

Engineering Mechanics: Statics in SI Units, 12e

3

Equilibrium of a Particle

Chapter Objectives

- Concept of the free-body diagram for a particle
- Solve particle equilibrium problems using the equations of equilibrium

Chapter Outline

1. Condition for the Equilibrium of a Particle
2. The Free-Body Diagram
3. Coplanar Systems
4. Three-Dimensional Force Systems

3.1 Condition for the Equilibrium of a Particle

- Particle at *equilibrium* if
 - At rest
 - Moving at constant a constant velocity
- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle

3.1 Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

$$\mathbf{a} = 0$$

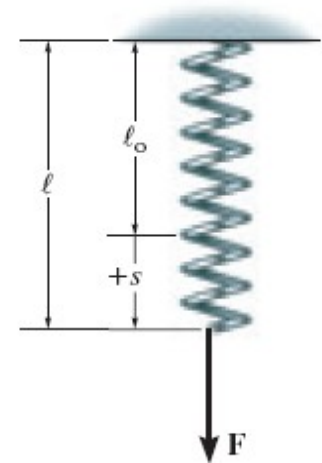
therefore, the particle is moving in constant velocity or at rest

3.2 The Free-Body Diagram

- Best representation of all the unknown forces ($\Sigma \mathbf{F}$) which acts on a body
- A sketch showing the particle “free” from the surroundings with all the forces acting on it
- Consider two common connections in this subject –
 - Spring
 - Cables and Pulleys

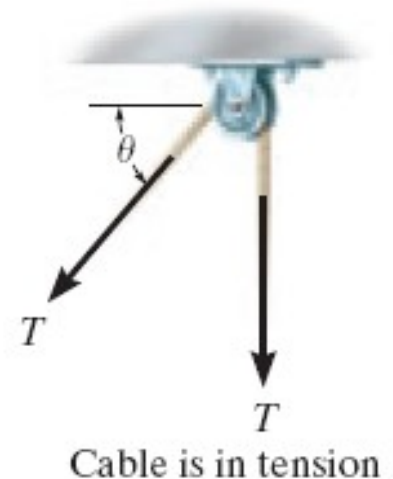
3.2 The Free-Body Diagram

- Spring
 - Linear elastic spring: change in length is directly proportional to the force acting on it
 - *spring constant or stiffness* k : defines the elasticity of the spring
 - Magnitude of force when spring is elongated or compressed
 - $F = ks$



3.2 The Free-Body Diagram

- Cables and Pulley
 - Cables (or cords) are assumed negligible weight and cannot stretch
 - Tension always acts in the direction of the cable
 - Tension force must have a constant magnitude for equilibrium
 - For any angle θ , the cable is subjected to a constant tension T



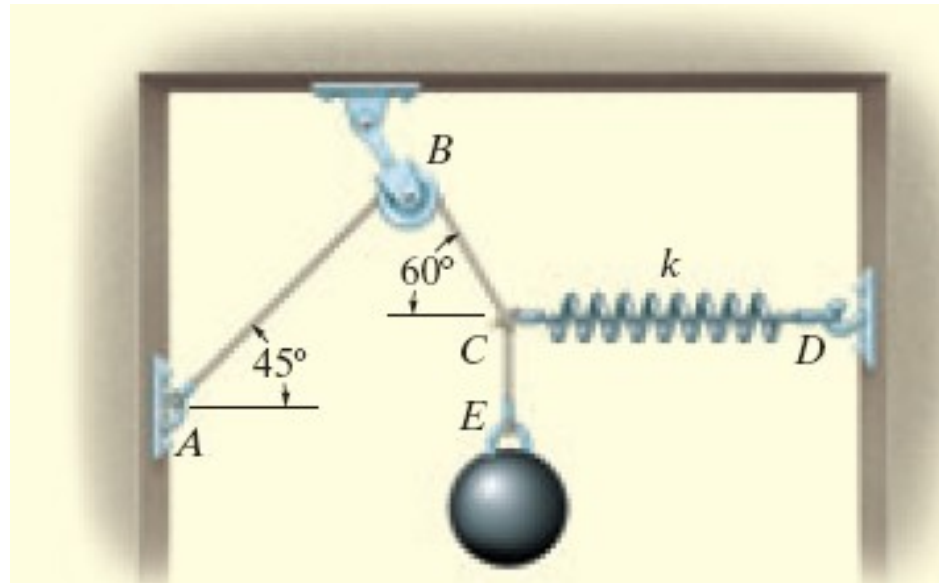
3.2 The Free-Body Diagram

Procedure for Drawing a FBD

1. Draw outlined shape
2. Show all the forces
 - Active forces: particle in motion
 - Reactive forces: constraints that prevent motion
3. Identify each forces
 - Known forces with proper magnitude and direction
 - Letters used to represent magnitude and directions

Example 3.1

The sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere, the cord CE and the knot at C.

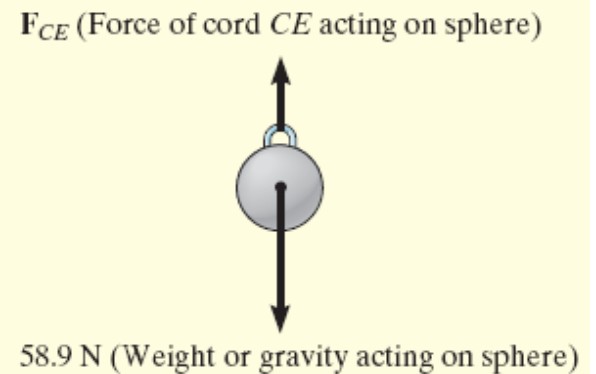


Solution

FBD at Sphere

Two forces acting, weight and the force on cord CE.

Weight of 6kg (9.81m/s^2) = 58.9N



Cord CE

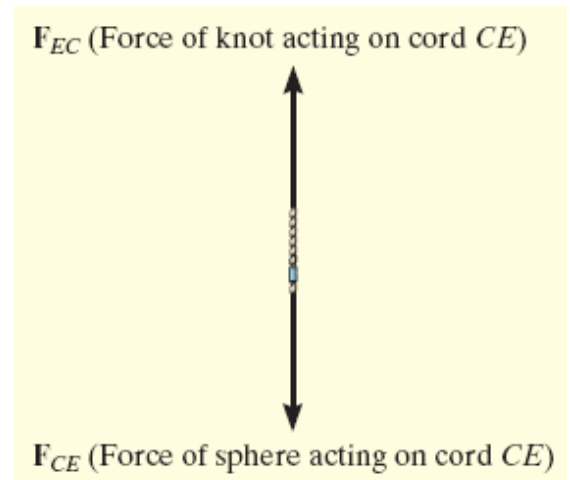
Two forces acting: sphere and knot

Newton's 3rd Law:

F_{CE} is equal but opposite

F_{CE} and F_{EC} pull the cord in tension

For equilibrium, $F_{CE} = F_{EC}$

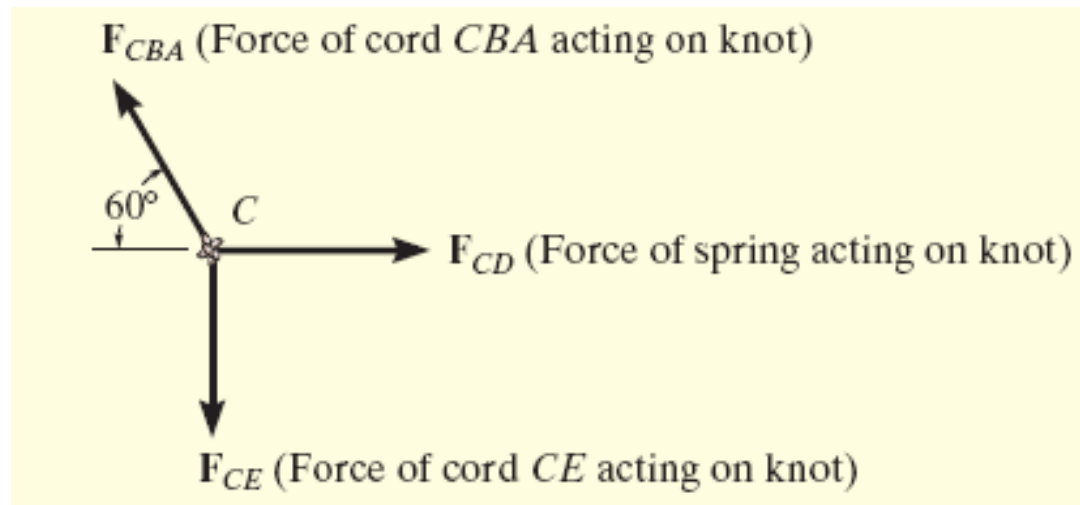


Solution

FBD at Knot

3 forces acting: cord CBA, cord CE and spring CD

Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE



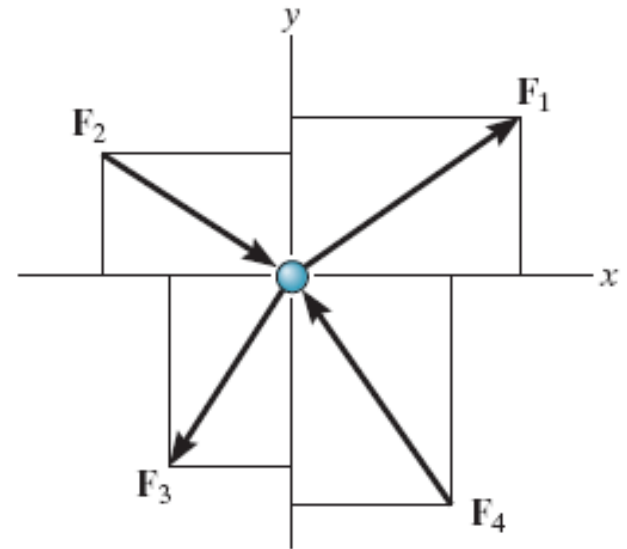
3.3 Coplanar Systems

- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

$$\sum \mathbf{F}_x = 0$$

$$\sum \mathbf{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero



3.3 Coplanar Systems

- Procedure for Analysis

1. Free-Body Diagram

- Establish the x, y axes
- Label all the unknown and known forces

2. Equations of Equilibrium

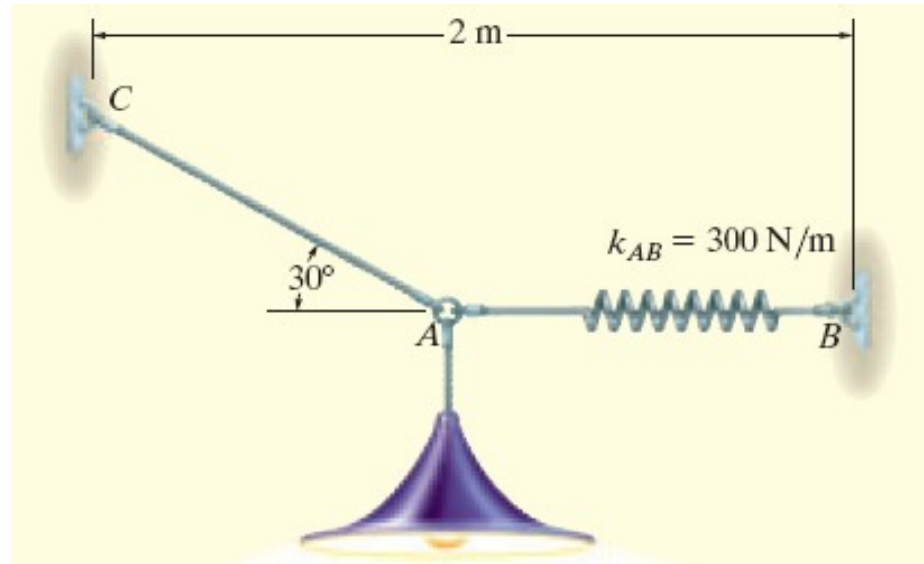
- Apply $F = ks$ to find spring force
- When negative result force is the reserve
- Apply the equations of equilibrium

$$\sum \mathbf{F}_x = 0$$

$$\sum \mathbf{F}_y = 0$$

Example 3.4

Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is $l'_{AB} = 0.4\text{m}$, and the spring has a stiffness of $k_{AB} = 300\text{N/m}$.



Solution

FBD at Point A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by $F = ks$.

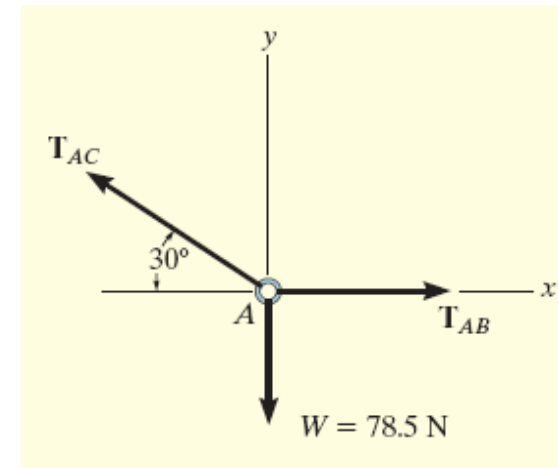
$$+\rightarrow \quad \sum \mathbf{F}_x = 0; \quad T_{AB} - T_{AC} \cos 30^\circ = 0$$

$$+\uparrow \quad \sum \mathbf{F}_y = 0; \quad T_{AB} \sin 30^\circ - 78.5 \text{ N} = 0$$

Solving,

$$T_{AC} = 157.0 \text{ kN}$$

$$T_{AB} = 136.0 \text{ kN}$$



Solution

$$T_{AB} = k_{AB}s_{AB}; \quad 136.0\text{N} = 300\text{N/m}(s_{AB})$$

$$s_{AB} = 0.453\text{N}$$

For stretched length,

$$l_{AB} = l'_{AB} + s_{AB}$$

$$\begin{aligned} l_{AB} &= 0.4\text{m} + 0.453\text{m} \\ &= 0.853\text{m} \end{aligned}$$

For horizontal distance BC,

$$2\text{m} = l_{AC}\cos 30^\circ + 0.853\text{m}$$

$$l_{AC} = 1.32\text{m}$$

3.4 Three-Dimensional Force Systems

- For particle equilibrium

$$\sum \mathbf{F} = 0$$

- Resolving into $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components

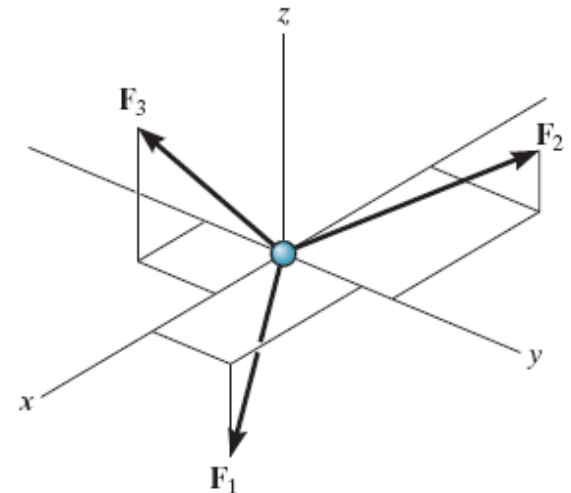
$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0$$

- Three scalar equations representing algebraic sums of the x, y, z forces

$$\sum F_x \mathbf{i} = 0$$

$$\sum F_y \mathbf{j} = 0$$

$$\sum F_z \mathbf{k} = 0$$



3.4 Three-Dimensional Force Systems

- Procedure for Analysis

Free-body Diagram

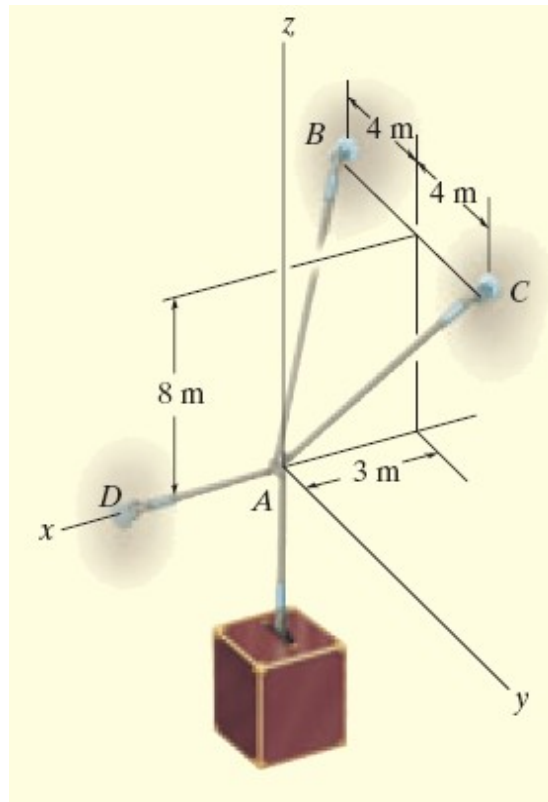
- Establish the x, y, z axes
- Label all known and unknown force

Equations of Equilibrium

- Apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$
- Substitute vectors into $\sum \mathbf{F} = 0$ and set i, j, k components = 0
- Negative results indicate that the sense of the force is opposite to that shown in the FBD.

Example 3.7

Determine the force developed in each cable used to support the 40kN crate.



Solution

FBD at Point A

To expose all three unknown forces in the cables.

Equations of Equilibrium

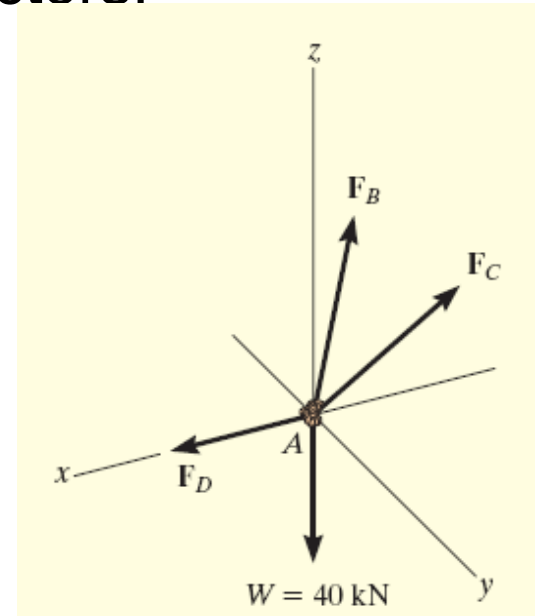
Expressing each forces in Cartesian vectors.

$$\begin{aligned}\mathbf{F}_B &= F_B(\mathbf{r}_B / r_B) \\ &= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{F}_C &= F_C(\mathbf{r}_C / r_C) \\ &= -0.318F_C\mathbf{i} - 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}\end{aligned}$$

$$\mathbf{F}_D = F_D\mathbf{i}$$

$$\mathbf{W} = -40\mathbf{k}$$



Solution

For equilibrium,

$$\begin{aligned}\sum \mathbf{F} = 0; \quad & \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0 \\ & -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_C\mathbf{i} \\ & \quad - 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = 0\end{aligned}$$

$$\sum F_x = 0; \quad -0.318F_B - 0.318F_C + F_D = 0$$

$$\sum F_y = 0; \quad -0.424F_B - 0.424F_C = 0$$

$$\sum F_z = 0; \quad 0.848F_B + 0.848F_C - 40 = 0$$

Solving,

$$F_B = F_C = 23.6\text{kN}$$

$$F_D = 15.0\text{kN}$$

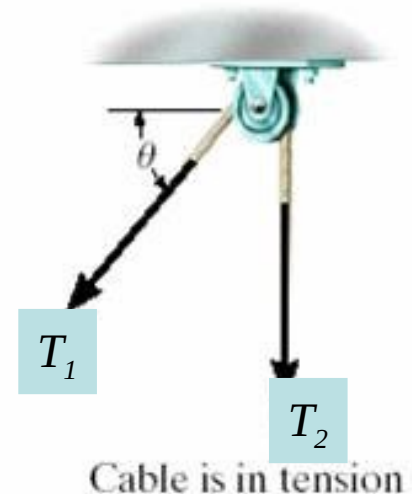
QUIZ

1. When a particle is in equilibrium, the sum of forces acting on it equals ____ . (Choose the most appropriate answer)

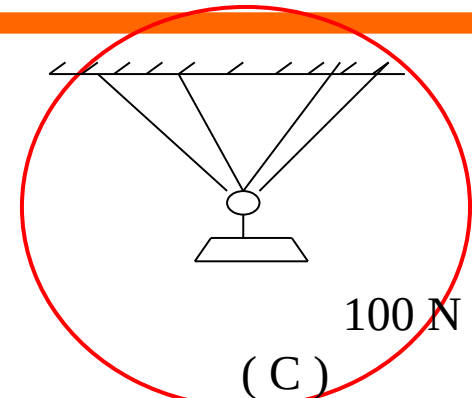
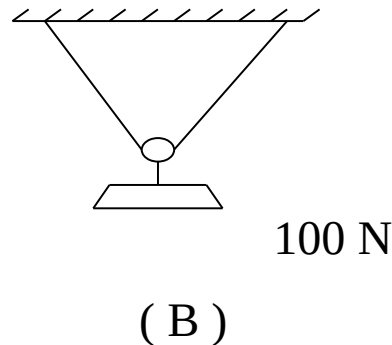
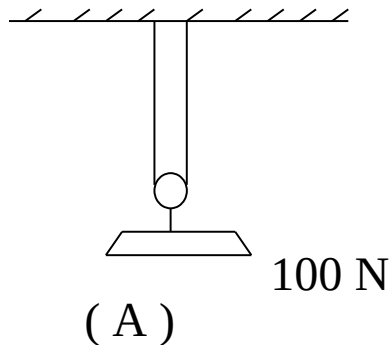
- A) A constant B) A positive number C) Zero
D) A negative number E) An integer

2. For a frictionless pulley and cable, tensions in the cables are related as

- A) $T_1 > T_2$
B) $T_1 = T_2$
C) $T_1 < T_2$
D) $T_1 = T_2 \sin \theta$



QUIZ



3. Assuming you know the geometry of the ropes, you cannot determine the forces in the cables in which system above?

4. Why?

A) The weight is too heavy.

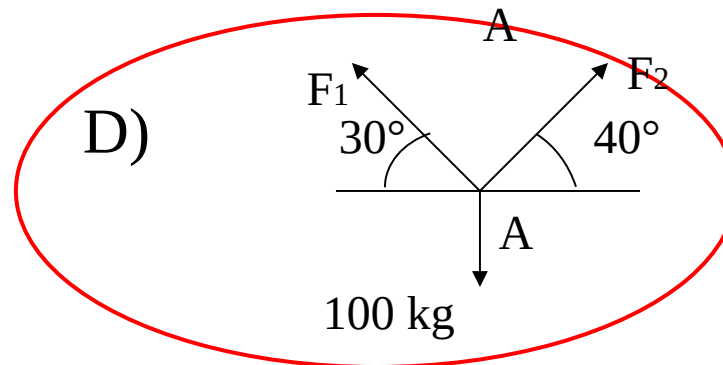
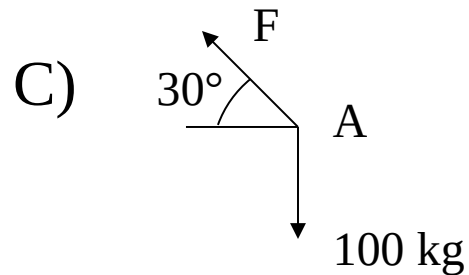
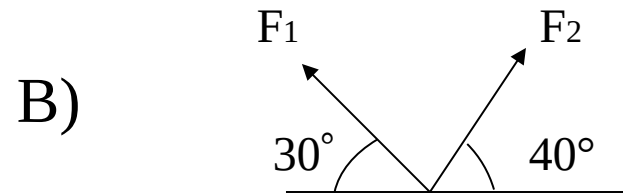
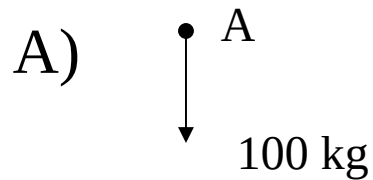
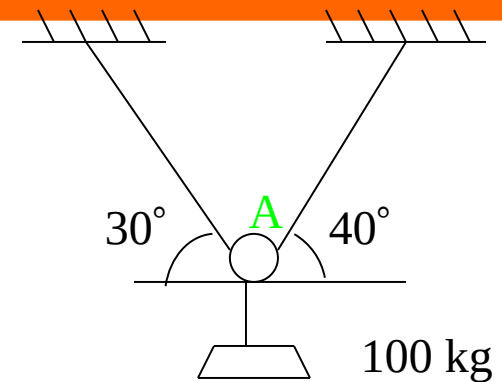
B) The cables are too thin.

C) There are more unknowns than equations.

D) There are too few cables for a 100 kg weight.

QUIZ

5. Select the correct FBD of particle A.



QUIZ

6. Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is ____ . Use a sign convention of + $\rightarrow$.

A) $F_2 \sin 50^\circ - 20 = 0$

B) $F_2 \cos 50^\circ - 20 = 0$

C) $F_2 \sin 50^\circ - F_1 = 0$

D) $F_2 \cos 50^\circ + 20 = 0$

QUIZ

7. Particle P is in equilibrium with five (5) forces acting on it in 3-D space. How many scalar equations of equilibrium can be written for point P?

- A) 2 B) 3 C) 4 D) 5 E) 6

8. In 3-D, when a particle is in equilibrium, which of the following equations apply?

A) $(\sum F_x) \mathbf{i} + (\sum F_y) \mathbf{j} + (\sum F_z) \mathbf{k} = 0$

B) $\sum \mathbf{F} = 0$

C) $\sum F_x = \sum F_y = \sum F_z = 0$

D) All of the above.

E) None of the above.

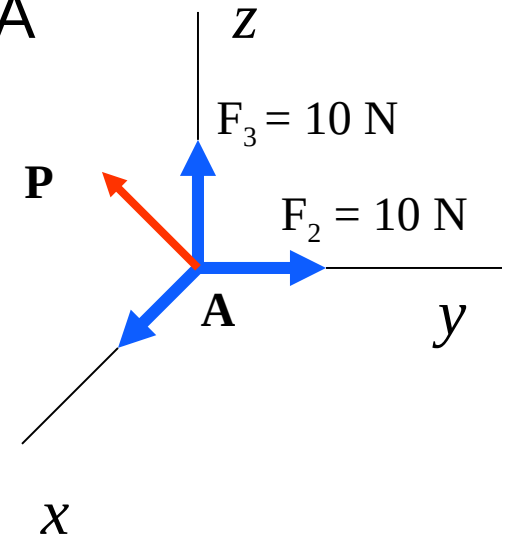
QUIZ

9. In 3-D, when you know the direction of a force but not its magnitude, how many unknowns corresponding to that force remain?
- A) One B) Two C) Three D) Four
10. If a particle has 3-D forces acting on it and is in static equilibrium, the components of the resultant force ____ .
- A) have to sum to zero, e.g., $-5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- B) have to equal zero, e.g., $0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$
- C) have to be positive, e.g., $5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$
- D) have to be negative, e.g., $-5\mathbf{i} - 5\mathbf{j} - 5\mathbf{k}$

QUIZ

11. Four forces act at point A and point A is in equilibrium. Select the correct force vector ***P***.

- A) $\{-20 \mathbf{i} + 10 \mathbf{j} - 10 \mathbf{k}\} \text{lb}$
- B) $\{-10 \mathbf{i} - 20 \mathbf{j} - 10 \mathbf{k}\} \text{lb}$
- C) $\{+ 20 \mathbf{i} - 10 \mathbf{j} - 10 \mathbf{k}\} \text{lb}$
- D) None of the above.



12. In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

- A) One
- B) Two
- C) Three
- D) Four