

SOLVING LPP USING GRAPHIC METHOD

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GRAPHICAL REPRESENTATION AND SOLUTION

conditions for Graphical Method

1. Is the problem an LP? Yes, if and only if:

All variables have power of 1, and they are added or subtracted (not divided or multiplied). The constraint must be of the following forms ($\geq$, $\leq$, or $=$, that is, the LP-constraints are always closed)

2. the objective must be either maximization or minimization.

3. Can I use the graphical method? Yes, if the number of decision variables is either 1 or 2.

CONT...

- **Steps for Graphical Solution**

A. Corner Point Method

- 1. Define the problem mathematically**
- 2. Graph by constraints by treating each inequality as equality.**
- 3. Locate the feasible region and the corner points.**
- 4. Find out the value of objective function at these points.**
- 5. Find out the optimal solution and the optimal value of O.F. if it exists.**

CONT...

- **Steps for Graphical Solution**
 - **B. Iso-Profit or Iso-Cost Line Method**
1. Define the problem mathematically
 2. Graph by constraints by treating each inequality as equality.
 3. Locate the feasible region and the corner points.
 4. Draw out a line having the slope of Objective Function Equation (this is called Iso-Cost / Profit Line in Minimization and Maximization problems respectively) somewhere in the middle of the feasible region

CONT...

- **Steps for Graphical Solution**
Iso-Profit or Iso-Cost Line Method ctd..
- 5. Move this line away from origin (in case of Maximization) or towards Origin (in case of Minimization) until it touches the extreme point of the feasible region.
- 6.If a single point is encountered, that reflects optimality and its coordination the solution. If Iso-Profit/ Cost line coincides with any constraint line at the extreme, then this is the case of multiple optimum solutions.

CONT...

Maximization Problem

Constraints	Model A (X_1)	Model B (X_2)	Maximum Available hrs.
Labor hr.	2	1	40
Machine hr.	1	3	45
Marketing hr.	1	0	12
Profit	\$300	\$250	

CONT...

A Maximization Problem

Solution

1. Formulation of mathematical modeling of LPP

$$\text{Max } Z = 300X_1 + 250X_2$$

St:

- $2X_1 + X_2 \leq 40$
- $X_1 + 3X_2 \leq 45$
- $X_1 \leq 12$
- $X_1, X_2 \geq 0$

2. Convert constraints inequalities into equalities

- $2X_1 + X_2 = 40$
- $X_1 + 3X_2 = 45$
- $X_1 = 12$

CONT...

A Maximization Problem

- *Solution*

3. Draw the graph by intercepts

$$2X_1 + X_2 = 40 \implies (0, 40) \text{ and } (20, 0)$$

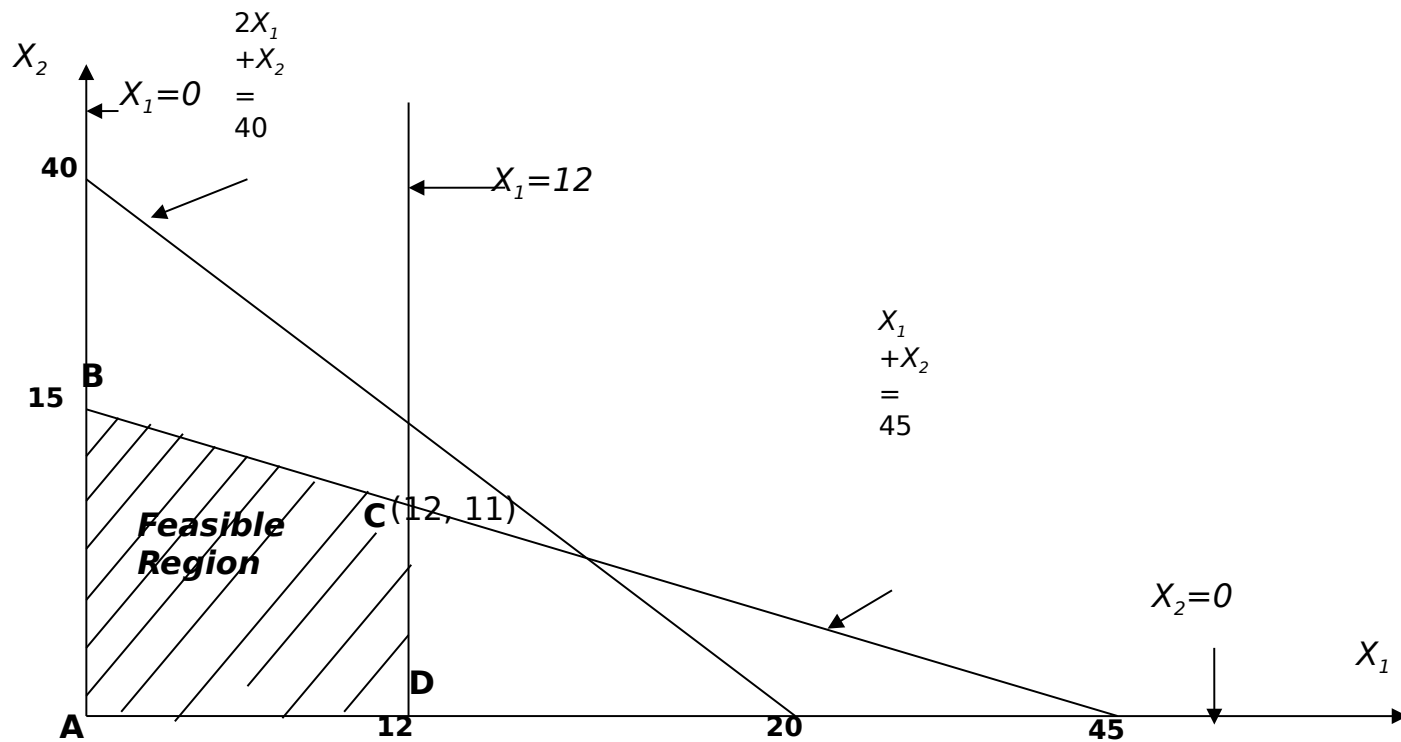
$$X_1 + 3X_2 = 45 \implies (0, 15) \text{ and } (45, 0)$$

$$X_1 = 12 \implies (12, 0)$$

$$X_1, X_2 = 0$$

CONT...

- Solution



CONT...

- **A Maximization Problem**

Solution

- 4. *Identify the feasible area of the solution which satisfies all constraints.*
- 5. *Identify the corner points in the feasible region*
- *A (0, 0), B (0, 15), C (12, 11) and D (12, 0)*
- 6. *Identify the optimal point*
- 7. *Interpret the result*

<u>Corners</u>	<u>Coordinates</u>	<u>MaxZ=300 X1 +250X2</u>
A	(0, 0)	\$0
B	(0, 15)	\$3750
C	(12, 11)	\$6350
D	(12, 0)	\$3600

Interpretation:

12 units of product A and 11 units of product B should be produced so that the total profit will be \$6350.

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Example ABE 2.1

Solve using Graphical method

Maximize $\{w = 2X_1 + 5X_2\}$

Subj. to:

$$X_1 \leq 4$$

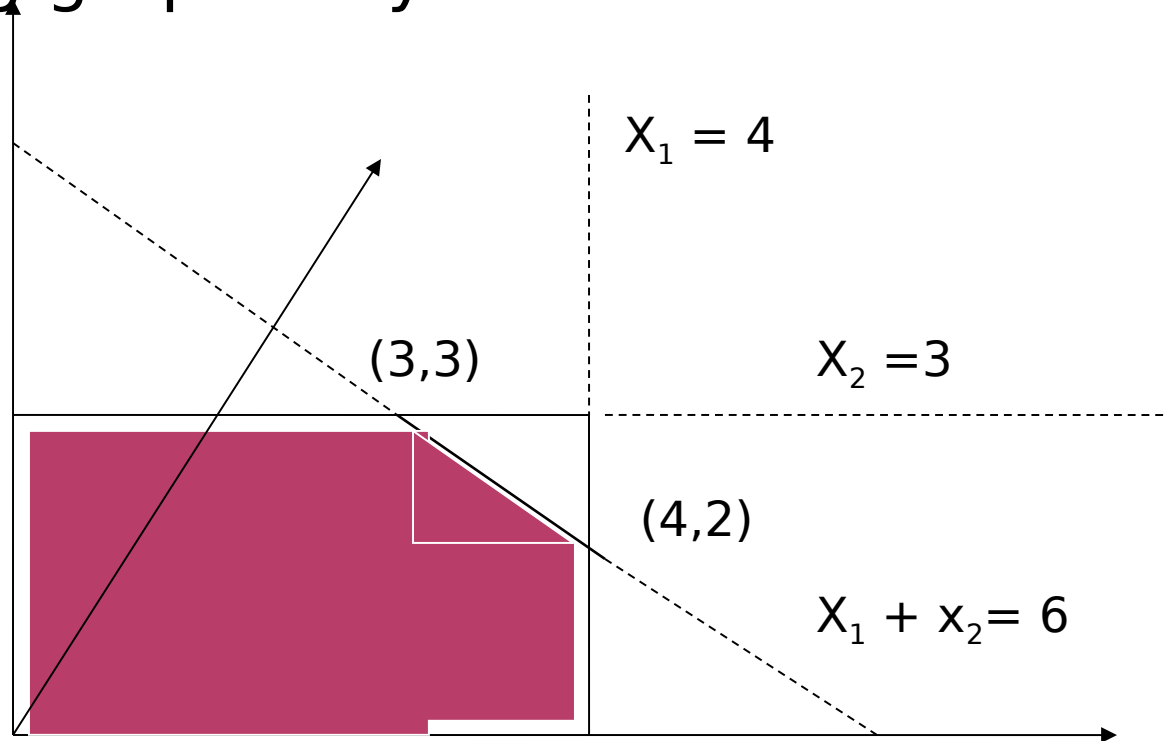
$$X_2 \leq 3$$

$$X_1 + X_2 \leq 6$$

$$X_1 \geq 0, X_2 \geq 0$$

CONT...

- Solving graphically



CONT...

- **Minimization Problem**
- **Minimize Z with inequalities of constraints in $\geq$ form**

Example:

Suppose that a machine shop has two different types of machines; machine 1 and machine 2, which can be used to make a single product. These machines vary in the amount of product produced per hr., in the amount of labor used and in the cost of operation.

Assume that at least a certain amount of product must be produced and that we would like to utilize at least the regular labor force. How much should we utilize each machine in order to utilize total costs and still meet the requirement? The data are given below.

Resource used				
	Machine 1 (X1)	Machine (X2)	Mini. requirement	
hrs _____				
Product produced/hr	20	15	100	
Labor/hr	2	3	15	
Operation Cost	\$25	\$30		

FORMULATE THE LP

$$\text{Min} Z = 25X_1 + 30X_2$$

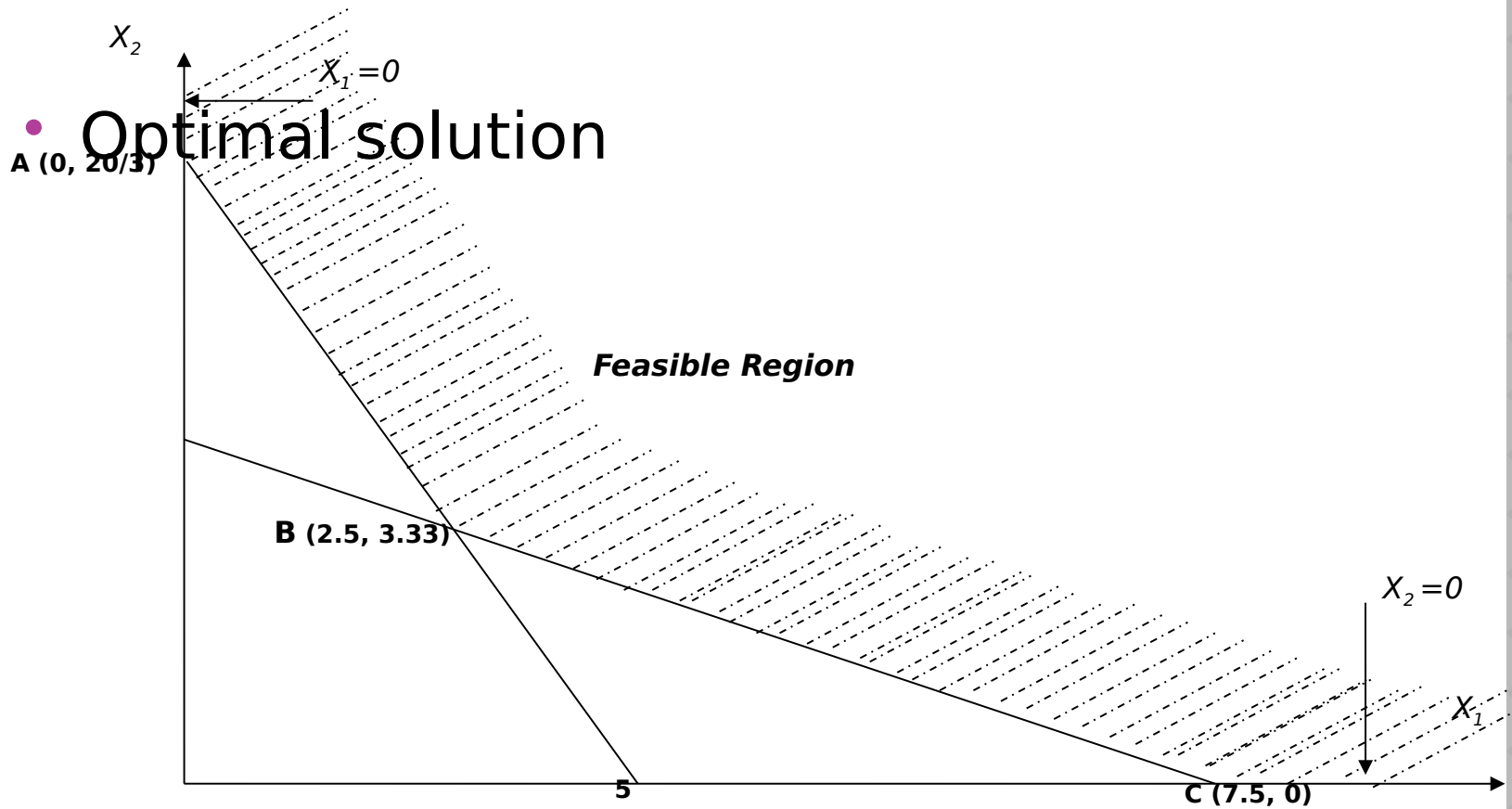
St:

$$20X_1 + 15X_2 \geq 100$$

$$2X_1 + 3X_2 \geq 15$$

$$X_1, X_2 \geq 0$$

CONT...



CONT...

- **Final solutions**

- | <u>Corners</u> | <u>Coordinates</u> | <u>MinZ=25 X1 + 30X2</u> |
|-----------------|---------------------------|--------------------------|
| A | (0, 20/3) | 200 |
| | | |
| <u>B</u> | <u>(2.5, 3.33)</u> | <u>162.5</u> |
| C | (7.5, 0) | 187.5 |
- $X1 = 2.5$
- $X2 = 3.33$ and
- $\text{MinZ} = 162.5$

CONT...

- **SPECIAL CASES IN GRAPHICS METHODS**

1. Redundant Constraint

- **If a constraint when plotted on a graph doesn't form part of the boundary making the feasible region of the problem that constraint is said to be redundant.**

Example:

A firm is engaged in producing two products A and B .Each unit of product A requires 2Kg of raw material and 4 labor-hrs for processing. Where as each unit of product B requires 3Kg of raw materials and 3hrs of labor. Every nit of product A needs 4 hrs and B needs 3.5hrs for packaging. Every week the firm has availability of 60Kg of raw material, 96 labor-hours and 105 hrs I the packaging department. 1 unit of product A sold yields \$40 profit and 1 unit of B sod yields \$35 profit.

Required:

- Formulate this problem as a LPP**
- Find the optimal solution**

CONT....

- **SPECIAL CASES IN GRAPHICS METHODS**

- ***Solution***

- | available
Resources
week | Products | | Resource
per |
|--------------------------------|----------|------|-----------------|
| | A | B | |
| Raw materials (Kg) | 2 | 3 | 60 |
| Labor (hr) 4 | 3 | 96 | |
| Packaging (hr)4 | 3.5 | 105 | |
| Profit per unit | \$40 | \$35 | |
- **Let X_1 =The No of units f product A produced per week**
- **X_2 =The No of units f product B produced per week**
- **LPP Model:**

CONT...

$$\text{Max } Z = 40X_1 + 35X_2$$

St:

$$2X_1 + 3X_2 \leq 60$$

$$4X_1 + 3X_2 \leq 96$$

$$4X_1 + 3.5X_2 \leq 105$$

$$X_1, X_2 \geq 0$$

CONT...

- **SPECIAL CASES IN GRAPHICS METHODS**

- After drawing the graph, The packaging hr is redundant.

- | <u>Corners</u> | <u>Coordinates</u> | <u>$Z_{max} = 40X1 + 35X2$</u> |
|-----------------------|---------------------------|--|
| • A | (0, 0) | 0 |
| • B | (0, 20) | 700 |
| • C | (18, 8) | 1000 |
| • D | (24, 0) | 960 |
- **$X1 = 18$**
- **$X2 = 8$ and**
- **$Z_{max} = 1000$**
- **Interpretation:**
- **The company should produce and sale 18 units of product A and 8 units of product B per week so as to get a maximum profit of 1000.**

CONT....

- Possible solutions to LP Problems

Alternative optimal solutions

Unbounded solution

No feasible solution

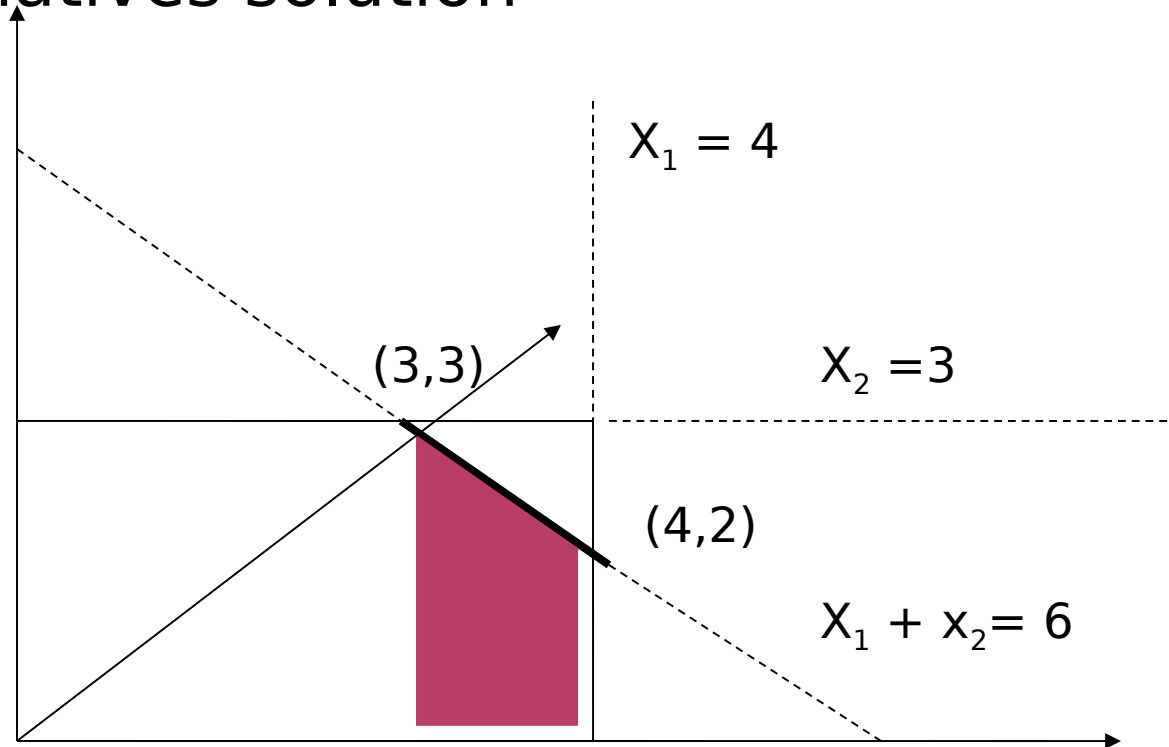
Degeneracy solution (optional) we will see it latter

CONT...

- **Alternative solution**
- (line segment of alternative optimal solution)
- **E.g.** Maximize $\{w = 2X_1 + 2X_2\}$
- Subj. to:
- $X_1 \leq 4$
- $X_2 \leq 3$
- $X_1 + X_2 \leq 6$
- $X_1 \geq 0, X_2 \geq 0$

CONT...

- Alternatives solution



CONT...

Unbounded solution

Eg. Max $\{W = 4x_1 + x_2\}$

Subj. to:

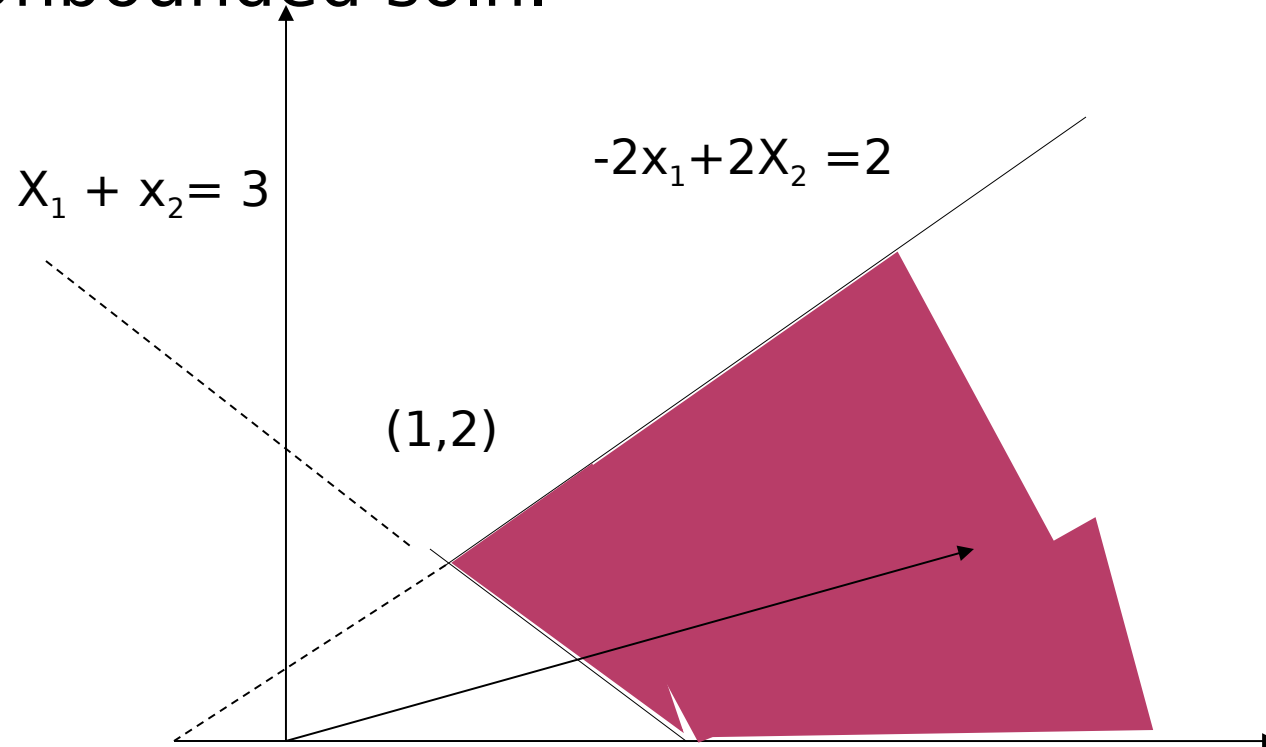
$$x_1 + x_2 \geq 3$$

$$-2x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

CONT...

- Unbounded soln.



CONT...

An LP problem with unbounded feasible area depends on the objective function and can be;

- one (finite) solution
- a line segment or half-line
alternative optimal solution
- unbounded

CONT...

No feasible solution

Eg. Max $\{W = x_1 + 4x_2\}$

Subj. to:

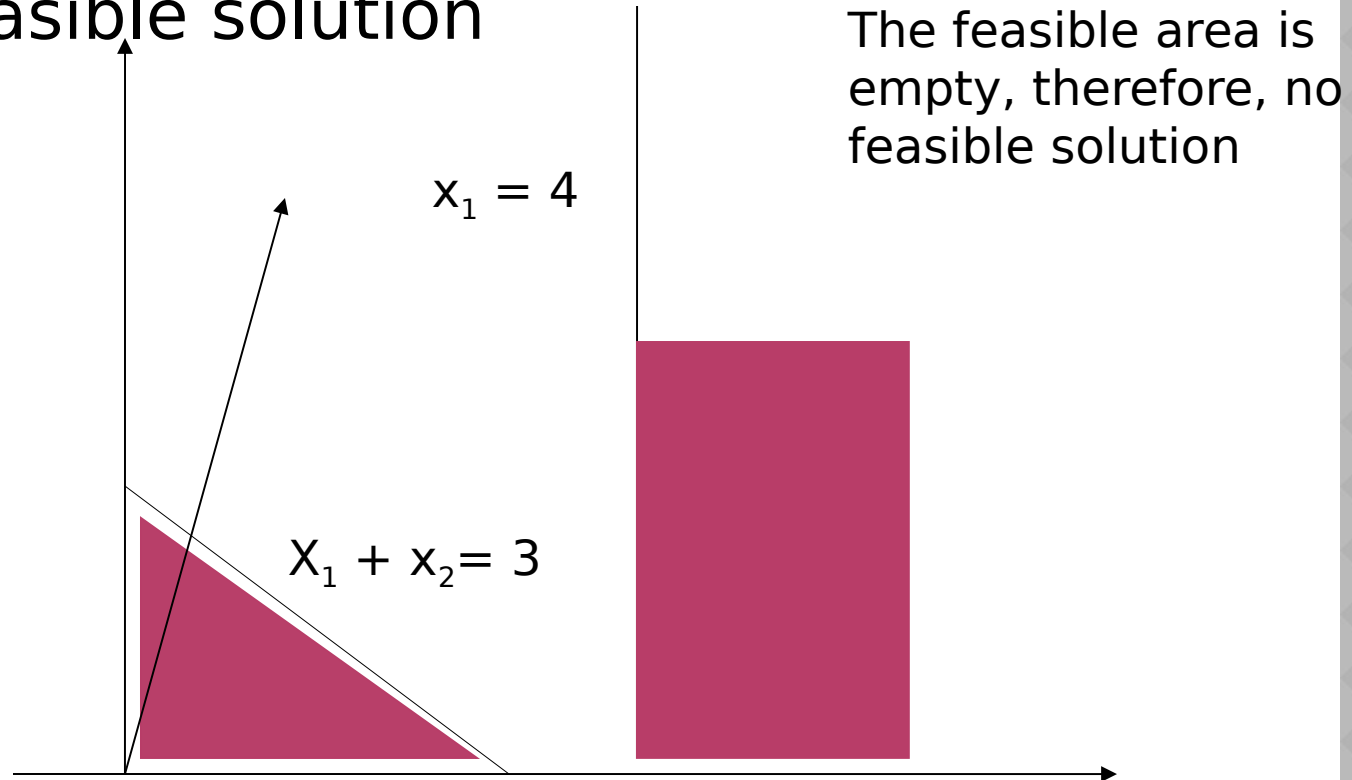
$$x_1 \geq 4$$

$$x_2 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

CONT...

- No feasible solution



CONT...

Binding (active) and non binding (inactive) constraints

From the ABE example,

Constraint $x_1 \leq 4$ is non binding

The other two are binding namely:

$$x_2 \leq 3$$

$$x_1 + x_2 \leq 6 \text{ are}$$

binding