

Engineering Mechanics: Statics in SI Units, 12e

2

Force Vectors

Chapter Objectives

- Parallelogram Law
- Cartesian vector form
- Dot product and angle between 2 vectors

Chapter Outline

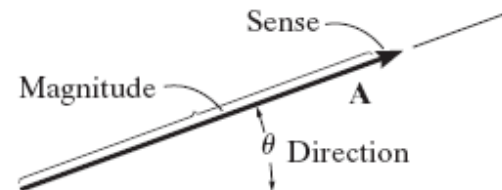
1. Scalars and Vectors
2. Vector Operations
3. Vector Addition of Forces
4. Addition of a System of Coplanar Forces
5. Cartesian Vectors
6. Addition and Subtraction of Cartesian Vectors
7. Position Vectors
8. Force Vector Directed along a Line
9. Dot Product

2.1 Scalars and Vectors

- Scalar
 - A quantity characterized by a positive or negative number
 - Indicated by letters in italic such as *A*
e.g. Mass, volume and length

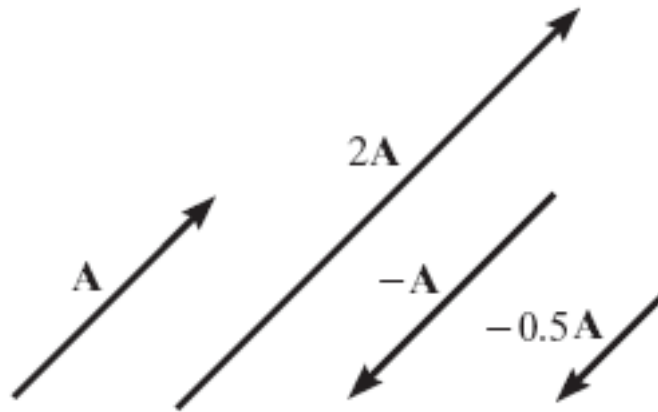
2.1 Scalars and Vectors

- Vector
 - A quantity that has magnitude and direction
e.g. Position, force and moment
 - Represent by a letter with an arrow over it, $\vec{A}$
 - Magnitude is designated as $|\vec{A}|$
 - In this subject, vector is presented as **A** and its magnitude (positive quantity) as A



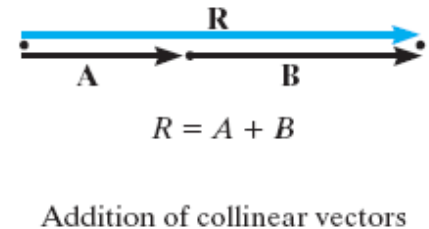
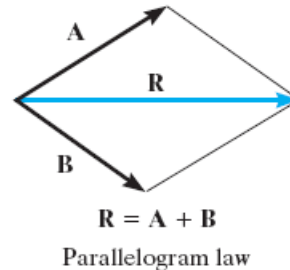
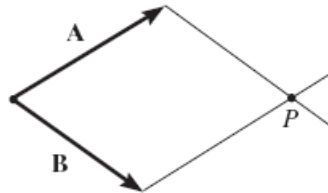
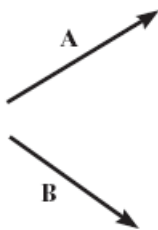
2.2 Vector Operations

- Multiplication and Division of a Vector by a Scalar
 - Product of vector **A** and scalar $a = a\mathbf{A}$
 - Magnitude = $|a\mathbf{A}|$
 - Law of multiplication applies e.g. $\mathbf{A}/a = (1/a) \mathbf{A}$, $a \neq 0$



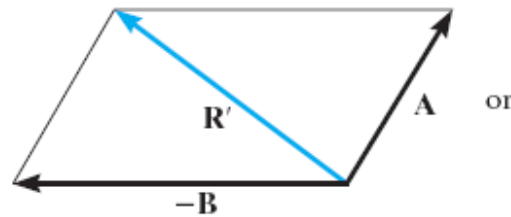
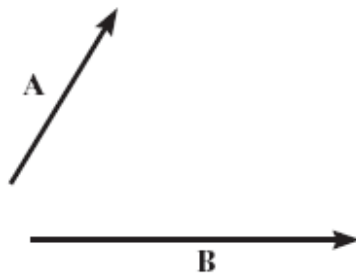
2.2 Vector Operations

- Vector Addition
 - Addition of two vectors **A** and **B** gives a resultant vector **R** by the *parallelogram law*
 - Result **R** can be found by *triangle construction*
 - Commutative e.g. $\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
 - Special case: Vectors **A** and **B** are *collinear* (both have the same line of action)

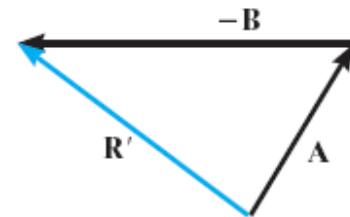


2.2 Vector Operations

- Vector Subtraction
 - Special case of addition
 - e.g. $\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
 - Rules of Vector Addition Applies



Parallelogram law
Vector subtraction

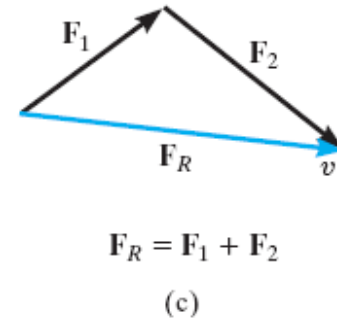
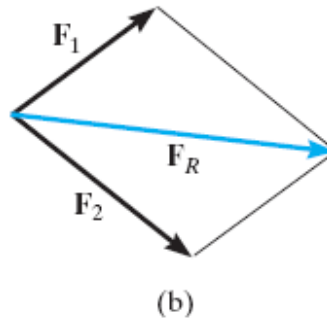
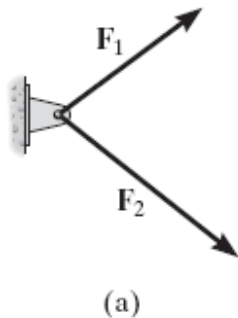


Triangle construction

2.3 Vector Addition of Forces

Finding a Resultant Force

- *Parallelogram law* is carried out to find the resultant force

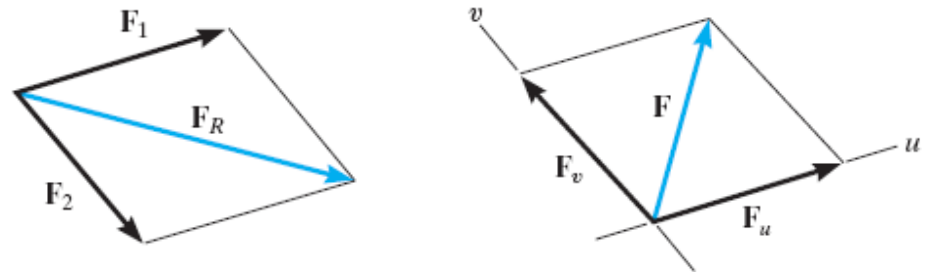


- Resultant,
$$\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2)$$

2.3 Vector Addition of Forces

Procedure for Analysis

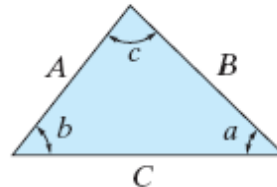
- Parallelogram Law
 - Make a sketch using the *parallelogram law*
 - 2 components forces add to form the resultant force
 - Resultant force is shown by the diagonal of the parallelogram
 - The components is shown by the sides of the parallelogram



2.3 Vector Addition of Forces

Procedure for Analysis

- Trigonometry
 - Redraw half portion of the parallelogram
 - Magnitude of the resultant force can be determined by the *law of cosines*
 - Direction of the resultant force can be determined by the *law of sines*
 - Magnitude of the two components can be determined by the *law of sines*



Cosine law:

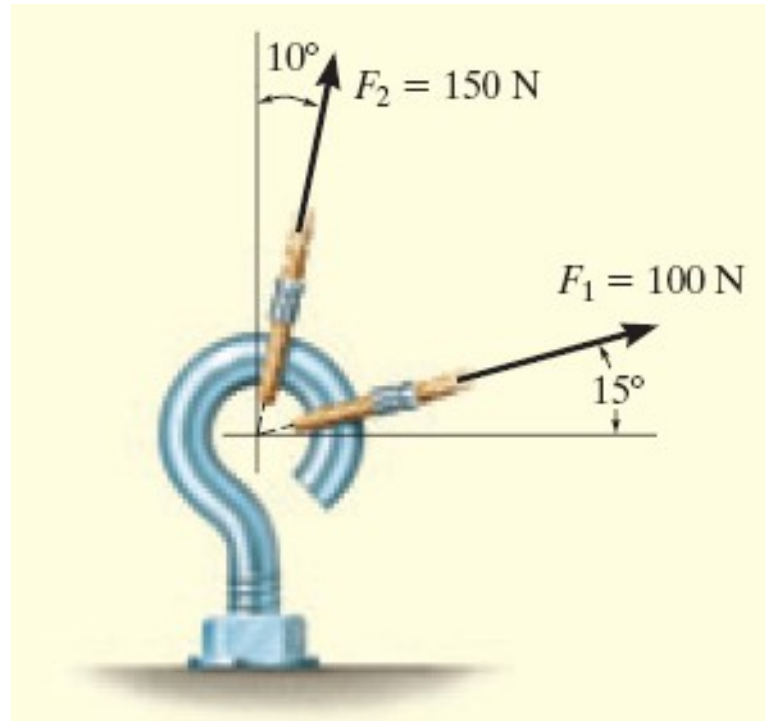
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Example 2.1

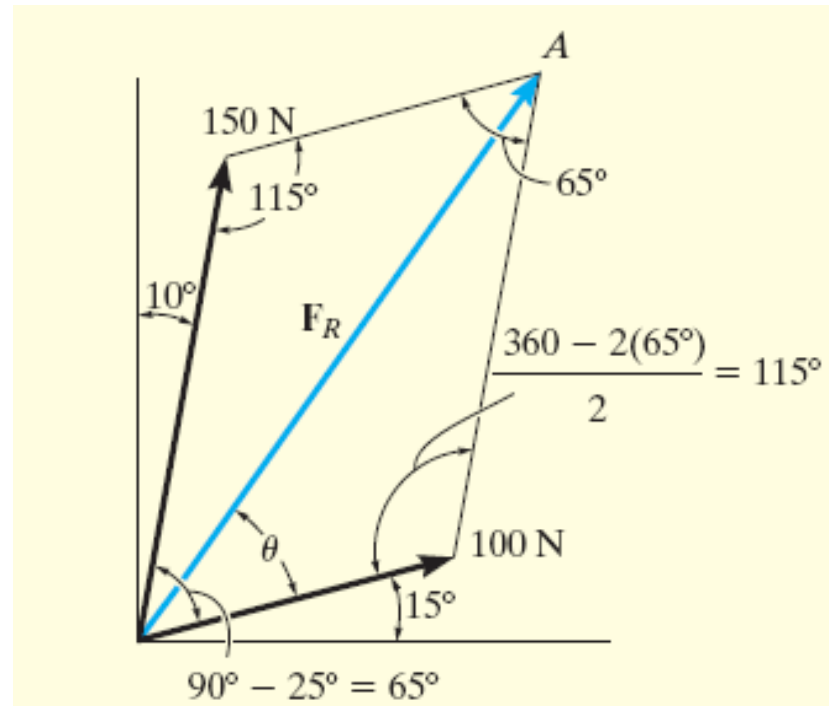
The screw eye is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



Solution

Parallelogram Law

Unknown: magnitude of $\mathbf{F}_R$ and angle θ



Solution

Trigonometry

Law of Cosines

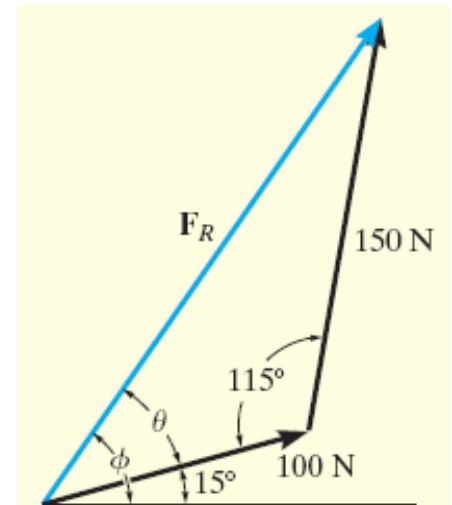
$$\begin{aligned} F_R &= \sqrt{(100N)^2 + (150N)^2 - 2(100N)(150N)\cos 115^\circ} \\ &= \sqrt{10000 + 22500 - 30000(-0.4226)} = 212.6N = 213N \end{aligned}$$

Law of Sines

$$\frac{150N}{\sin \theta} = \frac{212.6N}{\sin 115^\circ}$$

$$\sin \theta = \frac{150N}{212.6N} (0.9063)$$

$$\theta = 39.8^\circ$$



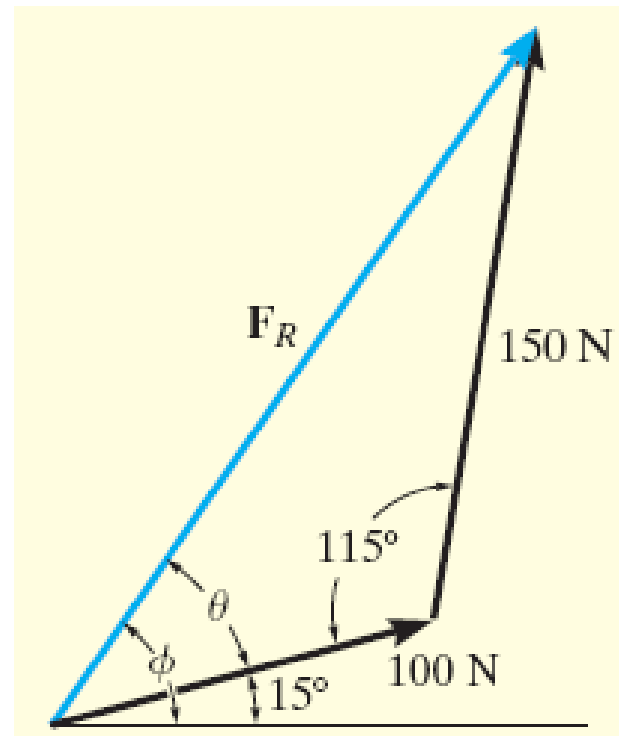
Solution

Trigonometry

Direction ϕ of $\mathbf{F}_R$ measured from the horizontal

$$\phi = 39.8^\circ + 15^\circ$$

$$= 54.8^\circ \angle \phi$$

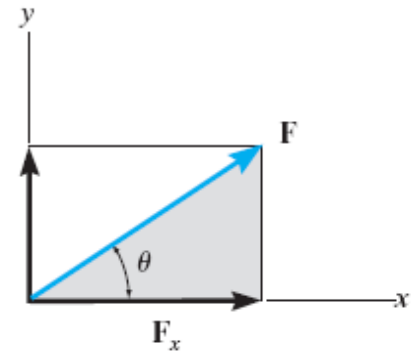


2.4 Addition of a System of Coplanar Forces

- Scalar Notation
 - x and y axes are designated positive and negative
 - Components of forces expressed as algebraic scalars

$$F = F_x + F_y$$

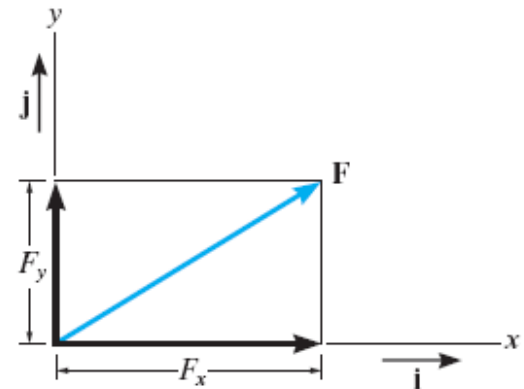
$$F_x = F \cos \theta \quad \text{and} \quad F_y = F \sin \theta$$



2.4 Addition of a System of Coplanar Forces

- Cartesian Vector Notation
 - Cartesian unit vectors **i** and **j** are used to designate the x and y directions
 - Unit vectors **i** and **j** have dimensionless magnitude of unity (= 1)
 - Magnitude is always a positive quantity, represented by scalars F_x and F_y

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$



2.4 Addition of a System of Coplanar Forces

- Coplanar Force Resultants

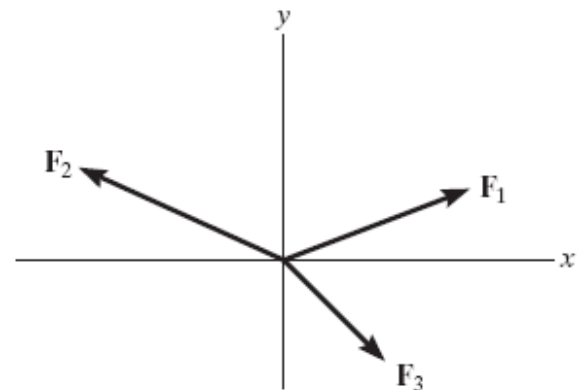
To determine resultant of several coplanar forces:

- Resolve force into x and y components
- Addition of the respective components using scalar algebra
- Resultant force is found using the parallelogram law
- Cartesian vector notation:

$$F_1 = F_{1x}i + F_{1y}j$$

$$F_2 = -F_{2x}i + F_{2y}j$$

$$F_3 = F_{3x}i - F_{3y}j$$



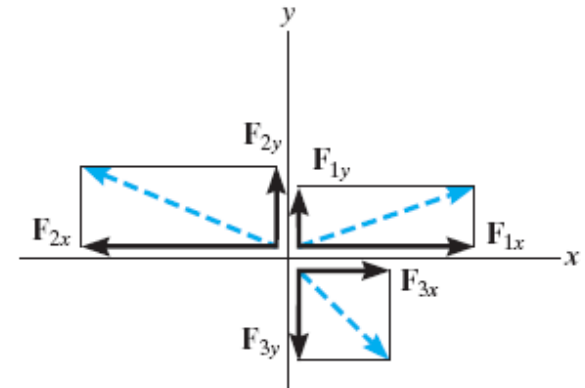
2.4 Addition of a System of Coplanar Forces

- Coplanar Force Resultants
 - Vector resultant is therefore

$$\begin{aligned} F_R &= F_1 + F_2 + F_3 \\ &= (F_{Rx})i + (F_{Ry})j \end{aligned}$$

- If scalar notation are used

$$\begin{aligned} F_{Rx} &= F_{1x} - F_{2x} + F_{3x} \\ F_{Ry} &= F_{1y} + F_{2y} - F_{3y} \end{aligned}$$



2.4 Addition of a System of Coplanar Forces

- Coplanar Force Resultants

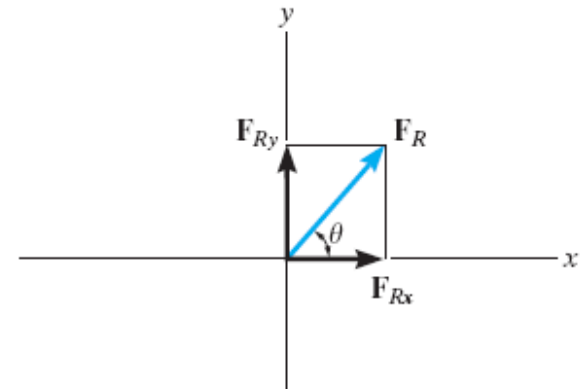
- In all cases we have

$$F_{Rx} = \sum F_x$$

$$F_{Ry} = \sum F_y \quad * \text{ Take note of sign conventions}$$

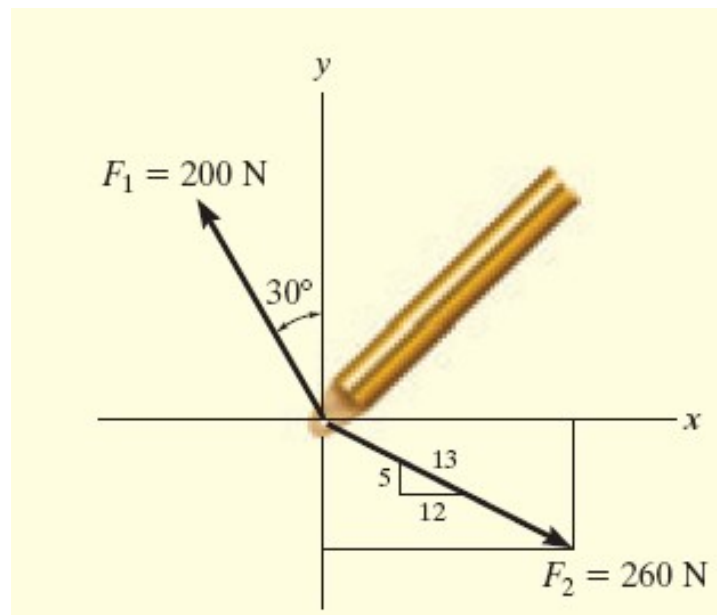
- Magnitude of $\mathbf{F}_R$ can be found by Pythagorean Theorem

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{and } \theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$



Example 2.5

Determine x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom. Express each force as a Cartesian vector.



Solution

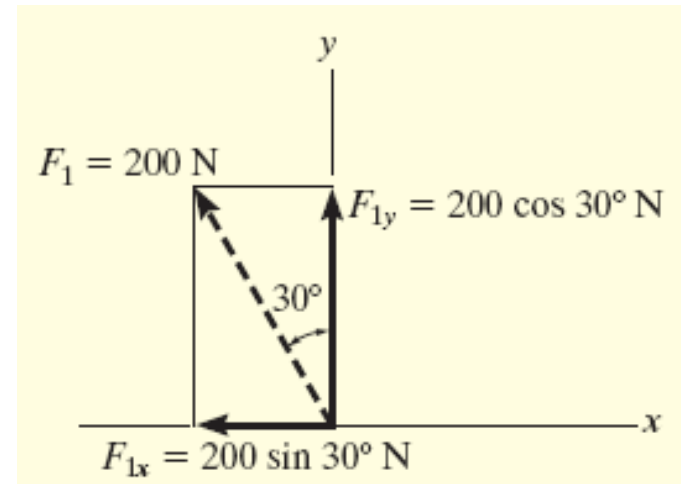
Scalar Notation

$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$

$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

Hence, from the slope triangle, we have

$$\theta = \tan^{-1} \left(\frac{5}{12} \right)$$



Solution

By similar triangles we have

$$F_{2x} = 260 \left(\frac{12}{13} \right) = 240 \text{ N}$$

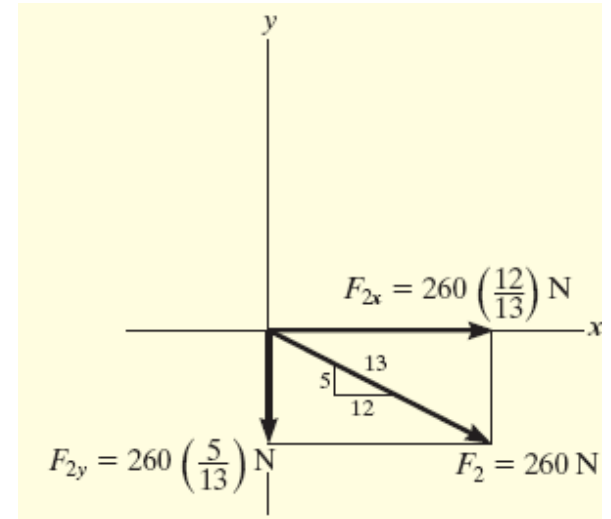
$$F_{2y} = 260 \left(\frac{5}{13} \right) = 100 \text{ N}$$

Scalar Notation: $F_{2x} = 240 \text{ N} \rightarrow$

$$F_{2y} = -100 \text{ N} = 100 \text{ N} \downarrow$$

Cartesian Vector Notation: $F_1 = \{-100i + 173j\} \text{ N}$

$$F_2 = \{240i - 100j\} \text{ N}$$



Solution

Scalar Notation

$$F_{1x} = -200 \sin 30^\circ N = -100 N = 100 N \leftarrow$$

$$F_{1y} = 200 \cos 30^\circ N = 173 N = 173 N \uparrow$$

Hence, from the slope triangle, we have:

$$\theta = \tan^{-1} \left(\frac{5}{12} \right)$$

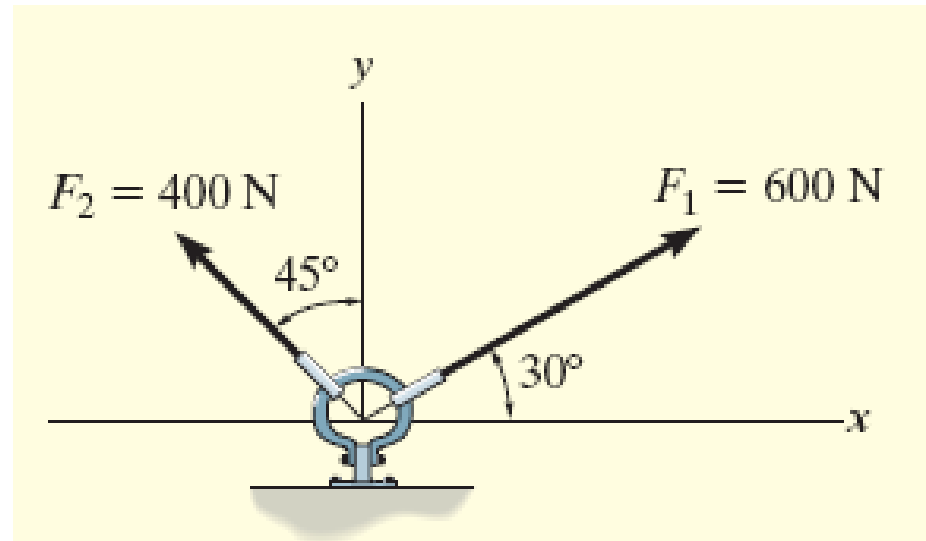
Cartesian Vector Notation

$$F_1 = [-100i + 173j] N$$

$$F_2 = [240i - 100j] N$$

Example 2.6

The link is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and orientation of the resultant force.



Solution I

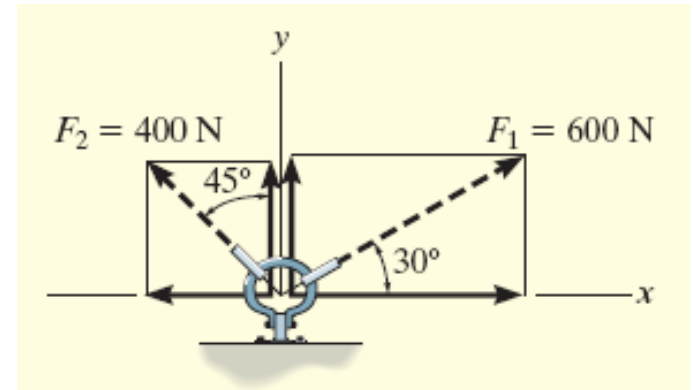
Scalar Notation:

$$F_{Rx} = \Sigma F_x :$$

$$\begin{aligned} F_{Rx} &= 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N} \\ &= 236.8 \text{ N} \rightarrow \end{aligned}$$

$$F_{Ry} = \Sigma F_y :$$

$$\begin{aligned} F_{Ry} &= 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N} \\ &= 582.8 \text{ N} \uparrow \end{aligned}$$



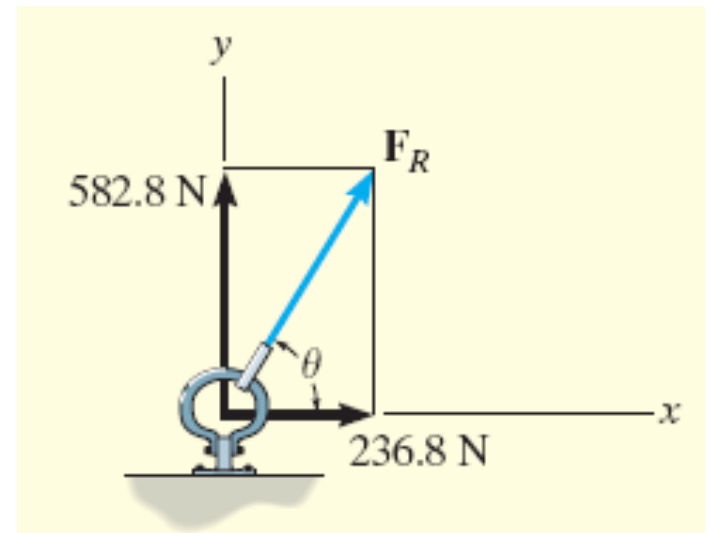
Solution I

Resultant Force

$$F_R = \sqrt{(236.8N)^2 + (582.8N)^2}$$
$$= 629N$$

From vector addition, direction angle θ is

$$\theta = \tan^{-1} \left(\frac{582.8N}{236.8N} \right)$$
$$= 67.9^\circ$$



Solution II

Cartesian Vector Notation

$$\mathbf{F}_1 = \{ 600\cos 30^\circ \mathbf{i} + 600\sin 30^\circ \mathbf{j} \} \text{ N}$$

$$\mathbf{F}_2 = \{ -400\sin 45^\circ \mathbf{i} + 400\cos 45^\circ \mathbf{j} \} \text{ N}$$

Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= (600\cos 30^\circ \text{N} - 400\sin 45^\circ \text{N})\mathbf{i} \\ &\quad + (600\sin 30^\circ \text{N} + 400\cos 45^\circ \text{N})\mathbf{j} \\ &= \{236.8\mathbf{i} + 582.8\mathbf{j}\}\text{N}\end{aligned}$$

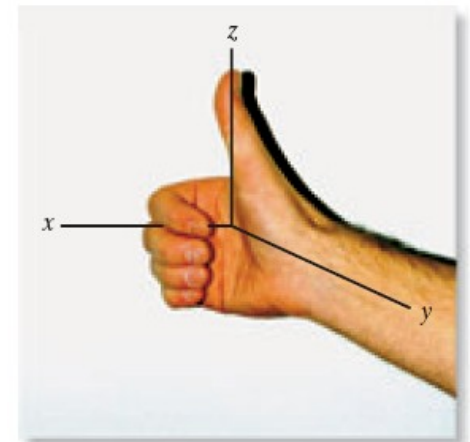
The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

2.5 Cartesian Vectors

- Right-Handed Coordinate System

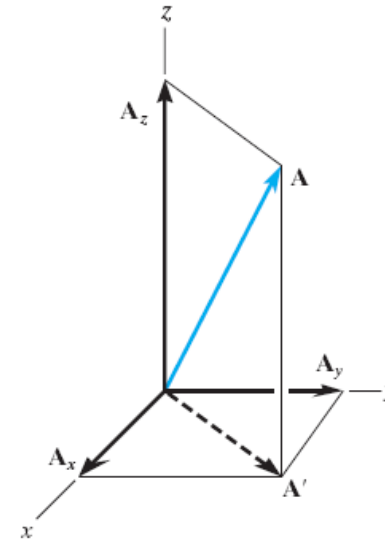
A rectangular or Cartesian coordinate system is said to be right-handed provided:

- Thumb of right hand points in the direction of the positive z axis
- z -axis for the 2D problem would be perpendicular, directed out of the page.



2.5 Cartesian Vectors

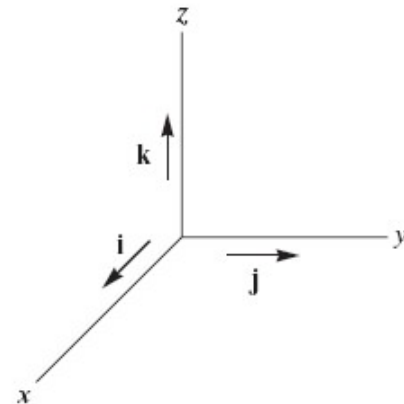
- Rectangular Components of a Vector
 - A vector **A** may have one, two or three rectangular components along the x, y and z axes, depending on orientation
 - By two successive application of the parallelogram law
$$\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$$
$$\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$$
 - Combining the equations, A can be expressed as
$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$



2.5 Cartesian Vectors

- Unit Vector
 - Direction of **A** can be specified using a unit vector
 - Unit vector has a magnitude of 1
 - If **A** is a vector having a magnitude of $A \neq 0$, unit vector having the same direction as **A** is expressed by $\mathbf{u}_A = \mathbf{A} / A$. So that

$$\mathbf{A} = A \mathbf{u}_A$$

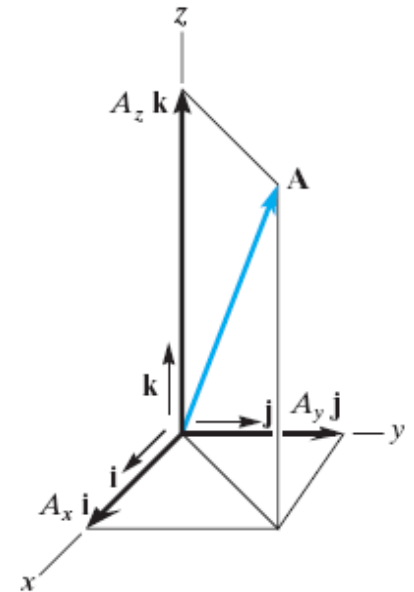


2.5 Cartesian Vectors

- Cartesian Vector Representations
 - 3 components of **A** act in the positive **i**, **j** and **k** directions

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

*Note the magnitude and direction of each components are separating vector algebraic operations.



2.5 Cartesian Vectors

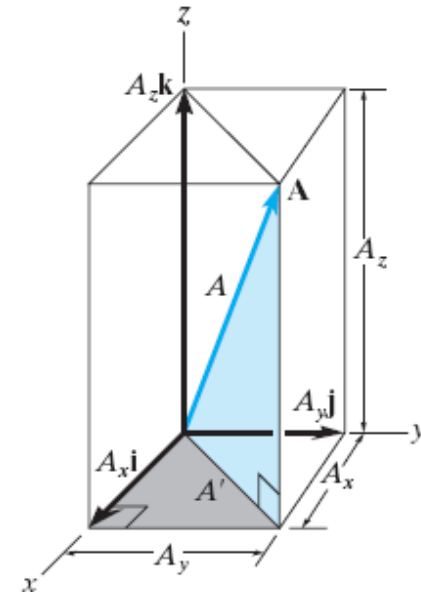
- Magnitude of a Cartesian Vector

- From the colored triangle, $A = \sqrt{A'^2 + A_z^2}$

- From the shaded triangle, $A' = \sqrt{A_x^2 + A_y^2}$

- Combining the equations gives magnitude of **A**

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

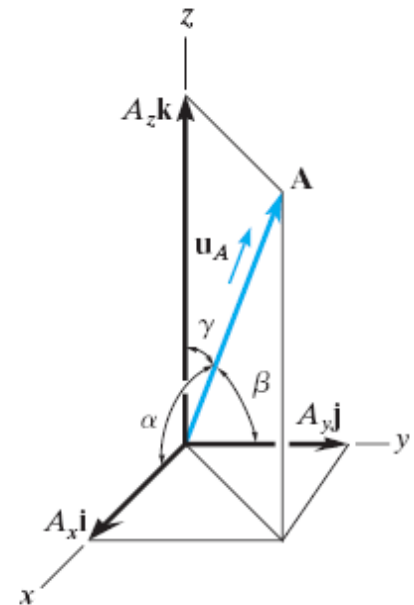


2.5 Cartesian Vectors

- Direction of a Cartesian Vector
 - Orientation of A is defined as the coordinate direction angles α , β and γ measured between the tail of A and the positive x , y and z axes
 - $0^\circ \leq \alpha, \beta \text{ and } \gamma \leq 180^\circ$
 - The *direction cosines* of $\mathbf{A}$ is

$$\cos \alpha = \frac{A_x}{A} \quad \cos \gamma = \frac{A_z}{A}$$

$$\cos \beta = \frac{A_y}{A}$$



2.5 Cartesian Vectors

- Direction of a Cartesian Vector
 - Angles α , β and γ can be determined by the inverse cosines

Given

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

then,

$$\mathbf{u}_A = \mathbf{A} / A = (A_x/A) \mathbf{i} + (A_y/A) \mathbf{j} + (A_z/A) \mathbf{k}$$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

2.5 Cartesian Vectors

- Direction of a Cartesian Vector

- $\mathbf{u}_A$ can also be expressed as

$$\mathbf{u}_A = \cos\alpha \mathbf{i} + \cos\beta \mathbf{j} + \cos\gamma \mathbf{k}$$

- Since $\sqrt{A_x^2 + A_y^2 + A_z^2} = A$ and $\mathbf{u}_A = \frac{\mathbf{A}}{A}$ we have

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

- $\mathbf{A}$ as expressed in Cartesian vector form is

$$\mathbf{A} = A\mathbf{u}_A$$

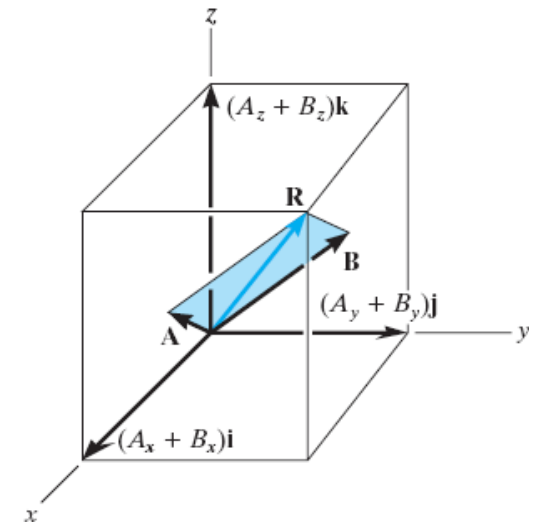
$$= A\cos\alpha \mathbf{i} + A\cos\beta \mathbf{j} + A\cos\gamma \mathbf{k}$$

$$= A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

2.6 Addition and Subtraction of Cartesian Vectors

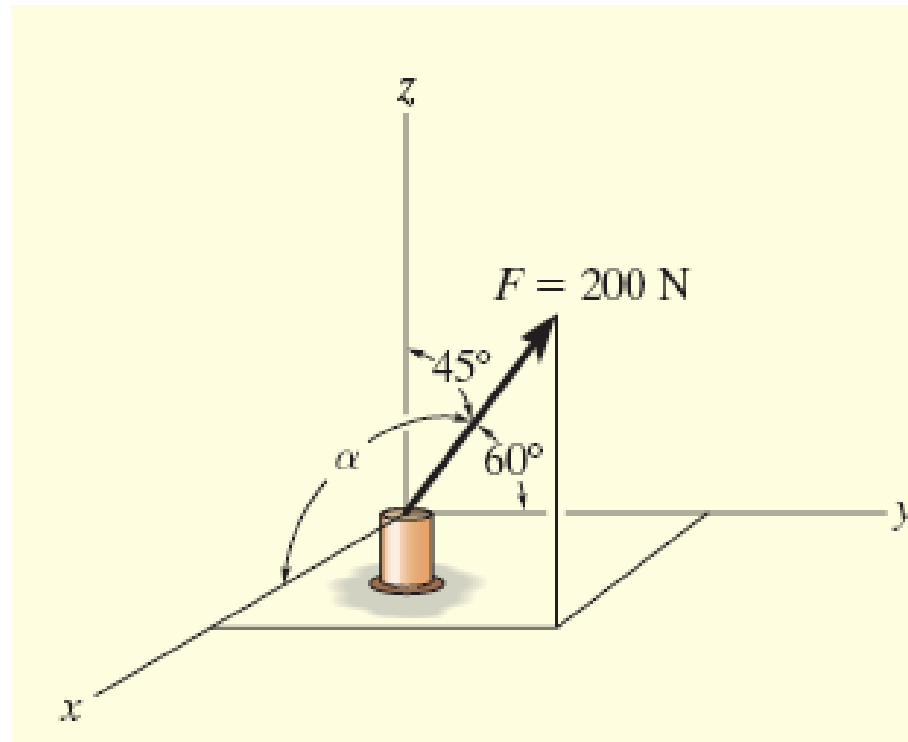
- Concurrent Force Systems
 - Force resultant is the vector sum of all the forces in the system

$$\mathbf{F}_R = \sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k}$$



Example 2.8

Express the force $\mathbf{F}$ as Cartesian vector.



Solution

Since two angles are specified, the third angle is found by

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 \alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos \alpha = \sqrt{1 - (0.5)^2 - (0.707)^2} = \pm 0.5$$

Two possibilities exist, namely

$$\alpha = \cos^{-1}(0.5) = 60^\circ$$

$$\alpha = \cos^{-1}(-0.5) = 120^\circ$$

Solution

By inspection, $\alpha = 60^\circ$ since $\mathbf{F}_x$ is in the +x direction

Given $F = 200\text{N}$

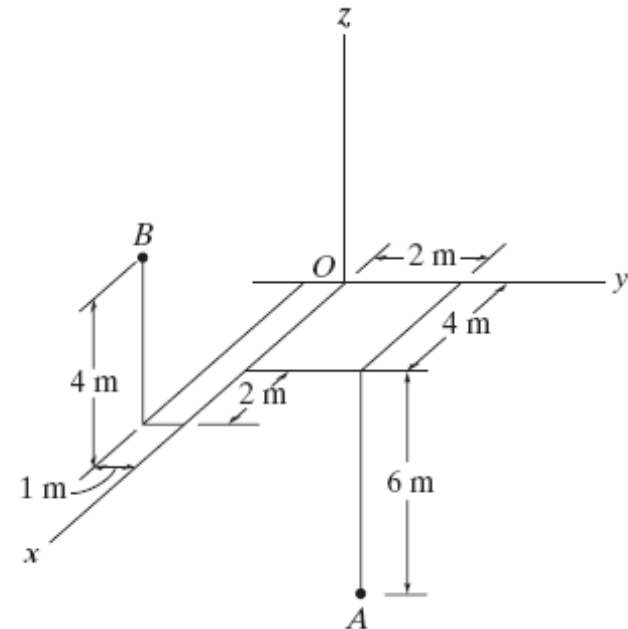
$$\begin{aligned}\mathbf{F} &= F\cos\alpha\mathbf{i} + F\cos\beta\mathbf{j} + F\cos\gamma\mathbf{k} \\ &= (200\cos 60^\circ\text{N})\mathbf{i} + (200\cos 60^\circ\text{N})\mathbf{j} \\ &\quad + (200\cos 45^\circ\text{N})\mathbf{k} \\ &= \{100.0\mathbf{i} + 100.0\mathbf{j} + 141.4\mathbf{k}\}\text{N}\end{aligned}$$

Checking:

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(100.0)^2 + (100.0)^2 + (141.4)^2} = 200\text{N}\end{aligned}$$

2.7 Position Vectors

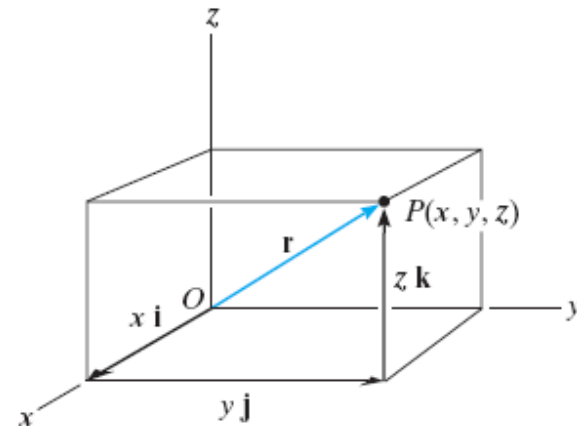
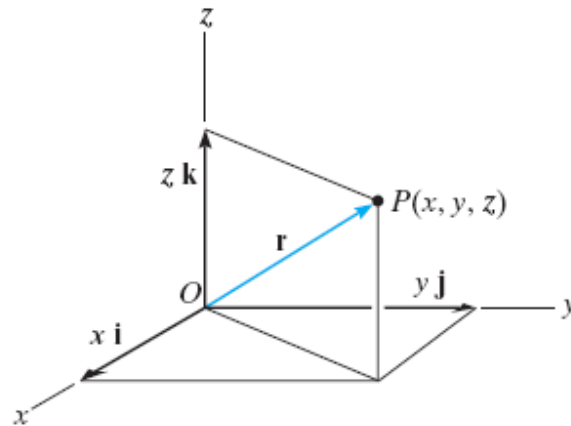
- x, y, z Coordinates
 - Right-handed coordinate system
 - Positive z axis points upwards, measuring the height of an object or the altitude of a point
 - Points are measured relative to the origin, O .



2.7 Position Vectors

Position Vector

- Position vector $\mathbf{r}$ is defined as a fixed vector which locates a point in space relative to another point.
- E.g. $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$



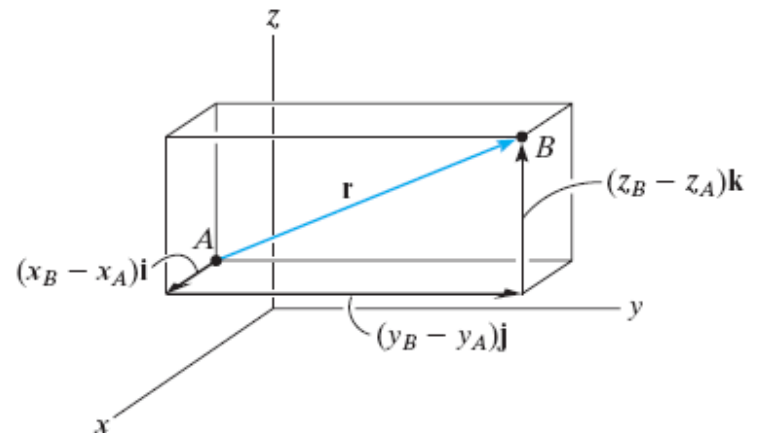
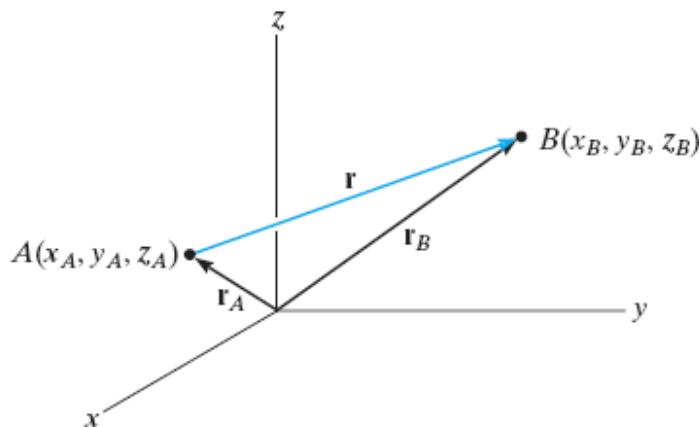
2.7 Position Vectors

Position Vector

- Vector addition gives $\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$
- Solving

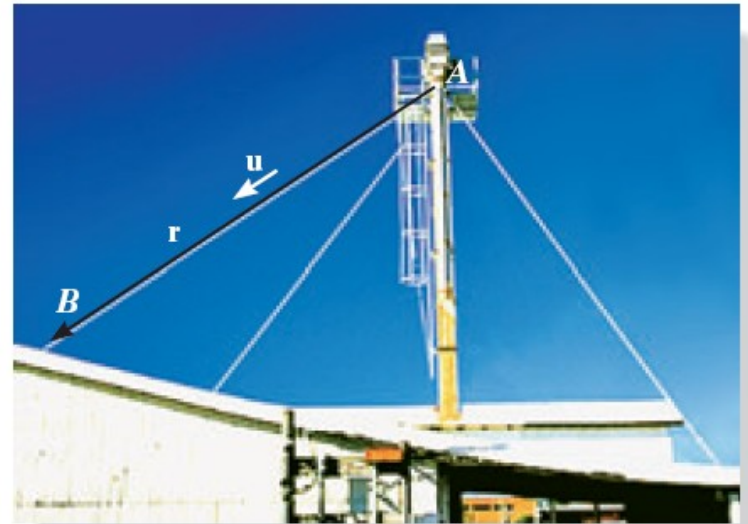
$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

$$\text{or } \mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$



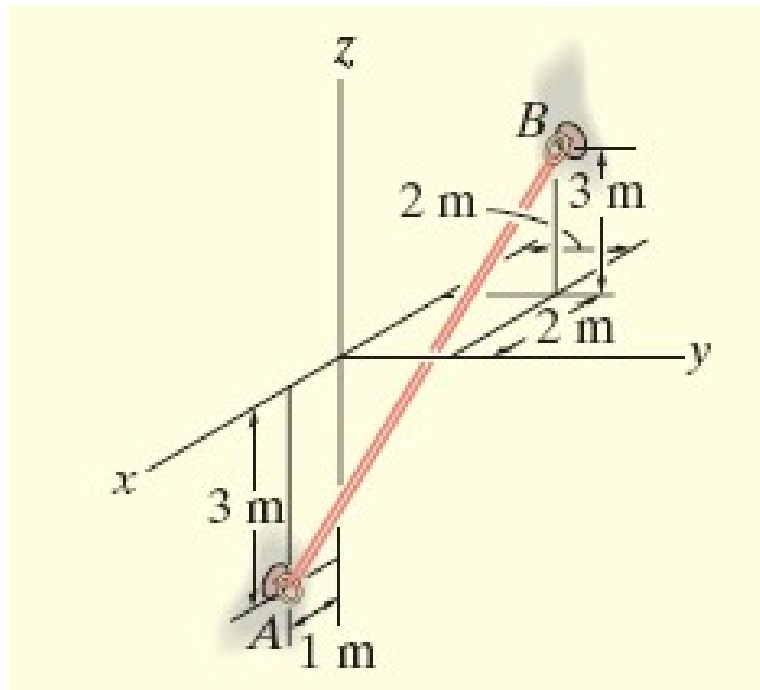
2.7 Position Vectors

- Length and direction of cable AB can be found by measuring A and B using the x, y, z axes
- Position vector $\mathbf{r}$ can be established
- Magnitude r represent the length of cable
- Angles, α , β and γ represent the direction of the cable
- Unit vector, $\mathbf{u} = \mathbf{r}/r$



Example 2.12

An elastic rubber band is attached to points A and B. Determine its length and its direction measured from A towards B.



Solution

Position vector

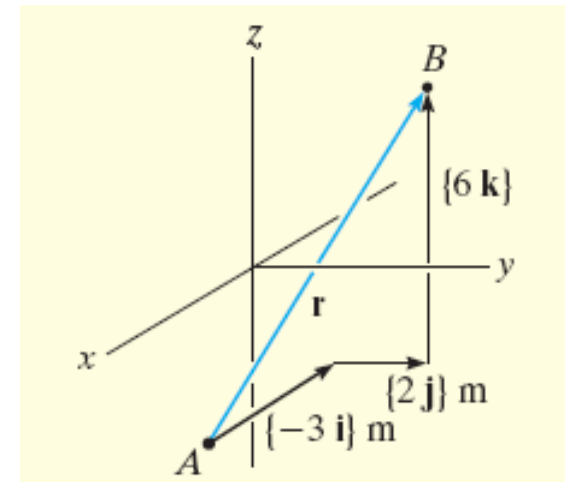
$$\begin{aligned} \mathbf{r} &= [-2\text{m} - 1\text{m}]\mathbf{i} + [2\text{m} - 0]\mathbf{j} + [3\text{m} - (-3\text{m})]\mathbf{k} \\ &= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{m} \end{aligned}$$

Magnitude = length of the rubber band

$$r = \sqrt{(-3)^2 + (2)^2 + (6)^2} = 7\text{m}$$

Unit vector in the direction of $\mathbf{r}$

$$\begin{aligned} \mathbf{u} &= \mathbf{r} / r \\ &= -3/7\mathbf{i} + 2/7\mathbf{j} + 6/7\mathbf{k} \end{aligned}$$

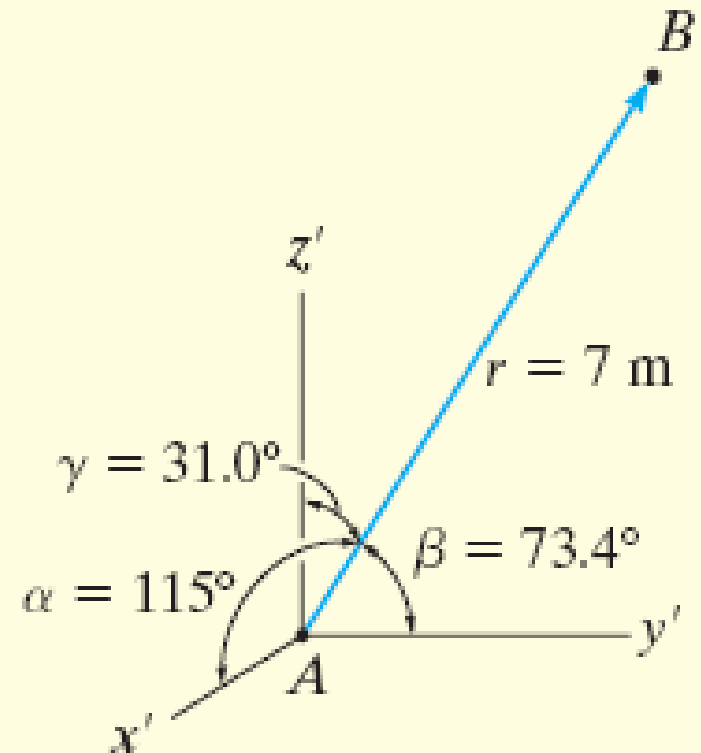


Solution

$$\alpha = \cos^{-1}(-3/7) = 115^\circ$$

$$\beta = \cos^{-1}(2/7) = 73.4^\circ$$

$$\gamma = \cos^{-1}(6/7) = 31.0^\circ$$

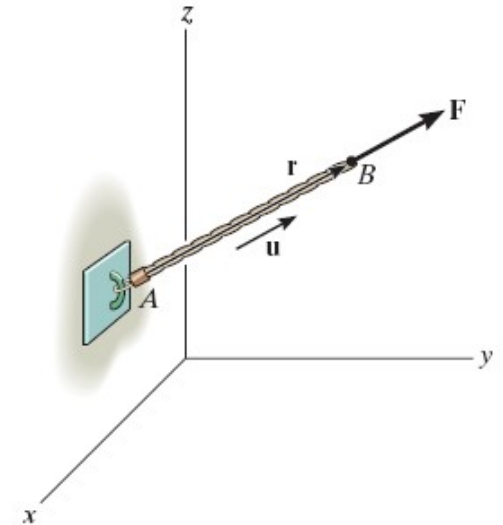


2.8 Force Vector Directed along a Line

- In 3D problems, direction of $\mathbf{F}$ is specified by 2 points, through which its line of action lies
- $\mathbf{F}$ can be formulated as a Cartesian vector

$$\mathbf{F} = F \mathbf{u} = F (\mathbf{r}/r)$$

- Note that $\mathbf{F}$ has units of forces (N) unlike $\mathbf{r}$, with units of length (m)



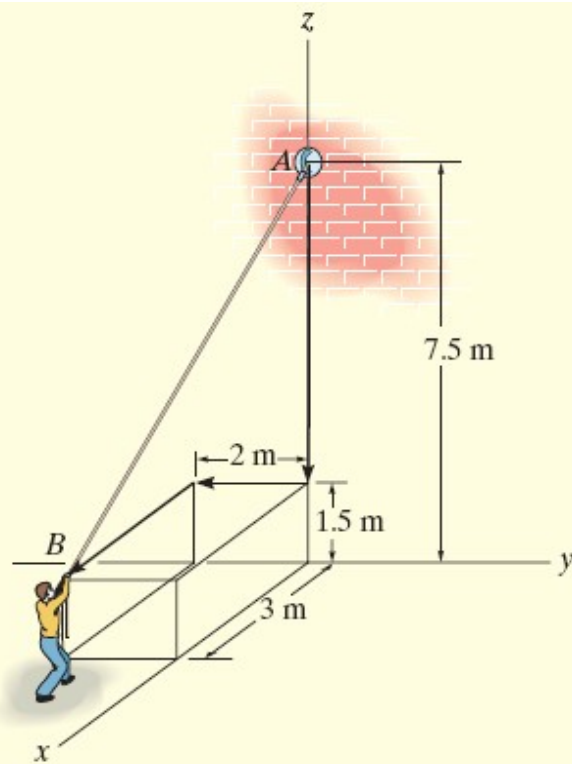
2.8 Force Vector Directed along a Line

- Force $\mathbf{F}$ acting along the chain can be presented as a Cartesian vector by
 - Establish x, y, z axes
 - Form a position vector $\mathbf{r}$ along length of chain
- Unit vector, $\mathbf{u} = \mathbf{r}/r$ that defines the direction of both the chain and the force
- We get $\mathbf{F} = Fu$



Example 2.13

The man pulls on the cord with a force of 350 N. Represent this force acting on the support A, as a Cartesian vector and determine its direction.



Solution

End points of the cord are A (0m, 0m, 7.5m) and B (3m, -2m, 1.5m)

$$\begin{aligned}\mathbf{r} &= (3\text{m} - 0\text{m})\mathbf{i} + (-2\text{m} - 0\text{m})\mathbf{j} + (1.5\text{m} - 7.5\text{m})\mathbf{k} \\ &= \{3\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}\}\text{m}\end{aligned}$$

Magnitude = length of cord AB

$$r = \sqrt{(3\text{m})^2 + (-2\text{m})^2 + (-6\text{m})^2} = 7\text{m}$$

Unit vector,

$$\begin{aligned}\mathbf{u} &= \mathbf{r} / r \\ &= 3/7\mathbf{i} - 2/7\mathbf{j} - 6/7\mathbf{k}\end{aligned}$$

Solution

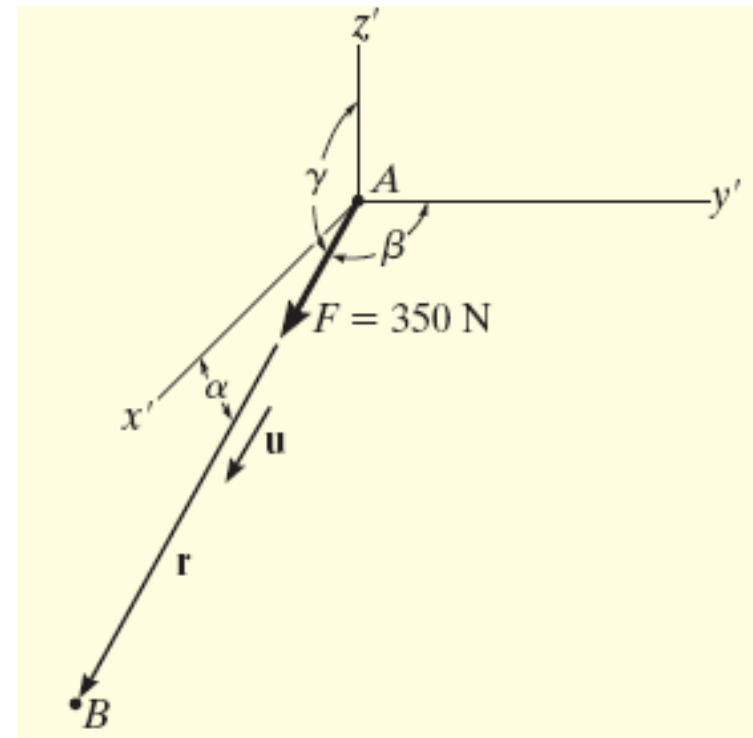
Force $\mathbf{F}$ has a magnitude of 350N, direction specified by $\mathbf{u}$.

$$\begin{aligned}\mathbf{F} &= F\mathbf{u} \\ &= 350\text{N}\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) \\ &= \{150\mathbf{i} - 100\mathbf{j} - 300\mathbf{k}\} \text{ N}\end{aligned}$$

$$\alpha = \cos^{-1}\left(\frac{3}{7}\right) = 64.6^\circ$$

$$\beta = \cos^{-1}\left(-\frac{2}{7}\right) = 107^\circ$$

$$\gamma = \cos^{-1}\left(-\frac{6}{7}\right) = 149^\circ$$

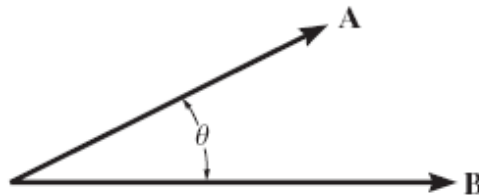


2.9 Dot Product

- Dot product of vectors **A** and **B** is written as **A·B** (Read **A** dot **B**)
- Define the magnitudes of **A** and **B** and the angle between their tails

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \quad \text{where } 0^\circ \leq \theta \leq 180^\circ$$

- Referred to as scalar product of vectors as result is a scalar



2.9 Dot Product

- Laws of Operation

1. Commutative law

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

2. Multiplication by a scalar

$$a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})a$$

3. Distribution law

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$$

2.9 Dot Product

- Cartesian Vector Formulation
 - Dot product of Cartesian unit vectors

$$\mathbf{i} \cdot \mathbf{i} = (1)(1)\cos 0^\circ = 1$$

$$\mathbf{i} \cdot \mathbf{j} = (1)(1)\cos 90^\circ = 0$$

- Similarly

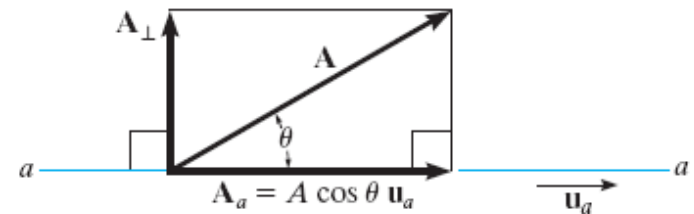
$$\mathbf{i} \cdot \mathbf{i} = 1 \quad \mathbf{j} \cdot \mathbf{j} = 1 \quad \mathbf{k} \cdot \mathbf{k} = 1$$

$$\mathbf{i} \cdot \mathbf{j} = 0 \quad \mathbf{i} \cdot \mathbf{k} = 0 \quad \mathbf{j} \cdot \mathbf{k} = 0$$

2.9 Dot Product

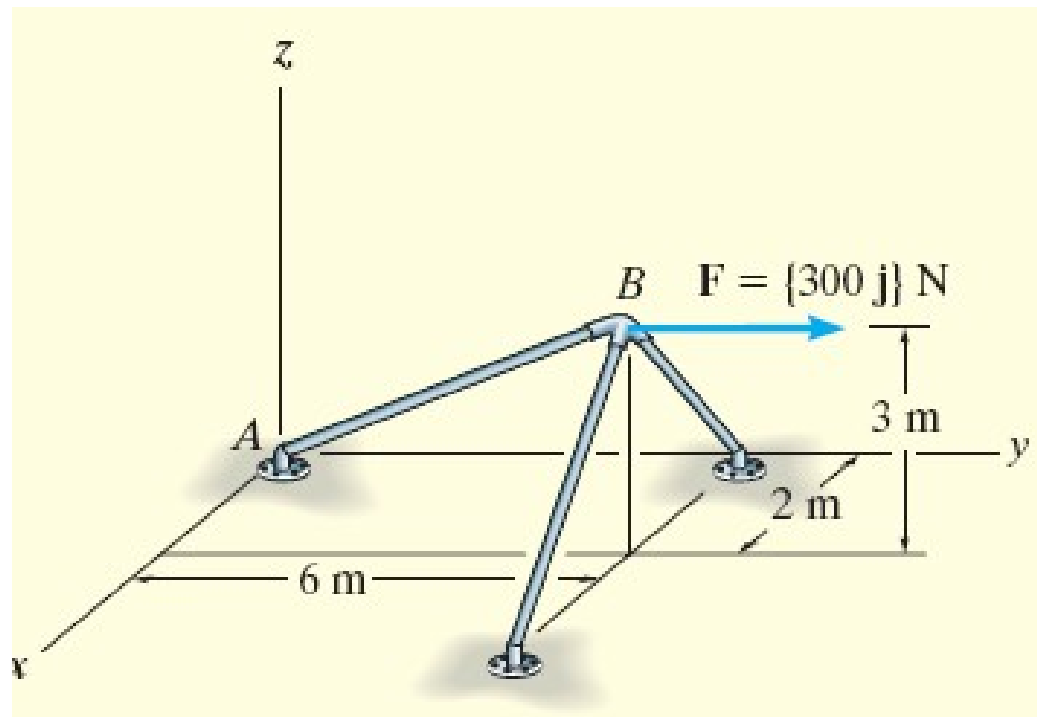
- Cartesian Vector Formulation
 - Dot product of 2 vectors **A** and **B**
$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$
- Applications
 - ***The angle formed between two vectors or intersecting lines.***
$$\theta = \cos^{-1} [(\mathbf{A} \cdot \mathbf{B}) / (AB)] \quad 0^\circ \leq \theta \leq 180^\circ$$
 - ***The components of a vector parallel and perpendicular to a line.***

$$A_a = A \cos \theta = \mathbf{A} \cdot \mathbf{u}_a$$



Example 2.17

The frame is subjected to a horizontal force $\mathbf{F} = \{300\mathbf{j}\}$ N. Determine the components of this force parallel and perpendicular to the member AB.



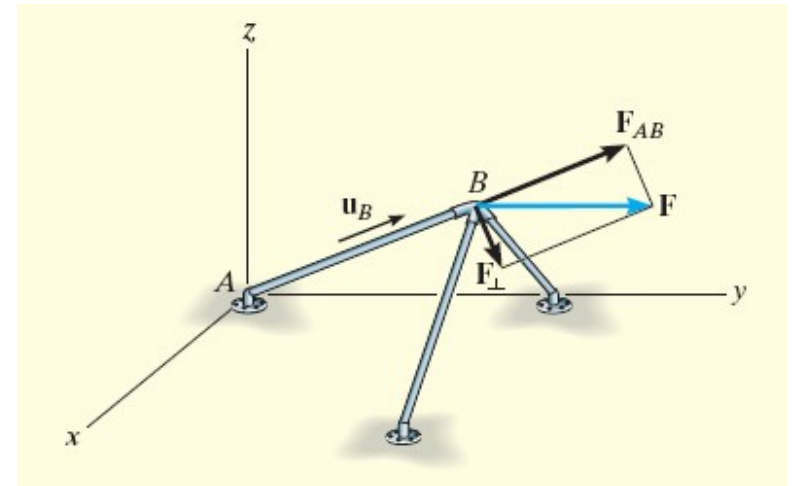
Solution

Since

$$\begin{aligned} \vec{u}_B &= \frac{\vec{r}_B}{|\vec{r}_B|} = \frac{2\vec{i} + 6\vec{j} + 3\vec{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} \\ &= 0.286\vec{i} + 0.857\vec{j} + 0.429\vec{k} \end{aligned}$$

Thus

$$\begin{aligned} |F_{AB}| &= |F| \cos \theta \\ &= F \cdot \vec{u}_B = (300\vec{j}) \cdot (0.286\vec{i} + 0.857\vec{j} + 0.429\vec{k}) \\ &= (0)(0.286) + (300)(0.857) + (0)(0.429) \\ &= 257.1\text{N} \end{aligned}$$



Solution

Since result is a positive scalar, $\mathbf{F}_{AB}$ has the same sense of direction as $\mathbf{u}_B$. Express in Cartesian form

$$\begin{aligned}\vec{F}_{AB} &= |\vec{F}_{AB}| \vec{u}_{AB} \\ &= (257.1\text{N})(0.286\vec{i} + 0.857\vec{j} + 0.429\vec{k}) \\ &= \{73.5\vec{i} + 220\vec{j} + 110\vec{k}\}\text{N}\end{aligned}$$

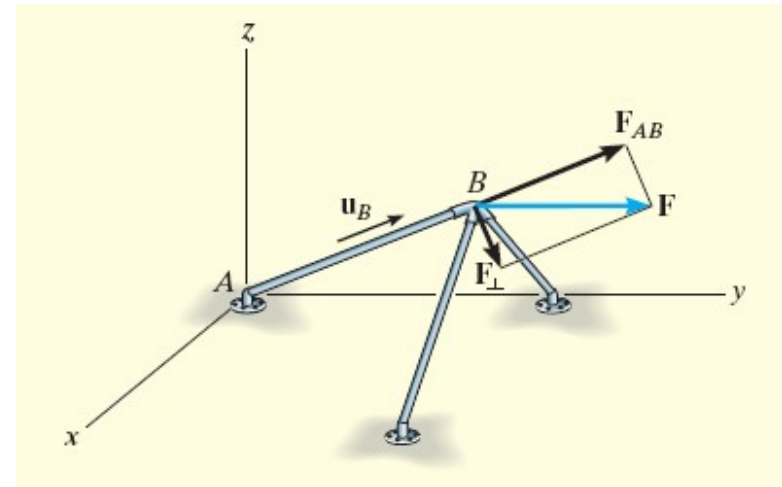
Perpendicular component

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{AB} = 300\vec{j} - (73.5\vec{i} + 220\vec{j} + 110\vec{k}) = \{-73.5\vec{i} + 80\vec{j} - 110\vec{k}\}\text{N}$$

Solution

Magnitude can be determined from $\mathbf{F}_\perp$ or from Pythagorean Theorem,

$$\begin{aligned} |\vec{F}_\perp| &= \sqrt{|\vec{F}|^2 - |\vec{F}_{AB}|^2} \\ &= \sqrt{(300\text{N})^2 - (257.1\text{N})^2} \\ &= 155\text{N} \end{aligned}$$



QUIZ

1. Which one of the following is a scalar quantity?
A) Force B) Position C) Mass D) Velocity
2. For vector addition, you have to use _____ law.
A) Newton's Second
B) the arithmetic
C) Pascal's
D) the parallelogram

QUIZ

3. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?

A) Yes, but not uniquely.

B) No.

C) Yes, uniquely.

4. Can you resolve a 2-D vector along three directions (say at 0 , 60 , and 120°)?

A) Yes, but not uniquely.

B) No.

C) Yes, uniquely.

QUIZ

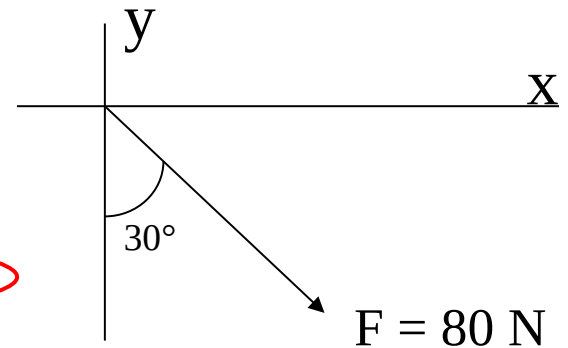
5. Resolve $\mathbf{F}$ along x and y axes and write it in vector form. $\mathbf{F} = \{ \text{_____} \}$ N

A) $80 \cos (30^\circ) \mathbf{i} - 80 \sin (30^\circ) \mathbf{j}$

B) $80 \sin (30^\circ) \mathbf{i} + 80 \cos (30^\circ) \mathbf{j}$

C) $80 \sin (30^\circ) \mathbf{i} - 80 \cos (30^\circ) \mathbf{j}$

D) $80 \cos (30^\circ) \mathbf{i} + 80 \sin (30^\circ) \mathbf{j}$



6. Determine the magnitude of the resultant $(\mathbf{F}_1 + \mathbf{F}_2)$ force in N when $\mathbf{F}_1 = \{ 10\mathbf{i} + 20\mathbf{j} \}$ N and $\mathbf{F}_2 = \{ 20\mathbf{i} + 20\mathbf{j} \}$ N .

A) 30 N

B) 40 N

C) 50 N

D) 60 N

E) 70 N

QUIZ

7. Vector algebra, as we are going to use it, is based on

a _____ coordinate system.

- A) Euclidean B) Left-handed
C) Greek D) Right-handed E) Egyptian

8. The symbols α , β , and γ designate the _____ of

a 3-D Cartesian vector.

- A) Unit vectors B) Coordinate direction angles
C) Greek societies D) X, Y and Z components

QUIZ

9. What is not true about an unit vector, $\mathbf{u_A}$?

A) It is dimensionless.

B) Its magnitude is one.

C) It always points in the direction of positive X- axis.

D) It always points in the direction of vector $\mathbf{A}$.

10. If $\mathbf{F} = \{10 \mathbf{i} + 10 \mathbf{j} + 10 \mathbf{k}\}$ N and

$\mathbf{G} = \{20 \mathbf{i} + 20 \mathbf{j} + 20 \mathbf{k}\}$ N, then $\mathbf{F} + \mathbf{G} = \{ \text{_____} \}$ N

A) $10 \mathbf{i} + 10 \mathbf{j} + 10 \mathbf{k}$

B) $30 \mathbf{i} + 20 \mathbf{j} + 30 \mathbf{k}$

C) $-10 \mathbf{i} - 10 \mathbf{j} - 10 \mathbf{k}$

D) $30 \mathbf{i} + 30 \mathbf{j} + 30 \mathbf{k}$

QUIZ

11. A position vector, $\mathbf{r}_{PQ}$, is obtained by

- A) Coordinates of Q minus coordinates of P
- B) Coordinates of P minus coordinates of Q
- C) Coordinates of Q minus coordinates of the origin
- D) Coordinates of the origin minus coordinates of P

12. A force of magnitude F , directed along a unit vector $\mathbf{U}$, is given by $\mathbf{F} = \underline{\hspace{2cm}}$.

- A) $F(\mathbf{U})$
- B) $\mathbf{U} / F$
- C) $F / \mathbf{U}$
- D) $F + \mathbf{U}$
- E) $F - \mathbf{U}$

QUIZ

13. **P** and **Q** are two points in a 3-D space. How are the position vectors $\mathbf{r}_{PQ}$ and $\mathbf{r}_{QP}$ related?

- A) $\mathbf{r}_{PQ} = \mathbf{r}_{QP}$ B) $\mathbf{r}_{PQ} = -\mathbf{r}_{QP}$
C) $\mathbf{r}_{PQ} = 1/\mathbf{r}_{QP}$ D) $\mathbf{r}_{PQ} = 2\mathbf{r}_{QP}$

14. If $\mathbf{F}$ and $\mathbf{r}$ are force vector and position vectors, respectively, in SI units, what are the units of the expression $(\mathbf{r} * (\mathbf{F} / F))$?

- A) Newton B) Dimensionless
C) Meter D) Newton - Meter
E) The expression is algebraically illegal.

QUIZ

15. Two points in 3 – D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector $\mathbf{r}_{QP}$ is given by

A) $\{3 \mathbf{i} + 3 \mathbf{j} + 3 \mathbf{k}\} \text{ m}$

B) $\{-3 \mathbf{i} - 3 \mathbf{j} - 3 \mathbf{k}\} \text{ m}$

C) $\{5 \mathbf{i} + 7 \mathbf{j} + 9 \mathbf{k}\} \text{ m}$

D) $\{-3 \mathbf{i} + 3 \mathbf{j} + 3 \mathbf{k}\} \text{ m}$

E) $\{4 \mathbf{i} + 5 \mathbf{j} + 6 \mathbf{k}\} \text{ m}$

16. Force vector, $\mathbf{F}$, directed along a line PQ is given by

A) $(\mathbf{F}/F) \mathbf{r}_{PQ}$

B) $\mathbf{r}_{PQ}/r_{PQ}$

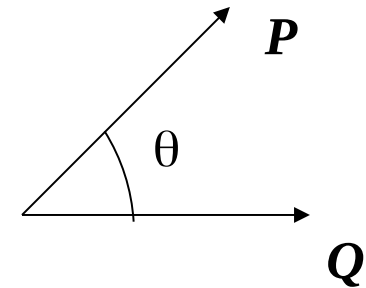
C) $F(\mathbf{r}_{PQ}/r_{PQ})$

D) $F(\mathbf{r}_{PQ}/r_{PQ})$

QUIZ

17. The dot product of two vectors ***P*** and ***Q*** is defined as

- A) $P Q \cos \theta$ B) $P Q \sin \theta$
C) $P Q \tan \theta$ D) $P Q \sec \theta$



18. The dot product of two vectors results in a _____ quantity.

- A) Scalar B) Vector
C) Complex D) Zero

QUIZ

19. If a dot product of two non-zero vectors is 0, then the two vectors must be _____ to each other.

- A) Parallel (pointing in the same direction)
- B) Parallel (pointing in the opposite direction)
- C) Perpendicular
- D) Cannot be determined.

20. If a dot product of two non-zero vectors equals -1, then the vectors must be _____ to each other.

- A) Parallel (pointing in the same direction)
- B) Parallel (pointing in the opposite direction)
- C) Perpendicular
- D) Cannot be determined.

QUIZ

1. The dot product can be used to find all of the following except _____.

A) sum of two vectors

B) angle between two vectors

C) component of a vector parallel to another line

D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors ***P*** and ***Q***.

$$\mathbf{P} = \{5 \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k}\} \text{ m}$$

$$\mathbf{Q} = \{-2 \mathbf{i} + 5 \mathbf{j} + 4 \mathbf{k}\} \text{ m}$$

A) -12 m

B) 12 m

C) 12 m²

D) -12 m²

E) 10 m²