

ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY



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Engineering Mechanics II (Dynamics)

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(Group Assignment on work sheet 9-12) For section A students

GROUP 1

SECTION A

Submitted to: Mr. Maeregu

Addis Ababa-Ethiopa

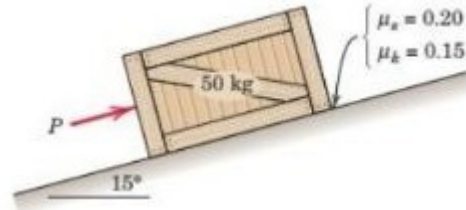
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WORKSHEET 9

1. The 50kg crate is stationary when the force P is applied. Determine resulting acceleration of P the crate if

a) $P=0$ b) $P=150\text{N}$ and c) $P=300\text{N}$



Given: $M=50\text{kg}$; $\mu_s=0.20$; $\mu_k=0.15$

Required: $a = ?$

Solution

Frictional force, $f_s = \mu_s \times mg \cos(\theta) = 0.2 \times 50 \times 9.8 \times \cos(15) = 94.66\text{ N}$

$f_k = \mu_k \times mg \times \cos(\theta) = 0.15 \times 50 \times 9.8 \times \cos(15) = 71\text{ N}$

$F = m g \sin(\theta)$ -----weight force along X- axis

$F = 50 \times 9.8 \times \sin(15) = 126.82\text{ N}$

Let assume the system is at equilibrium

$$\sum F_x = 0$$

$$P - mg \times \sin(\theta) - f = 0 \dots\dots\dots 1$$

- a. Case 1 when $P = 0\text{ N}$

Using equation 1 we get: - the downward force $F > f_s$

so, the acceleration, $a = \text{net force} / \text{effective mass}$

$$a = (F - f_k) / (m) = (126.82 - 71) / 50 = 1.12\text{ m/s}^2 \text{ downwards}$$

- b. Case 2 when $P = 150\text{ N}$

Using equation 1 we get: - $(P - F) < f_s$ So, there is no acceleration

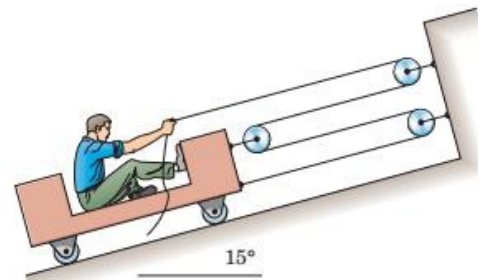
- c. Case 3 when $P = 300\text{ N}$

Using equation 1 we get: - $(P - F) > f_s$

The acceleration, $a = \text{net force} / \text{effective mass}$

$$a = (P - F - f_k) / (m) = (300 - 126.82 - 71) / 50 = 2.04\text{ m/s}^2 \text{ upwards}$$

2. A man pulls himself up the 15° incline by the method shown. If the combined mass of the man and cart is 100 kg , determine the acceleration of the cart if the man exerts a pull of 175 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.



Given: $\theta = 15^\circ$; $m = 100\text{kg}$; $T = 175\text{N}$ (since the friction between the pulley and rope is neglected)

Required: $a = ?$

Solution:

$$T = W \sin(\theta) + ma$$

But because there is a pulley system that makes the ropes have four tension points then the formula will be evolving to,

$$4T = W \sin(\theta) + ma, \text{ where } W = mg$$

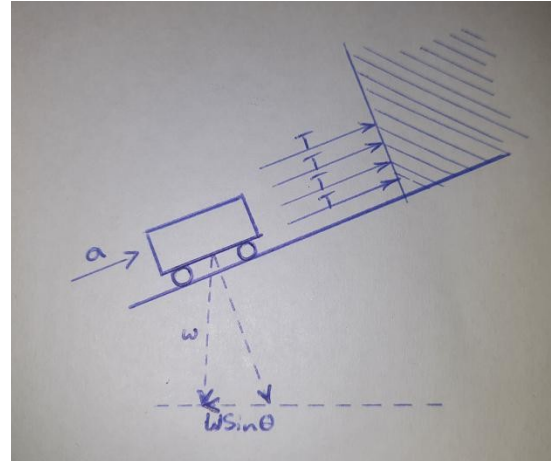
$$4T = mg \sin(\theta) + ma, \text{ then after}$$

replacing all the terms with its value,

$$4 * 175\text{N} = 100\text{kg} * 9.81\text{m/s}^2 \sin(15^\circ)$$

$$+ 100\text{kg} * a$$

$$a = \underline{4.4609\text{m/s}^2}$$



3. A skier starts from rest on the 40° slope at time $t = 0$ and is clocked at $t = 2.58$ s as he passes a speed checkpoint 20 m down the slope. Determine the coefficient of kinetic friction between the snow and the skis. Neglect wind resistance.

Given

Initially at rest point the velocity and the time will be zero $U=0$ at $t=0$

At the distance of $S=20\text{m}$ in $t=2.58\text{sec}$; $\theta=40^\circ$

Required: The coefficient of kinetic friction between the snow and the skis ($\mu_k = ?$)

Solution

From the equation of distance:

$$S = ut + \frac{1}{2} at^2$$

$$20 = \frac{1}{2} \times a (2.58)^2$$

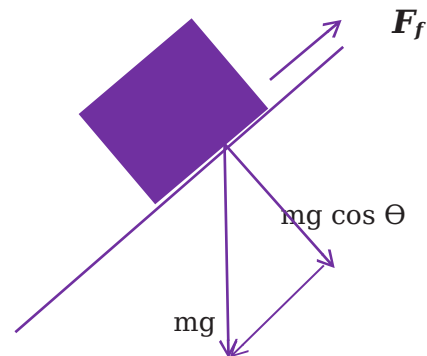
$$a = 6 \text{ m/s}^2$$

$$ma = mg \sin \theta - mg \cos \theta$$

$$a = g \sin \theta - \mu_k g \cos \theta$$

$$6 = 9.81 \sin 40^\circ - \mu_k \times 9.81 \cos 40^\circ = 6.31 - 7.515 \mu_k$$

$$\mu_k = \frac{0.31}{7.515} = 0.0413; \text{ the coefficient of kinetic friction will be } 0.0413$$



4. A skier starts from rest on the 40° slope at time $t = 0$ and is clocked at $t = 2.58$ s as he passes a speed checkpoint 20 m down the slope. Determine the coefficient of kinetic friction between the snow and the skis. Neglect wind resistance.



Given: Initially at rest point the velocity and the time will be zero $U=0$ at $t= 0$

At the distance of $S = 20\text{m}$ in $t= 2.58\text{sec}$; $\Theta = 40^\circ$

Required: The coefficient of kinetic friction between the snow and the skis ($\mu_k = ?$)

Solution

From the equation of distance:

$$S = ut + \frac{1}{2}at^2;$$

$$20 = \frac{1}{2} \times a(2.58)^2$$

$$a = 6 \text{ m/s}^2$$

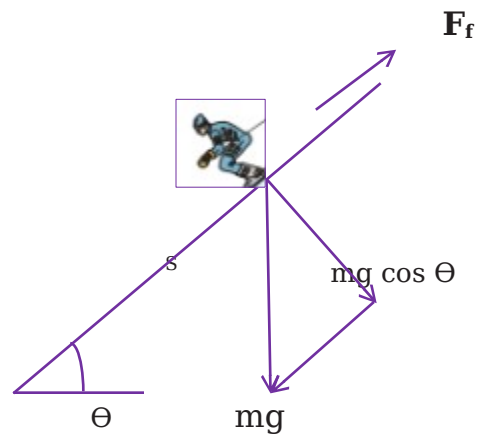
$$ma = mg \sin \Theta - mg \cos \Theta$$

$$a = g \sin \Theta - \mu_k g \cos \Theta$$

$$6 = 9.81 \sin 40^\circ - \mu_k \times 9.81 \cos 40^\circ$$

$$6 = 6.31 - 7.515\mu_k$$

$$\mu_k = \frac{0.31}{7.515} = 0.0413; \text{ The coefficient of kinetic friction will be } 0.0413$$



5. A small package is deposited by the conveyor belt onto the 30° ramp at A with a velocity of 0.8 m/s . Calculate the distance s on the level surface BC at which the package comes to rest. The coefficient of kinetic friction for the package and supporting surface from A to C is 0.30 .

Given: $V = 0.8$; $F_k = 0.3$; $g = 9.81 \text{ m/s}$; $\theta = 30^\circ$

Required: distance s on the level surface BC

Solution

Here we split the problem in to two parts

First we find the acceleration of the body by using the FBD and then by using the kinematic relations we find out the velocities at the required locations.

i. From A to B

$$\sum F_y = 0$$

$$N = mg \cos \theta = 0.866mg$$

$$\sum F_x = 0$$

$$ma = mg \sin \theta = mg \sin \theta - 0.3 \times (0.866mg) = 0.5mg - 0.259mg$$

$$a = 2.356 \text{ m/s}^2, \text{ by using } v^2 - u^2 = 2as$$

$$v_B - (0.8)^2 = 2(2.356)(2)$$

$$v_B = 3.12 \text{ m/s}$$

ii. From B to C

$$\sum F_y = 0; N = mg$$

$$\sum F_x = a_{1x} m$$

$$ma_1 = -f_k = -(0.3)mg$$

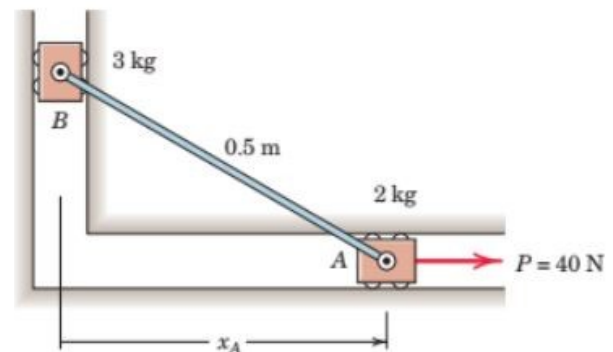
$$a_1 = -2.943 \text{ m/s}^2$$

$$\text{by using } v^2 - u^2 = 2as; v_C^2 - v_B^2 = 2as$$

$$s = 1.707 \text{ m}$$

From the above calculations we get the distance travelled by the box on the level surface BC = 1.707m

6. The sliders A and B are connected by a light rigid bar of length $l = 0.5 \text{ m}$ and move with negligible friction in the slots, both of which lie in a horizontal plane. For the position where $x_A = 0.4 \text{ m}$, the velocity of A is $v_A = 0.9 \text{ m/s}$ to the right. Determine the acceleration of each slider and the force in the bar at this instant.



Given: $Y_B = 0.3$; $X_A = 0.4 \text{ m}$; $L = 0.5 \text{ m}$; $V_A = 0.9$

$$\therefore \cos \theta = \frac{0.4}{0.5} = 0.8 \quad \theta = 37^\circ$$

Required: $a = ?$ $T = ?$

Solution;

Angular velocity of the rod

$$\omega = \frac{v_A}{r_{y_B}} = \frac{0.9}{0.3} = 3 \text{ rad/s}$$

Applying relative equation

$$a_A = a_B + \left(\alpha * r_{\frac{A}{B}} \right) - \left(\omega^2 r_{\frac{A}{B}} \right)$$

$$a_A i = a_B j + \alpha k * [0.4i - 0.3j] - [(3^2) * (0.4i - 0.3j)]$$

$$a_A i = a_B j + 0.3\alpha i + 0.4\alpha j - 3.6i + 2.7j$$

Equating the components

$$a_A i = 0.3\alpha i - 3.6i$$

$$a_B j = 0.4\alpha j + 2.7j$$

7. The slotted arm OA rotates about a fixed axis through O. At the instant under consideration, $\theta = 30^\circ$, $\dot{\theta} = 45 \text{ deg/s}$, and $\ddot{\theta} = 20 \text{ deg/s}^2$. Determine the forces applied by both arm OA and the sides of the slot to the 0.2-kg slider B. Neglect all friction, and let $L = 0.6 \text{ m}$. The motion occurs in a vertical plane.

Given: $\theta = 30^\circ$; $\dot{\theta} = 45 \text{ deg/s}$; $\ddot{\theta} = 20 \text{ deg/s}^2$; $L = 0.6 \text{ m}$; 0.2-kg

Required: The motion occurs in a vertical plane.

Solution

$$\dot{\theta} = 45 \text{ deg/sec} = 45 \times (\pi/180) \text{ rad/sec} = 0.785 \text{ rad/sec}$$

Convert angular acceleration of the arm from deg/s^2 to rad/sec^2

$$\ddot{\theta} = 20 \text{ deg/sec}^2 = 20 \times (\pi/180) \text{ rad/sec}^2 = 0.349 \text{ rad/sec}^2$$

Calculate the radial distance of the slider from o.

$$\cos = L/r$$

$$r = L/\cos = 0.6/\cos 30^\circ = 0.693 \text{ m}$$

Calculate the velocity of the slider.

$$V_\theta = V \cos \theta$$

$$V = v_\theta / \cos \theta = r\dot{\theta} / \cos \theta = 0.693 \times 0.785 / \cos = 0.628 \text{ m/s}$$

Calculate the velocity of the arm

$$r = v \sin = 0.628 \times \sin 30^\circ = 0.314 \text{ m/s}$$

Calculate the acceleration of the slider in tangential direction.

$$a = r\ddot{\theta} + 2r\dot{\theta} = a \cos = 0.693 \times 0.349 + 0.314 \times 0.785 = 0.849 \text{ m/s}^2$$

Consider the equilibrium forces acting in the radial direction from the free body diagram.

$$\sum F_r = m a_r$$

$$N \cos 30^\circ - mg \sin 30^\circ = m a \sin 30^\circ$$

$$N \cos 30^\circ - 0.2 \times 9.81 \times \sin 30^\circ = 0.2 \times 0.849 \times \sin 30^\circ$$

$$N=1.231N$$

Consider the equilibrium of forces acting in direction from the free body diagram.

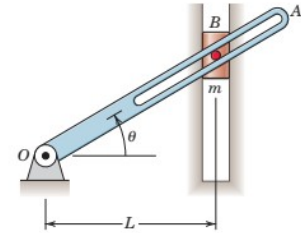
$$\Sigma F = ma$$

$$-N\sin 30^\circ + F - mg\cos 30^\circ = ma\cos 30^\circ$$

$$-1.231\sin 30^\circ + F - 0.2 \times 9.81\cos 30^\circ = 0.2 \times 0.849\cos 30^\circ$$

$$F = 2.46N$$

8. The slotted arm OA rotates about a fixed axis through O . At the instant under consideration, $\theta = 30^\circ$, $\dot{\theta} = 45 \text{ deg/s}$, and $\ddot{\theta} = 20 \text{ deg/s}^2$. Determine the forces applied by both arm OA and the sides of the slot to the 0.2-kg slider B . Neglect all friction, and let $L = 0.6 \text{ m}$. The motion occurs in a vertical plane.



Required: The force that is applied on both OA and slot to the slider B , given mass of 0.2kg

Solution:

Since the force is always calculated using the two components of acceleration the radial and angular and its magnitude is resolved by Pythagorean Theorem. So, the value of r from OB

We need to convert it into radian $\theta = 0.535 \text{ rad}$; $\dot{\theta} = 0.785 \text{ rad/s}$; $\ddot{\theta} = 0.349 \text{ rad/s}^2$

$$\cos \theta = L/R$$

$$R = 0.693 \text{ rad} \dots \dots \dots \dot{R} = 0$$

$$F_r = m a_r = 0.2 \text{ kg} (\ddot{r} - r \dot{\theta}^2) = 0.2 \text{ kg} (-0.693 \text{ rad} (0.785 \text{ rad/s})^2) = -0.085 \text{ N}$$

$$F_\theta = m a_\theta = 0.2 \text{ kg} (r \ddot{\theta} - 2 \dot{r} \dot{\theta}) = 0.2 \text{ kg} (0.693 \text{ rad} (0.349 \text{ rad/s}^2)) = 0.04837 \text{ N}$$

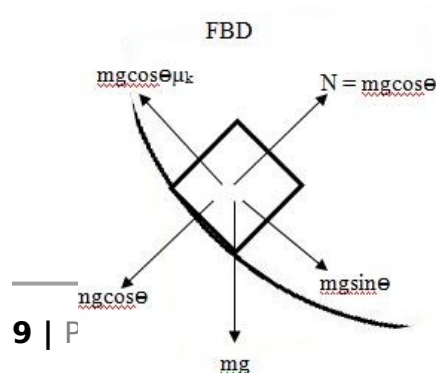
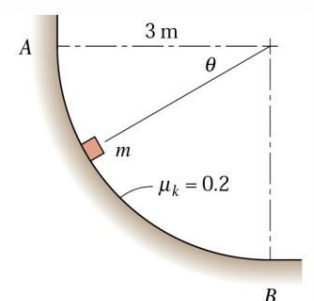
The applied force will be the magnitude and direction of these forces :-

$$F = (0.085 \text{ N})^2 + (0.04837 \text{ N})^2 = 0.097 \text{ N}$$

$$\tan \theta = (0.04837 \text{ N} / 0.085 \text{ N})$$

$$\theta = 3.3^\circ$$

9. A small object is released from rest at A and slides with friction down the circular path. If the coefficient of friction is 0.20 , determine the velocity of the object as it passes B .



➤ By Using Work and Energy Principle

Given: $R = 3m$; $\mu_k = 0.2$

Required: $v_f = ?$

Solution:

$$E_o = E_f$$

$$W + PE_o + KE_o = PE_f + KE_f + Heat_{lost}$$

$$0 + PE_o + 0 = 0 + KE_f + W_{overcome\ friction}$$

$$mgR = \frac{1}{2}mv_f^2 + dW$$

By integration we can calculate work done to overcome friction

$$dW = mg \cos \theta \mu R d\theta \cos \phi; \int dW = \int_{90^\circ}^{0^\circ} mg \cos \theta \mu R d\theta \cos \phi$$

$$W = \mu mg \cos \phi R \int_{90^\circ}^{0^\circ} \cos \theta d\theta = \mu mg R [\sin \theta]_{90^\circ}^{0^\circ} = \mu mg R$$

Substitution

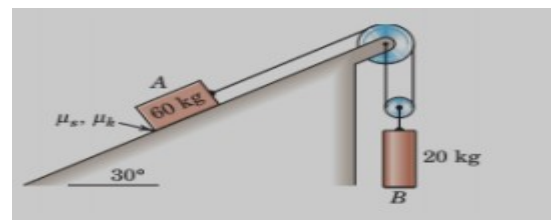
$$mgR = \frac{1}{2}mv_f^2 + W = \frac{1}{2}mv_f^2 + \mu mgR$$

$$gR = \frac{1}{2}v_f^2 + \mu gR = \frac{1}{2}v_f^2 + 0.2gR; 0.8gR = \frac{1}{2}v_f^2$$

$$v_f^2 = 1.6gR$$

$$v_f = \sqrt{1.6gR} = \sqrt{1.6 \times 9.81 \times 3} = 6.86 \text{ m/s}$$

10. The system is released from rest with the cable taut. For the kinetic friction coefficients 0.25 and static coefficient of friction 0.2, calculate the acceleration of each body and the tension T in the cable. Neglect the small mass and friction of the pulleys.

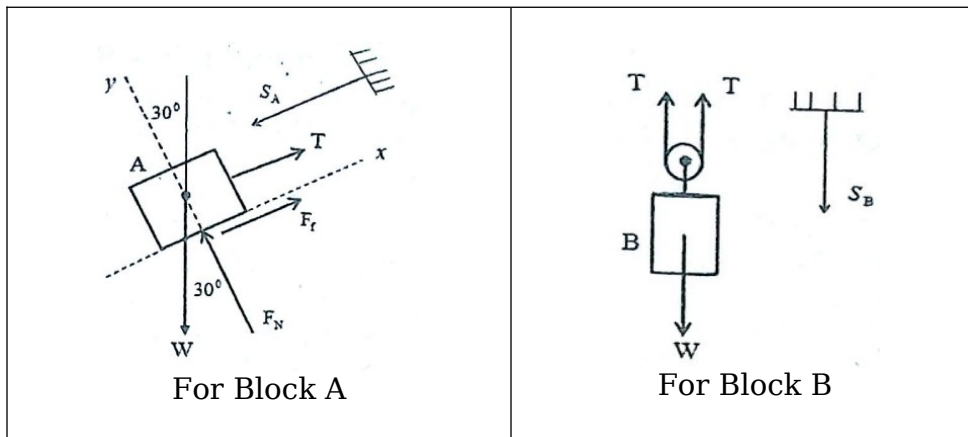


Given: $m_a = 60 \text{ kg}$; $m_b = 20 \text{ kg}$; kinetic friction coefficients = 0.25; static friction coefficient = 0.2; angle = 30 degree; $g = 9.8 \text{ m/s}^2$

Required: a_A & a_B ? T_a & T_b ?

Solution:

The free body diagram for block A and B is shown as follows:



Test of motion:

For block A:

$$\sum F_x = M_A a_A$$

$$-T - \mu_k W_A \cos 30^\circ + \mu_k W_A \sin 30^\circ = M_A a_A$$

$$T = W_A \sin 30^\circ - W_A \cos 30^\circ - M_A a_A$$

$$= (60 \text{ kg} (9.8 \text{ m/s}^2) \sin 30^\circ) - 0.2(60 \text{ kg} (9.8 \text{ m/s}^2) \cos 30^\circ) - (60 \text{ kg}) a_A$$

$$T = 192.155 - 60 a_A$$

For block B

$$\sum F_y = M_B a_B$$

$$2T - W_B = M_B a_B$$

$$T - 88 \text{ N} = 10 a_B$$

From the kinematics

$$a_A = 2a_B$$

Considering the cases of static equilibrium i.e acceleration is zero for both blocks

For Block A:

$$\sum F_x = 0, T + F - w_A \sin 30^\circ = 0$$

$$F = w_A \sin 30^\circ - 98$$

$$= 60(9.81) \sin 30^\circ - 98$$

$$= 196 \text{ N}$$

For Block B:

$$\sum F_y = 0, 2T - w_B = 0$$

$$T = \frac{w_B}{2} = \frac{m_B g}{2}$$

$$\therefore \frac{20(9.81)}{2} = 98 \text{ N}$$

Maximum frictional force:

$$F_{\max} = \mu_s N \quad \therefore 0.25(60)(9.8) \cos 30^\circ = 127. \text{ N}$$

Since, $F > F_{\max}$ the motion is impending.

Hence, using the above equation,

$$T = 192.155 - 60a_A = 192.155 - 60(2a_B) = 192.155 - 120a_B$$

Solving the above two equations, we get;

$$\begin{cases} T + 120a_B = 192.155 \\ T + 120a_B = 88 \end{cases}$$

$$a_B = 0.725 \text{ m/s}^2$$

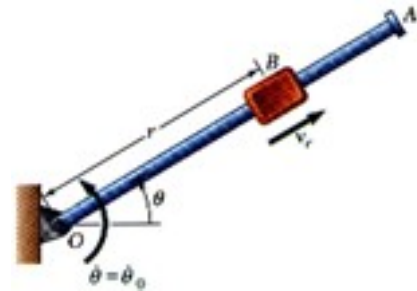
$$T = 105.35 \text{ N}$$

The acceleration a_A is calculated as;

$$a_A = 2(0.725 \text{ m/s}^2) = 1.45 \text{ m/s}^2$$

11. A block B with a mass m can slide freely on a frictionless arm OA that rotates in a horizontal plane at a constant rate $\dot{\theta} = 0$. Knowing that B is released at a distance r_0 from O, express as a function of r ,

- The component v_r of the velocity of B along OA,
- The magnitude of the horizontal force F exerted on B by the arm OA



Strategy

- ☐ Write the radial and transverse equation of the block
- ☐ Integrate the radial equation to find an expression for the radial velocity
- ☐ Substitute known information in to the transverse equation to find an expression for the force on the block

Given:

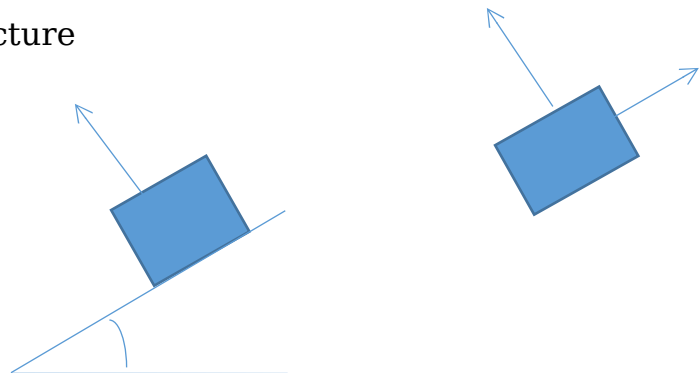
$$m_B = m$$

$r = r_0$ and other shown on above picture

Required: a) v_r and b) F

Solution:

FBD



$$\sum F_r = m a_r : 0 = (r - r \dot{\theta}^2)$$

$$\sum F_\theta = m a_\theta : F = m(r \ddot{\theta} + 2r \dot{\theta})$$

Integrate the radial equation to find an expression for the radial velocity

$$r'' = v' = \frac{dv_r}{dt} = \frac{dv_r}{dr} \cdot \frac{dr}{dt} = v_r \frac{dv_r}{dr}$$

$$\ddot{r} = \dot{v}_r = \frac{dv_r}{dt} = \frac{dv_r}{dr} \frac{dr}{dt} = v_r \frac{dv_r}{dr}$$

$$v_r dv_r = r \dot{\theta}^2 dr = r \dot{\theta}_0^2 dr$$

$$\int_0^v v_r dv_r = \dot{\theta}_0^2 \int_{r_0}^r r dr$$

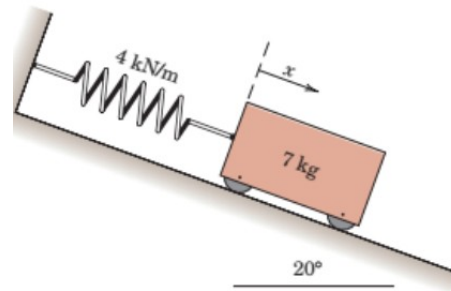
$$v_r^2 = \dot{\theta}_0^2 (r^2 - r_0^2)$$

Substituting known information into the traverse equation, we find an expression for the force on the block.

$$F = 2m\theta_o^2 \dot{\theta}$$

WORKSHEET 10

1. The spring is unstretched when $x = 0$. If the body moves from the initial position $x_1 = 100\text{mm}$ to the final position $x_2 = 200\text{mm}$, (a) determine the work done by the spring on the body and (b) determine the work done on the body by its weight.



Required:

- a. determine the work done by the spring on the body and
- b. Determine the work done on the body by its weight

Solution:

Since the work done by the spring is half of the product of the stiffness and the square of change in displacement. We use the following formula

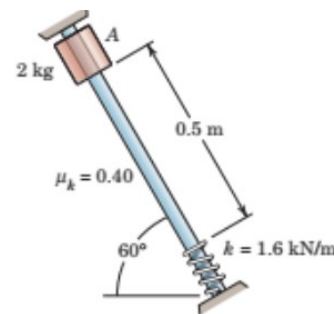
- Work done by the spring on the body can be calculated as

$$W_k = \frac{1}{2}K\Delta X^2 = \frac{1}{2} \cdot 4000 \cdot (0.2 - 0.1)^2 = \mathbf{20J}$$

- Work done on the body by its weight can be calculated using the following formula

$$W_w = mgsin(\beta) \cdot (\Delta x) = 7\text{kg} \cdot 9.81 \cdot \sin(20) \cdot (0.2 - 0.1) = \mathbf{2.35J}$$

2. The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.40. Calculate (a) the velocity v of the collar as it strikes the spring and (b) the maximum deflection x of the spring.



Required:

- A. Velocity v of the collar as it strikes the spring and
- B. Maximum deflection x of the spring

Solution:

To solve this kind of problem it is preferred to use the conservation of energy which states “energy neither created nor destroyed”.

Which means the energy we have initially must be equal to energy we have at final stage.

- A. First we have to find the corresponding alternate side of each.

$$X = 0.5 * \sin(60) = 0.43\text{m}$$

$$Y = 0.5 * \cos(60) = 0.25\text{m}$$

$$E_o = E_f$$

Work done by frictional force.

$$F_n = Mg \cos(\beta) = 2\text{kg} * 9.81 * \cos(60) = 9.81\text{N}$$

$$W_f = F_n * 0.5 = 9.81\text{N} * 0.5 = -4.9\text{J}$$

$$PE(\text{initial}) = mgsin(\beta)*l = 2\text{kg} * 9.81 * \sin(60) * 0.5 = 8.5\text{J}$$

Now we can see the conservation of energy to solve the rest.

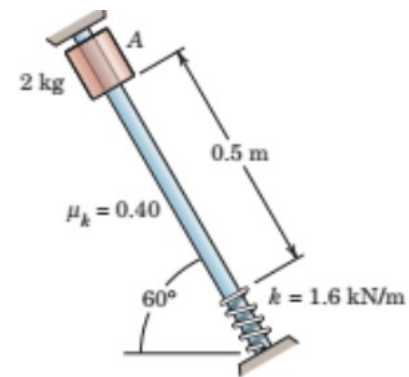
KE (initial) + PE(initial) - W(by friction) = KE(final)

Since the initial velocity at initial = 0m/s, KE (initial) = 0

$$0 + 8.5\text{J} - 4.9\text{J} = \frac{1}{2} * m * v(\text{final})^2$$

$$3.6\text{J} = 2\text{kg} * V^2 ; 3.6 = V^2$$

V = 1.99m/s



B. in this case the work done by the spring is also included. And when the collar is momentarily at rest the final velocity will be 0m/s.

PE(initial) + KE (initial) = KE(final) + W(by friction) + EPE

$$8.5\text{J} + 0\text{J} = F_n * (l+x) + \frac{1}{2} * kx^2$$

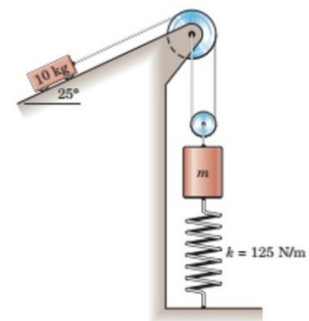
$$8.5\text{J} = 9.81(0.5 + x) + 0.5(1600x^2)$$

$$3.6 = 9.81x + 800x^2$$

$$800x^2 - 9.81x + 3.6 = 0$$

X = 6mm

3. The system is released from rest with no slack in the cable and with the spring stretched 200 mm. Determine the distance s traveled by the 10-kg cart before it comes to rest (a) if m approaches zero and (b) if $m = 2\text{kg}$. Assume no mechanical interference.



Required

This question requires to solve the inclined distance the block travels as it pulls up the cylinder of mass m and the extension of the spring as it does so.

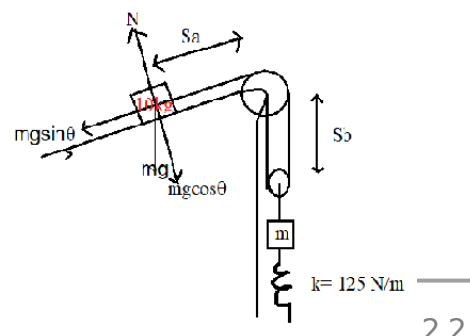
Solution

Given that: spring stretched (x_1) = 200mm = 0.2m

$$M_1 = 10\text{kg}$$

$$M_2 = 0\text{kg} \text{ and } m_2 = 2\text{kg}$$

$$K = 125 \text{ N/m}$$



$$\theta = 25^\circ$$

First draw the free body diagram

The system is released from rest therefore we can assume that $v_a = 0\text{m/s}$, $v_b = 0\text{m/s}$

A. When $m = 2\text{kg}$

From position analysis

$$2s_b = s_a, s_b = s_a/2$$

Using Work-Energy principle:

Work done by all forces = change in kinetic energy

$$W_{m_1g} + W_{mg} + W_{\text{spring}} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

$$-(mg\sin\theta) s_a + mg*s_b + [\frac{1}{2} k\Delta(x_i)^2 - \frac{1}{2} k(\Delta x_f)^2] = 0-0$$

Spring from the given is initially stretched by 0.2m , $\Delta x_i = 0.2\text{m}$

Then it moves by s_b , making the new extension $\Delta x_f = 0.2\text{m} - s_b$

$$-(10*9.81*\sin 25^\circ) s_a + 2*9.81* (s_a/2) + \frac{1}{2} * 125 [(0.2)^2 - (0.2 - s_b)^2]$$

$$-31.65 s_a + 62.5 [-(0.25 s_a^2 + 0.2 s_a) = 0$$

$$-15.625 s_a^2 - 19.15 s_a = 0$$

$s_a = -1.226\text{m}$, where the negative sign indicates that the block moves backwards or down the incline.-----ANS

B. When $m \approx 0\text{kg}$

Using the equation,

$$W_{m_1g} + W_{mg} + W_{\text{spring}} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$$

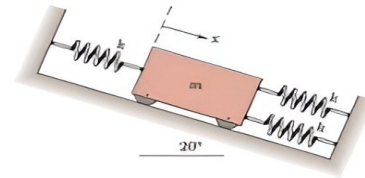
$$-(mg\sin\theta) s_a + mg*s_b + [\frac{1}{2} k\Delta(x_i)^2 - \frac{1}{2} k(\Delta x_f)^2] = 0-0$$

$$-(10*9.81*\sin 25^\circ) s_a + \frac{1}{2} * 125 (-0.25 s^2 + 0.2 s) = 0$$

$$-15.625 s^2 - 28.96 s = 0$$

$s_a = -1.85\text{m}$, the negative sign indicates that the block will move backwards or down the incline. -----ANS

4. The three springs of equal moduli are unstretched when the cart is released from rest in the position $x=0$. If $k=120\text{N/m}$ and $m=10\text{kg}$, determine



- The speed v of the cart when $x=50\text{mm}$,
- The maximum displacement X_{max} of the cart, and
- The steady-state displacement X_{ss} that would exist after all oscillations cease

Solution:

- For body of mass 10kg

P.E for 10kg mass + spring energy = R.E of 10kg mass

$$Mgh - [1/2 \cdot (k+k+k)x^2] = 1/2 mv^2$$

h = height through which PE change has occurred $x=0.005\text{m}$;

$$g=9.81\text{m/s}^2; m=10\text{kg}$$

$$mgsin 20^\circ - x(k+k+k)x^2 = 1/2 mv^2$$

$$[10 \cdot 9.81 \cdot (0.005 \sin 20^\circ)] - [1/2 \cdot (3 \cdot 120) \cdot 0.005^2] = 1/2 \cdot 10 \cdot v^2$$

$$\mathbf{V=0.496\text{m/s}}$$

- Spring = spring energy

$$mg \cdot m \sin 20^\circ = 1/2 (3 \cdot 120) \cdot x m^2$$

$$\mathbf{xm = 186.4\text{mm}}$$

$$\text{maximum displacement of cart } X_m = 186.4\text{mm}$$

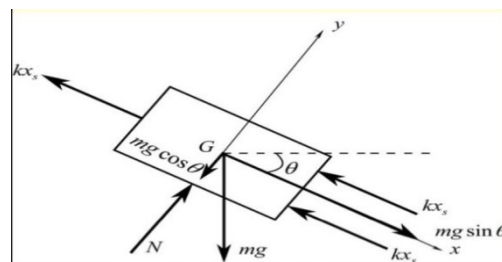
- FBD

Steady state displacement of spring

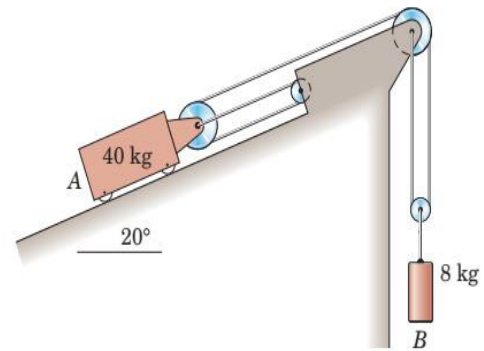
$$Mg \cdot \sin 20^\circ = 3 \cdot KX_s$$

$$10 \cdot 9.81 \cdot \sin 20^\circ = 3 \cdot 120 \cdot X_s$$

$$\mathbf{X_s = 93.2\text{mm}}$$

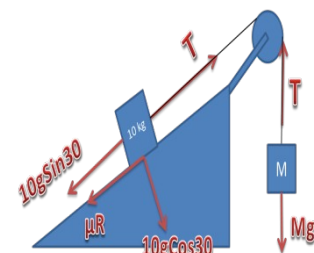


5. If the system is released from rest, determine the speeds of both masses after B has moved 1m. Neglect friction and the masses of the pulleys



Required: The required in this problem is the velocities of both masses, mass A and mass B

Solution: The last phrase neglecting frictional force and mass of the pulleys will deduct some of the terms in the equation. So, the question has two loads, which are connected by two rigid cables. We need to find an equation that links the tension and acceleration, set a datum reference in which we take measurement, since it is a constrained motion we need to find analysis for that too.



For mass a: $mg \sin \theta - 3T = ma$ (Friction = 0)

For mass B: $2T - mg = ma$

All this expression will be deducted because the mass is negligible so we take another approach

Position analysis: Let the first cable to be that holds mass A by L_1 and for mass B is L_2 assuming the datum is the diagonal pulley._

$$2L_1 + 2L_2 + C_1 = L_T, \quad l_2 = 1 \text{ meter}$$

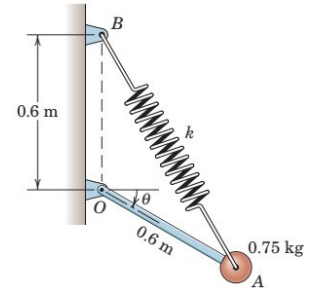
Velocity analysis: We know that velocity can be defined as the rate of change of position. That is:-

$$v = \frac{dl}{dt} \quad \therefore \frac{dl}{dt} (3(l_1) + 2(l_2) + C_1) = \dot{C}_1$$

$$v = 3\dot{l}_1 + 2\dot{l}_2 = 0 \quad \text{But } (\dot{} \text{ means velocity})$$

$$\Rightarrow v_1 = -\frac{2}{3} v_2 = -0.66 \frac{m}{s}$$

6. The 0.75-kg particle is attached to the light slender rod OA which pivots freely about a horizontal axis through point O. The system is released from rest while in the position $\theta=0$ where the spring is unstretched. If the particle is observed to stop momentarily in the position $\theta=50^\circ$, determine the spring constant k . For your computed value of k , what is the particle speed v at the position $\theta=25^\circ$?



Given:

Mass of the particle, $m = 0.75 \text{ kg}$

Initial velocity, $v = 0 \text{ m/s}$

Position, $\theta = 50^\circ$

Required: i) $k = ?$ and ii) V at $\theta = 25^\circ$

Solution:

Applying Conservation of Energy

Initial total mechanical energy of the system = Final mechanical energy of the system

Initially: $T_1 = 0$, $(V_g)_1 = 0$, $(V_e)_1 = 0$

Finally: $T_2 = 0$,

$(V_g)_2 = -mg\Delta h$, but $\Delta h = 0.6 \sin(50^\circ) = 0.46 \text{ m}$

$(V_g)_2 = -(0.75)(9.81)(0.46) = -3.38 \text{ J}$

$(V_e)_2 = \frac{1}{2}k\Delta x^2$, but $\Delta x = 2(0.6)\sin(70^\circ) - 2(0.6)\sin(45^\circ) = 0.279 \text{ m}$

Hence, $(V_e)_2 = \frac{1}{2}k(0.279)^2 = 0.0389k$

Now, $T_1 + (V_g)_1 + (V_e)_1 = T_2 + (V_g)_2 + (V_e)_2$

$0 = 0.0389k - 3.38 \text{ J}$

$k = 86.8 \text{ N/m}$

ii) At $\theta = 25^\circ$

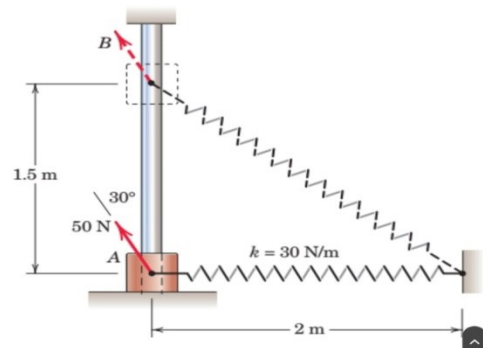
Again by Conservation of energy but we have KE here, i.e, $0 = \frac{1}{2}k\Delta x^2 -$

$mg\Delta h + \frac{1}{2}mv^2$

$$v^2 = \frac{2mg\Delta h - k\Delta x^2}{m} = \frac{2(0.75)(9.81)(0.46) - 86.8(0.279)^2}{0.75} = 1.8811 \text{ m}^2/\text{s}^2$$

$$V = \sqrt{1.8811} = 1.372 \text{ m/s}$$

7. The collar has a mass of 2 kg and is attached to the light spring, which has a stiffness of 30 N/m and an unstretched length of 1.5 m. The collar is released from rest at A and slides up the smooth rod under the action of the constant 50-N force. Calculate the velocity v of the collar as it passes position B.



Given:

Mass of collar, $M = 2$ kg

Unstretched length, $L = 2$ m

Stiffness of the spring, $K = 30$ N/m

Solution:

Find the deflection spring A :

$$\delta_A = (X_{\text{original}} - X_{\text{unstretched}}) = 2 - 1.5 = 0.5 \text{ m}$$

Find the deflection spring B:

$$\delta_B = (X_{\text{original}} - X_{\text{unstretched}}) = \sqrt{(2^2 + 1.5^2)} - 1.5 = 1 \text{ m}$$

From the work energy theorem (datum A):

$$T_A + V_A + U_{1-2} = T_B + V_B$$

$$1/2 m V_A^2 + 1/2 k \delta_A^2 + (F \cos 30^\circ) 1.5 = 1/2 m v_B^2 + mg (h_B) + 1/2 k \delta_B^2$$

$$0 + 1/2 (30) (0.5)^2 + (50 \cos 30^\circ) 1.5 = 1/2 (2) v^2 + (2) (9.81) (1.5) +$$

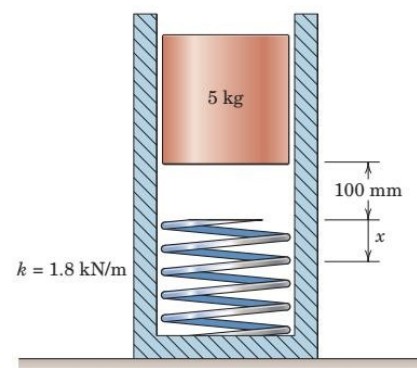
$$1/2 (30) (1)^2$$

$$3.75 + 64.952 = v^2 + 29.43 + 15$$

$$V^2 = 24.272$$

$$v = 4.927 \text{ m/s}$$

8. The 5-kg cylinder is released from rest in the position shown and compresses the spring of stiffness $k = 1.8 \text{ kN/m}$. Determine the maximum compression x_{max} of the spring and the maximum velocity V_{max} of the cylinder along with the corresponding deflection x of the spring.



Given:

Mass of the cylinder, $m = 5 \text{ kg}$

Stiffness constant of the Spring $k = 1.8 \text{ kN/m} = 1800 \text{ N/m}$

Heigh $h = 100 \text{ mm} = 0.1 \text{ m}$, $v_o = 0 \text{ m/s}$; $g = 9.81 \text{ m/s}^2$

Required:

- A. The maximum compression $x_{\max}$ of the spring
- B. $V_{\max}$ of the cylinder along with the corresponding deflection x of the spring

Solution: since the system starts from rest, $T_1=0$. Two forces are acting on the cylinder:

Spring force: $F_{sp}=k\Delta l=1800x$

Weight force: $w=mg=5(9.81)=49.05\text{ N}$

Hence, we'll have two types of work here namely; work done by spring force and work done by weight

Principle of Work and Energy: When the block is momentarily at rest, **W** which displaces downward $h=(0.1+x_{max})m$, does positive work, whereas F_{sp} does negative work.

Hence, using $T_1 + \sum U_{1-2} = T_2$

$$0 + mg(0.1 + x_{max}) - \frac{1}{2}kx_{max}^2 = 0$$

$$mg(0.1 + x_{max}) = \frac{1}{2}kx_{max}^2$$

$$5 \times 9.81 \times (0.1 + x_{max}) = \frac{1}{2} \times 1800x_{max}^2$$

$$900x_{max}^2 - 49.05x_{max} - 4.905 = 0$$

$$x_{max}^2 - 0.0545x_{max} - 0.00545 = 0$$

Solving the quadratic equation, we get; $x_{max}=0.1059=105.9\text{ mm}$

Therefore, the maximum compression of the spring is 105.9 mm

C. To find the V_{max} of the cylinder along with the corresponding deflection x of the spring, let's apply the principle of Work and Energy again; thus, this time our height would be $h=(0.1+x)m$.

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + mgh - \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$$

$$5 \times 9.81 \times (0.1 + x) - \frac{1}{2}(1800)x^2 = \frac{1}{2}(5)v_{max}^2$$

$$49.05(0.1 + x) - 900x^2 = 2.5v_{max}^2$$

$$4.905 + 49.05x - 900x^2 = 2.5v_{max}^2$$

$$v_{max}^2 = 1.962 + 19.62x - 360x^2$$

$$v_{max} = \sqrt{1.962 + 19.62x - 360x^2}$$

To get the corresponding value of x let's set the first derivative of the function 0.

$$\frac{d(v_{max})}{dx} = \frac{d(\sqrt{1.962 + 19.62x - 360x^2})}{dx}$$

$$v_{max} = 19.62 - 720x = 0$$

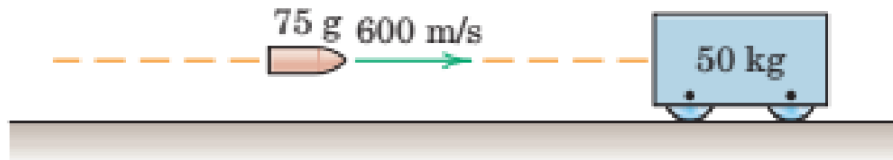
Hence, $x = 0.0273m = 27.3mm$

It follows that, $v_{max} = \sqrt{1.962 + 19.62(0.0273) - 360(0.0273)^2} = 1.493m/s$

Therefore, the V_{max} of the cylinder along with the corresponding deflection x of the spring are $1.493m/s$ and $27.3mm$, respectively.

WORKSHEET 11

1. A 75-g projectile traveling at 600 m/s strikes and becomes embedded in the 50-kg block, which is initially stationary. Compute the energy lost during the impact. Express your answer as an absolute value and as a percentage n of the original system energy E.

**Given:**

M_{of projectile} (m_p) = 75g = 0.075kg

M_{of block} (m_b) = 50kg

Initial velocity of the projectile (V_p) = 600m/s

Initial velocity of the block (V_b) = 0 (since it starts from rest)

Required: Energy lost = ?

Solution:

Since the question says that, after collision they become embedded they will have common final velocity V. So, first we will find the final velocity by using the conservation of momentum in the x coordinate, since the motion is in the x - axis.

$$m_p V_{p1x} + m_b V_{b1x} = (m_p + m_b) V_{2x}$$

Solving for V_{2x} gives us,

$$V_{2x} = \frac{m_p V_{p1x} + m_b V_{b1x}}{m_p + m_b}$$
$$V_{2x} = \frac{0.075 \text{ kg} \left(\frac{600 \text{ m}}{\text{s}} \right) + 50 \text{ kg} (0 \text{ m/s})}{50 \text{ kg} + 0.075 \text{ kg}}$$
$$V_{2x} = 0.899 \text{ m/s}$$

Now let us find the kinetic energy before and after collision to compute the energy lost;

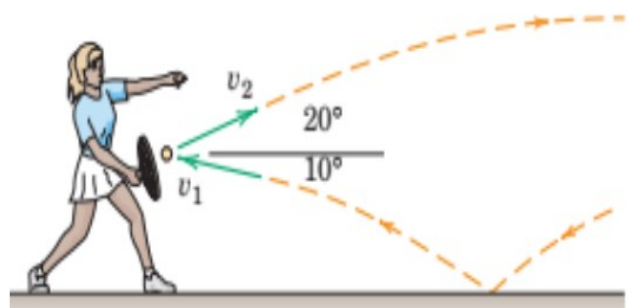
$$\text{Initial energy, } T_1 = \frac{1}{2} m_p (V_{p1x})^2 + \frac{1}{2} m_b (V_{b1x})^2 = \frac{1}{2} (0.075 \text{ kg}) (600 \text{ m/s})^2 + \frac{1}{2} (50 \text{ kg}) (0 \text{ m/s})^2$$
$$= 13500 \text{ J}$$

$$\text{Final energy, } T_2 = \frac{1}{2} (m_p + m_b) V_{2x} = \frac{1}{2} (0.075 \text{ kg} + 50 \text{ kg}) (0.899 \text{ m/s})^2 = 20.2 \text{ J}$$

Now we can compute the absolute energy lost, $|\Delta E| = T_1 - T_2 = 13,480 \text{ J}$

$$\text{Percent loss : } n = \frac{|\Delta E|}{T_1} = \frac{13480 \text{ J}}{13500 \text{ J}} (100)\% = 99.9\%$$

2. A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is $V_1 = 15 \text{ m/s}$ and after impact its speed is $V_2 = 22 \text{ m/s}$, with directions as shown in the figure. If the 60 g ball is in contact with the racket for 0.05 s , determine the magnitude of the average force R exerted by the racket on the ball. Find the angle made by R with the horizontal. Comment on the treatment of the ball weight during impact.



Given:

$$V_1 = 15 \text{ m/s}$$

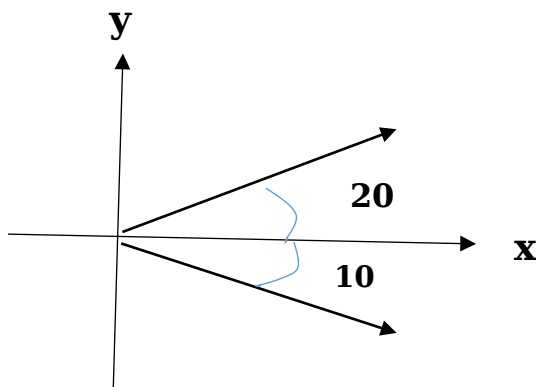
$$V_2 = 22 \text{ m/s}$$

$$m = 60 \text{ g} = 0.06 \text{ Kg}$$

$$t = 0.05 \text{ s}$$

Required: $R = ?$; $\theta = ?$

Solution



$$\int_0^t \sum F_x dt = (mv_2)_x - (mv_1)_x$$

Substituting the above expression,

$$\int_0^t R_x dt = 0.06 * (22 \cos 20^\circ) - 0.06 * (-15 \cos 10^\circ)$$
$$R_x [t]_0^{0.05} = 0.06 * (22 \cos 20^\circ) + 0.06 * (15 \cos 10^\circ)$$
$$R_x * 0.05 = 2.127$$
$$\mathbf{R_x = 42.53N}$$

Applying the principle of conservation of linear momentum in y direction,

$$\int_0^t \sum F_x dt = (mv_2)_y - (mv_1)_y$$

Impact force R_y considerably larger than the weight of the ball. Therefore, weight of the ball is a non-impulsive force and can be neglected.

Substituting values in the above expression,

$$\begin{aligned}\int_0^t (R_y - mg) dt &= 0.06 * (22 \sin 20^\circ) - 0.06 * (15 \sin 10^\circ) \\ \int_0^t (R_y - 0.6 * 9.81) dt &= 0.06 * (22 \sin 20^\circ) - 0.06 * (15 \sin 10^\circ) \\ [(R_y)t - (0.06 * 9.81) t]_0^{0.05} &= 0.06 * (22 \sin 20^\circ) - 0.06 * (15 \sin 10^\circ) \\ \mathbf{R_y} &= \mathbf{6.49N}\end{aligned}$$

Then, Magnitude becomes,

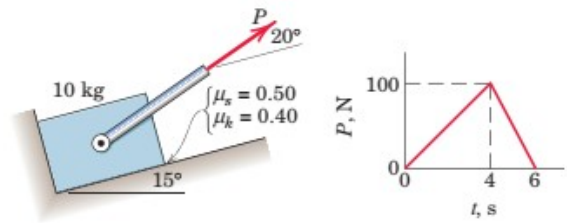
$$R = \sqrt{42.53^2 + 6.49^2} = 43.02N$$

The value of β made by R with horizontal,

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{6.49}{42.53}\right) = 8.68^\circ$$

Hence, the magnitude of average force R is **43.02N** and the angle β made by R with the horizontal is **8.68°**.

3. The 10-kg block is at rest on the rough incline at time $t = 0$ and then it is subjected to the force of constant direction and time-varying magnitude P given in the plot. Determine the velocity of the block at times $t = 1, 3, 5$, and 7 s. Note that the force P is zero after $t = 6$ s.



Given:

Mass of the block = 10 Kg; Weight = 10g

Force varying as shown in the figure

From the graph

$t = 0$ s, $P = 0$

$t = 1$ s, $P = 25$ N

$t = 2$ s, $P = 50$ N

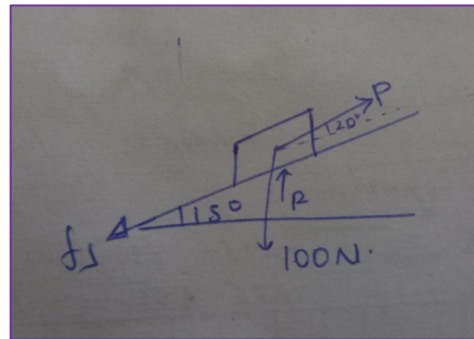
$t = 3$ s, $P = 75$ N

$t = 4$ s, $P = 100$ N

$t = 5$ s, $P = 50$ N

$t = 6$ s, $P = 0$ N

Free body diagram



Where: f_s and R are frictional force and normal reactions

A body is ready to move when all forces are in equilibrium.

Force along perpendicular to AB plane is $(100)\sin 75^\circ - R = 0$; $R = 93.59$ N

We know that $f_s = \mu N$, $(0.5)(93.59) = 48.29$

Force along AB plane

$100\cos 75^\circ + f_s - P = 0$; $F_s = 74.86$

When load applying reach to 81.18 N only body tends to move, the time at 74.86 is

$t_{at 2} = \frac{\text{Load at 2}}{74.86}$, $t = 2.99$ sec

$t = 2.99$ sec

At 1 sec: The body is at rest condition.

At 3 sec

$$\sum F_x = 0$$

$$100\sin 75^\circ - R_{3s} = 0; R_{3s} = 93.59$$

$$f_{3s} = \mu_k R_{3s}, (0.4)(93.59) = 37.03$$

Resultant along AB plane

$$100\cos 75^\circ + f_{3s} - 75 = R$$

$R = -11.68$ (The negative sign indicates the body is moving upward.)

We know that $V = U + at$

$$F=ma; 11.68=100a; a= 0.1168\text{m/s}^2$$

We know that the initial velocity $U=0$

$$V=0 + (0.1168) (3-2.99) = 0.201\text{m/s}$$

At 5sec

$$\sum F_x = 0$$
$$100\sin 75' - R_{5s} = 0$$
$$R_{5s} = 93.59\text{N}$$
$$f_{5s} = (0.4) R_{5s} = 37.43$$

Resultant force

$$100 \cos 75' + f_{5s} - 50 = \text{Resultant}$$

Resultant force = 18.31 (+ve sign indicate body moving downward)

$$F=ma; 13.31= 100a; a= 0.1331\text{m/s}^2$$

From the graph max velocity of the block is at $t=4\text{s}$

At t=4s

Resultant force = $100\cos 75' + f_{5s} - 150 = -36.68\text{N}$
where (-ve sign indicates body moving upward)

At that point of acceleration

$$F=ma; 36.68=100a; a= 0.3668\text{m/s}^2$$
$$V= U + at$$
$$V_{4s} = 0 + 0.3668(4)$$
$$V_{4s} = 1.4672\text{m/s}$$

At t=5sec

$$a= 0.1331 \text{ m/s}^2$$
$$V_{5s} = U + at \text{ where, } U= V_{4s}$$
$$V= 1.4672 + (0.1331)(1)$$
$$V= 1.59\text{m/s}$$

At t=6sec

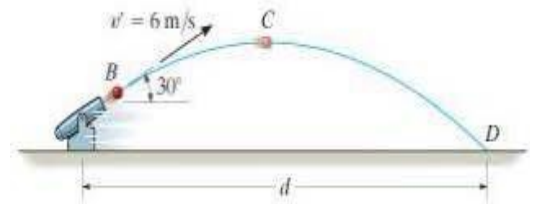
Resultant force = $100 \cos 75' + f_{7s} = 63.31 \text{ N}$

$$F= ma; 63.31= 100a; a= 0.6331\text{m/s}^2$$
$$V_{6s} = U + at \text{ where } U= V_{4s}$$
$$V_{6s} = 1.4672 + (0.633) (2)$$
$$V_{6s} = 2.733 \text{ m/s}$$

From 6s there is no force acting on the body, after the velocity body remains same until it reaches to original ground.

At 7sec: velocity = 2.733m/s

4. The 5-KG spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1-KG with velocity of 6m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the distance the ball is from the initial position of the gun at the instant the ball reaches its highest elevation C. Neglect the size of the gun.



Given: $M_g = 5 \text{ kg}$; $M_b = 1 \text{ kg}$; $V = 6 \text{ m/s}$; $U = 0 \text{ m/s}$

Required: $D_c = ?$

Solution:

We need to find velocity of ball with respect to ground to find range of ball. In vertical velocity will be same as that of ball w.r.t gun, because velocity of gun in vertical direction is zero.

In horizontal, gun will have some velocity backward so we need to find absolute velocity of ball.

$$6 \sin \theta = (u_{Bg})_v = 3$$

$$(u_{Bg})_H = u \cos \theta = 3\sqrt{3}$$

$$(u_{Bg})_v + (u_g)_v = (u_B)_v$$

$$(v_{Bg})_H + (U_g)_H = (U_B)_H$$

$$(u_B)_v = 3 \text{ m/s}$$

For horizontal, conserve momentum in horizontal direction as no friction is present.

Let gun goes backward with velocity x . The velocity of ball in horizontal to ground = $(3\sqrt{3} - x)$ also momentum conservation.

$$5x = (3\sqrt{3} - x)$$

$$x = \sqrt{3}/2$$

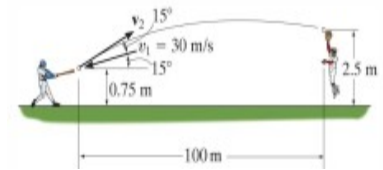
$$(u_B)_H = 3\sqrt{3} - \sqrt{3}/2$$

$$(u_B)_H = 5\sqrt{3}/2$$

$$T = 3/10 \times 2 = 0.6 \text{ sec}$$

$$D = (u_B)_H \times T = 9.5 \times 0.6 \times \sqrt{3} = \underline{2.595 \text{ m}}$$

5. The 0.15kg baseball has a speed of 30m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.

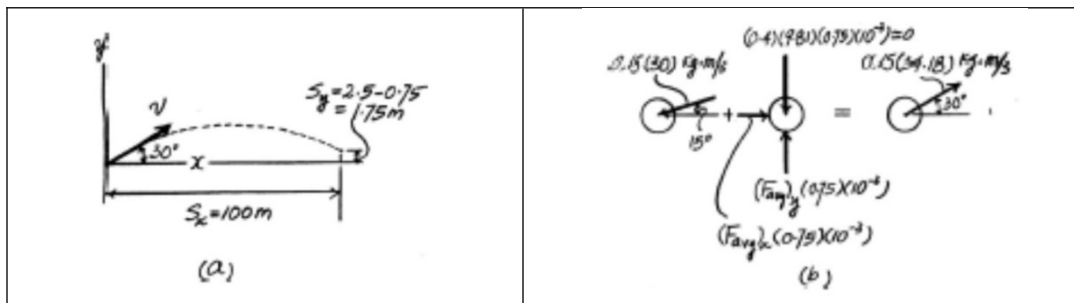


Given: $M=0.15\text{kg}$; $V_0=30\text{ m/s}$; $Y=0.75\text{ m}$; $G=10\text{ m/s}^2$

Required: Average Impulsive Force=?

Solution: _

FBD



$$(\rightarrow) x_x = (x_0)_x + (v_0)_x t$$

$$100 = 0 = v \cos 30^\circ t$$

$$t = 100 / v \cos 30^\circ$$

$$(\uparrow) y_y = (y_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$$

$$1.75 = 0 + v \sin 30^\circ (100 / v \cos 30^\circ) + \frac{1}{2} (-9.81) (100 / v \cos 30^\circ)^2$$

$$v = 34.18 \text{ m/s}$$

From principle of impulse and momentum: here the impulse generated by the weight of the base bar is very small compared to that generated by the force of the impact. Hence, it can be neglected by referring to the impulse and momentum diagram shown in fig b.

$$(\rightarrow) m(v_1)_x + \sum \int_{t_1}^{t_2} f_x dt = m(v_2)_x$$

$$-0.15(30) \cos 15^\circ + (F_{arg})_x (0.75)(10^{-3}) = 0.15(34.18) \cos 30^\circ$$

$$(F_{arg})_x = 11715.7 \text{ N}$$

$$M(v_1)_y + \sum \int F_y dr = m(v_2)_y$$

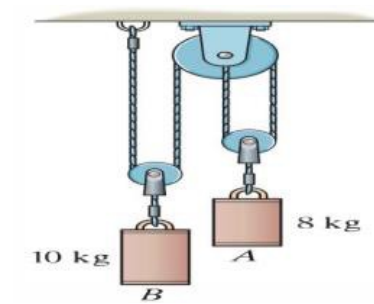
$$-0.15(30) \sin 15^\circ + (F_{arg})_y (0.75)(10^{-3}) = 0.15(34.18) \sin 30^\circ$$

$$(F_{arg})_y = 4970.9 \text{ N}$$

Thus

$$(F_{arg})^2 = (F_{arg})_x^2 + (F_{arg})_y^2 = 11715.7^2 + 4970.9^2 = 12.7 \text{ KN}$$

6. If cylinder A is given an initial downward speed of 2 m/s, determine the speed of each cylinder when $t = 3$ s. Neglect the mass of the pulleys



Solution

Principle of linear impulse and momentum

$$MV_1 + \sum \int_{t_1}^{t_2} F \times dt = MV_2$$

For cylinder A

$$MV_1 + 2Tt - wt = MV_2$$

$$-16 + 6T - 3mg = -8V_2$$

$$6T - 251.3 = -8V_2 \dots\dots\dots(1)$$

For cylinder B

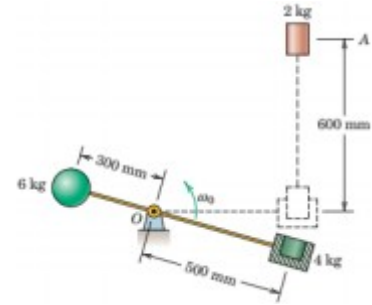
$$MV_1 + 2Tt - wt = MV_2$$

$$20 + 6T - 294 = 10V_2 \dots\dots\dots(2)$$

Solving for equation (1) and (2)

$$V_2 = 1.26 \text{ m/s}, V_A = -1.26 \text{ m/s and } V_B = 1.26 \text{ m/s}$$

7. The 6-kg sphere and 4-kg block (shown in section) are secured to the arm of negligible mass which rotates in the vertical plane about a horizontal axis at O. The 2-kg plug is released from rest at A and falls into the recess in the block when the arm has reached the horizontal position. An instant before engagement, the arm has an angular velocity 2rad/s. Determine the angular velocity of the arm immediately after the plug has wedged itself in the block.

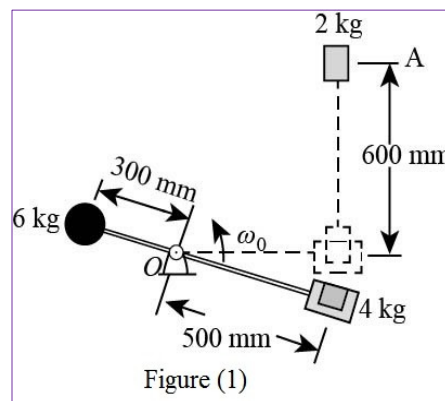


Given: Mass(b)=6kg; Mass(s)=4kg; Mass(p)=2kg; V(i)=0m/s;
W(o)=2rad/sec; G=9.81m/s; h=600mm; L(s)=300mm; L(p)=500mm

Required: Wf=?

Solution:

FBD (figure 1) =



- I. Calculate the velocity of the plug just before the engagement takes place.

$$V(p) = \sqrt{2gh}$$

$$V = \sqrt{2 \times 9.81 \times (600 \times 1/1000)}$$

$$V = \sqrt{2 \times (9.81) \times (0.6)}$$

$$V = 3.14 \text{ m/s}$$

- II. Calculate the initial angular momentum of the system about point O

$$(H_o) = M_b L_b W_o + M_s L_s W_o - M_p V_p L_p$$

$$(H_o) = (4)(0.5)(0.5)(2 \text{ rad/sec}) + (6 \text{ kg})(3)(3)(2 \text{ rad/sec})$$

$$(H_o) = -0.351 \text{ kg} \cdot \text{m}^2/\text{s}$$

Final angular momentum of the complete mass-system about point O

$$(H_o)_f = M_b L_b W_f + M_s L_s W_f + M_p L_p W_f$$

So when we substitute the numbers given

$$(H_o)_f = (4)(500)(500) + (6)(300)(300) + (2)(500)(500)Wf$$

$$H_o = 2.04Wf \text{ kg}\cdot\text{m}^2/\text{sec}$$

and finally the principle of conservation of angular momentum, which is

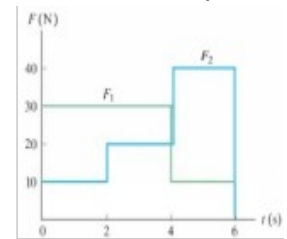
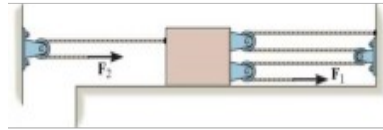
$$(H_o)_I = (H_o)_f$$

We found both the initial and the final in our above calculations, we substitute the numbers

$$-0.351 \text{ kgm/s for the initial } H(o)$$

So the angular velocity of the arm immediately after the lug wedged in the block is 0.1721 rad/sec

8. The 40-kg slider block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces F_1 and F_2 . If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.



Given

Mass of the block, $m = 40\text{ kg}$

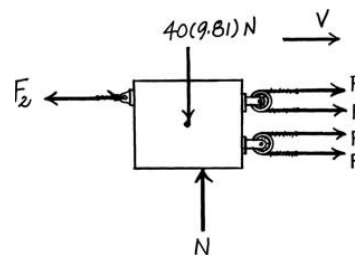
Initial Speed, $V_1 = 1.5\text{ m/s}$

Time, $t = 6\text{ sec}$

Required

Speed of the block at $t = 6\text{ s}$ $v_f = ?$

Solution:



Principle of Impulse and Momentum: The impulses produced by F_1 and F_2 are equal to the area under the respective $F-t$ graph. Referring to the FBD of the block above:

$$m(v_1)_x + \sum \int F_x dt = m(v_2)_x$$

$$40(1.5) + 4[4(30) + 10(6-4)] = 40(v_2)_x$$

$$(v_2)_x = 12\text{ m/s}$$

Thus, speed of the block at $t = 6\text{ s}$ is 12 m/s

9. A pendulum consists of two 3.2-kg concentrated masses positioned as shown on a light but rigid bar. The pendulum is swinging through the vertical position with a clockwise angular velocity 6 rad/s when a 50-g bullet traveling with velocity of 300 m/s in the direction shown strikes the lower mass and becomes embedded in it. Calculate the angular velocity which the pendulum has immediately after impact and find the maximum angular deflection of the pendulum.

Given:

Concentrated mass of the pendulum, $m_p = 3.2\text{ kg}$

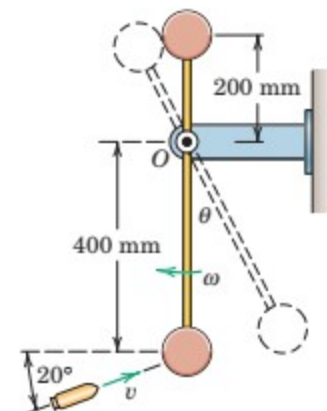
Clockwise angular velocity $\omega_i = -6\text{ rad/s}$

Mass of the bullet, $m_b = 50\text{ g} = 0.05\text{ kg}$

Linear velocity of the bullet, $v_b = 300\text{ m/s}$

Dimensions,

Required:



- i. The angular velocity which the pendulum has immediately after impact, ω_f
- ii. the maximum angular deflection of the pendulum, θ

Solution:

Using the law of conservation of angular momentum:

$\hookrightarrow$

$$r_1 m_b (v_b)_i + r_1 m_p (r_1 \omega_i) + r_2 m_p \dot{\theta}$$

Substituting the value in the above equation, we have;

$$(0.05 \text{ kg})(300 \text{ m/s}) \dot{\theta}$$

$$\dot{\theta} (0.05 \text{ kg} + 3.2 \text{ kg})(0.4 \text{ m})^2 \omega_f + (3.2 \text{ kg})(0.2 \text{ m})^2 \omega_f 5.638 - 0.768 - 3.072$$

$$\dot{\theta} 0.52 \omega_f + 0.128 \omega_f$$

$$\dot{\theta} \omega_f = \frac{1.798}{0.648}$$

$$\dot{\theta} 2.775 \text{ rad/s}$$

Hence, the angular velocity immediately after impact is $\omega_f = 2.775 \text{ rad/s}$

The positive value of the final angular velocity tells us that it is acting in the counterclockwise direction. Considering the energy after impact and choosing datum at O and applying conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$\hookrightarrow$

$$2 + 0.493 - 6.475 = -6.475 \cos \theta$$

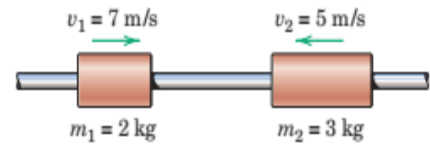
$$\dot{\theta} \cos \theta = 0.615$$

$$\dot{\theta} \theta = \cos^{-1}(0.615 \dot{\theta}) = 52.1^\circ \dot{\theta}$$

Therefore, the maximum angular deflection of the pendulum, $\theta = 52.1^\circ$

WORKSHEET 12

1. Compute the final velocities and after collision of the two cylinders which slide on the smooth horizontal shaft. The coefficient of restitution is 0.6.



Given:

Mass of the first and the second object is given $m_1 = 2$, $m_2 = 3$

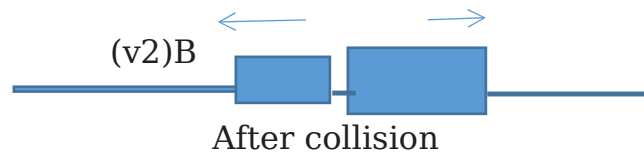
Volume of the first and second object also given $(v_1)_1 = 7$, $(v_2) = -5$

Required

$(v_1)_B$

$(v_1)_2 = ?$

$(v_2)_2 = ?$



Solution:

This is question of central impact from conservation of momentum equation of,

$$m_1(v_1)_A + m_2(v_2)_A = m_1(v_1)_B + m_2(v_2)_B$$

$$2(7) + 3(-5) = 2(v_1)_B + 3(v_2)_B$$

$$14 - 15 = 2(v_1)_B + 3(v_2)_B$$

$$-1 = 2(v_1)_B + 3(v_2)_B$$

$$2(v_1)_B + 3(v_2)_B = -1 \dots \dots \dots \text{EQUATION ONE (1)}$$

Again, from coefficient of restitution:

$$e = \frac{(v_2)_B - (v_1)_B}{(v_1)_A - (v_2)_A}$$

$$e = (v_2)_B - (v_1)_B$$

$$(v_1)_A - (v_2)_A$$

$$(v_2)_B - (v_1)_B = 7.2$$

$$(v_2)_B = (v_1)_B + 7.2 \dots \dots \dots \text{EQUATION (2)}$$

Thus using $(v_2)_B = (v_1)_B + 7.2$, in the first equation, which is from equation (1)

$$2(v_1)_B + 3(v_2)_B = -1$$

$$2(v_1)_B + 3((v_1)_B + 7.2) = -1$$

$$2(v_1)_B + 3(v_1)_B + 21.6 = -1$$

$$5(v_1)_B = -22.6$$

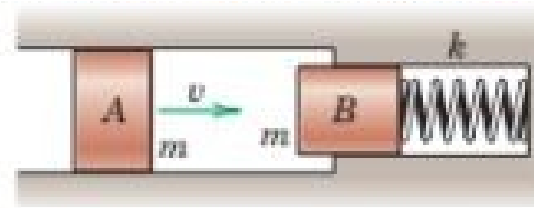
$$(v_1)_B = -4.52$$

$$(v_2)_B = (v_1)_B + 7.2$$

$$(v_2)_B = -4.52 + 7.2$$

$$(v_2)_B = 2.68 \text{ m/s}$$

2. Cylinder A is moving to the right with speed v when it impacts the initially stationary cylinder B. Both cylinders have mass m , and the coefficient of restitution for the collision is e . Determine the maximum deflection of the spring of modulus k . Neglect friction.



Given: $v_A = v$; $v_B = 0$; $m_A = m_B = m$; $\frac{u}{v} = e$

Required: $x = ?$

Solution;

$$KE_A + EPE_B = 0$$

$$\frac{1}{2} m v_B^2 - \frac{1}{2} k x^2 = 0$$

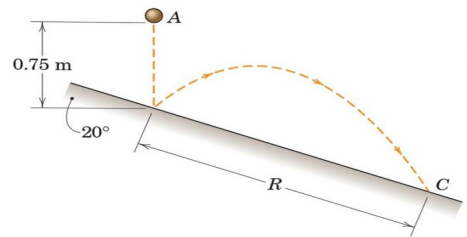
$$\frac{1}{2} (m v^2) = \frac{1}{2} k x^2$$

$$m v^2 = k x^2$$

$$\frac{m v^2}{k} = x^2$$

$$x = \sqrt{\frac{m v^2}{k}}$$

3. The ball is released from position A and drops 0.75 m to the incline. If the coefficient of restitution in the impact is $e = 0.85$, determine the slant range R .



Required;

The required in this problem is the free body diagram and the decomposition of velocity parameters. The slant distance 'R'.

Solution

This problem has a ball, which is released from position A (we will assume it was initially at rest) and collides with the incline. We are given an angle and a coefficient of restitution which we can use to derive necessary formulas to compute this question.

Analysis of position, velocity and acceleration are not needed.

- ❑ Construct a free body diagram
- ❑ Calculate the velocity at which the ball hits the incline:
Substitute, 9.81 m/s for g , 0.75 m for h

$v = \sqrt{2gh}$, from the relationship between kinetic and potential energy

$$v = \sqrt{2 \cdot 9.81 \cdot 0.75} = \sqrt{14.715} = 3.84 \text{ m/s}$$

- ❑ Calculate the velocity (v_t) of ball for the chosen direction of incline:

$$V_t = v \cdot \sin 20^\circ; \text{ Substitute, } 3.84 \text{ m/s for } v$$

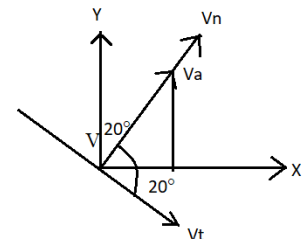
$$V_t = 3.84 \sin 20^\circ = 1.312 \text{ m/s}$$

- ❑ Calculate the velocity (v) of ball in the perpendicular direction of incline:

$$v_n = (v \cdot \cos 20^\circ) \cdot e$$

From our given and calculated we know

$$v = 3.84 \text{ m/s and } e = 0.85$$



e is the coefficient of restitution that can be determined by the change in the departed velocities to approached velocities or the change in the final velocities divided by the change in the initial velocities.

$$e = \frac{v_{B2} - v_{A2}}{(v_{B1} - v_{A1})}$$

Substitute, 3.84 m/s for v , 0.85 for e

$$V_n = (3.84 * \cos 20^\circ) \times 0.85 = 3.06 \text{ m/s}$$

- Calculate absolute velocity for rebound:

$v_a = \sqrt{(v_t)^2 + (v_n)^2}$; Substitute, 1.312 m/s for (v_t) , 3.06m/s for (v_n)

$$V_a = \sqrt{(1.312)^2 + (3.06)^2} = 3.33 \text{ m/s}$$

And the angle θ absolute velocity for rebound maintains with inclination plane is

$$\tan \theta = (v_n/v_t); \text{ Substitute known parameters } \tan \theta = (3.06/1.312)$$

$$\theta = \tan^{-1}(3.06/1.312) = 66.8^\circ$$

The displacement along X - coordinate direction is:

$$x = x_o + (v_a * \cos (\theta - 20^\circ)) * t$$

$$(R * \cos 20^\circ) = 0 + (3.33 * \cos 46.8^\circ) * t \quad R = 2.425 * t$$

$$R = 2.425 * t$$

$$t = R/2.425$$

Displacement in along Y-coordinate direction is:

$$y = y_o + (v_a * \sin (\theta - 20^\circ)) * t - 0.5gt^2$$

$$-R \sin 20^\circ = 0 + (3.33 * \sin 46.8^\circ) * t$$

$$R \sin 20^\circ = (9.81/2)t^2 - (3.33 \sin 46.8^\circ) * t$$

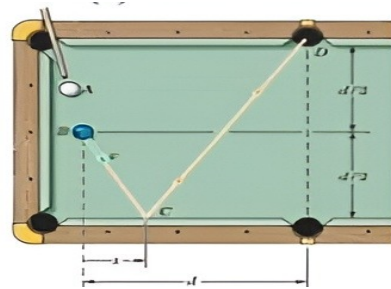
Substitute, $R/2.425$ for t

$$R \sin 20^\circ = (9.81/2) * (R/2.425)^2 - (3.33 * \sin 46.8^\circ) * (R/2.425)$$

$$\sin 20^\circ = (9.81/2) * (1/2.425)^2 * R - (3.33 \sin 46.8^\circ) * (1/2.425)$$

$$R = 1.613 \text{ m} \text{-----ANS}$$

4. Pool ball B is to be shot into the side pocket D banking it off the cushion at C specify the location x of the cushion impact for coefficient of restitution (a) $e=1$ and (b) $e=0.8$

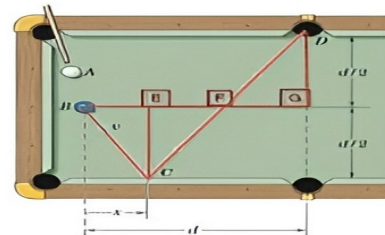


Solution:

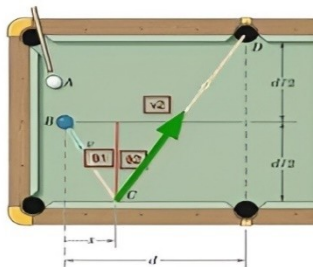
- a. When $e=1$, the situation is of reflecting from mirror

$$x = BE = FE = FG = DG/3 = d/3$$

$$x = d/3$$



- b. When $e=0.8$ we have the situation,



$$V_2 \cos(\theta_2) = e v \cos(\theta_1) \quad v_2 \sin(\theta_2) = v \sin(\theta_1)$$

$$\tan(\theta_2) = 1/e \tan(\theta_1)$$

Hence we have the relations,

$$BE = CE \tan(\theta_1) = d/2 \tan(\theta_1) = xBF = CE$$

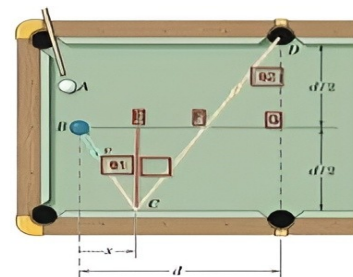
$$\tan(\theta_2) = d/2e \tan(\theta_1) = x/e \quad FG = DG \tan(\theta_2) = d/2e \tan(\theta_1) = x/e$$

Hence we have

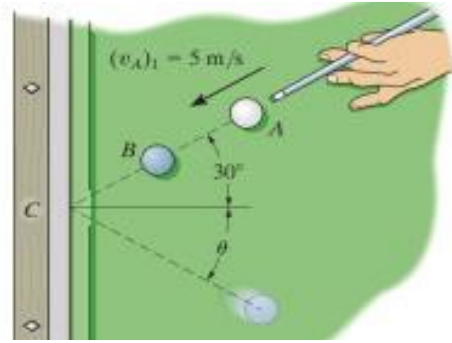
$$BG = BE + EF + FGd = x + x/e + x/e \quad x = e/2 + e$$

$$*d = 0.8/2.8 \quad d = 2/7 \quad d = 0.2857d$$

$$\mathbf{X = 0.286d}$$



5. The cue ball A given an initial velocity $V_{a1} = 5 \frac{m}{s}$, If it makes a direct collision with ball B ($e=0.8$), Determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 Kg. Neglect the size of each ball.



Required;

The final velocity of B

The angle that it has after rebounding at point C

Solution:

This type of problems are solved using the two basic laws: - law of conservation of momentum and law of restitution considering all mass have the same m

From law of conservation of momentum

$$m(v_{a1}) + m(v_{b1}) = m(v_{a2}) + m(v_{b2}) \dots \dots \dots V_{b2} = 0$$

$$0.4 \text{ kg} \left(5 \frac{m}{s} \right) = 0.4 \text{ kg} (v_{a2}) + 0.4 \text{ kg} (v_{b2})$$

$$5 \frac{m}{s} = (v_{a2}) + (v_{b2}) \dots \dots \dots \text{Equation 1}$$

From coefficient of restitution equation

$$e = \frac{(v_{b2}) - (v_{a2})}{(v_{a1}) - (v_{b1})} = \frac{(v_{b2}) - (v_{a2})}{5 - 0} = 0.8$$

$$v_{b2} = 4 \frac{m}{s} + v_{a2} \dots \dots \dots \text{Equation 2}$$

Equating both equation 1 and equation 2

$$v_{a2} = 0.5 \frac{m}{s} \text{ and } v_{b2} = 4.5 \frac{m}{s}$$

In order to determine the when ball B strikes point C

$$m_b (v_{b2}) = m_b (v_{b3})$$

$$0.4 \text{ kg} (4.50 \sin 30^\circ) = 0.4 \text{ kg} (v_{b3}) \sin \theta$$

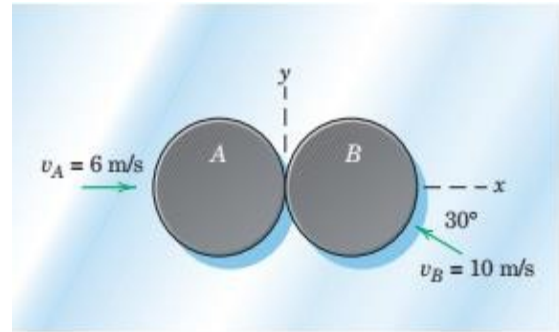
$$2.25 = (v_{b3}) \sin \theta$$

Using the second restitution

$$e = \frac{(v_{b3}) - (v_{a3})}{(v_{a2}) - (v_{b2})} = \frac{0 - (-v_{b3})}{4.5 \cos 30^\circ - 0} = 0.6$$

$$(v_{b3}) = 3.24 \frac{m}{s} \text{ and } \theta = 43.9^\circ$$

6. Two identical hockey pucks moving with initial velocities v_A and v_B collide as shown. If the coefficient of restitution is 0.75, determine the velocity (magnitude and direction with respect to the positive x-axis) of each puck just after impact. Also calculate the percentage loss n of system kinetic energy.



Given:

$$V_A = 6 \text{ i} \frac{\text{m}}{\text{s}}$$

$$V_B = 10 \frac{\text{m}}{\text{s}}$$

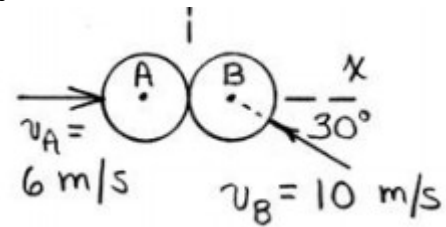
$$e = 0.75$$

Required:

- The magnitude and direction of their velocities after impact
- The percentage loss n of system kinetic energy

Solution:

- Since the x-axis is along the line of impact, we can apply the conservation of momentum along x-axis. That is along the x-axis; As the balls are identical, $m_A = m_B$



$$m_A(v_{A1}) + m_B(v_{B1})_x = m_A(v_{A2})_x + m_B(v_{B2})_x$$

$$6 = (v_{A2})_x + (v_{B2})_x \quad (1)$$

Again, from coefficient of restitution:

$$e = \frac{(v_{B2})_x - (v_{A2})_x}{(v_{A1})_x - (v_{B1})_x} = 0.75$$

$$(v_{B2})_x - (v_{A2})_x = 10.995 \quad (2)$$

Thus, adding equations (1) and (2)

$$\begin{cases} (v_{A2})_x + (v_{B2})_x = 6 \\ (v_{B2})_x - (v_{A2})_x = 10.995 \end{cases}$$

$$2(v_{B2})_x = 16.995 \Rightarrow (v_{B2})_x = 8.4975 \text{ m/s}$$

$$\dot{\varphi} (v_{A2})_x = -6.83 \text{ m/s}$$

For y-direction: since there is no force along the y-direction, the impulse of each object is zero.

That is
$$\sum_{t_1}^{t_2} F_y dt = 0$$

so,
$$\begin{cases} m_A (v_{A1})_y = m_A (v_{A2})_y = 0 \\ m_B (v_{B1})_y = m_B (v_{B2})_y \end{cases} \begin{cases} (v_{A1})_y = (v_{A2})_y = 0 \\ (v_{B1})_y = (v_{B2})_y = 10 \sin 30^\circ = 5 \text{ m/s} \end{cases}$$

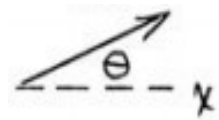
Magnitudes: 0

Directions:

For A: $\tan \theta_A = \frac{(v_{A2})_y}{(v_{A2})_x} = \frac{0}{-6.827} = 0 = 0 \Rightarrow \theta_A = \tan^{-1}(0) = 0^\circ \vee \theta_A = 180^\circ$

But the correct value of the direction angle is $\theta_A = 180^\circ$

For B: $\tan \theta_B = \frac{(v_{B2})_y}{(v_{B2})_x} = \frac{5}{4.17} = 1.2 \Rightarrow \theta_B = \tan^{-1}(1.2) = 50.2^\circ$



A. Energy Loss: To calculate the percentage loss of system kinetic energy, we need to calculate the kinetic energy before and after impact. Let the kinetic energy before impact be T_1 and after impact be T_2 .

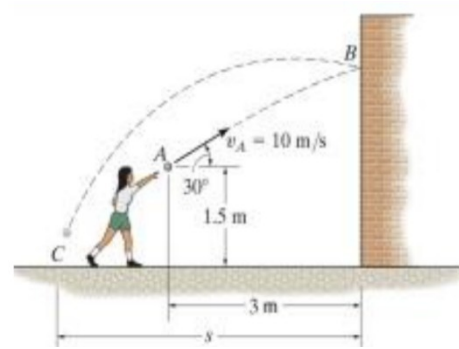
Now, calculate the initial and final energies

Initial energy: $T_1 = \frac{1}{2} m_A 0$

Initial energy: $T_2 = \frac{1}{2} m_A 0$

Percentage lost: $n = \frac{|T_1 - T_2|}{T_1} \cdot (100\%) = \frac{|68 m_A - 44.51 m_A|}{68 m_A} \cdot 100 = 34.54\%$

7. The girl throws the 0.5-kg ball toward the wall with an initial velocity $V_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



Given

Mass of the ball = 0.5 kg

Initial velocity of A (V_A) = 10 m/s

Coefficient of restitution (e) = 0.5

Required

- (a) The velocity at which it strikes the wall at B
- (b) The velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$
- (c) The distance s from the wall to where it strikes the ground at C.

Solution

By considering the horizontal motion of the ball before the impact, we have

$$S_x = (s_o)x + v_x t$$

$$3 = 0 + 10 \cos(30^\circ) t$$

$$t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$v_y = (v_o)_y + (a_c)_y t = 10 \sin 30^\circ + (-9.81 \text{ m/s}^2)(0.3464 \text{ s}) = 1.602 \text{ m/s}$$

The vertical position of point B above the ground is given by

$$S_y = (S_o)y + (V_o)_y t + \frac{1}{2}(ac)_y t^2$$

$$(S_B)_y = 1.5 + 10 \sin 30^\circ (0.3464) + \frac{1}{2}(-9.81) t^2$$

Thus, the magnitude of the velocity and its directional angle are

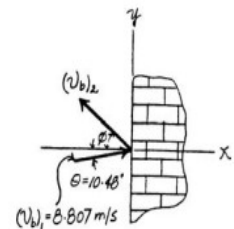
$$(V_b)_1 = \sqrt{(10 \cos(30^\circ))^2 + 1.602^2} = 8.807 \text{ m/s} \approx 8.81 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{1.602}{10 \cos(30^\circ)} = 10.48^\circ \approx 10.5^\circ$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of

$(V_b)_1 = 8.807$ m/s, it rebounds with a speed of $(V_b)_2$.

$$m_b(V_{b_y})_1 = m_b(V_{b_y})_2$$



$$m_b(1.602) = m_b(V_b)_2 \sin \phi$$
$$(V_b)_2 \sin \phi = 1.602 \dots \dots \dots (\text{equ-1})$$

Coefficient of restitution (x);

$$e = \frac{(v_w)_2 - (v_{b_x})_2}{(v_{b_x})_1 - \dots}$$

$$0.5 = 0 - \dots \text{ (equ-2)}$$

Solving equation 1 and 2;

$$\theta = 20.3^\circ (V_{b_x})_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s}$$

By considering the vertical motion of the ball after the impact, we have

$$S_y = \dots$$

$$-2.643 = 0 + 0.617 \sin 20.3^\circ t_1 + \frac{1}{2}(-9.81 t^2)$$

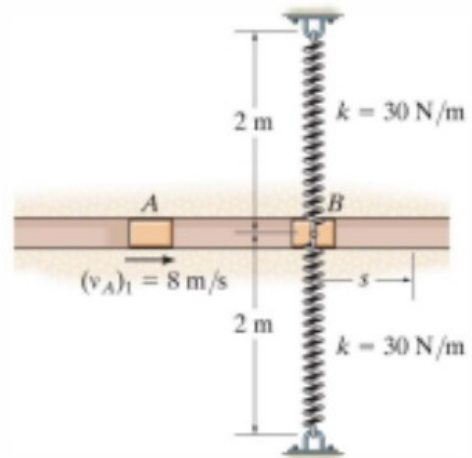
$$t_1 = 0.9153 \text{ sec}$$

By considering the horizontal motion of the ball after the impact, we have

$$s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30^\circ (0.9153) = 3.96 \text{ m}$$

8. The slider block B is confined to move within the smooth slot. It is connected to two springs, each of which has a stiffness of $k = 30 \text{ N/m}$. They are originally stretched 0.5 m when $S = 0$ as shown. Determine the maximum distance, $S_{\max}$ block B moves after it is hit by block A which is originally traveling at $(V_A)_i = 8 \text{ m/s}$. Take $e = 0.4$ and the mass of each block to be 1.5 kg .



Given: $K = 30 \text{ N/m}$; $e = 0.5$; $(V_A)_i = 8 \text{ m/s}$; $m = 1.5 \text{ Kg}$; $l = 0.5 \text{ m}$; $L = 2 \text{ m}$

Required: $S_{\max} = ?$

Solution;

$$e = \frac{(V_b)_f - (V_a)_f}{(V_a)_i - (V_b)_i}$$

$$0.5 = \frac{(V_b)_f - (V_a)_f}{8 - 0}$$

$$(V_b)_f = 4 \text{ m/s} + (V_a)_f \dots\dots\dots(1)$$

Expression for initial conservation of momentum is $P_i = m((V_a)_i + (V_a)_i)$

Expression for final conservation of momentum is $P_f = m((V_b)_f + (V_b)_f)$

Expression for conservation of momentum is $P_i = P_f$

$$m((V_a)_i + (V_a)_i) = m((V_b)_f + (V_b)_f)$$

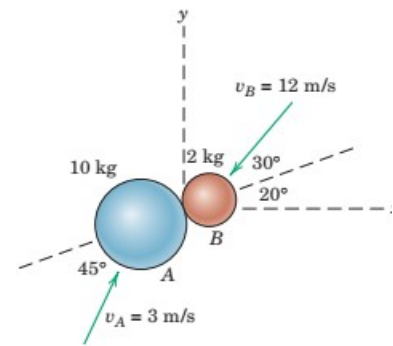
Substituting the value, $m(8 + 0) = m(4 + (V_a)_f + (V_a)_f)$; **$(V_a)_f = 2 \text{ m/s}$**

Substituting $(V_a)_f = 2 \text{ m/s}$ into equation (1); **$(V_b)_f = 4 \text{ m/s} + 2 \text{ m/s} = 6 \text{ m/s}$**

$$\frac{1}{2} * 1.5 * 6^2 * 30 * 0.3^2 = (30 * (\sqrt{S_{\max}^2 + 2^2} - (2 - 0.3)^2)$$

$$\mathbf{S_{\max} = 1.53 \text{ m}}$$

9. Sphere A collides with sphere B as shown in the figure. If the coefficient of restitution is 0.5, determine the x- and y-components of the velocity of each sphere immediately after impact. Motion is confined to the x-y plane



Given: $V_A = 3 \text{ m/s}$; $V_B = 12 \text{ m/s}$; $M_A = 10 \text{ kg}$; $M_B = 2 \text{ kg}$; $e = 0.5$

Required: Find the x and y components of each sphere after the impact

Solution:

Let's calculate the tangential components of v' for sphere A and B

$$V'_{At} = V_{At} = 3 \sin 45^\circ = 2.12 \text{ m/s}$$

$$V'_{Bt} = V_{Bt} = -12 \sin 30^\circ = -6 \text{ m/s}$$

As there is no external force applied to the system we assume that energy is conserved.

$$M_A V_{An} + M_B V_{Bn} = M_A V'_{An} + M_B V'_{Bn}$$

$$10 (3 \cos 45^\circ) + 2(-12 \cos 30^\circ) = 10V'_{An} + 2V'_{Bn} \quad (1)$$

The equation of $e = \frac{V'_{Bn} - V'_{An}}{V_{Bn} - V_{An}}$;

$$0.5 = \frac{V'_{An} - V_{Bn}}{(3 \cos 45^\circ) - (-12 \cos 30^\circ)} \quad (2)$$

Solving from equation 1 and 2, we get the result $V'_{An} = -1.007 \text{ m/s}$, $V'_{Bn} = 5.25 \text{ m/s}$. Then we can solve the rest,

$$V'_{Ax} = -2.12 \sin 20^\circ - 1.007 \cos 20^\circ = -1.672 \text{ m/s}$$

$$V'_{Ay} = 2.12 \cos 20^\circ - 1.007 \sin 20^\circ = 1.649 \text{ m/s}$$

$$V'_{Bx} = (-6 \sin 20^\circ) + 5.25 \cos 20^\circ = 6.99 \text{ m/s}$$

$$V'_{By} = -6 \cos 20^\circ + 5.25 \sin 20^\circ = -3.84 \text{ m/s}$$