Rates, Ratios and Proportions
and Measures of Disease Frequency

Nancy D. Barker, M.S.

Rates, Ratios, Proportions

- Three general classes of mathematical parameters.

- Often used to relate the *number of cases* of a disease or health outcome to the *size of the source population* in which they occurred.

Hypothetical data on the frequency of Hepatitis in two cities

Location	New cases	Year	Population
City A	62	2002	25,000
City B	35	2002	7,000

Hypothetical data on the frequency of Hepatitis in two cities

Location	New cases	Year	Population
City A	62	2002	25,000
City B	35	2002	7,000

Annual occurrence of Hepatitis:

City A: 62 / 25,000 = 2.5 per 1,000

City B: 35 / 7,000 = 5.0 per 1,000

Ratio

- Obtained by dividing one quantity by another. These quantities may be related or may be totally independent.
- Usually expressed as: $\frac{x}{y} \times 10^n$

Example: Number of stillbirths per thousand live births.

$$\frac{\#\text{stillbirths}}{\#\text{livebirths}} \times 1000$$

- General term that includes Rates and Proportions.

Ratio

Example: A foodborne epidemic occurred in an elementary school. The attack rate in the first grade was 24% while the attack rate in the second grade was 16%. Compare these two attack rates.

$$\frac{24\%}{16\%} = \frac{1.5}{1}$$

- Risk Ratio or Relative Risk

Proportion

- A ratio in which the numerator is included in the denominator.
- Expressed as: $\frac{x}{y} \times 10^n$ where, 10^n is often 100.

Example: The number of fetal deaths out of the total number of births.

$$\frac{\#\text{ of fetal deaths}}{\text{live births} + \text{fetal deaths}} \times 100$$

- Answer often read as a percent.

Proportion

Example: 500 people attended a company picnic and 50 became ill with acute enteric disease. Calculate the proportion of ill persons.

$$\frac{50}{500} \times 100 = 10.0\%$$

Rate

- A measure of how quickly something of interest happens.
- Expressed as: $\frac{x}{y} \times 10^n$

Example: The number of new cases of Parkinson's disease which develops per 1,000 person-years of follow-up.

$$\frac{\#\text{ of new cases of Parkinson's disease}}{\text{Total time disease-free subjects observed}} \times 1000$$

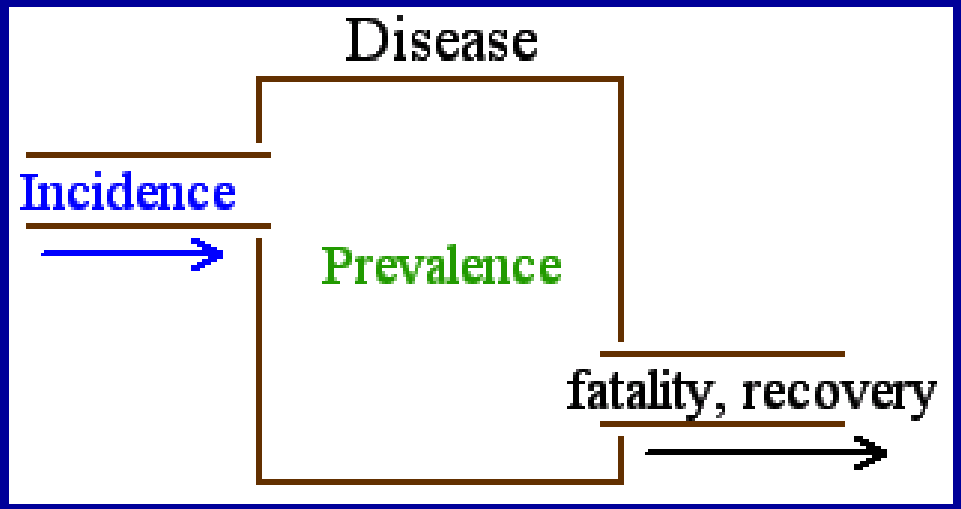
- Time, place and population must be specified for each type of rate.

Measures of Disease Frequency

- Incidence (I): Measures new cases of a disease that develop over a period of time.

- Prevalence (P): Measures existing cases of a disease at a particular point in time or over a period of time.

Prevalence vs. Incidence



- Prevalence can be viewed as describing a pool of disease in a population.
- Incidence describes the input flow of new cases into the pool.
- Fatality and recovery reflects the output flow from the pool.

Cumulative Incidence

$$CI = \frac{I}{N}$$

I = # of new cases during follow-up

N = # of disease-free subjects at start of follow-up

Measures the frequency of addition of new cases of disease and is always calculated for a given period of time.

Example

N=1000 men age 45

I=50 developed prostate cancer

Follow-up = 10 years

No one lost to follow-up

No one withdrew from study

$$\hat{CI} = \frac{I}{N} = \frac{50}{1000} = 0.05 = 5\%$$

10-year risk:

$$\hat{CI} = \frac{I}{N} = 5\%$$

Cumulative Incidence

- Most common way to estimate risk.
- Always a proportion
- Assumes a fixed cohort
- For brief specified periods of time, e.g. an outbreak, commonly called an Attack “Rate”

- Beware of attrition
- Formula does not reflect continually changing population size for dynamic cohorts.
- Does not allow subjects to be followed for different time periods.

Prevalence

- Measures existing cases of a health condition
- Primary feature of a cross-sectional study
- Two types of Prevalence
  - Point prevalence
  - Period prevalence

Point Prevalence

$$P = \frac{C}{N}$$

C = # of observed cases at time t

N = Population size at time t

Measures the frequency of disease at a given point in time

Point Prevalence Example

Suppose there are 150 individuals in a population and, on a certain day, 15 are ill with the flu. What is the estimated prevalence for this population?

$$P = \frac{15}{150} = 10\%$$

Period Prevalence

$$PP = \frac{C + I}{N}$$

- C = the # of prevalent cases at the beginning of the time period.
- I = the # of incident cases that develop during the period.
- N = size of the population for this same time period.

Prevalence

Useful for:

- Assessing the health status of a population.
- Planning health services.

Not Useful for:

- Identifying risk factors

Example

Suppose we followed a population of 150 persons for one year, and 25 had a disease of interest at the start of follow-up and another 15 new cases developed during the year.

1) What is the period prevalence for the year?

2) What is the point prevalence at the start of the period?

3) What is the cumulative incidence for the one year period?

Example

Suppose we followed a population of 150 persons for one year, and 25 had a disease of interest at the start of follow-up and another 15 new cases developed during the year.

1) What is the period prevalence for the year?

PP = (25 + 15) / 150 = 0.27 or 27%

2) What is the point prevalence at the start of the period?

P = 25/150 = 0.17 = 17%

3) What is the cumulative incidence for the one year period?

CI = 15/125 = 0.12 = 12%

Risk (proportion)

$0 \leq \text{RISK} \leq 1$

$0\% \leq \text{percentage} \leq 100\%$

Specify risk period

Example:

The 10-year risk that a 45-year-old male will develop prostate cancer is 5%.

Rate

- Describes how rapidly health events are occurring in a population of interest.
- In Epidemiologic studies, we typically measure the average rate at which a disease is occurring over a period of time.
- Example:

  50 new cases per 10,000 person-years

  Interpretation:

  An average of 50 cases occurs for every 10,000 years of disease free follow-up time observed on a cohort of subjects.

Rate

Which of these describes a rate?

1) 5 new cases per 100 person days
2) 40 miles per hour
3) 10 new cases out of 100 disease-free persons
4) 60 new murders per year
5) 60 deaths out of 200 lung cancer patients

Incidence Rate

$$IR = \frac{I}{PT}$$

I = # of new cases during follow-up

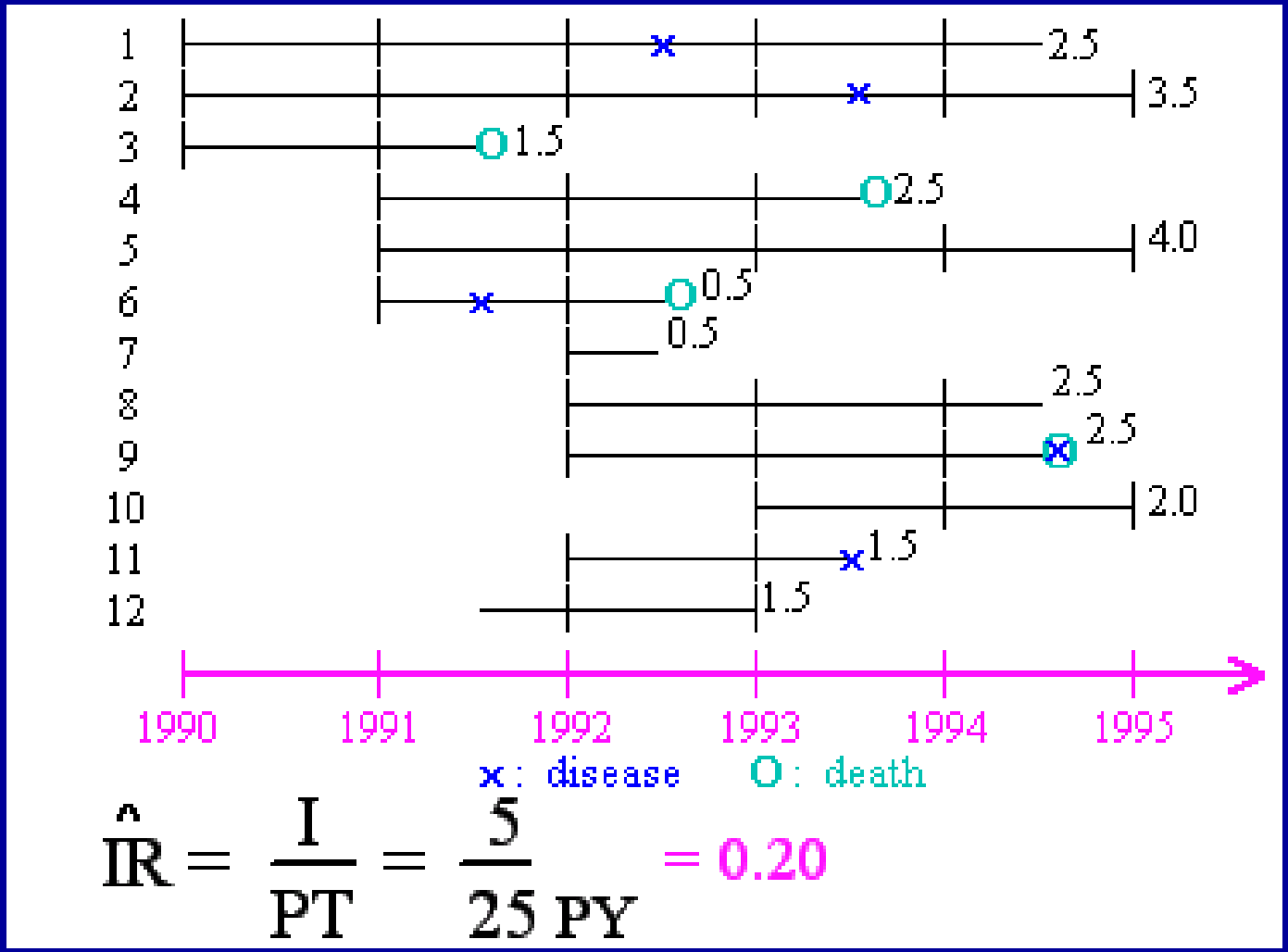
PT = total time that disease–free individuals in the cohort are observed over the study period.

Synonyms: hazard rate, incidence density rate.

Measures the rapidity with which new cases are occurring in a population

Example

Hypothetical cohort of 12 initially disease-free subjects followed over a 5-year period from 1990 to 1995.



$$\hat{IR} = \frac{I}{PT} = \frac{5}{25\ PY} = 0.20$$

Example, cont.

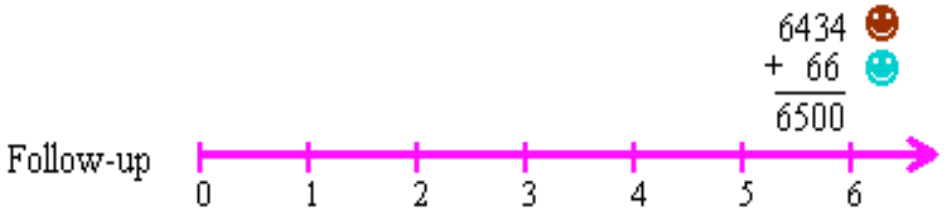
$$\hat{IR} = \frac{I}{PT} = \frac{5}{25 PY} = 0.20$$

$$= 20 \text{ new cases per } 100 \text{ person-years}$$

Study questions:

1) Is the value of 0.20 a proportion?
2) Does the value of 0.20 represent the risk of developing disease?

Parkinson's Disease



Total # disease-free person-years = 38,458 disease-free person-years

$$\hat{IR}_{\text{6-year study period}} = \frac{66 \text{ cases}}{38{,}458 \text{ person years}}$$

$$\hat{IR}_{\text{6-year study period}} = 0.0017 \frac{\text{cases}}{\text{person-year}}$$

1.7 new cases per 1,000 person-years

Confusing Risk with Rate

- The term “Rate” is often been used incorrectly to describe a measure of risk.
  - e.g.,
    - Attack Rate
    - Death Rate
    - Case-Fatality Rate
- When reading Epidemiologic literature, one should be careful to determine the actual measure being reported.

Risk (proportion)

Probability that an individual with certain characteristics such as:

Age

Race

Sex

Smoking status

Will experience a health status change over a specified follow-up period.

Assumes:

Does not have disease at start of follow-up.

Does not die from other cause during follow-up.

Study Questions

1) Is point prevalence a proportion?

2) Suppose 13 people from a population of size 406,245 had a particular disease at time t. What is the point prevalence of this disease at time t?

3) Which of the following expressions is equivalent to the point prevalence estimate of 0.000032?
   a. 3.2 per 1,000
   b. 3.2 per 100,000
   c. 32 per 100,000

Mortality Risk

Disease-Specific Mortality Risk

All-causes Mortality Risk

Case-Fatality Risk

$$CI = \frac{I}{N}$$

Mortality Rate

Disease-Specific Mortality Rate

All-causes Mortality Rate

Case-Fatality Rate

$$IR = \frac{I}{PT}$$

References

- ActivEpi CD-ROM; Kleinbaum
- ActivEpi Companion Textbook; Kleinbaum, Sullivan, Barker
- Epidemiology in Medicine, Hennekens