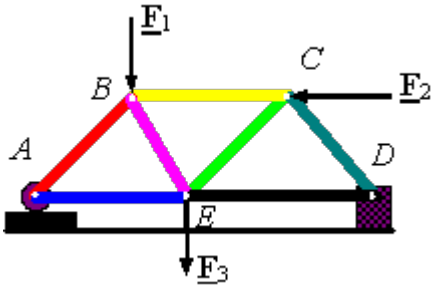
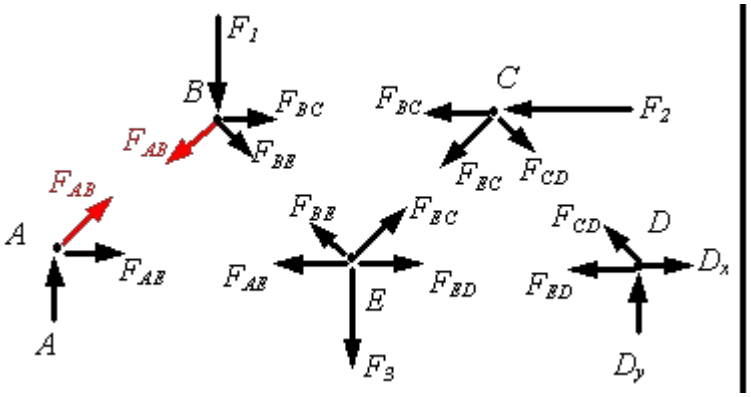
The analysis of trusses

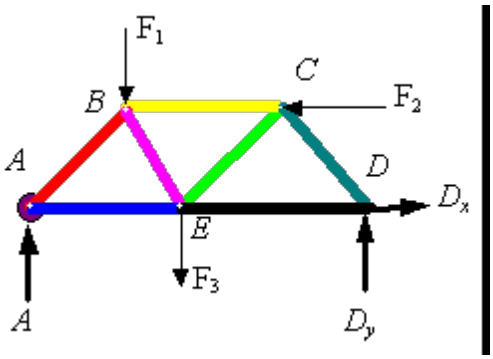
- **A truss**: A truss is a structure made of two force members all pin connected to each other.



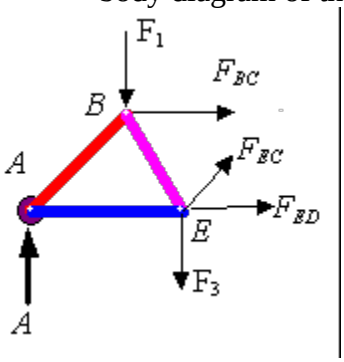
- **The method of joints**: This method uses the free-body-diagram of joints in the structure to determine the forces in each member. For example, in the above structure we have 5 joints each having a free body diagram as follows



- Note how Newton's third law controls how one introduces F_{AB} on the joints *A* and on the joint *B*. For each joint one can write two equations ($\sum F_x = 0, \sum F_y = 0$). The moment equation is trivially satisfied since all forces on a joint pass trough the joint. For example, for the above truss we have 5 joints, therefore we can write 10 equations of equilibrium (two for each joint). In the above example there are seven unknown member forces ($F_{AB,}$ F_{BC}, F_{CD}, F_{ED}, F_{EC}, F_{BE}, F_{AE}) plus three unknown support reactions (A, D_x, D_y), giving a total of 10 unknowns to solve for using the 10 equations obtained from equilibrium.



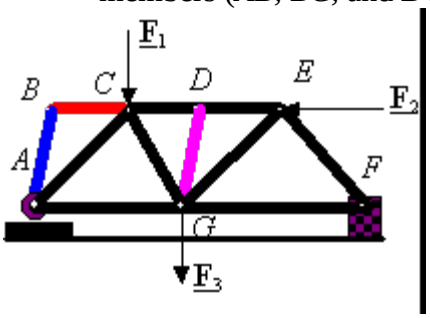
- **The method of sections**: This method uses free-body-diagrams of sections of the truss to obtain unknown forces. For example, if one needs only to find the force in BC, it is possible to do this by only writing two equations. First, draw the free body diagram of the full truss and solve for the reaction at *A* by taking moments about *D*. Next draw the free body diagram of the section shown and take moments about *E* to find the force in *BC*.



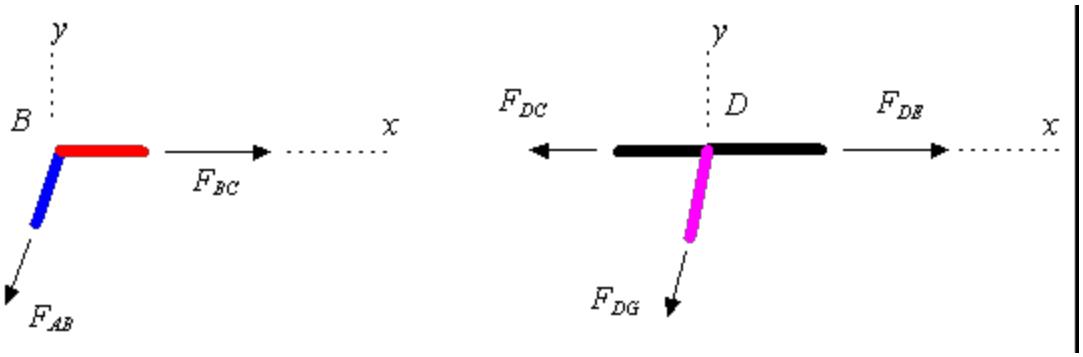
- In the method of sections one can write three equations for each free-body-diagram (two components of force and one moment equation).

Things to consider:

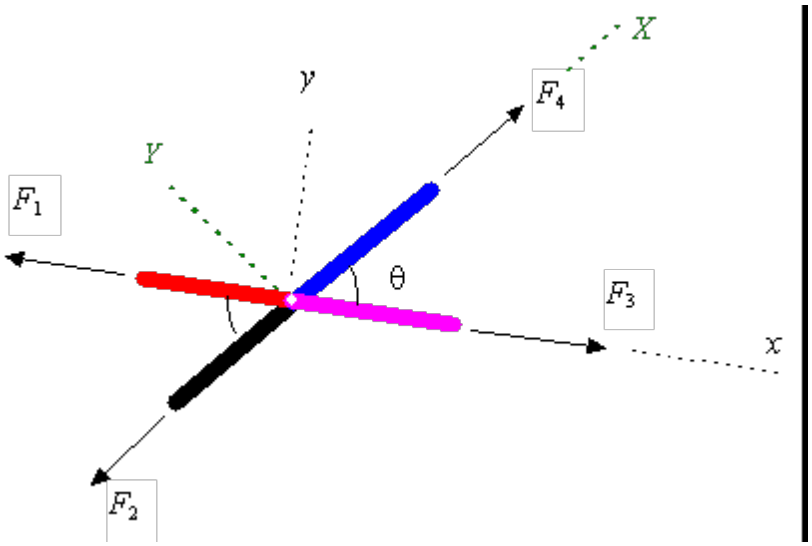
- **Zero force members**: Some members in a truss cannot carry load. These members are called zero force members. Examples of zero force members are the colored members (*AB, BC,* and *DG*) in the following truss.



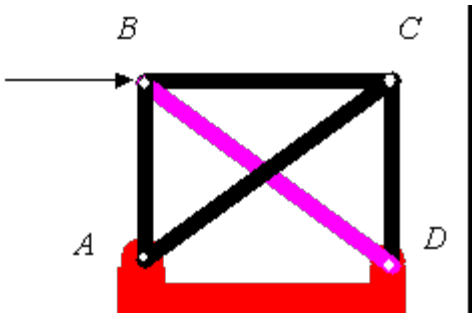
Consider the following free-body-diagrams



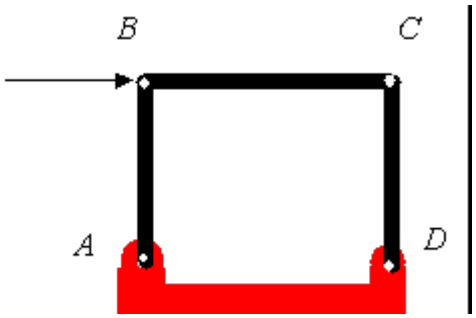
- If you sum the forces in the *y*-direction in the left free-body-diagram, you will see that F_{AB} must be zero since it is not balanced by another force. Then if you sum forces in the *y*-direction you will find that F_{BC} must also be zero. If you sum the forces in the *y* direction in the right free-body-diagram, you will see that F_{DG} must be zero since it is not balanced by another force.
  - **A redundant joint**: Sometimes a joint is redundant. For example, in the following free-body-diagram the load is directly transmitted from each member to the one opposite it without any interaction.

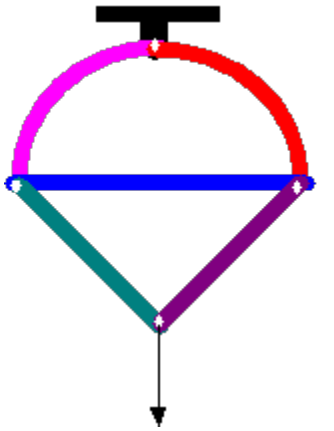


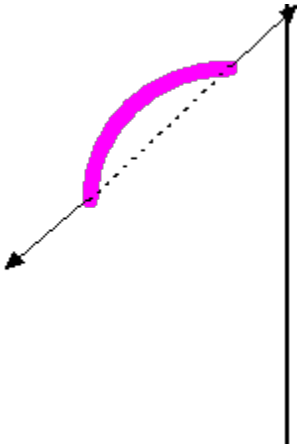
- By summing forces along the *y*-direction one will get $F_2=F_4$, and by summing forces along the *Y*-direction one will get $F_1=F_3$.
  - **Redundant members**: Sometimes a structure contains one or more redundant members. These members must be removed from the truss, otherwise one will have an insufficient number of equations to solve for the unknown member forces. Slender members are not very useful in compression since they buckle and, as a result, lose their load carrying capability. For example, in the following truss one of the two members *AC* or *BD* is redundant. To solve the problem, we remove member *BD* which will go into compression as a result of the applied loading (i.e., the diagonal *AC* will have to increase in length and the diagonal *BD* will have to decease in length for the structure to bend to the right). If we did not remove this member we would have 9 unknowns (five member loads and four support reactions) and only 8 equations (two for each joint).



- **Mechanisms**: Sometimes there is too much freedom in a structure. For example, the following structure cannot carry any load since it will collapse under the load.







- **Curved members**: Remember that the two forces acting on a two-force-member are along the line connecting the two points on which the loads are applied.

Frames and Machines

Frames and Machines represent problems that are applications of using the equilibrium equations. As such, they do not represent any new ideas.

Strategies and ideas for solving problems:

- Don't write down any equations until you have developed your plan of action.
- Develop your strategy for solving the problem by drawing free-body-diagrams of the entire frame or machine and each one of sub-units. Remember that the free-body-diagram of the complete body is equal to the sum of the free-body-diagrams of its sub-sections.
- For each free-body-diagram analyze the diagram in terms of how you can use the equations to directly solve for some unknowns (Don't write down the equations until you finish analyzing all the free body diagrams).
- Look for any two force members.
- Remember that you can write three equations for each free-body-diagram.
- Don't forget to take advantage of the moment equation to eliminate unknowns.
- After you have analyzed all the free body diagrams, look for the simplest way to get to the solution without writing any extra equations. This will become your plan of attack.
- Once you have formulated your plan, write down the necessary equations and solve for the unknowns.

Truss analysis

- ❑ Determines forces in each member of a truss structure



Static determinacy

❑ 3 classes of structures

• *Statically determinate*: structure is exactly restrained. # reactions = # equations

• *Not statically determine*: structure not sufficiently restrained and will move!
reactions > # equations

• *Statically indeterminate*: structure is overrestrained.
reactions < # equations

Can use Method of Joints for analysis of statically determinate trusses

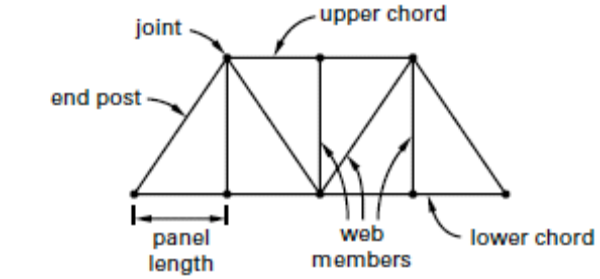
Assumptions in method of joints

- Truss members (bars) are straight and massless
- Bars are assumed rigid
- Bars are connected at their ends through joints
- External loads are applied only at joints
- Supports (pins, rollers) are frictionless
- Loads in a bar have lines of action collinear withthe bar axis; force is uniaxial and
  can be compressive or tensile

Steps in method of joints

- Draw FBDs – one with supports, one replaced with reaction forces
- Write the equations of equilibrium, find Determinacy
- Solve equilibrium equations, redraw FBD labeling quantities
- Determine the internal loads, label on diagram
- Check your solution to see that forces and moments sum to zero

Figure 11.1 Parts of a Bridge Truss



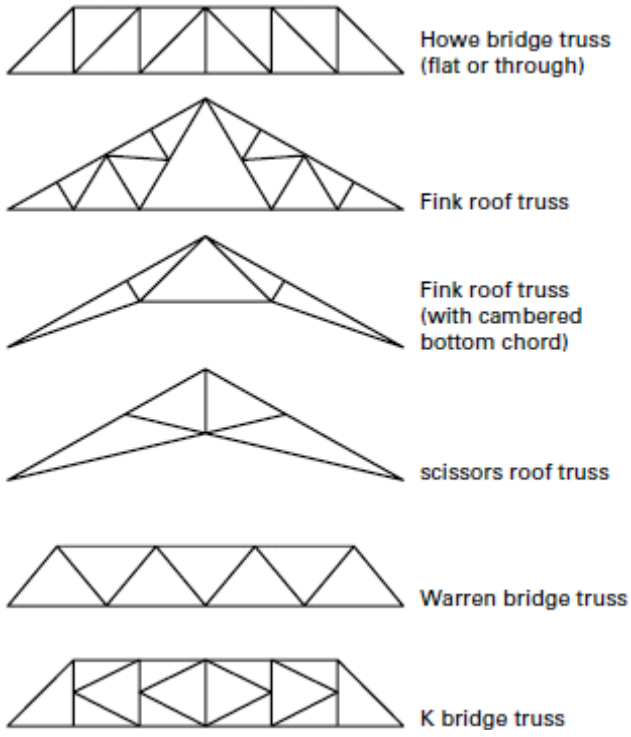


Figure 11.2 Special Types of Trusses

