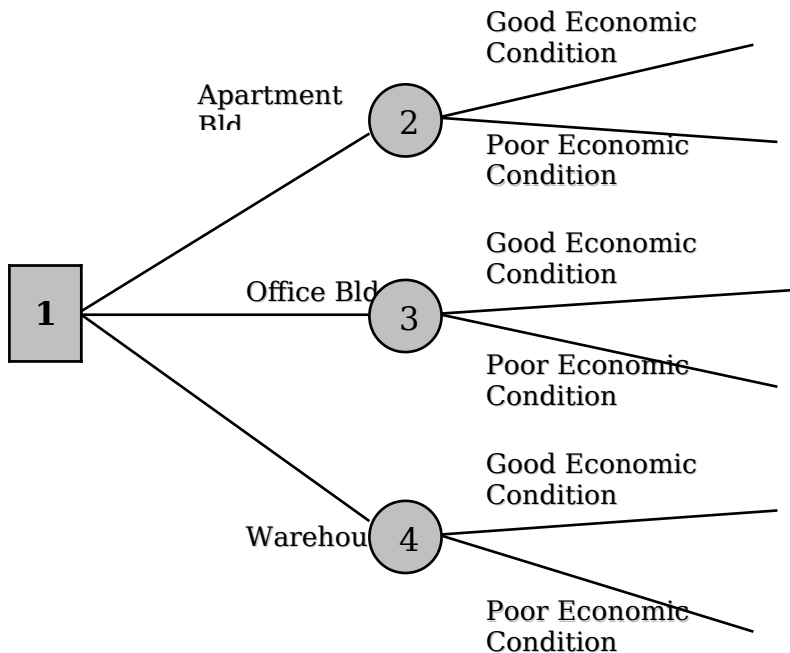
*Table 8.10 payoff table for real estate investment example*

| Decision<br>(Purchase) | States of Nature            |                             |
|------------------------|-----------------------------|-----------------------------|
|                        | Good economic<br>Conditions | Poor economic<br>Conditions |
|                        | 0.60                        | 0.40                        |

|                    |           |           |
|--------------------|-----------|-----------|
| Apartment building | \$ 50,000 | \$ 30,000 |
| Office building    | 100,000   | -40,000   |
| warehouse          | 30,000    | 10,000    |

The circle ( ) and squares ( ) in figure 8.1 are referred to as nodes. The squares are decision nodes, and the branches emanating from a decision node reflect the alternative decisions possible at that point. For example, in figure 8.1, node 1 signifies a decision to purchase an apartment building, office building, or warehouse. The circles are probability nodes, and the branches emanating from them indicate the states of nature that can occur: good economic conditions or poor economic conditions.

**Figure 8.1 Decision tree for real estate investment example**



The decision tree represents the sequence of events in a decision situation. First, one of the three decision choices is selected at node 1. Depending on the branch selected, the decision maker arrives at probability node 2, 3, or 4, where one of the states on nature will prevail, resulting in one of six possible payoffs.

Determining the best decision using a decision tree involves computing the expected value at each probability node. This is accomplished by starting with the final outcomes (payoffs) and working backward through the decision tree toward node 1. First, the expected value of the payoffs is computed at each probability node.

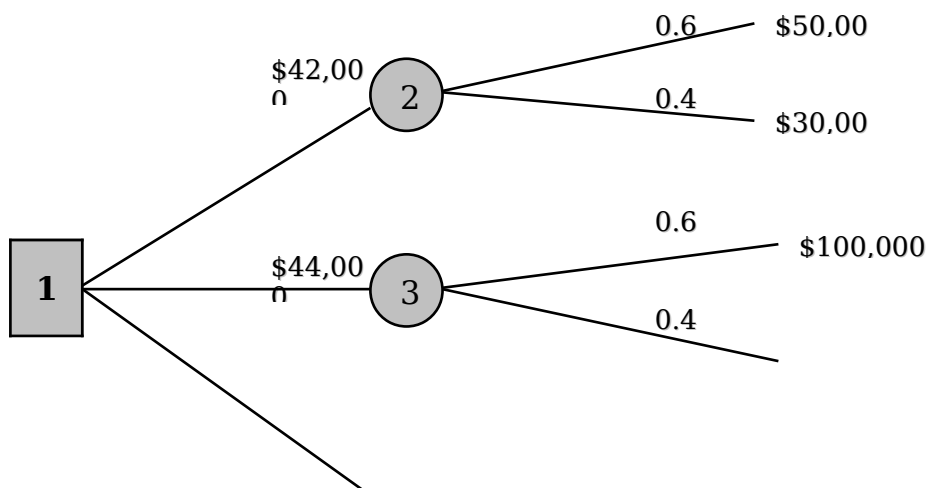
$$\text{EV (node 2)} = 0.60(\$50,000) + 0.40(\$30,000) = \$42,000$$

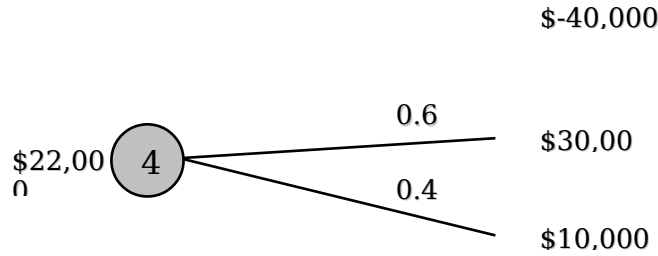
$$\text{EV (node 3)} = 0.60(\$100,000) + 0.40(\$-40,000) = \$44,000$$

$$\text{EV (node 4)} = 0.60(30,000) + 0.04(10,000) = \$22,000$$

These values are now shown as the expected payoff from each of the three branches emanating from node 1 in figure 8.2 each of these three expected values at nodes 2, 3, and 4 is the outcome of a possible decision that can occur at node 1. Moving toward node 1, we select the branch that comes from the probability node with the highest expected payoff. In figure 8.2 the branch corresponding to the highest payoff, \$44,000, is from node 1 to node 3. This branch represents the decision to purchase the office building. The decision to purchase the office building, with an expected payoff of \$44,000, is the same result we achieved earlier using the expected value criterion. In fact, when only one decision is to be made (i.e., there is no series of decisions), the decision tree will always yield the same decision and expected payoff as the expected value criterion. As a result, in these decision situations the decision tree is not very useful. However, when a sequence or series of decision is required, the decision tree can be very useful.

**Figure 8.2 Decision tree with expected values at probability nodes**





### Check your Progress Exercise

1. What are a payoff table and states of nature? Explain with examples.
2. Describe the difference between the maximax and maximin criteria.
3. Explain the concept of regret.
4. What is the expected value of perfect information and how is it computed? Explain with example.
5. **GY Plc.** has acquired a textile company and is contemplating the future of one of its major plants located in southern Addis. Three alternative decisions are being considered: (1) expand the plant and produce lightweight, durable materials for possible sales to the military, a market with little foreign competition; (2) maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition; or (3) sell the plant now. If one of the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends on foreign market conditions. The following payoff table describes this decision situation.

|  | <b>States of Nature</b> |                |
|--|-------------------------|----------------|
|  | <b>Good</b>             | <b>Poor</b>    |
|  | <b>foreign</b>          | <b>foreign</b> |

| <b><i>Decision</i></b> | <b><i>competitive conditions</i></b> | <b><i>competitive conditions</i></b> |
|------------------------|--------------------------------------|--------------------------------------|
| Expand                 | \$ 800,000                           | \$ 500,000                           |
| Maintain status quo    | 1,300,000                            | -150,000                             |
| Sell now               | 320,000                              | 320,000                              |

**Required:**

A. Determine the best decision using the following decision criteria.

1. Maximax
2. Maximin
3. Minimax regret
4. Hurwicz ( $Q = 0.30$ )
5. Equal likelihood

B. Assume it is now possible to estimate a probability of 0.70 that good foreign competitive conditions will exist and a probability of 0.30 that poor conditions will. Determine the best decision using expected value and expected opportunity loss.

C. Compute the expected value of perfect information.

D. Develop a decision tree, with expected values at the probability nodes.

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## **8.4 SUMMARY**

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The purpose of this unit was to demonstrate the concepts and fundamentals of decision making when uncertainty exists. Within this context, several decision making criteria were presented. The maximax, maximin, minimax regret, equal likelihood, and Hurwicz decision criteria were demonstrated for cases in which probabilities could not be attached to the occurrence of outcomes. The expected value criterion and decision

trees were discussed for cases in which probabilities could be assigned to the states of nature of a decision situation.

All of the decision criteria presented in this unit were demonstrated via rather simplified examples: actual decision making situations are usually more complex. Nevertheless, the process of analyzing decisions presented in this unit is the logical method that most decision makers follow in order to make a decision.

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### **8.5 ANSWER TO CHECK YOUR PROGRESS EXERCISE**

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1. The actual events that may occur in the future are known as states of nature. At the time the decision is made, the decision maker is uncertain which states of nature will occur in the future and has no control over them.

- A payoff table is a means of organizing and illustrating the payoffs from the different decisions, given the various states of nature in a decision problem.

(For more on these refer to Section 8.2)

2. With the maximax criterion, the decision maker selects the decision that will result in the maximum of the maximum payoffs. (In fact, this is how this criterion derives its name—a maximum of maximum.) On the other hand, the maximin criterion is *pessimistic*. With the maximin criterion, the decision maker selects the decision that will reflect the maximum of the minimum payoffs.

(For more on these refer to part 1 and 2 of Section 8.3)

3. Minimax regret is a decision criterion where the decision maker attempts to avoid regret by selecting the decision alternative that minimizes the maximum regret.

(For more on this refer to part 3 of Section 8.3)

4. Hint: Expected value of perfect information is the difference between expected value with information and without information.

- 5. A. 1. Decision: Maintain the status quo
- 2. Decision: Expand
- 3. Decision: Expand
- 4. Decision: Expand
- 5. Decision: Expand