

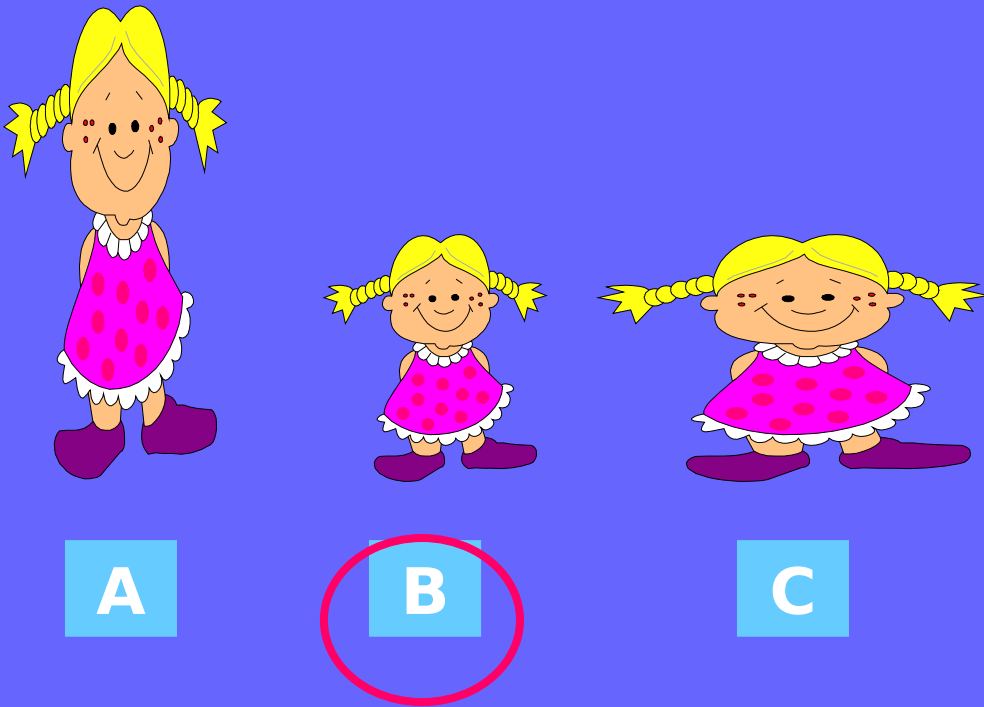
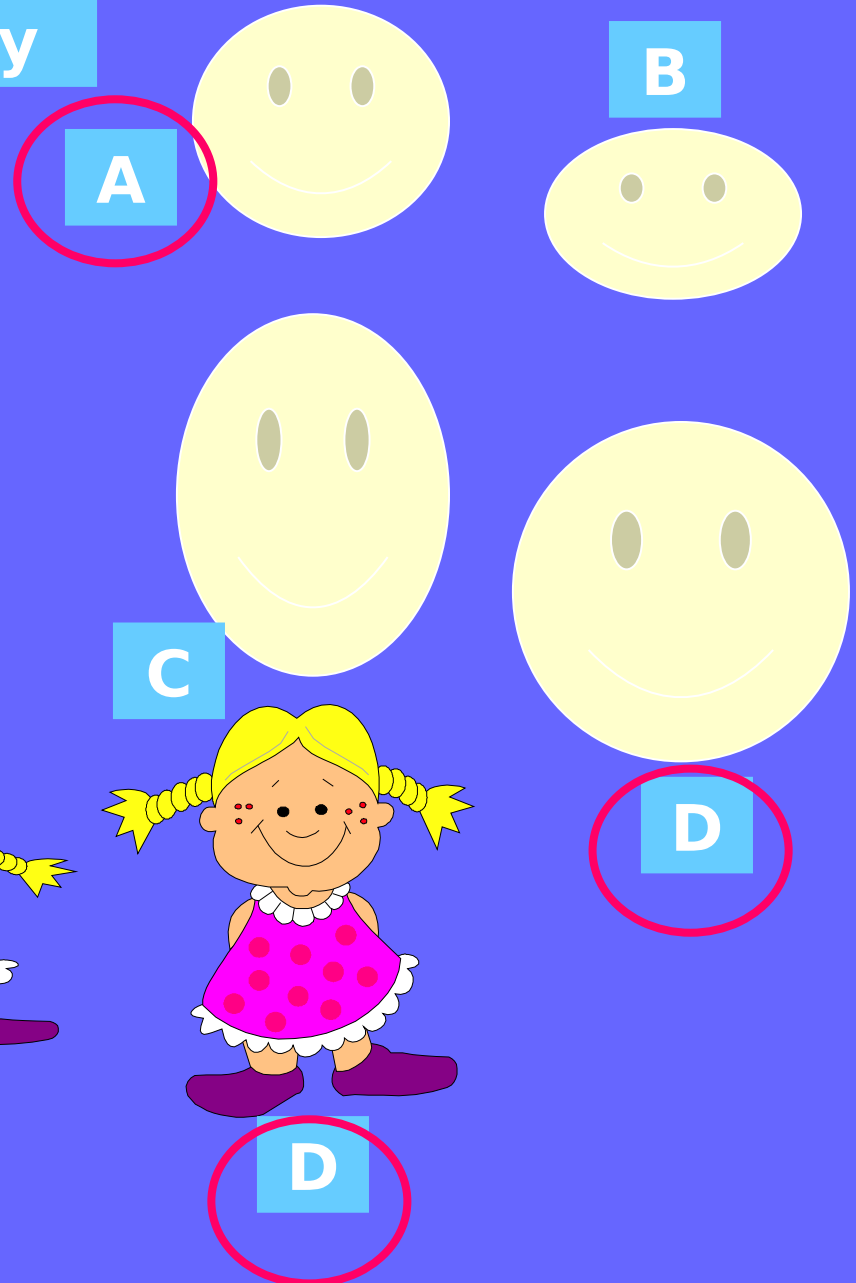
Similar Shapes

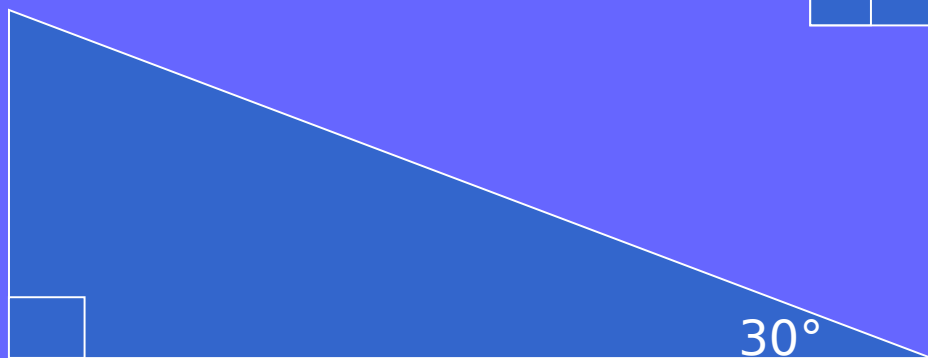
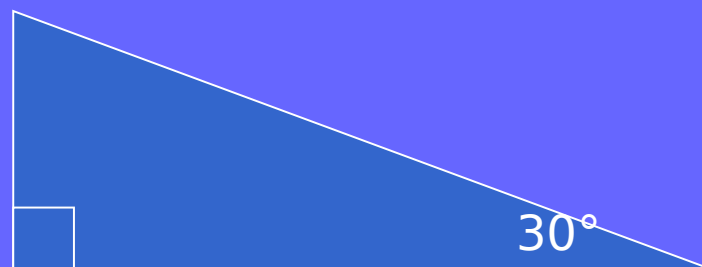
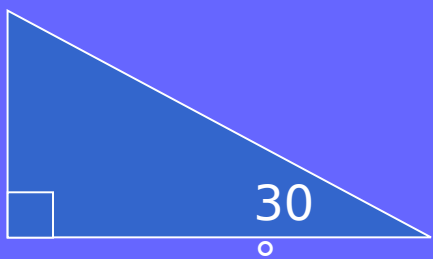
Outcomes

To understand the principles of similar shapes (especially triangles) and apply these to a range of questions at GCSE level

Similarity

Remember, similar shapes are always in proportion to each other. There is no distortion between them.



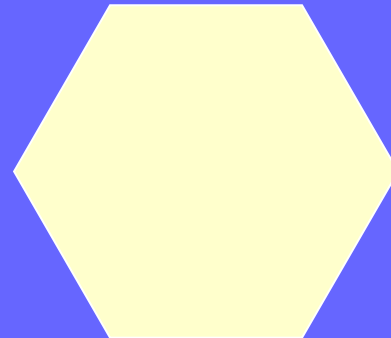
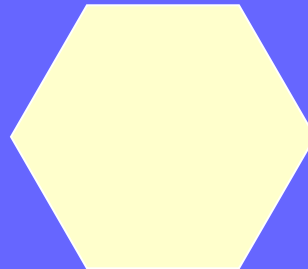
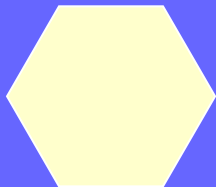


Conditions for similarity

Two shapes are similar only when:

- Corresponding sides are in proportion **and**
- Corresponding angles are equal

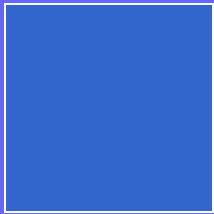
All regular
polygons are
similar



Conditions for similarity

Two shapes are similar only when:

- Corresponding sides are in proportion **and**
- Corresponding angles are equal

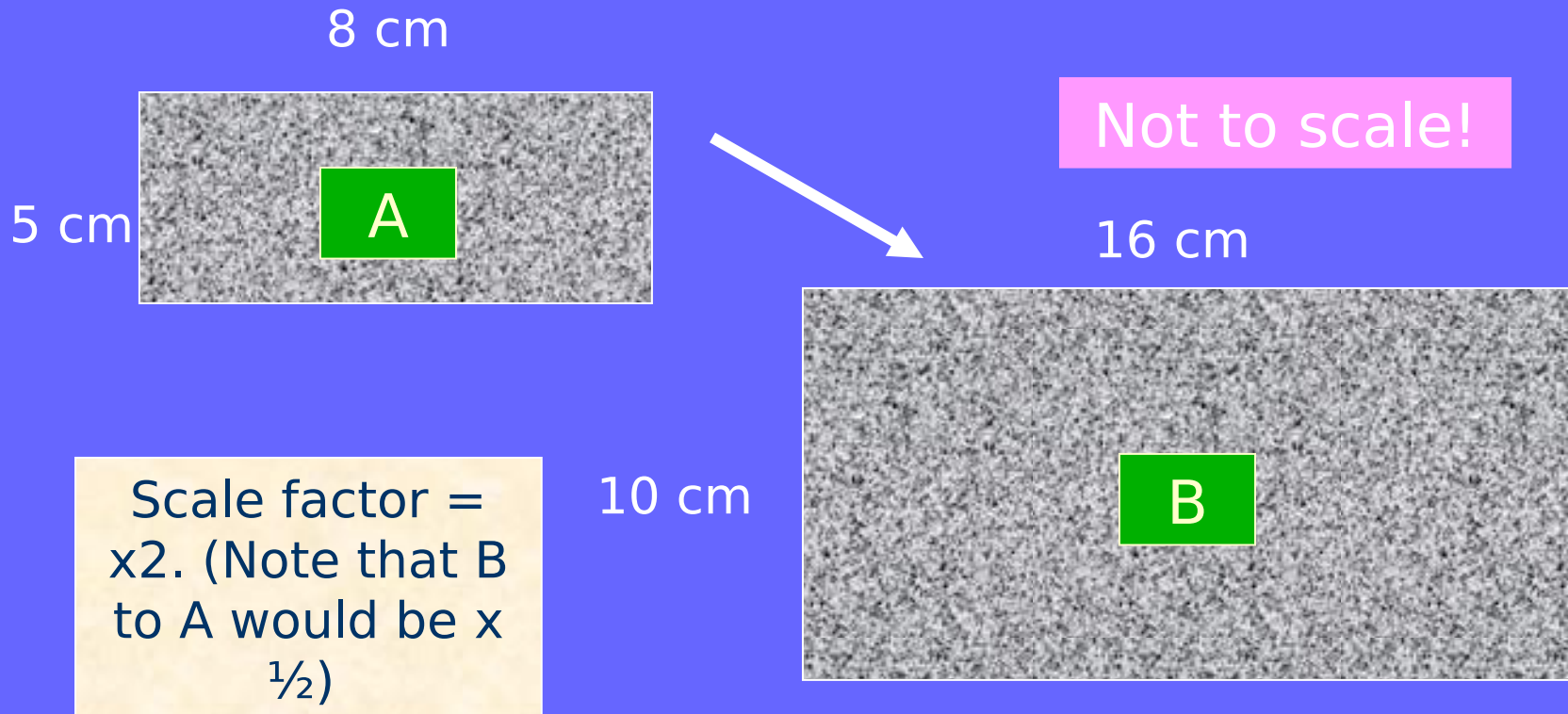


All rectangles are **not** similar to one another since only condition 2 is true.

If two objects are similar then one is an enlargement of the other

The rectangles below are **similar**:

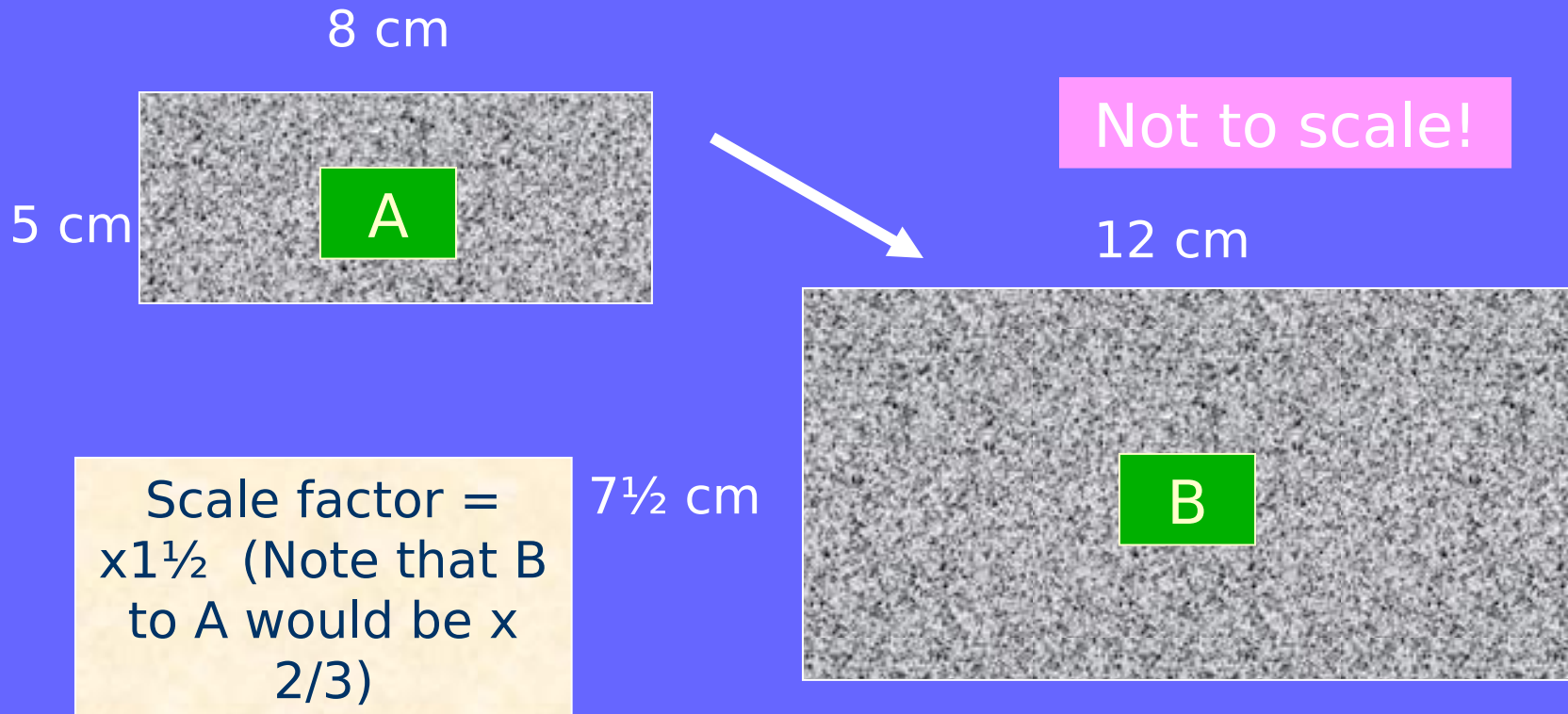
Find the scale factor of enlargement that maps A to B



If two objects are similar then one is an enlargement of the other

The rectangles below are **similar**:

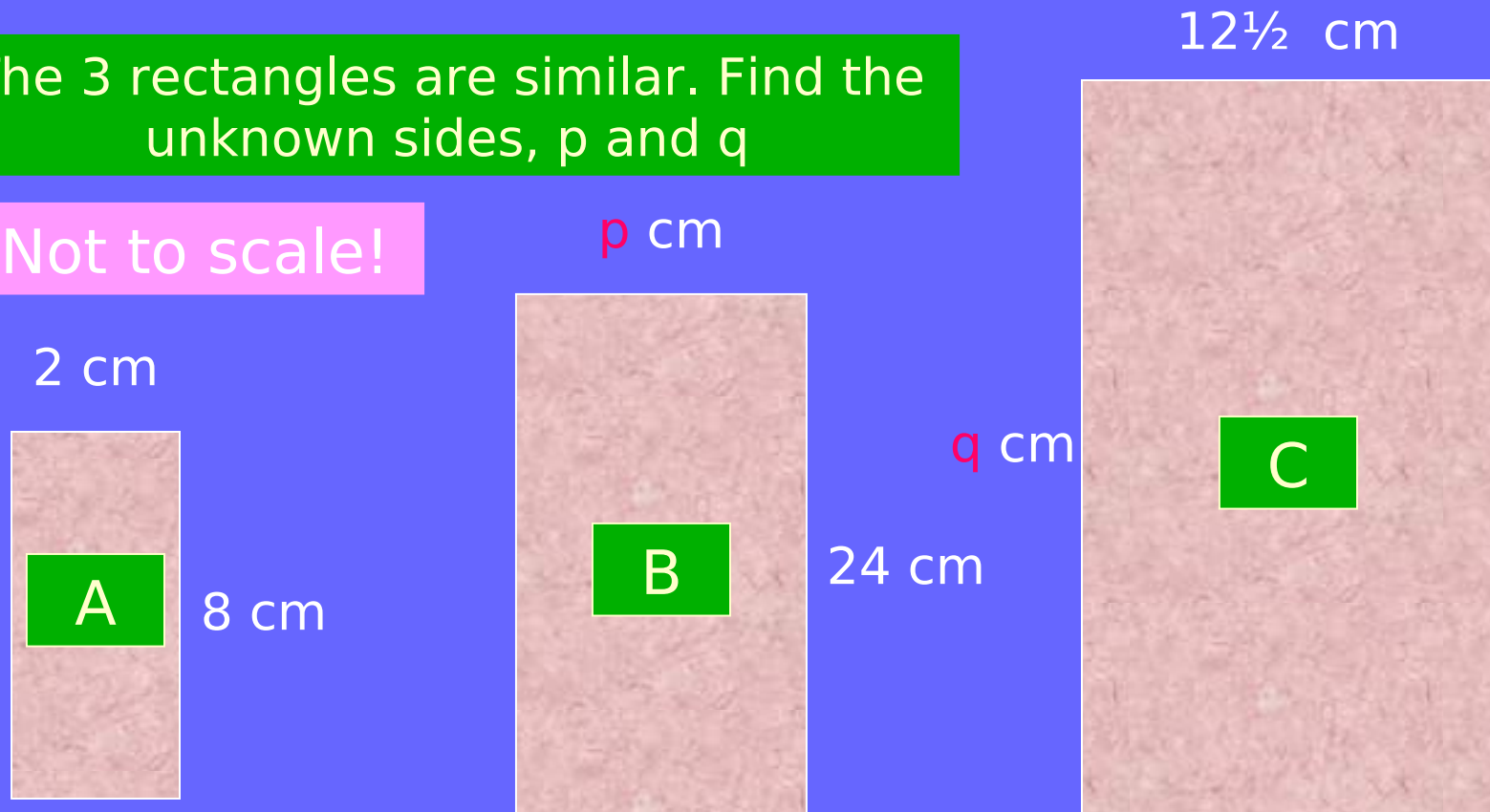
Find the scale factor of enlargement that maps A to B



If we are told that two objects **are similar** and we can find the **scale factor of enlargement** by **comparing corresponding sides** then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides, p and q

Not to scale!



1. Comparing corresponding sides in A and B.

$$SF = 24/8 = \times 3.$$

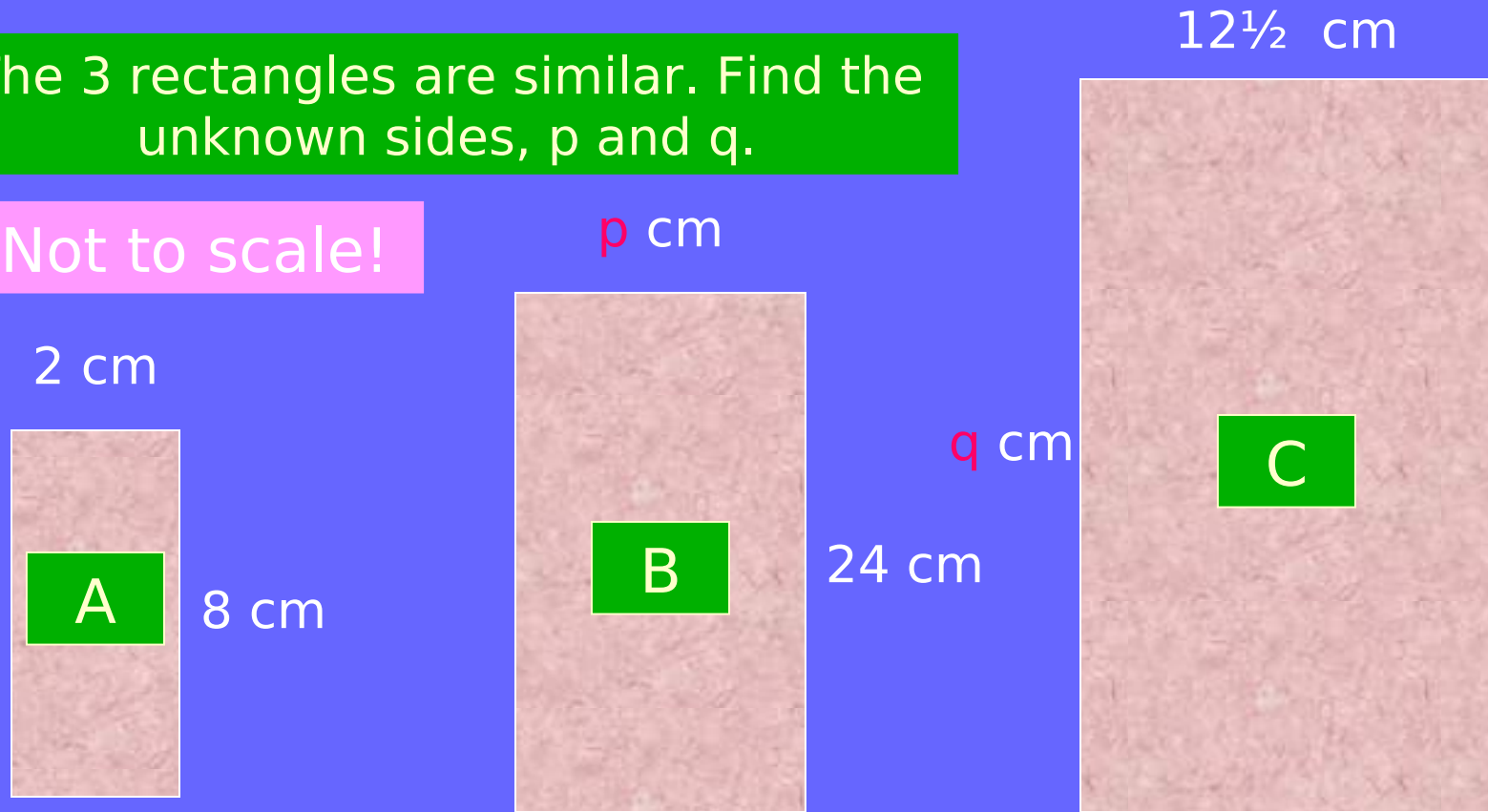
2. Apply the scale factor to find the unknown side.

$$p = 3 \times 2 = \underline{6} \text{ cm.}$$

If we are told that two objects **are similar** and we can find the **scale factor of enlargement** by **comparing corresponding sides** then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides, p and q.

Not to scale!



1. Comparing corresponding sides in A and C.

$$SF = 12.5/2 = 6.25$$

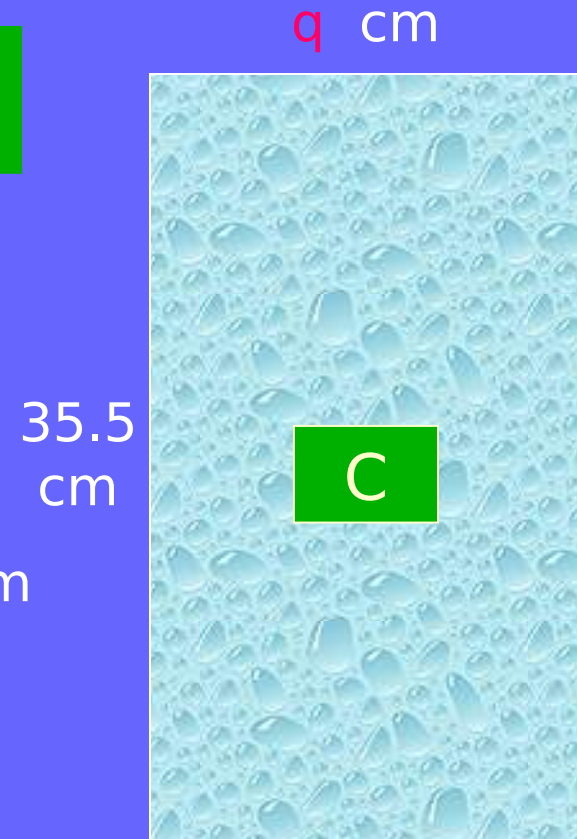
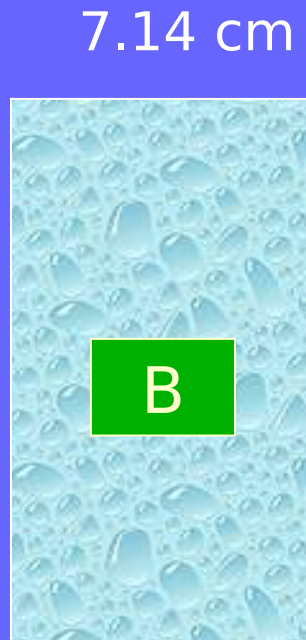
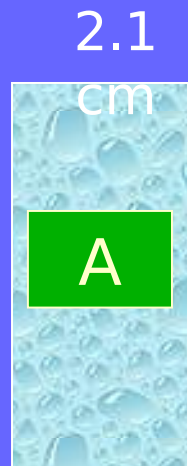
2. Apply the scale factor to find the unknown side.

$$q = 6.25 \times 8 = \underline{50} \text{ cm.}$$

If we are told that two objects **are similar** and we can find the **scale factor of enlargement** by **comparing corresponding sides** then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides p and q

Not to scale!



1. Comparing corresponding sides in A and B.

$$SF = 7.14/2.1 = \times 3.4.$$

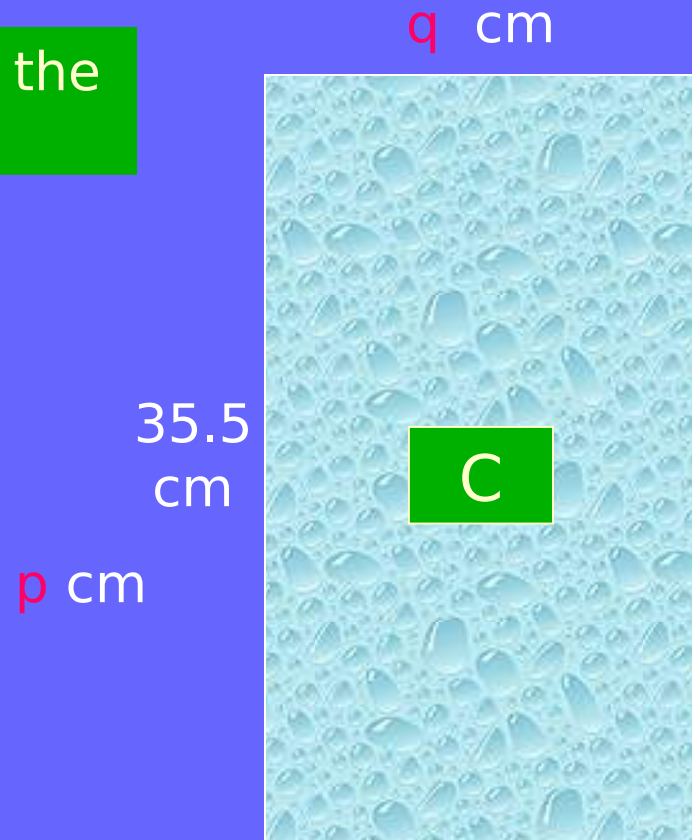
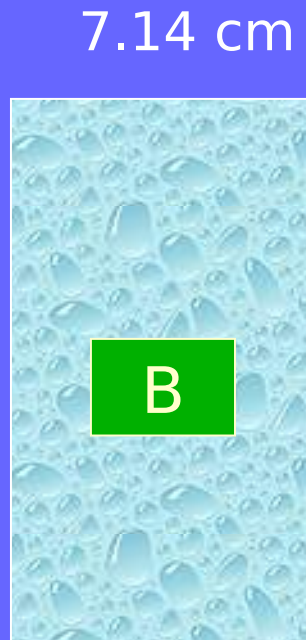
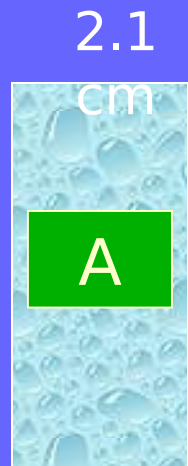
2. Apply the scale factor to find the unknown side.

$$p = 3.4 \times 5 = \underline{17 \text{ cm}}.$$

If we are told that two objects are similar and we can find the scale factor of enlargement by comparing corresponding sides then we can calculate the value of an unknown side.

The 3 rectangles are similar. Find the unknown sides p and q

Not to scale!

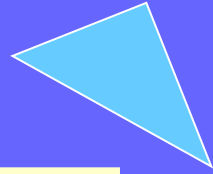
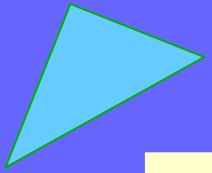


1. Comparing corresponding sides in A and C.

$$SF = 35.5/5 = \times 7.1.$$

2. Apply the scale factor to find the unknown side.

$$q = 7.1 \times 2.1 = \underline{14.91 \text{ cm}}$$

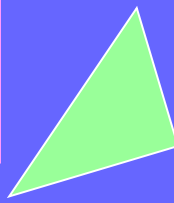
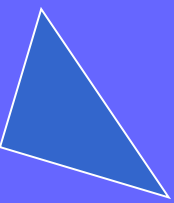


Similar Triangles

Similar triangles are important in mathematics and their application can be used to solve a wide variety of problems.

The two conditions for similarity between shapes as we have seen earlier are:

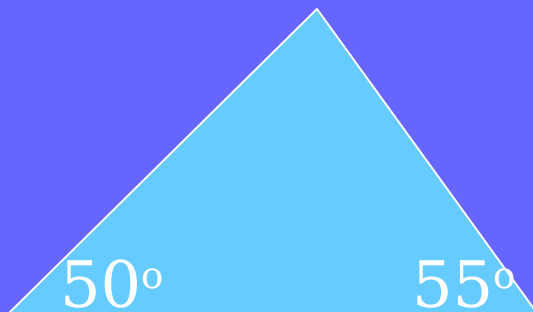
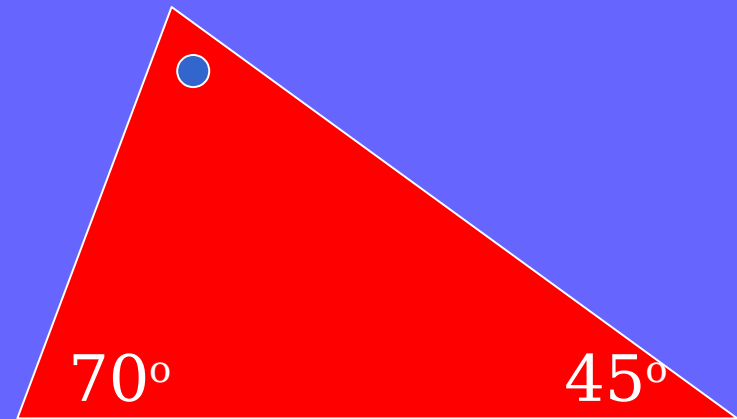
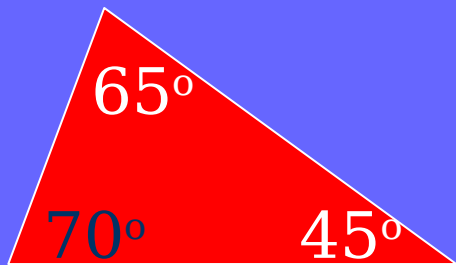
- Corresponding sides are in proportion **and**
 - Corresponding angles are equal
- 
- 



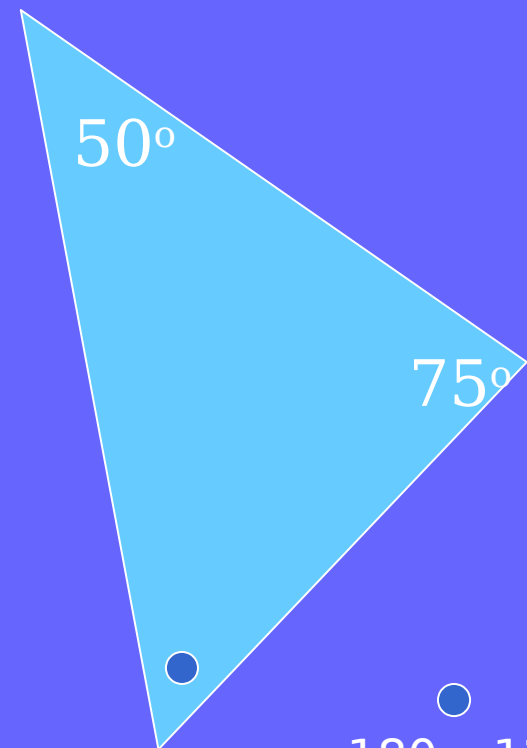
Two triangles are similar if their

- Corresponding angles are equal

These two triangles are similar since they are equiangular.



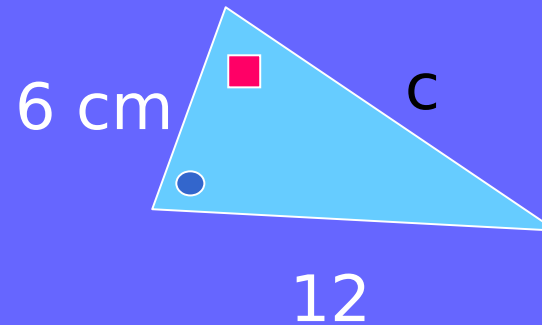
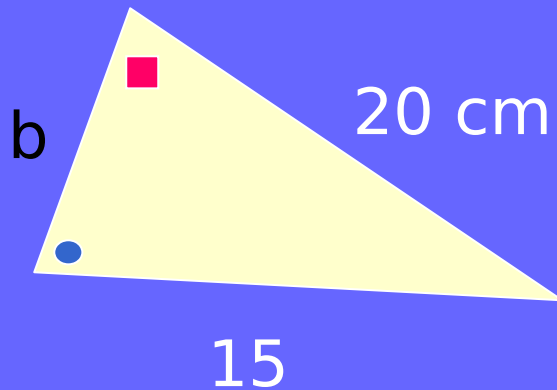
These two triangles are similar since they are equiangular.



If 2 triangles have 2 angles the same then they must be equiangular.

$$= 180 - 125 = 55$$

Finding Unknown Sides



Since the triangles are **equiangular** they are **similar**.

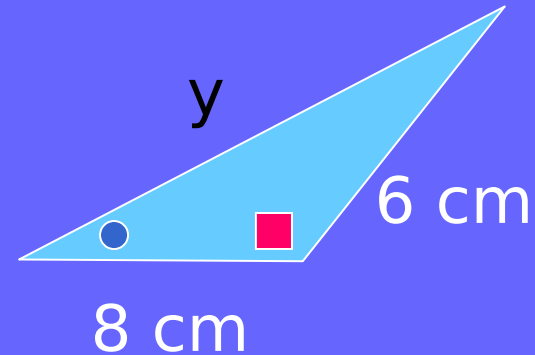
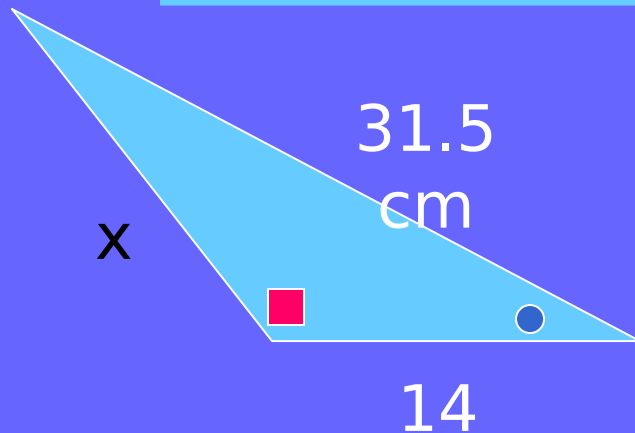
So comparing **corresponding** sides to find the scale factor of enlargement.

$$SF = 15/12 = \times 1.25.$$

$$b = 1.25 \times 6 = \underline{7.5 \text{ cm}}$$

$$c = 20/1.25 = \underline{16 \text{ cm}}$$

Finding Unknown Sides



Since the triangles are **equiangular** they are **similar**.

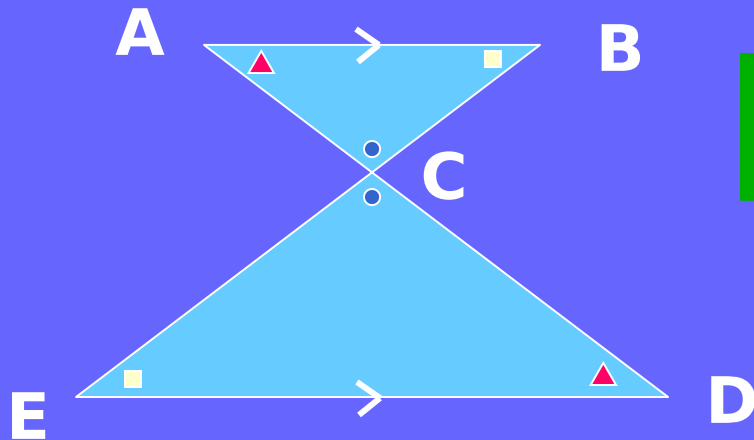
So comparing **corresponding** sides to find the scale factor of enlargement.

$$SF = 14/8 = x1.75.$$

$$x = 1.75 \times 6 = \underline{10.5} \\ \text{cm}$$

$$y = 31.5/1.75 = \underline{18} \\ \text{cm}$$

Determining similarity



Triangles ABC and DEC are similar. Why?

Angle ACB = angle ECD (Vertically Opposite)

Angle ABC = angle DEC (Alt angles)

Angle BAC = angle EDC (Alt angles)

Since **ABC** is similar to **DEC**

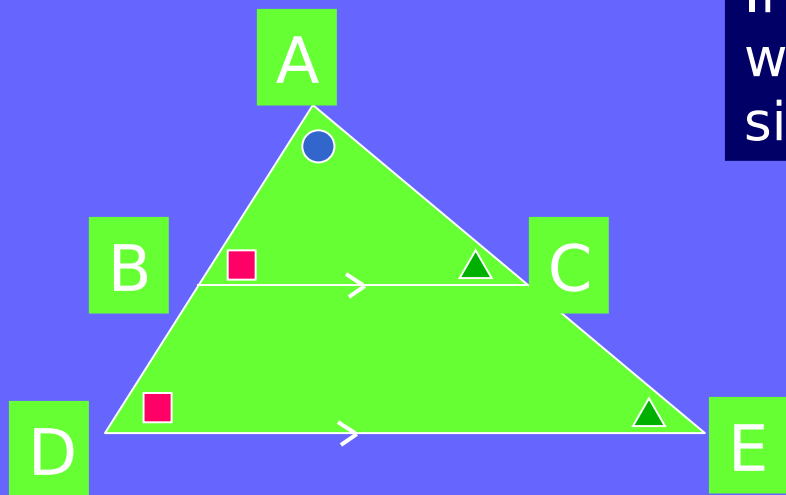
we know that corresponding sides are in proportion

AB→**DE**

BC→**EC**

AC→**DC**

The **order** of the lettering is important in order to show which pairs of sides correspond.



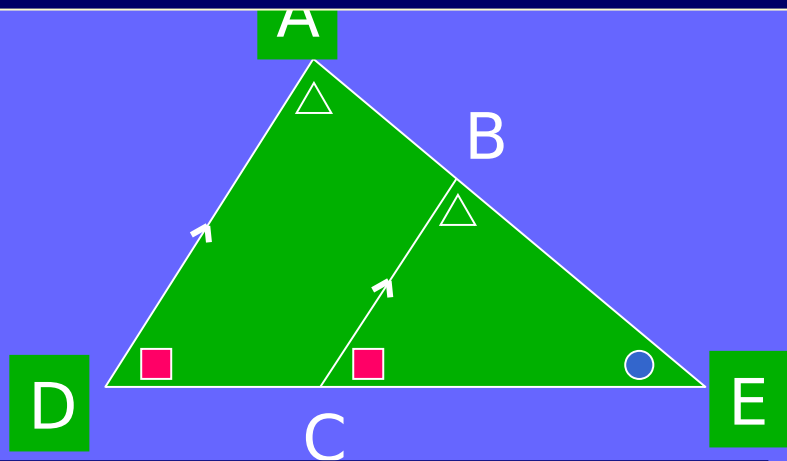
If BC is parallel to DE, explain why triangles ABC and ADE are similar

Angle BAC = angle DAE (common to both triangles)

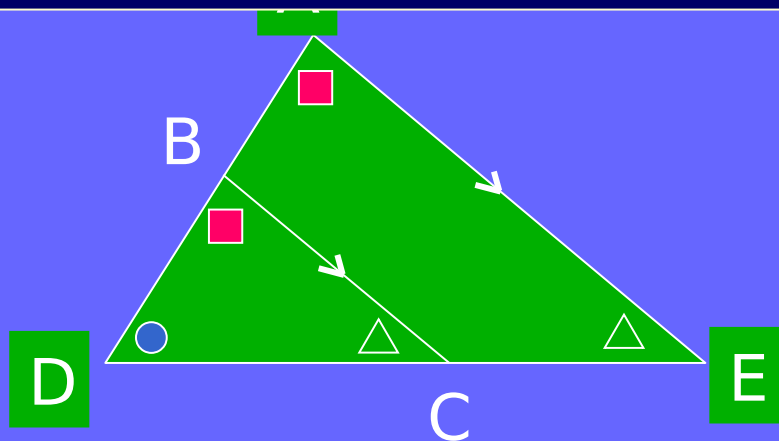
Angle ABC = angle ADE (corresponding angles between parallels)

Angle ACB = angle AED (corresponding angles between parallels)

A line drawn parallel to any side of a triangle produces 2 similar triangles.



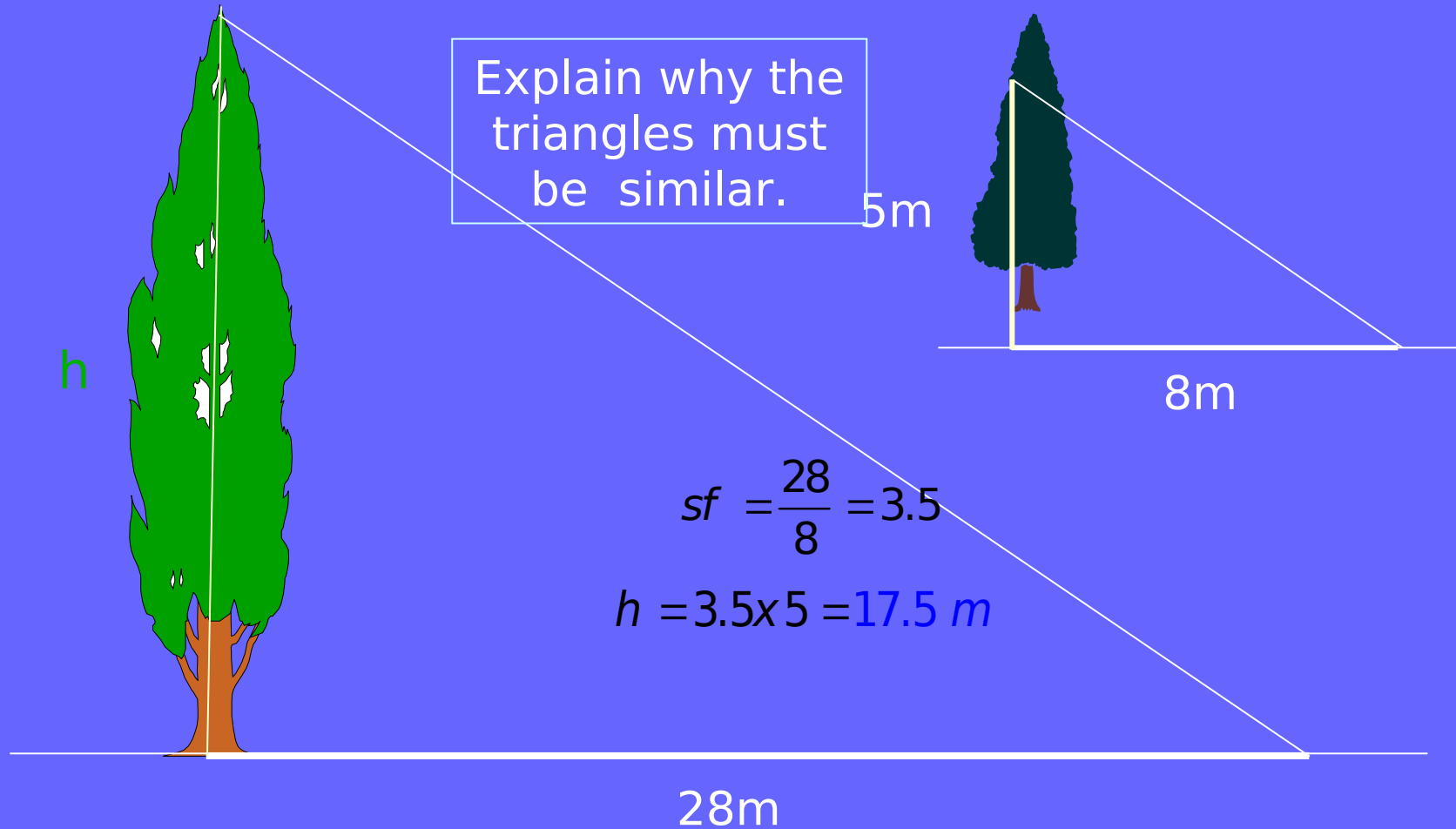
Triangles EBC and EAD are



Triangles DBC and DAE are similar

Example Problem 1

A tree 5m high casts a shadow 8 m long. Find the height of a tree casting a shadow 28 m long.

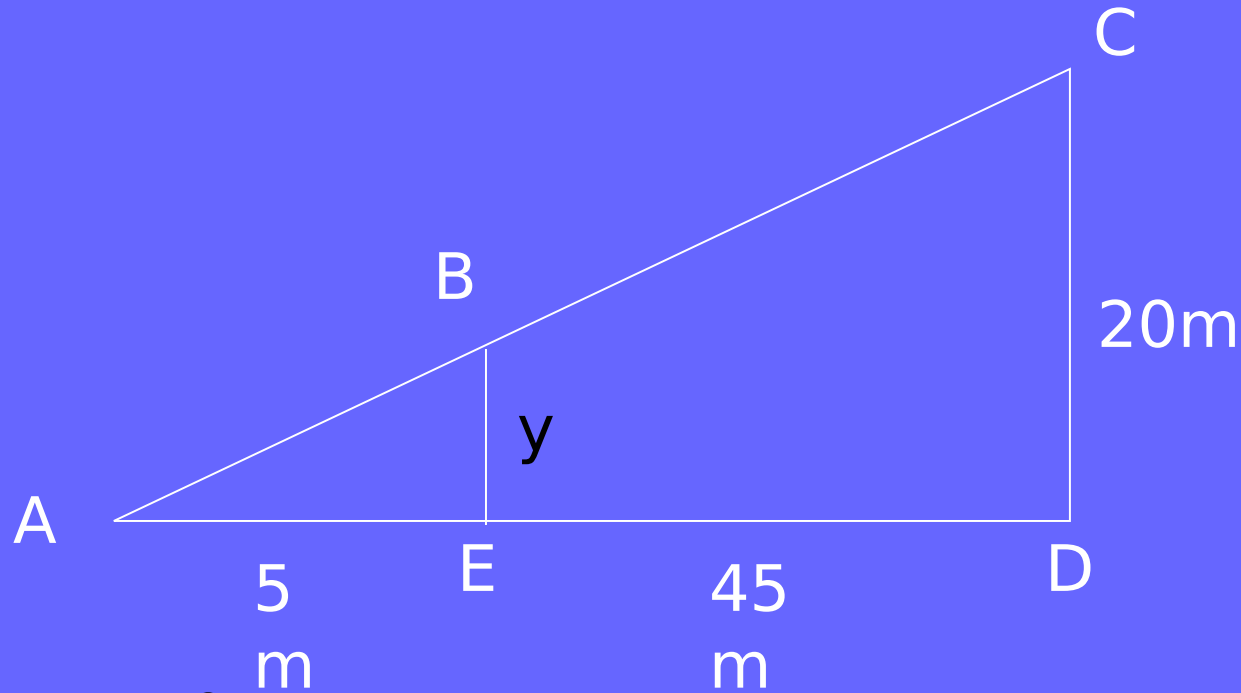


Example Problem

2

The two triangles below are similar:

Find the distance y .



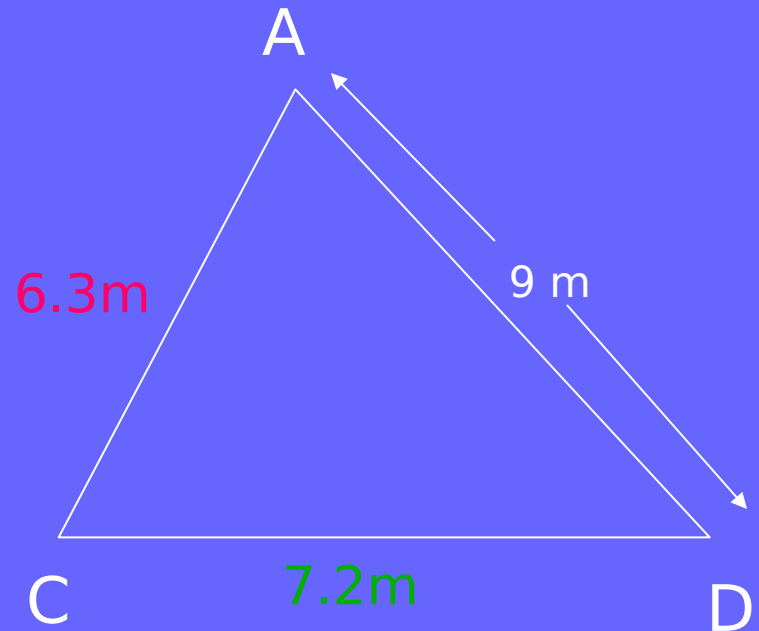
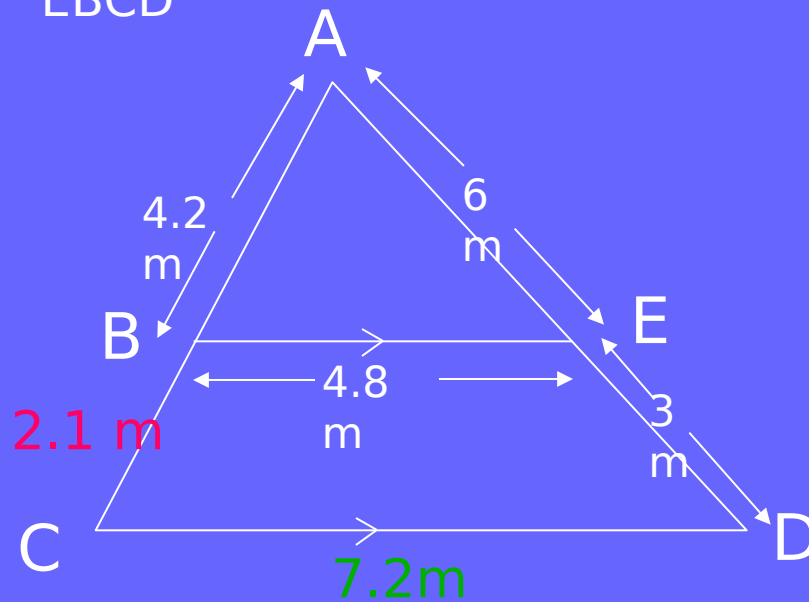
$$sf = \frac{50}{5} = 10$$

$$\Rightarrow y = \frac{20}{10} = 2 \text{ m}$$

Example Problem 3

In the diagram below BE is parallel to CD and all measurements are as shown.

- Calculate the length CD
- Calculate the perimeter of the Trapezium EBCD



$$(a) \text{ sf} = \frac{9}{6} = 1.5$$

$$\Rightarrow CD = 1.5 \times 4.8 = 7.2 \text{ m}$$

$$\text{So } AC = 1.5 \times 4.2 = 6.3 \text{ m}$$

$$BC = 6.3 - 4.2 = 2.1 \text{ m}$$

$$\text{Perimeter} = 7.2 + 3 + 4.8 + 2.1 = 17.1 \text{ m}$$