
UNIT 3: LINEAR PROGRAMMING: GRAPHICAL SOLUTIONS

Contents

- 3.0 Aims and Objectives
- 3.1 Introduction
- 3.2 Graphing The Model
 - 3.2.1 Plotting The Constraints
 - 3.2.2 Finding The Optimal Solution
- 3.3 Slack
- 3.4 A Minimization Example
- 3.5 Some Special Issues
 - 3.5.1 No Feasible Solutions
 - 3.5.2 Unbounded Problems
 - 3.5.3 Redundant Constraints
 - 3.5.4 Multiple Optimal Solutions
- 3.6 Summary
- 3.7 Answers to Check Your Progress Exercise

3.0 AIMS AND OBJECTIVES

After completing this unit, the student should be able to:

- identify LP problems that are amenable to graphical solution
- define or explain terms such as optimal solution, feasible solution space, corner point, redundant constraint, slack and surplus
- solve two variable LP problems graphically and interpret your answers
- recognize problems that have multiple solutions, problems that have no feasible solutions, and unbounded problems.

3.1 INTRODUCTION

Graphical linear programming is a relatively straight forward method for determining the optimal solution to certain linear programming problems. In practical terms, this method can be used only to solve problems that involve two decision variables, such as the agricultural research firm in Unit 2. However, most linear programming applications involve situations that have more than two decision variables, so graphical linear programming methods cannot be used to solve them. Even so, much can be gained from studying the graphical approach particularly in terms of the insight it generates about important concepts of models and solutions. Graphical methods provide a visual portrayal of many important concepts. Moreover, they provide a framework for intuitive explanations of other LP techniques that are covered in later units. Viewed from this perspective, graphical LP is a valuable learning tool.

In order to demonstrate the graphical method, consider the following illustrative example about Philips Corporation.

Illustration

Assume that Philips Corporation produces 2 types of TV sets called Model A and B. The company is in the market to make money; i.e., its objective is to maximize profit. The profit realized from Set A and B respectively are Birr 300 and Birr 250. The company wants to produce and sell thousands of sets daily but this is not possible due to some limitations. These limitations are:

- there are only 40 hours of labour each day in the production department
- there are only 45 hours of machine time each day
- the maximum that can be sold of Model A TV sets are 12

It is known that each set of Model A being higher in quality requires 2 hours of labor and each set of Model B requires only 1 hour of labor to be produced. The machine processing time for one unit of Model A and Model B are 1 hour and 3 hours respectively.

If the company's objective is to maximize profit, how many sets of each model should be produced each day. Describe the situation with linear programming model.

Required:

- A. Develop the linear programming model
- B. Solve the model using graphic approach

Model Formulation

Step 1: Problem definition:

Determining the number of Model A and Model B TV sets that must be produced and sold so as to maximize daily profits given the constraints.

Step 2: Variable representation

Let x_1 be the amount of model A TV sets that will be produced and sold per day

x_2 be the amount of Model B TV sets that will be produced and sold per day

Z be the maximum profit

Step 3: Objective function

$$Z_{\max} : 300x_1 + 250x_2$$

Step 4: Constraint identification

$$\text{Labour hour constraint } 2x_1 + x_2 \leq 40$$

$$\text{Machine time constraint } x_1 + 3x_2 \leq 45$$

$$\text{Marketing constraint } x_2 \leq 12$$

$$\text{All variable } \geq 0$$

3.2 GRAPHING THE MODEL

Graphing a model consists of several distinct steps:

1. Plot each of the constraints
2. Determine the region or area that contains all of the points that satisfy the entire set of constraints.
3. Determine the optimal solution

Let's examine each of these steps as we solve the Philips Corporation problem.

3.2.1 Plotting The Constraints

We begin by plotting the non-negativity constraints, as shown in the figure below. For purpose of illustration, the area of feasibility for these two constraints (i.e., the area that satisfies both constraints) has been shaded in. however, as a general rule, the shading would be done after all constraints have been plotted.

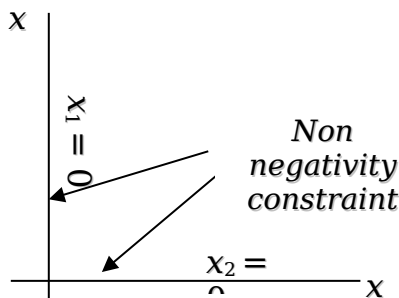


Fig 3.1 A graph showing the non-negativity constraints

The procedure for graphing each of the remaining constraints relies on the fact that a straight line can be plotted if any two points on that line can be identified. Of course, the constraints in this problem are all inequalities rather than lines. As such, they describe a region or area that will satisfy them. However, the boundary of that area of feasibility is represented by the "equal" portion of the "less than or equal to" constraint. Recognizing this, we can deal with the task of graphing a constraint into two parts. First, treat the constraint as an equality and

plot the straight line that is the boundary of the feasible region; then, shade in the feasible region.

Consider the labour hour constraint:

$$2x_1 + x_2 \leq 40$$

Removing the inequality portion of the constraint produces this straight line:

$$2x_1 + x_2 = 40$$

There are a number of different ways to determine the coordinates of two points on the line. Perhaps the easiest two points to identify and work with are the points where the line intersects each axis (i.e., where $x_1 = 0$ and where $x_2 = 0$). We can solve for those points, first by setting $x_2 = 0$ and solving for x_1 , and then setting $x_1 = 0$ and solving for x_2 . Thus, with, $x_2 = 0$ we find:

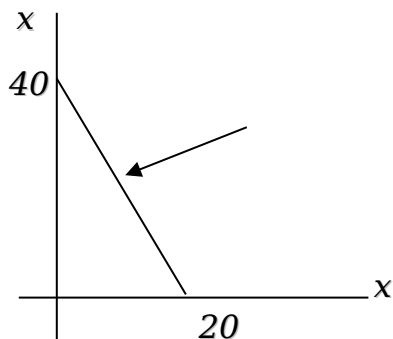
$$2x_1 + 1(0) = 40$$

Solving, we find that $2x_1 = 40$, so $x_1 = 20$ when $x_2 = 0$. Similarly, we can solve the equation for x_2 when $x_1 = 0$:

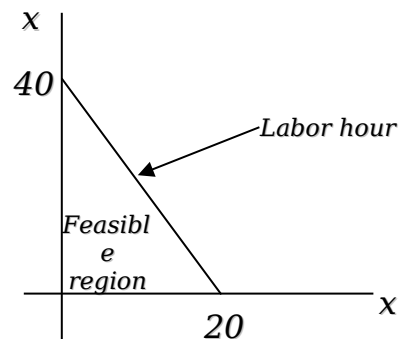
$$2(0) + x_2 = 40$$

Solving for x_2 , we find $x_2 = 40$.

Thus, we have two points: $x_1 = 0$, $x_2 = 40$, and $x_1 = 20$, $x_2 = 0$. We can now add this line to our graph of the non-negativity constraints by connecting these two points. See the figures below:



A plot of the first constraint (Labor hour)



The feasible region, given the first constraint and the non-negative constraint

Fig 3.2

Fig 3.3

Next we must determine which side of the line represent points that are less than 40. To do this, we can select a test point that is not on the line, and we can substitute the x_1 and x_2 values of that point into the left side of the equation of the line. If the result is less than 40, this tells us that all points on that side of the line are less than the value of the line. Conversely, if the result is greater than 40, this indicates that the other side of the line represents the set of points that yield value that are less than 40. A relatively simple test point to use is the origin (i.e., $x_1 = 0$ and $x_2 = 0$). Substituting these values into the equation yields:

$$2(0) + 1(0) = 0$$

Obviously this is less than 40. Hence, the side of the line closest to the origin represents the less than area.

The feasible region for this constraint and the non-negativity constraints then becomes the shaded portion show in figure 3-3.

Continuing with the problem, we can add the two remaining constraints to the graph. For machining time constraint:

1. Convert the constraint into the equation of a straight line by replacing the inequality sign with an equality sign:

$$x_1 + 3x_2 \leq 45 \text{ becomes } x_1 + 3x_2 = 45$$

2. Set x_1 equal to zero and solve for x_2

$$1(0) + 3x_2 = 45$$

Solving, we find $x_2 = 15$. Thus, the line will intersect the axis at 15.

3. Next, set x_2 equal to zero and solve for x_1 :

$$x_1 + 3(0) = 45$$

Solving, we find $x_1 = 45$. Thus, the other end of the line will intersect the x_1 axis at 45.

4. Add the line to the graph (See figure 3.4)
5. Note that the area of feasibility for this constraint is below the line. Again, the area of feasibility at this point is shaded in for illustration, although when graphing problems, it is more practical to refrain from shading in the feasible solution space until all constraint lines have been drawn. However, because constraints are plotted one at a time, using a small arrow for each to indicate the direction of feasibility can be helpful.

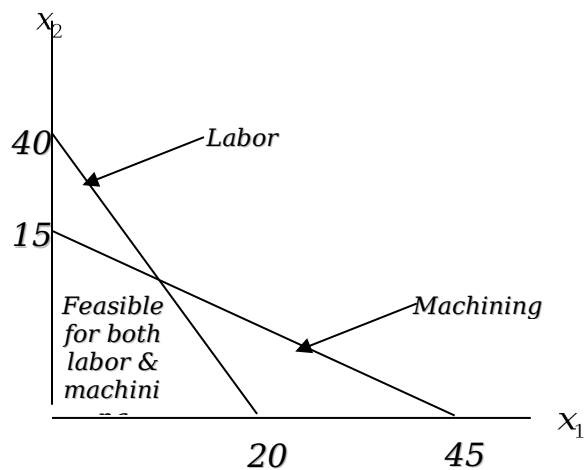


Fig 3.4

The marketing constraint is handled in the same manner:

1. Convert into an equality:

$$x_1 = 12$$
2. Since there is no x_2 variable we cannot find any value here. Rather we directly proceed to changing the expression $x_1 = 12$ into a vertical line, because the equation is a one variable expression.
3. Determine the feasible solution space - the area that satisfies all of the constraints, and shade it in. (See figure 3.5)

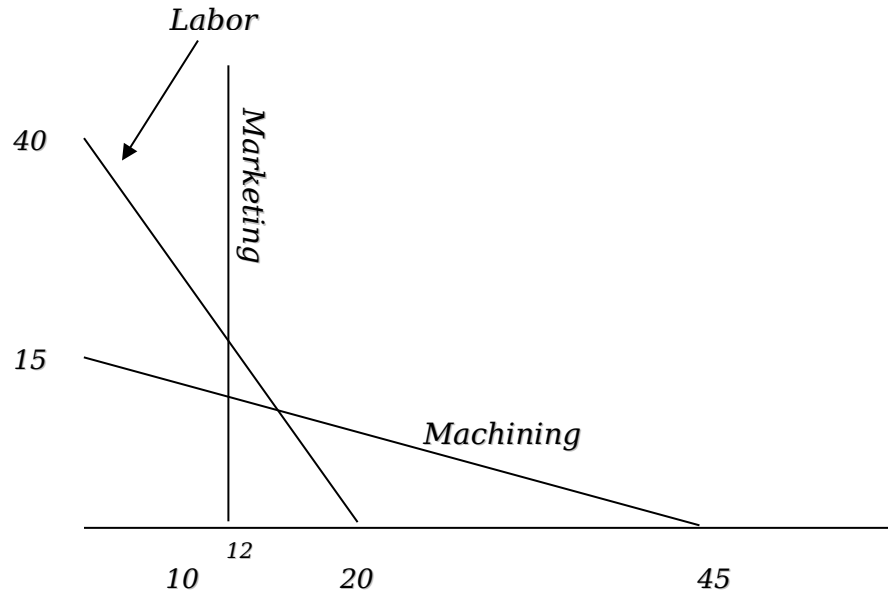


Fig 3.5

3.2.2 Finding The Optimal Solution

Once the constraints have been graphed and the area of feasibility has been identified, it is possible to obtain the optimal solution to the problem. Two different approaches to obtaining the solution will be demonstrated. One involves graphing the objective function, and the other involves examining points at the edge of the feasible solution space where constraints meet. However this text addresses the later approach (the extreme point) for the analysis.

The Extreme Point Approach

The extreme point theorem states that for problems that have optimal solutions, a solution will occur at an extreme, or corner point. Thus, if a problem has a single optimal solution, it will occur at a corner point. If it has multiple optimal solutions, at least one will occur at a corner point. Consequently, in searching for an optimal solution to a problem, we need only to consider the extreme points because one of those must be optimal. Further, by determining the value of the objective function at each corner point, we could identify the optimal solution by selecting the

corner point that has the best value (i.e., maximum or minimum, depending on which type of problem was involved) of the objective function. Hence, the necessary steps for the extreme point approach are:

1. Graph the problem
2. Determine the values of the decision variables at each corner point. Sometimes, this can be done by inspection; usually, it is done using simultaneous equations.
3. substitute the value of the decision variables at each corner point into the objective function to obtain its value at each corner point.
4. After all corner points have been so evaluated, select the one with the highest value of the objective function (for a maximization problem) or lowest value (for a minimization problem) as the optimal solution.

We can use the extreme point approach to verify the optimal solution for Philips Corporation problem.

The solution can be obtained by following the steps outlined above. Figure 3.6 repeats the graph of the problem that was constructed previously, and each of the extreme points are now labeled for discussion purposes. Note that 0,0 is included as an extreme point, even though it will not yield any profit because both decision variables are equal to zero.

Table 3.1 summarizes the approach used to obtain each point and its profit.

Note that the coordinates of points A, B, and D were determined by inspection. The coordinates of the origin are obvious. In addition, the x_1 intercept of the machining time constrain were known by having determined them in the course of plotting the constraints. The coordinates of point C, can be determined by solving simultaneously for the intersection of the marketing and machining lines. (See figure 3.6)

Machining $x_1 + 3x_2 = 45$

Marketing

$$x_1 = 12$$

As usual, there are a number of ways for obtaining a common solution to this set of equations. One way would be to substitute the second equation i.e., value of x_1 that is 12 in the first equation. Thus,

$$x_1 + 3x_2 = 45$$

$$12 + 3(x_2) = 45$$

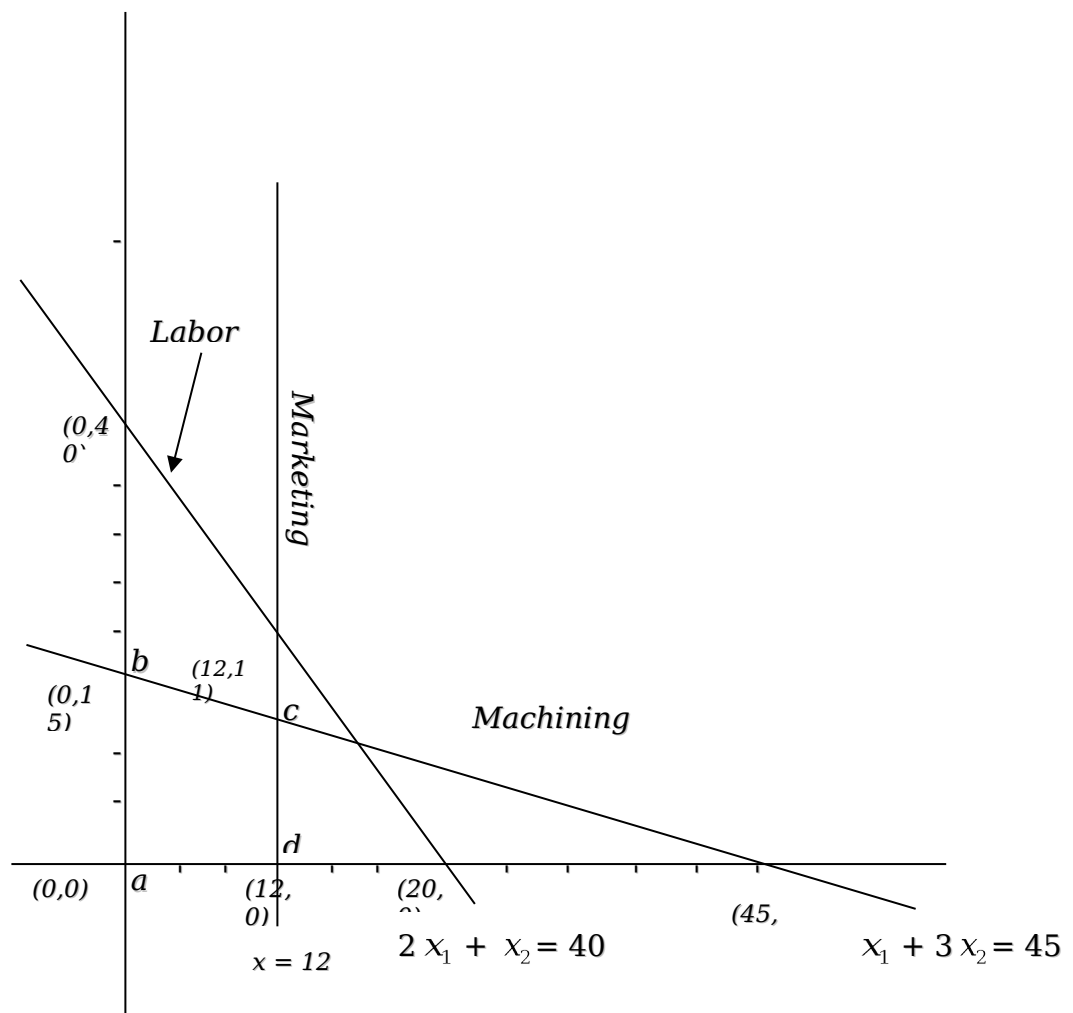
$$x_2 = 11$$

By substituting $x_2 = 11$ in to the machining constraint, the value of x_1 at intersection point C can be determined.

$$x_1 + 3(11) = 45$$

$$x_1 = 45 - 33$$

$$x_1 = 12$$



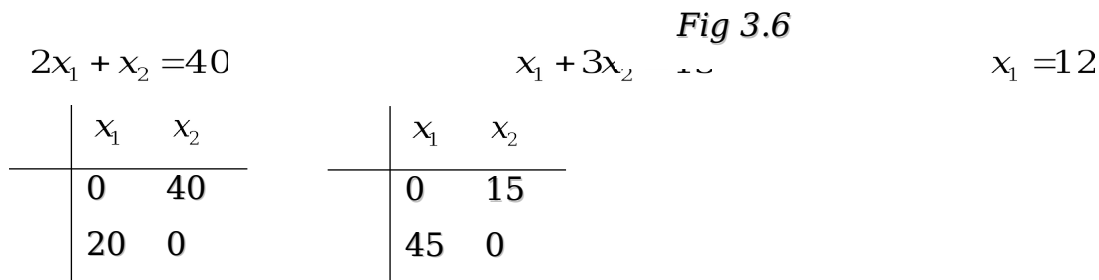


Table 3.1 Extreme point solutions for the Philips Corporation problem

Points	Coordinates (x_1 , x_2)	How determined	Value of obj. function $300 x_1 + 250 x_2$
a	(0,0)	Observation	0
b	(0,15)	Observation	3750
c	(12,11)	Simultaneous equation	6350 largest
d	(12,0)	Observation	3600

Determining Slack (Unused) Resources

Labor hour: $2(12) + 11 = 40$

$35 = 40$ there is 5 hours of labour time slack

Machine time: $12 + 3(11) = 45$

$45 = 45$ All have been used

Hence, the coordinates of point C are $x_1 = 12$ and $x_2 = 11$. The profit at each corner point can be computed by substituting the coordinates of the point into the objective function. Point C ($x_1 = 12$, $x_2 = 11$) represents the optimal solution because it yields the largest profit for the extreme points. (See table 3.1)

3.3 SLACK

Slack is the amount of scarce resource that is unused by a given solution. this amount can range from zero, for a case in which all of a particular resources is used, to the original amount of the resource that was available (i.e., none of it is used).

Let us see what slack remains for the optimal solution to the **Philips Corporation** problem. The amount of unused resource can be computed by substituting the values of the decision variables into each constrain and subtracting the result from the original right-hand side amount. Thus, the original constraints were:

$$\text{Labour hour} \quad 2x_1 + x_2 \leq 40$$

$$\text{Machining time} \quad x_1 + 3x_2 \leq 45$$

$$\text{Marketing} \quad x_1 \leq 12$$

The optimal values of the decision variables were:

$$x_1 = 12 \text{ and } x_2 = 11$$

Substituting these values in to the constraints yields the results shown in the table below (Table 3-2)

Table 3-2: Computing the amount of slack for the optimal solution to the Philips Corporation problem.

Constrai nt	Amount used with $x_1 = 12$ and $x_2 = 11$	Originally available	Amount of slack (available - use)
Labor	$2(12) + 1(11) = 35$	40	$40 - 35 = 5$ hours
Machine	$1(12) + 3(11) = 45$	45	$45 - 45 = 0$ hours
Marketin g	$1(12) = 12$	12	$12 - 12 = 0$ requirement

We can see that 5 hours of labour will be unused given the optimal solution, but that all of the machining time and marketing requirement will be satisfied or used. Recall our graphical solution to the problem (see figure 3.6). The optimal solution was at the corner point where the

marketing and machining constraint line intersected (i.e., at those boundaries), which implied that all of those resources and requirements would be used or met. Constraints that have not slack are sometimes referred to as binding constraints since they limit, or bind, the solution. Because the labor boundary was not involved, we would expect that some labour hours would remain, although we cannot easily tell from the graph how much labour would be unused.

Knowledge of unused capacity can be useful for planning. A manager may be able to use the remaining labor hours for purposes or perhaps, to schedule safety seminars, training sessions or other activities. In still other vein, it may be profitable to consider expansion or acquisition of additional amounts of the other resources to allow for greater use of the labour resource and, thereby, to increase profits by increasing the size of the feasible solution space. This approach is discussed in detail in a later unit which deals with sensitivity analysis.

Slack can potentially exist in a $\leq$ constraint. Moreover, it can be useful to represent slack in a constraint. This is accomplished using a slack variable, S , which carries a subscript that denotes which constraint it applies to. For instance, s_1 refers to the amount of slack in the first constraint, s_2 to the amount of slack in the second constraint, and so on. When slack variables are added to the constraints, they are no longer inequalities because the slack variable accounts for any difference between the left and right hand sides of the expression. Hence, once slack variables are added to the constraints they become equalities. Furthermore, every variable in a model must be represented in the objective function. However, since slack does not provide any real contribution to the objective, each slack variable is assigned a coefficient of zero in the objective function. With slack variables included, the Philips Corporation problem would appear as follows:

$$\begin{array}{llll}
 \text{Maximize} & & 300x_1 + 250x_2 & \\
 \text{Subject to:} & & & \\
 \text{Labour} & 2x_1 + x_2 + s_1 & = & 40 \\
 \text{Machine} & x_1 + 3x_2 + s_2 & = & 45 \\
 \text{Marketing} & x_1 + s_3 & = & 45 \\
 & \text{All variables} & \geq & 0
 \end{array}$$

Note that all variables, including slack variables, must be non negative. When all of the constraints are written as equalities, the linear program is said to be in standard form.

3.4 MINIMIZATION EXAMPLE

Graphical solutions to minimization problems are very similar to solutions to maximization problems. There are a few differences that will be highlighted in this section. The main differences are that the constraints, in general, are the greater than or equal to variety, which causes the feasible solution space to be restricted to an area away from the origin, instead of close to the origin, as in a maximization problem, and the optimum being the point with the smallest possible value of the objective function, instead of the largest. These contrasts are illustrated in figure 3-7 below.

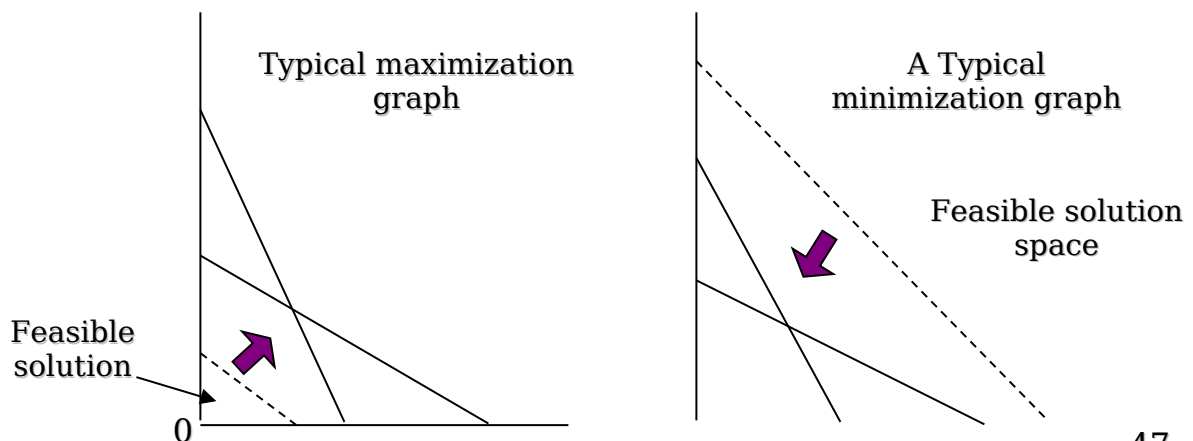


Fig. 3.7 A comparison of maximization and minimization problems

Example

Determine the values of decision variables that will yield the optimal cost in the following problem. Solve using the extreme point approach.

Minimization Case

A dietician has been asked by the athletic director of Unity University College to develop a snack that athletes can use in their training programs. The dietician intends to mix two separate products together to make the snack. The following information has been obtained by the dietician:

Nutrient	Minimum amount required (gr.)	Contribution per ounce (gr.)	
		Product A	Product B
Carbohydrates	20	2	5
Protein	12	6	1
Calories	450	90	50

Product A costs \$20 per ounce and products B costs \$10 per ounce.

Required:

- Formulate the linear programming model for the problem.
- Determine the optimal quantities of the two products for cost minimization.
- Are any requirements exceeded? If so, which ones, and by how much?

Model Formulation

1. Problem definition:

Determining the amount of ounce of Product A and B that must be purchased by the director to make the snack and minimize the cost of making the diet.

2. Variable representation:

Let x_1 be the amount of ounce of Product A

x_2 be the amount of ounce of Product B

z be the minimum cost

3. Objective function definition

$$z_{\min} : 20x_1 + 10x_2$$

4. Identify the possible constraints

$$2x_1 + 5x_2 \geq 20 \quad \text{Carbohydrates requirement}$$

$$6x_1 + x_2 \geq 12 \quad \text{Protein requirement}$$

$$90x_1 + 50x_2 \geq 450 \quad \text{Calories requirement}$$

$$\text{All variables} \geq 0$$

Solution

Graph the constraints using this producer:

1. treat each constraint as an equality
2. for all the constraints, find the x_1 intercept by setting x_2 equal to zero and solving for x_1 , and then find the x_2 intercept by setting x_1 equal to zero and solving for x_2
3. plot these constraints. Shade in the feasible solution space (see figure 3.8)

The extreme points can be determined either by inspection or from simultaneous equations. The results are summarized in table 3.3.

The minimum value of the objective function is $\frac{680}{7}$ i.e., about 97.14 which occurs when $x_1 = \frac{25}{7}$ and $x_2 = \frac{18}{7}$ and $x_1 = \frac{3}{7}$ and $x_2 = \frac{62}{7}$ as an alternative.

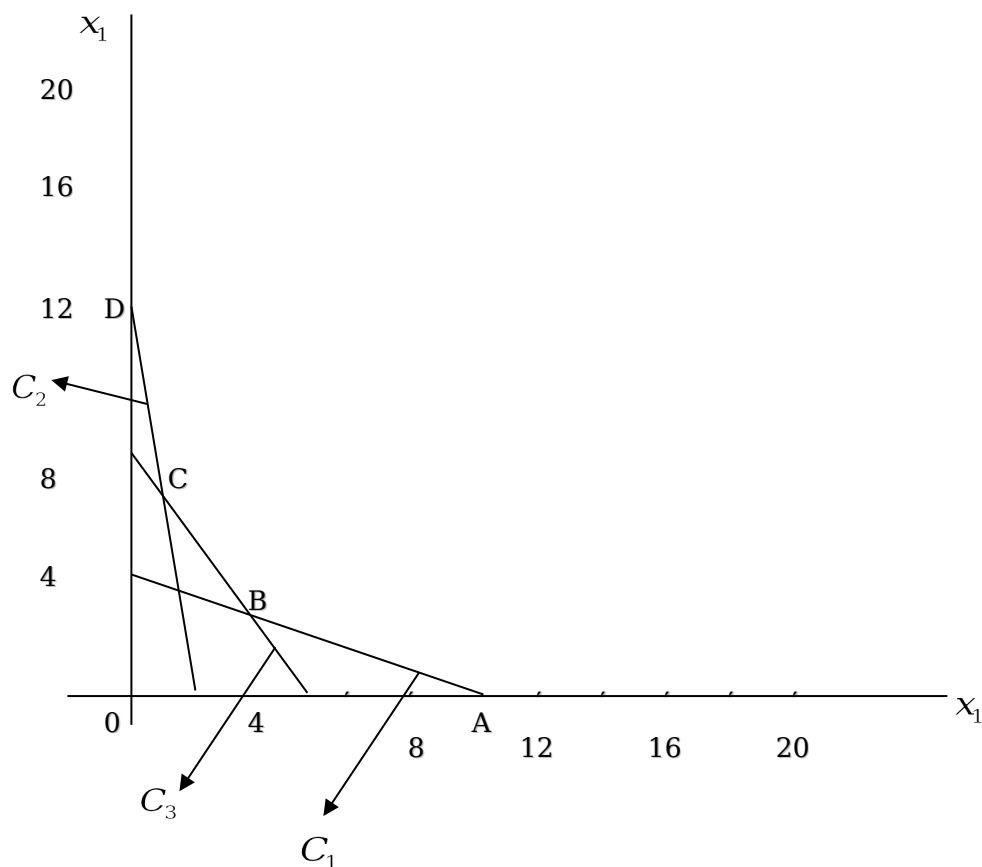


Figure 3.8

Table 3.3: Summary of extreme point analysis of minimization

	Extreme Point		How determined	Value of obg. Function $20x_1 + 10x_2$
	x_1	x_2		
A	10	0	Inspection	\$200
B	$\frac{25}{7}$	$\frac{18}{7}$	Simultaneous equation	$\frac{680}{7}$

C	$\frac{3}{7}$	$\frac{62}{7}$	Simultaneous equation	$\frac{680}{7}$
D	0	12	Inspection	120

Surplus

Surplus is the amount by which the optimal solution causes a $\geq$ constraint to exceed the required minimum amount. It can be determined in the same way that slack can: substitute the optimal values of the decision variables in to the left side of the constraint and solve. The difference between the resulting value and the original right hand side amount is the amount of surplus. For instance, in the above example, substituting the first optimal values in each of the first and the third constraints would show that there is no surplus. We can also see from the graph that the solution is on both lines, so there is no surplus. However, in the second constraint, with $x_1 = \frac{25}{7}$ and $x_2 = \frac{18}{7}$, we find that the surplus is 12, $(6(\frac{25}{7}) + 1(\frac{18}{7}) = 24)$.

Alternatively, you can consider a value of $x_1 = \frac{3}{7}$ and $x_2 = \frac{62}{7}$ in all the three constraints and see the results. The second and third constraints will show you that there is no surplus, where as the first constraint identify a surplus of $25\frac{1}{7}$ which is $(2(\frac{3}{7}) + 5(\frac{62}{7}) = 45\frac{1}{7}) - 20$.

Surplus can potentially occur in a $\geq$ constraint. Since surplus is defined as an amount by which the left hand side of a constraint exceeds the right hand side, surplus can be represented in a constraint by subtracting from the left hand side. The result is an equality. Also, surplus variables must be accounted for in the objective function. As with slack variables, this is done by using a coefficient of zero for each surplus variable. For the problem illustrated in a minimization example, the inclusion of surplus variables produces the following:

Minimize: $20x_1 + 10x_2 + 0S_1 + 0S_2 + 0S_3$

Subject to:

$$1. \quad 2x_1 + 5x_2 - S_1 = 20$$

$$2. \quad 6x_1 + x_2 - S_2 = 12$$

$$3. \quad 90x_1 + 50x_2 - S_3 = 450$$

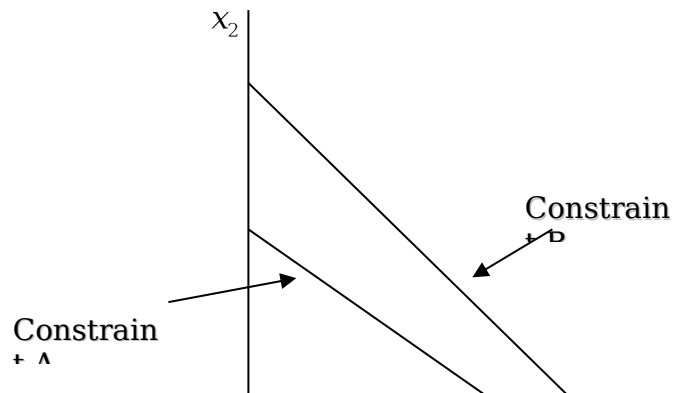
All variables ≥ 0

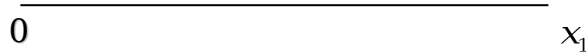
3.5 SOME SPECIAL ISSUES

This section points out some special issues that may arise during the formulation or solution of Linear Programming problems. These issues are:

3.5.1 No Feasible Solutions

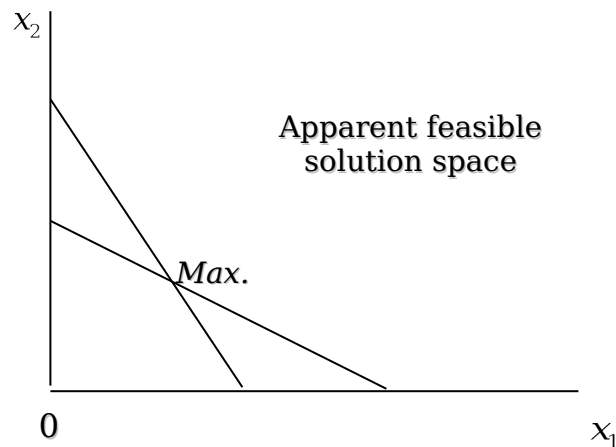
It is impossible to formulate a problem for which it is impossible to satisfy the set of all constraints. This situation sometimes occurs in problems that have a mix of greater than and less than constraints, where in order to satisfy one of the constraints, another constraint must be violated. consider the graph in the figure below, where constraint A places an upper bound on feasible solutions and constraint B places a lower bound on feasible solutions. Since the two constraints don't touch or overlap, there is no combination of decision variable values that can satisfy both constraints at the same time.





3.5.2 Unbounded Problems

It exists when the value of the objective function can be increased without limit. A graphical example of this is shown in the figure below. This difficulty is usually due to either incorrectly maximizing when the real goal is to minimize an objective function or using greater than constraints when less than constraints are called for. Although the latter error also may occur in a minimization problem, the problem will not be unbounded because of the non-negativity constraints (i.e., the resulting solution will be 0,0). Again, checking equalities or re-thinking the problem statement will resolve the problem. The latter case may uncover another constraint that should have been included in the original statement of the problem.

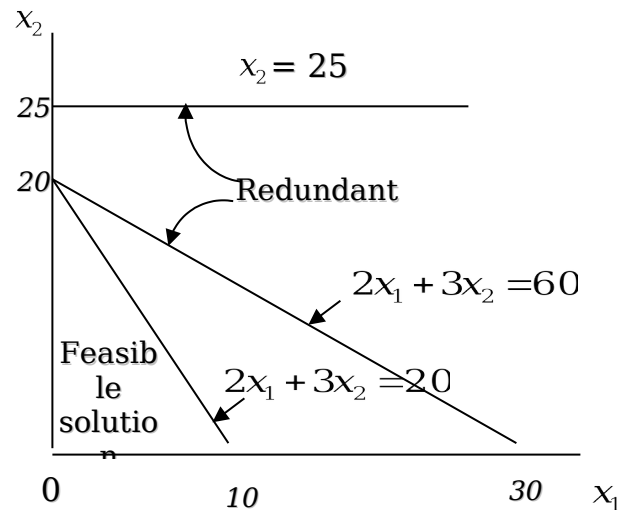


3.5.3 Redundant Constraints

In some cases, a constraint doesn't form a unique boundary of the feasible solution space. Such a constraint is called a redundant constraint. This concept is illustrated in the figure below. Note that a

constraint is redundant if it meets the following test. Its removal would not alter the feasible solution space.

When a problem has a redundant constraint, at least one of the other constraints in the problem is more restrictive than the redundant constraint.

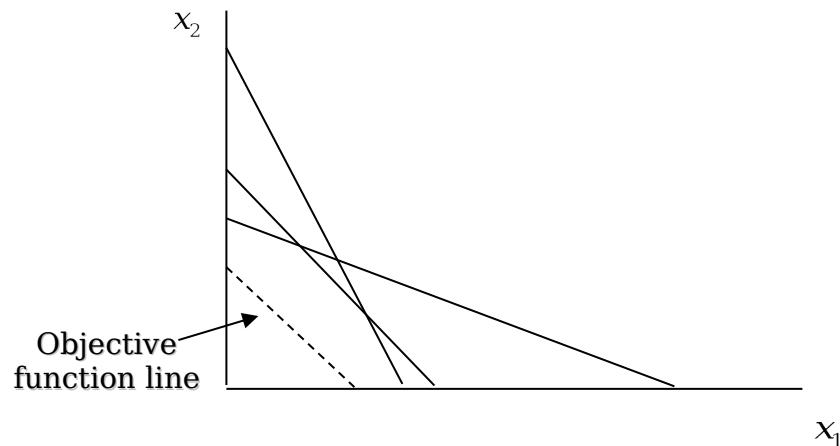


3.5.4 Multiple Optimal Solution

Most linear programming problems have a single optimal solution. However, in some instances, a problem will have multiple optimal solutions, which is to say that different combinations of values of the decision variables will yield the same optimal value for the objective function.

The figure below illustrates how this happens in a two variable problem. We can see that the objective function is parallel to a portion of the boundary of the feasible solution space. Because of this, it ends up last touching an entire segment of the boundary. Hence, every combination of x_1 and x_2 on this line will yield the same (optimal) solution.

One benefit of having multiple optimal solutions is that for other (perhaps qualitative) reasons, a manager may prefer one of these solutions over the others, even though each would achieve the same value of the objective function. Furthermore, if non-integer solutions are acceptable the manager has an infinity of optimal solutions to choose from. In practical terms, one of the two corner points is usually chosen because of ease in identifying its values.



Check Your Progress Exercise

1. Given this summarized linear programming model:

Maximize: $10x_1 + x_2$

Subject to:

1. $3x_1 + x_2 \leq 12$

2. $2x_1 + 2x_2 \leq 16$

3. $x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

- i) Graph the model
 - ii) Find the optimal point on the graph using the extreme point approach
 - iii) Use simultaneous equations to determine the optimal value of x_1 and x_2
 - iv) Determine the amount of slack for each constraint
2. Given the following summarized linear programming model:

Minimize: $3x_1 + 4x_2$

Subject to:

1. $6x_1 + x_2 \geq 120$

2. $4x_1 + 4x_2 \geq 320$

3. $3x_1 + 5x_2 \geq 300$

$$x_1, x_2 \geq 0$$

Required:

- i) Graph the model
 - ii) Determine the optimal point using the extreme point approach
 - iii) Determine the optimal values of x_1 and x_2 and find the minimum cost
 - iv) Determine the amount of slack or surplus for each constraint.
3. A wood products firm uses leftover time at the end of each week to make goods for stock. Currently, there are two products on the list of items that are produced for stock; a chopping board and a knife holder. Both items require three operations: cutting, gluing and finishing. The manager of the firm has collected the following data on these products:

Item	Profit per unit	Time per unit (minutes)		
		Cutting	Gluing	Finishing
Chopping board	\$2	1.4	5	12
Knife holder	\$6	0.8	13	3

The manager has also determined that during each week 56 minutes of cutting, 650 minutes of gluing and 360 minutes of finishing time are available.

Required:

- a) Formulate the problem as a linear programming model
- b) Determine the optimal quantities of each decision variables
- c) Which resources are not completely used by your solution?
how much of each resource is unused?

3.6 SUMMARY

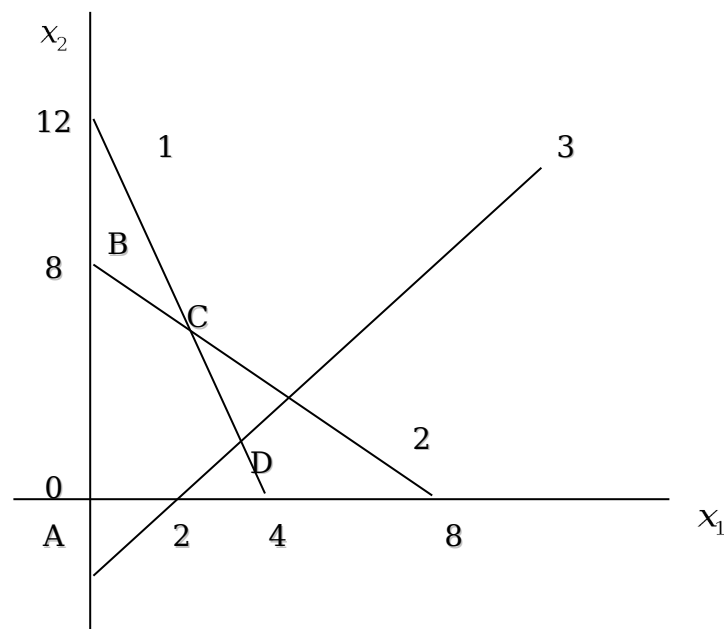
This unit illustrates how two variable linear programming problems can be approached using graphical techniques. Sometimes the solution can be read directly from the graph, but usually it must be found algebraically using simultaneous equations.

The graphical technique can only handle problems involving two variables; therefore, its value as a general tool for solving linear programming models is rather limited. Nonetheless, as a vehicle for illustrating important concepts related to linear programming models and solutions, it is invaluable.

The unit illustrates the solution for both maximization and minimization problems. In addition, the calculation of slack and surplus are demonstrated. Also, certain special issues that sometimes arise, including problems with multiple optimal solutions, and redundant constraints are discussed.

3.7 ANSWER TO CHECK YOUR PROGRESS EXERCISE

1. i) Hint: the graph will have the following structure



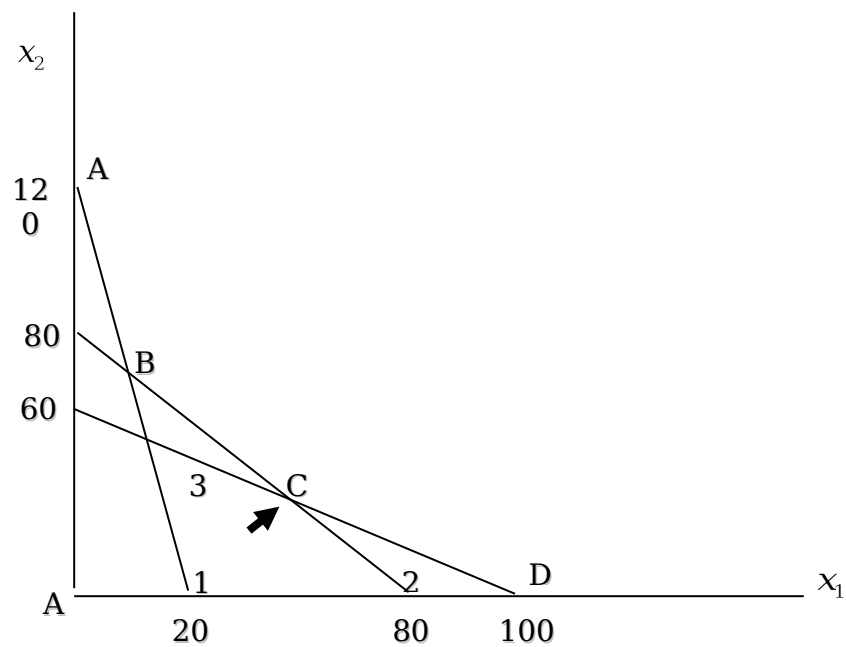
-2

ii) Corner point D

iii) $x_1 = 3.5$ and $x_2 = 1.5$

iv) Slack is only for the 2nd constraint and that is 6.

2. i) the graph will have the following structure:



ii) Corner point C

iii) $x_1 = 50$ and $x_2 = 30$

iv) the surplus is only for the first constraint that is 210.

3. The model for the problem in a summarized form is

Maximize: $2x_1 + 6x_2$

Subject to:

1. Cutting $1.4x_1 + 0.8x_2 \leq 56$

2. Gluing $5x_1 + 13x_2 \leq 65$

3. Finishing $12x_1 + 3x_2 \leq 360$

$$x_1, x_2 \geq 0$$