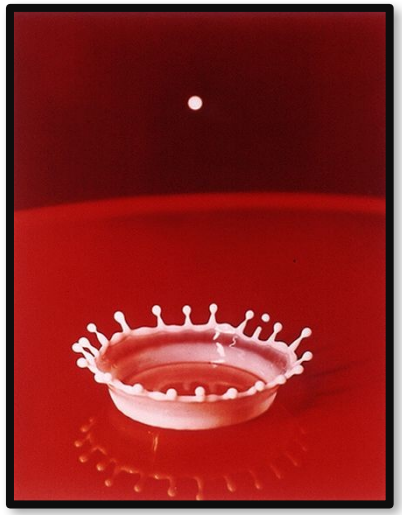
Chapter 3

Fluid Mechanics

- Density and Pressure in Static Fluid
- Buoyant Forces, Archimedes' Principle
- Moving Fluids and Bernoulli's Equation
- Properties of Bulk Matter, stress and Strain

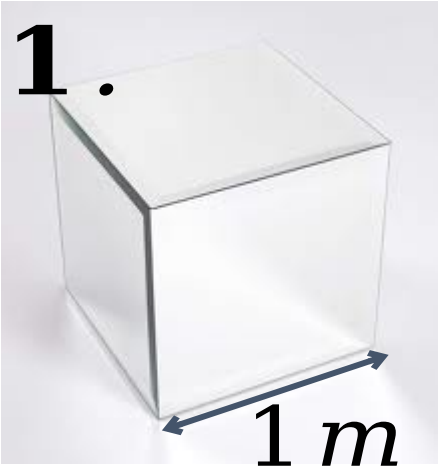


Density, specific weight, specific gravity

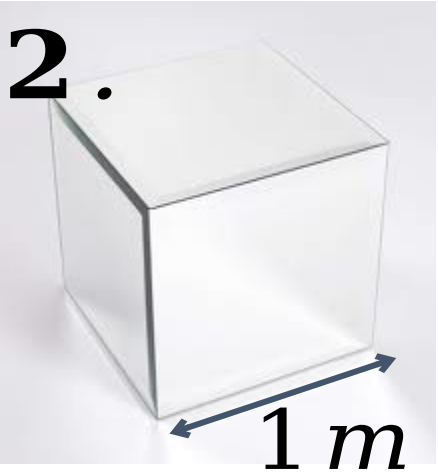
- **Density, ρ** (decreases with rise in T)
  - mass per unit volume (kg/m^3)
  - for water density = 1000 kg/m^3
- **Specific Weight, γ** (Heaviness of fluid)
  - weight per unit volume
  - for water
- **Specific Gravity = SG**
  - Ratio of the density of a fluid to the density of water

SG of Hg = 13.55

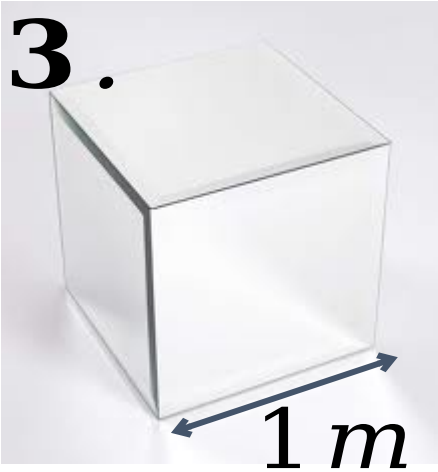
The cubic vessels contain the water at different temperatures. Which water has the **highest** density?



of water at



of water at



of water at

of water at

Example

A object has a mass of 36 grams and the volume of the water (reference material) is 3 mL. Find (a) the density, (b) the specific weight and (c) the specific gravity of the object. Also, specify if the object will sink or float in the water? The density of the water is 1 g/mL.

Solution:

(a) Density of object:

(b) Specific weight:

(C) Specific Gravity:

The specific gravity is greater than 1 so the object will sink in the water.

What is a Fluid?

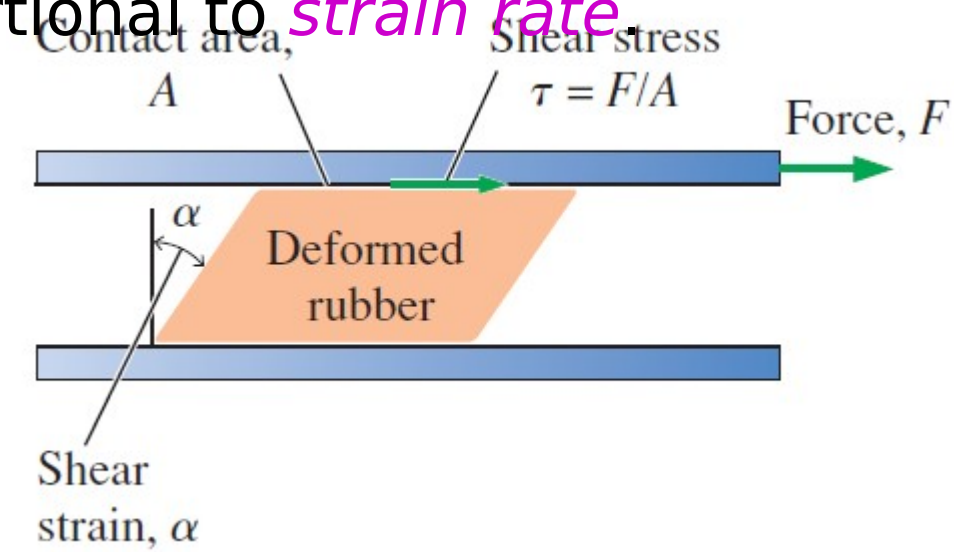
A fluid is a substance that deforms continuously under the influence of a shear stress, no matter how small.

Fluids exist in a liquid or gas phase.

In solids, stress is proportional to *strain*, e.g., Hooke's law

In fluids, stress is proportional to *strain rate*.

When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant



Stress and Strain

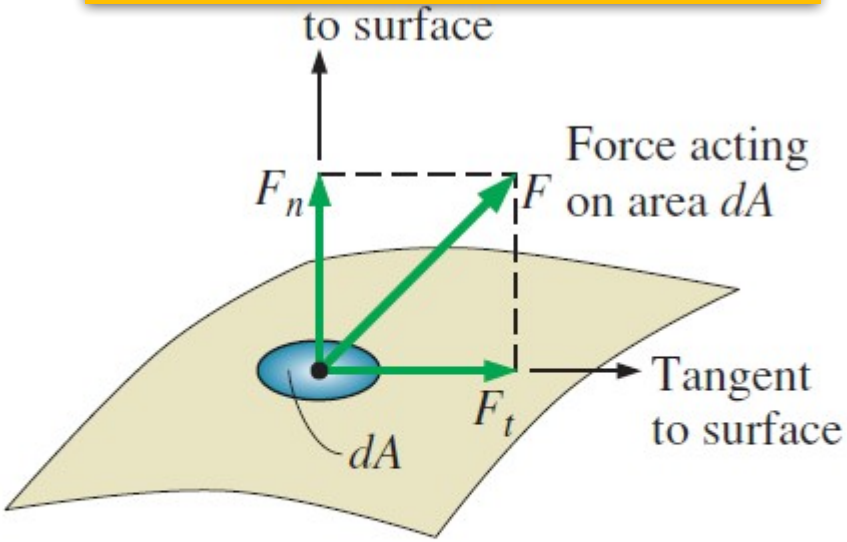
Stress: Force per unit area.

Normal stress: The normal component of a force acting on a surface per unit area.

Shear stress: The tangential component of a force acting on a surface per unit area.

Pressure: The normal stress in a fluid at rest.

Zero shear stress: A fluid at rest is at a state of zero shear stress.



$$\text{Normal stress: } \sigma = \frac{F_n}{dA}$$

$$\text{Shear stress: } \tau = \frac{F_t}{dA}$$

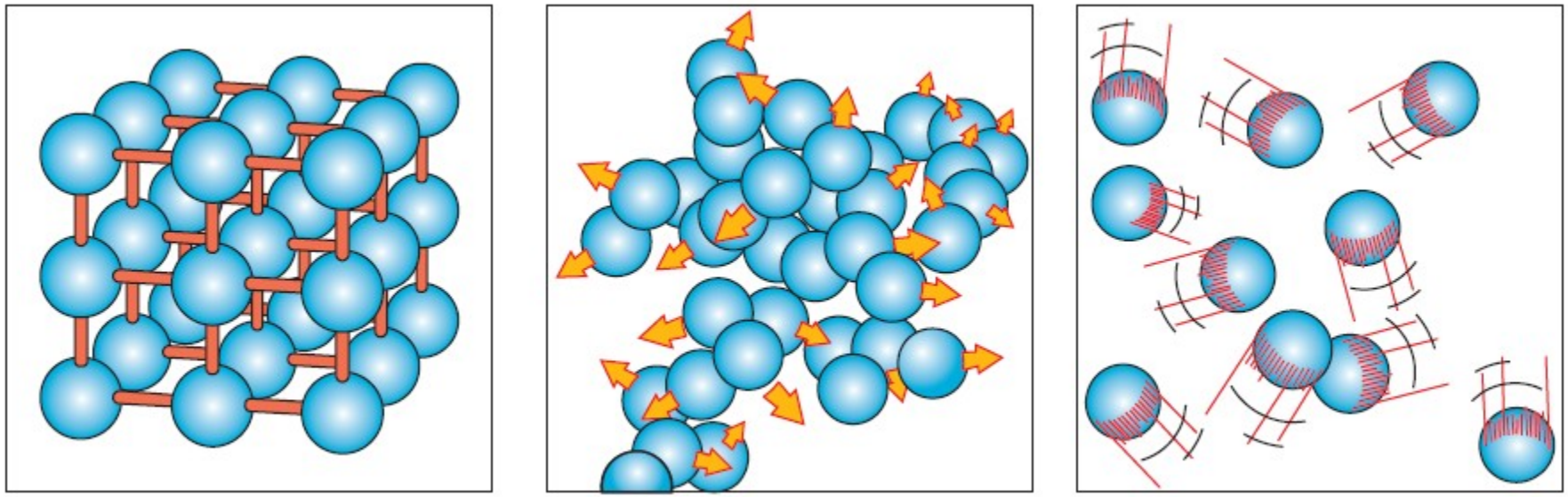
When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a horizontal free surface.

Intermolecular bonds are strongest in solids and weakest in gases.

Solid: molecules are at relatively fixed positions in some pattern

Liquid: molecules rotate and translate relative to each other

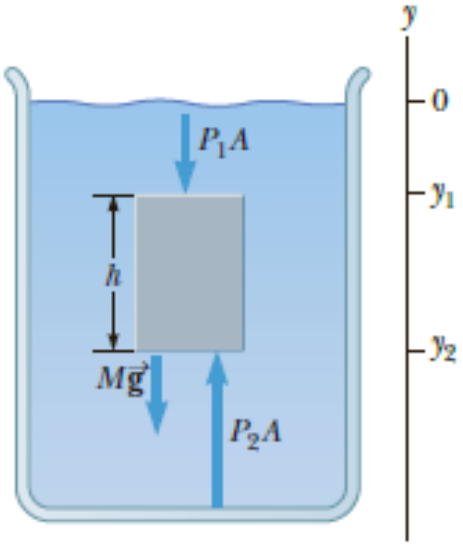
Gas: molecules are far apart from each other and move randomly



(a) (b) (c)

Pressure

- If an object is immersed in a fluid, the fluid exerts a force on every part of the object's surface.
- If the object and fluid are at **rest**, the force is perpendicular to the element of area on which it acts.



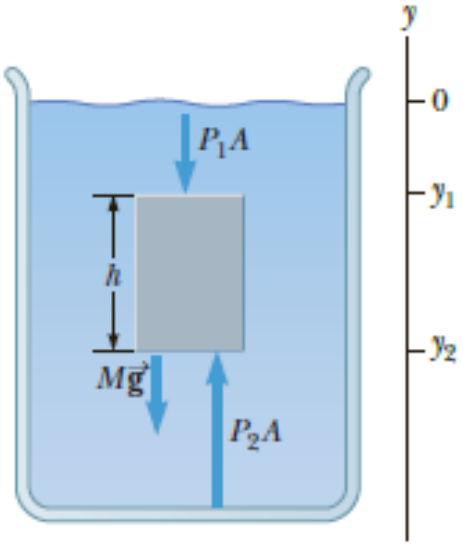
- The pressure is the force per unit area, measured in N/m^2, called Pascals

$$P_2A - P_1A - Mg = 0$$

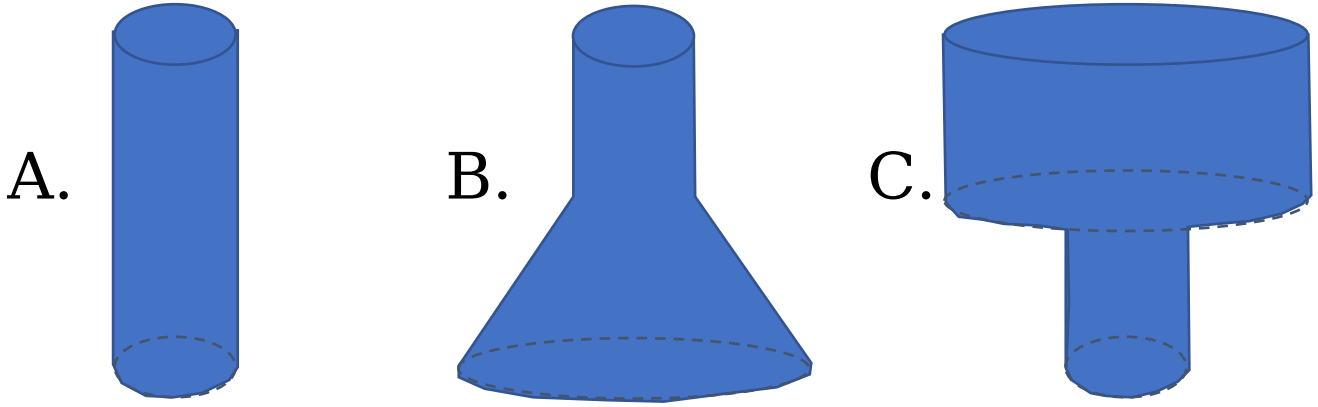
Pressure

- Let be the free surface of the liquid
- At the free surface the pressure is atmospheric pressure :

  Pa at sea level
- Then, the pressure at any depth will be

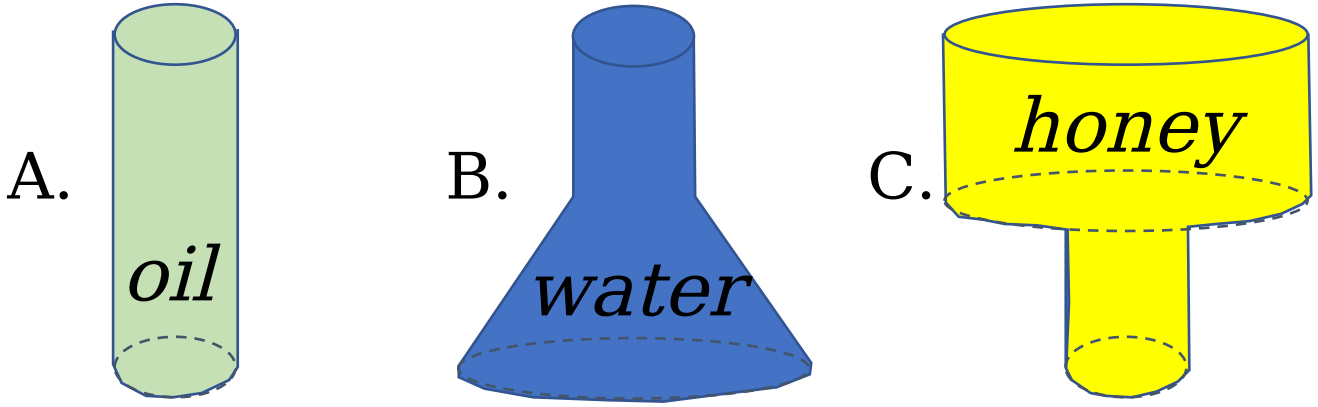


Consider the three open containers filled with water. How do the pressures at the bottoms compare ?



1. **Not enough information**

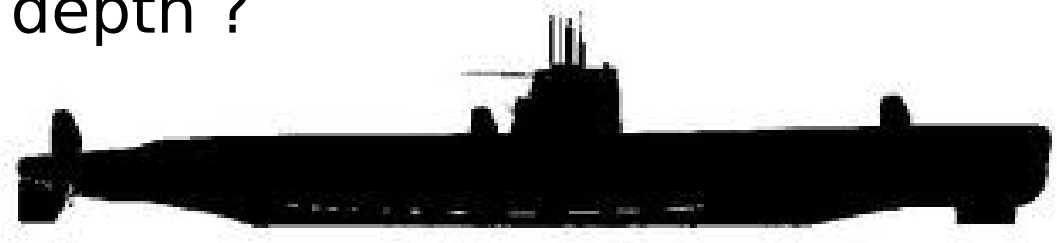
The three open containers are now filled with oil, water and honey respectively. How do the pressures at the bottoms compare ?



1. **Not enough information**

Calculating Crush Depth of a Submarine

A nuclear submarine is rated to withstand a pressure difference of before catastrophic failure. If the internal air pressure is maintained at , what is the maximum permissible depth ?



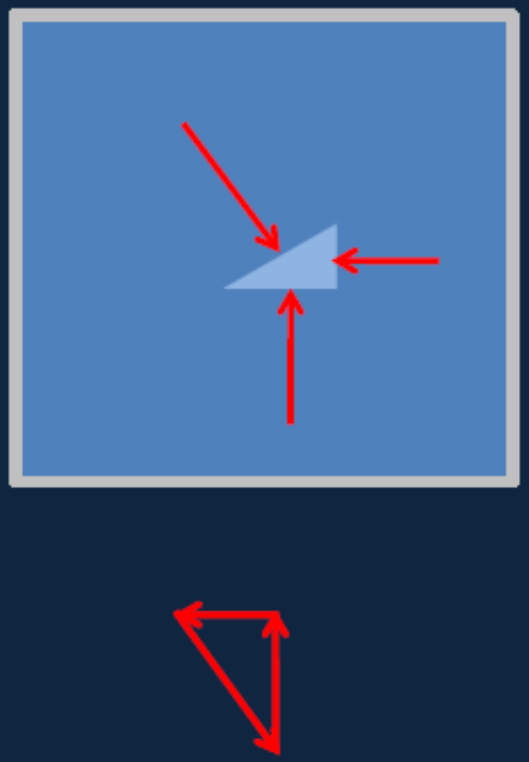
$$P = P0 + \rho gh$$

$$P - P0 = 70\,atm = 7.1 \times 10^{6}\,Pa\,;\,\rho = 1 \times 103\,kg/m3$$

$$h = \frac{P - P0}{\rho g} = \frac{7.1 \times 106}{1 \times 103 \times 9.8} = 720\,m$$

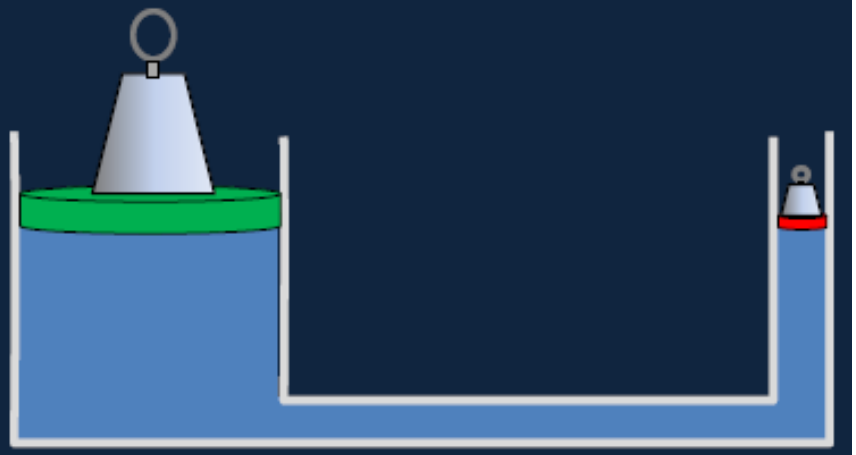
Pressure Same in all Directions

- At a point inside a fluid at rest, the pressure on a small area doesn't depend on which way the area is pointing.
- Imagine a small triangular wedge of the fluid, all at rest. The pressure forces on the sides must balance: they add to zero.
- The balance means the forces are <u>proportional</u> to the little areas, so the pressure is the same.



Pascal's Principle

- Added pressure to a fluid is transmitted through the fluid. This increased pressure is still equal at equal depths.
- The ratio of the balanced weights here is the ratio of the green/red areas.
- A small push on the small weight raises the big one—but not by much!

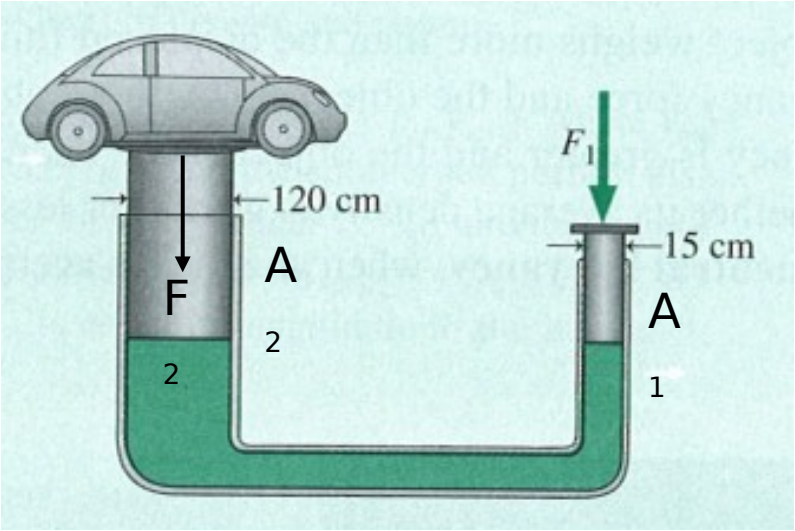


This is effectively a lever: the fluid is almost incompressible, so the distances traveled in small displacements are *inverse* to the areas, hence to the forces.

Pascal's Principle: Example

- Pressure force is transmitted through a fluid

Q. A large piston supports a car. The total mass of the piston and car is . What force must be applied to the smaller piston to support the car?



Pressure at the same height is the same!

$$\frac{F1}{A1}=\frac{F2}{A2}$$

$$F1=\frac{A1}{A2}mg=\frac{\pi\times 0.152}{\pi\times 1.202}\times 3200\times 9.8=490\,N$$

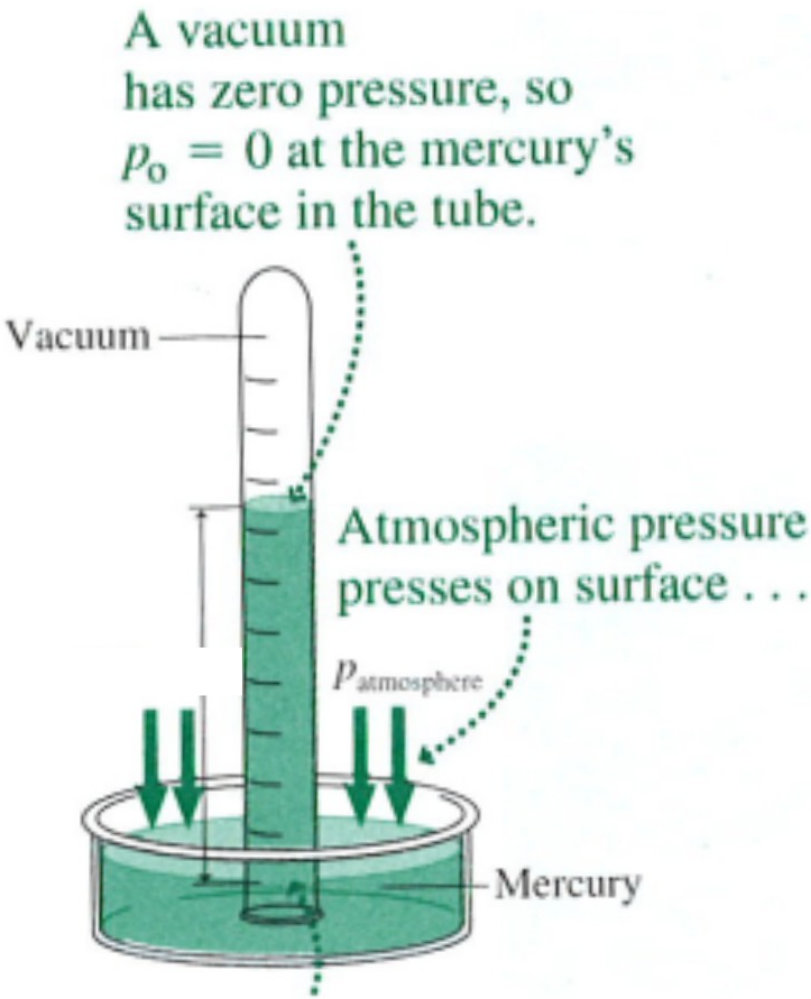
Measuring Pressure

Q. What is height of mercury (Hg) at ?

$$\rho_{Hg} = 13.6\, g/cm^3$$

$$P = P0 + \rho\, gh \rightarrow h = P/\rho\, g$$

$$h = \frac{1 \times 105}{1.36 \times 104 \times 9.8} = 0.75\, m$$

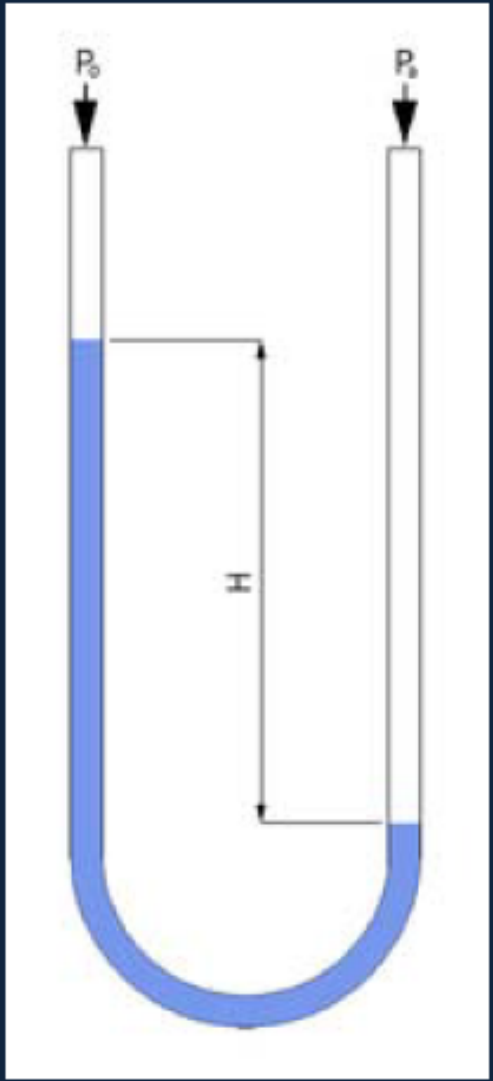


Atmospheric pressure can support a 10 meters high column of water. Moving to **higher density** fluids allows a table top **barometer** to be easily constructed

$$p = p_0 + \rho\, gh$$

Absolute Pressure and Gauge Pressure

- A common pressure gauge is the manometer, a U-tube with liquid. The pressure difference between the two sides is $\rho g H$.
- Tire pressures, for example, are measured relative to atmospheric pressure: this is called gauge pressure.
- Absolute pressure is relative to a vacuum. The absolute pressure in a swimming pool = $\rho g h$ + 1 atm.



Gauge Pressure

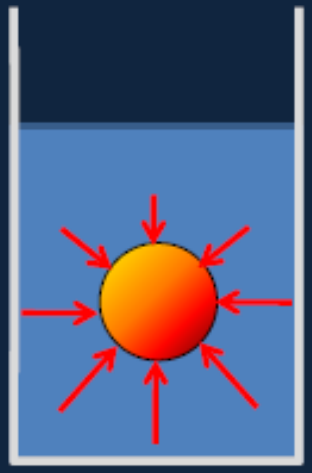
Gauge Pressure is the pressure difference from atmosphere.
(e.g. Tyres)

Archimedes' Principle and Buoyancy

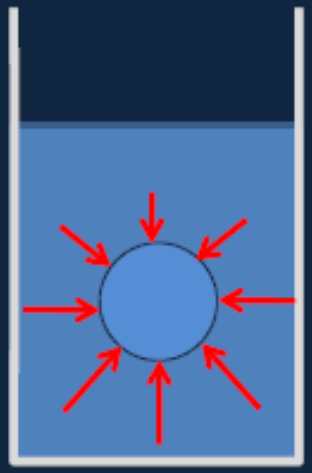
Why do some things **float** and other things **sink** ?

Buoyancy Force

- Any object immersed in a fluid feels pressure forces all around it. Since the pressure is greater at greater depth, there is a net upwards force—this is called buoyancy.



- Suppose the underwater object is replaced with a ghost: an extremely thin plastic sheet of the same identical shape, just enclosing the fluid.

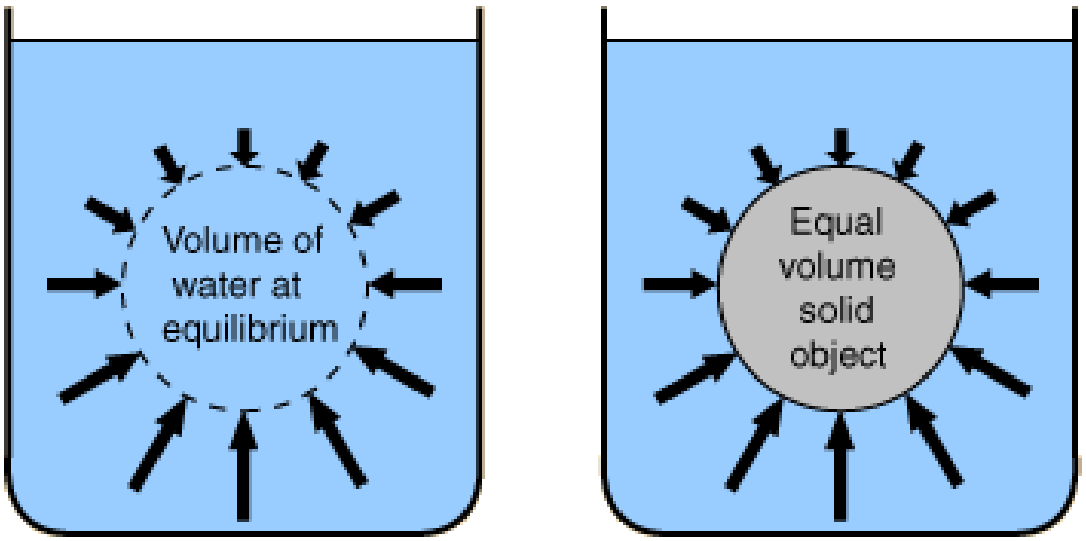


- The pressure forces on this ghost will be identical, and they will be just enough to support the weight of the fluid inside.

Archimedes' Principle and Buoyancy

Objects immersed in a fluid experience a Buoyant Force!

(up)



Partly immersed

Floating

Sinks

$$W_{solid} = m_{solid}\, g = \rho_{solid}\, V g \,(down)$$

The **Buoyant Force** is equal to the weight of the **displaced** fluid !

Archimedes' Principle

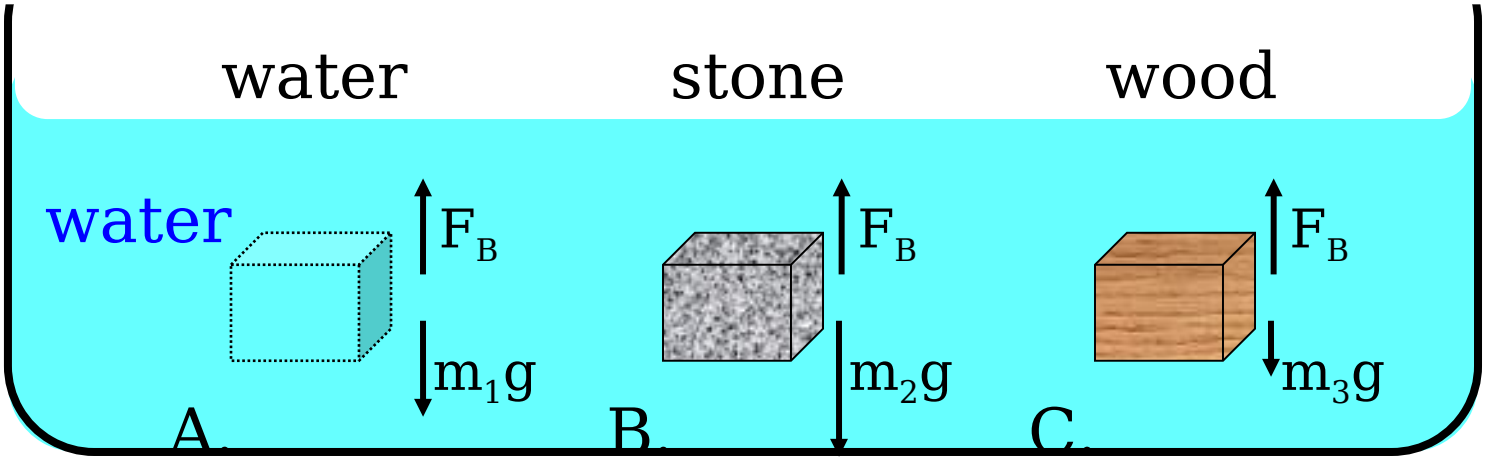
- A body immersed (or partially immersed) in a fluid experiences a net upward buoyancy force equal to the weight of the fluid displaced.

Archimedes' Principle and Buoyancy

The hot-air balloon floats because the **weight of air displaced** (= the buoyancy force) is greater than the **weight of the balloon**

The **Buoyant Force** is equal to the weight of the **displaced** fluid !

Which of the three cubes of length shown below has the largest buoyant force?



1. Not enough information

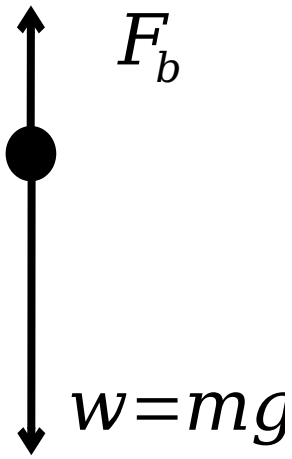
Example Archimedes' Principle and Buoyancy

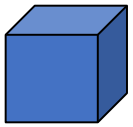
Q. Find the apparent weight of a concrete block when you lift it under water,

Solution

ter provides a buoyancy force

parent weight should be less





$$\rho_{con} = \frac{m}{V}$$

$$V = \frac{m}{\rho_{con}}$$

$$F_{net} = mg - F_b = w_{apparen}$$

$$F_b = m_{dispwater} g = \rho_{water} V g$$

$$w_{app} = mg - \frac{\rho_{water} mg}{\rho_{con}}$$

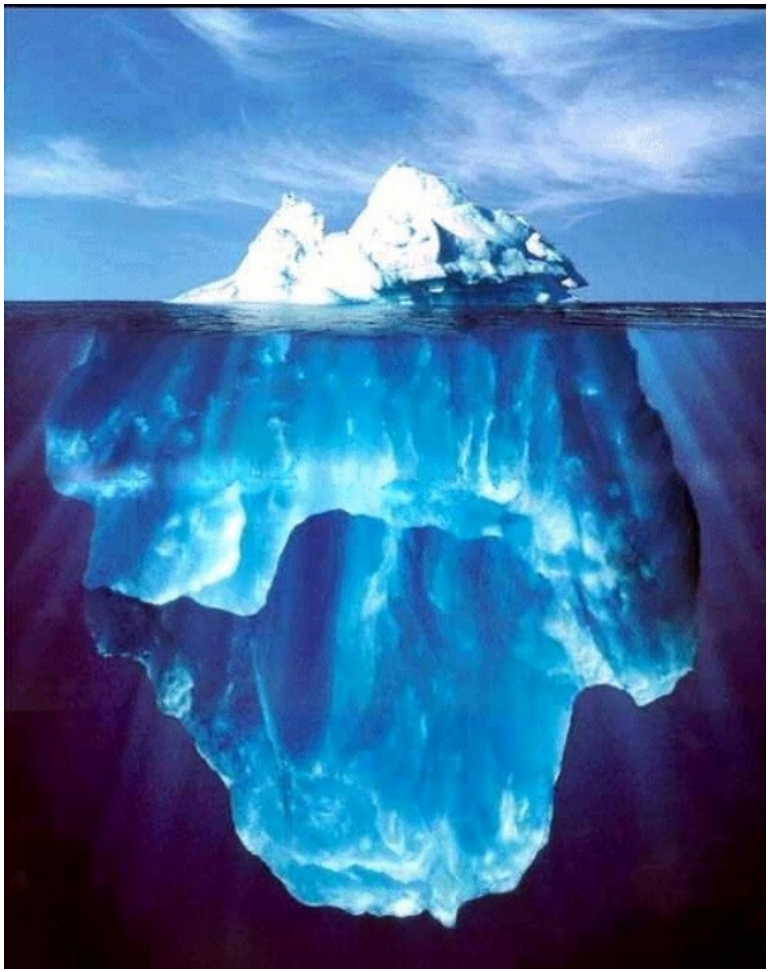
$$w_{app} = mg\left(1 - \frac{\rho_{water}}{\rho_{con}}\right)$$

$$= 60 \times 9.8 \times \left(1 - \frac{1000}{2200}\right) = 321 N$$

Floating Objects

Q. If the density of an iceberg is 0.86 that of seawater, how much of an iceberg's volume is below the sea?

Buoyancy force FB=weight of water displaced



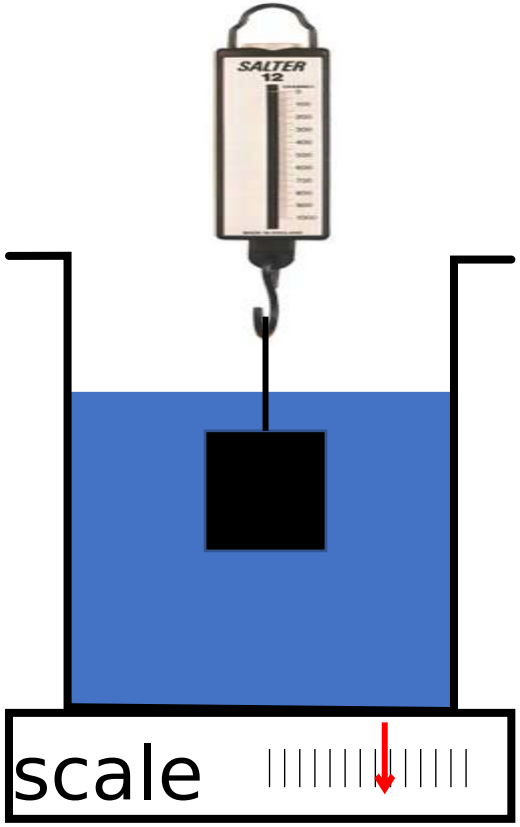
$$V\,sub = submerged\,volume$$

$$F\,B = mwater\,g = \rho water\,V\,sub\,g$$

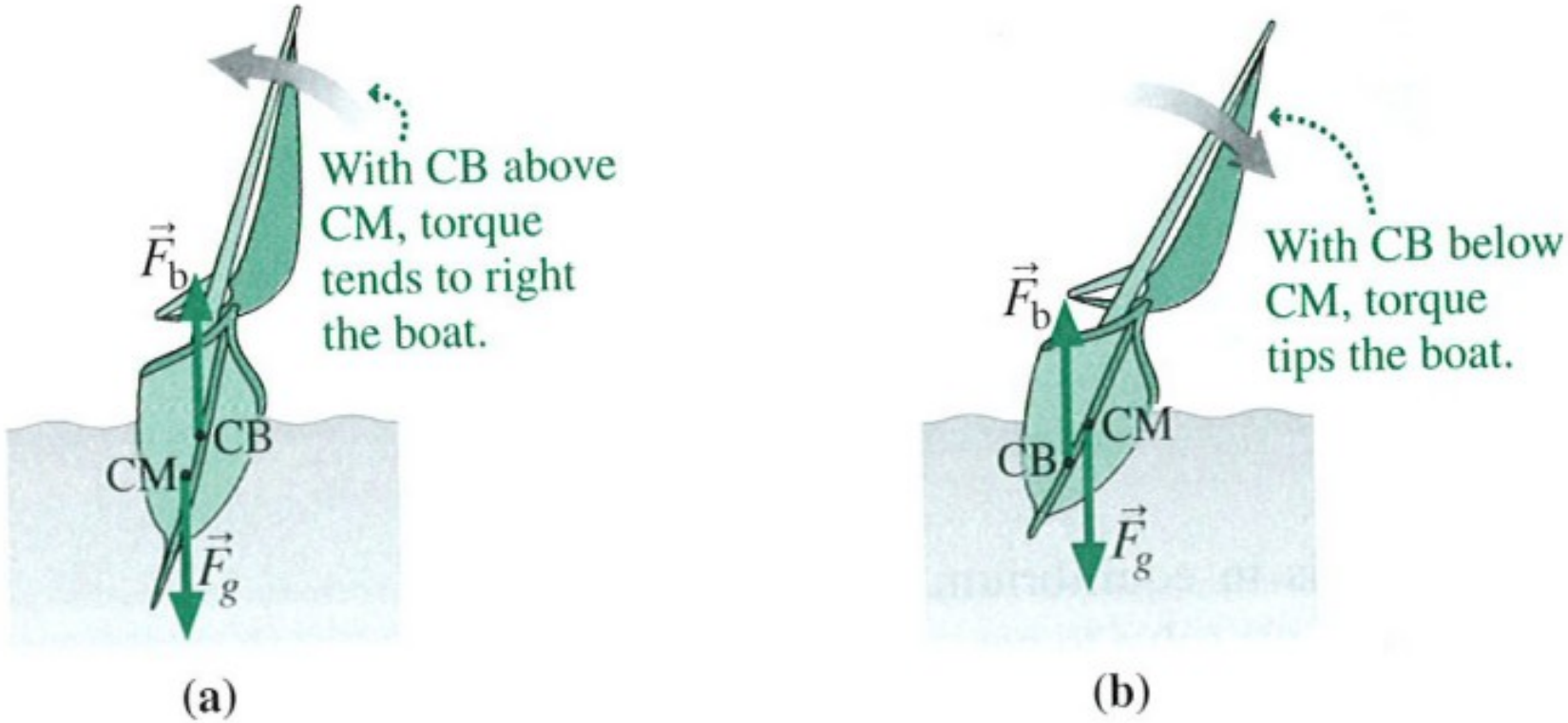
$$F\,B = mice\,g = \rho ice\,Vice\,g$$

$$\rho water\,V\,sub\,g = \rho ice\,V\,ice\,g \rightarrow \frac{V\,sub}{V\,ice} = \frac{\rho\,water}{\rho\,ice} = 0.86$$

A beaker of water weighs . A block of weight is suspended in the water by a spring balance reading . What does the scale read?



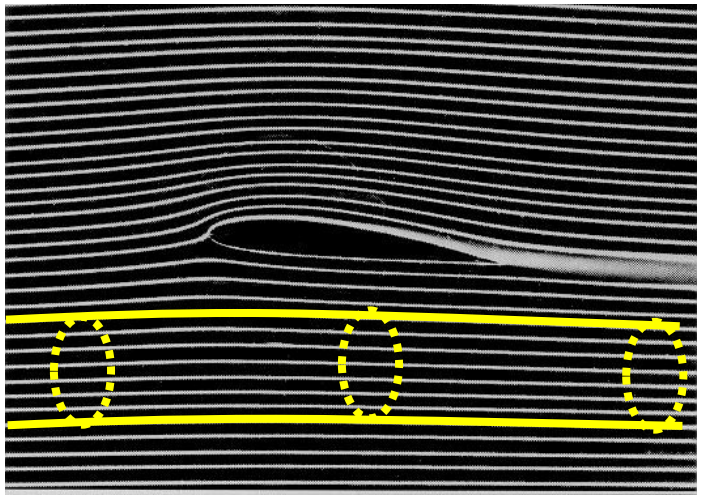
Centre of Buoyancy: Example



The **Centre of Buoyancy** is given by the Centre of Mass of the **displaced** fluid. For objects to float with stability the Centre of Buoyancy must be **above** the Centre of Mass of the object. Otherwise Torque yield Tip !

Fluid Dynamics

Laminar (steady) flow is where each particle in the fluid moves along a smooth path, and the paths do not cross.

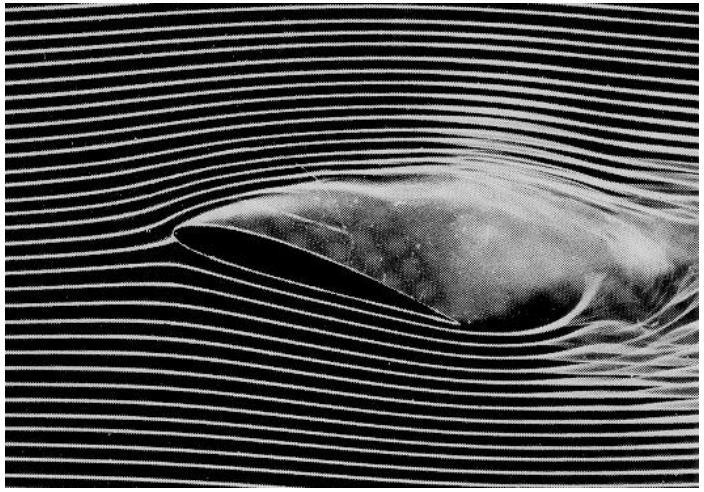


The path is called a **streamline.**

Velocity is **tangent** to the streamline for steady flow don't cross.

Spacing of streamlines measures **speed**.

A set of streamlines act as a pipe for an **incompressible** fluid.

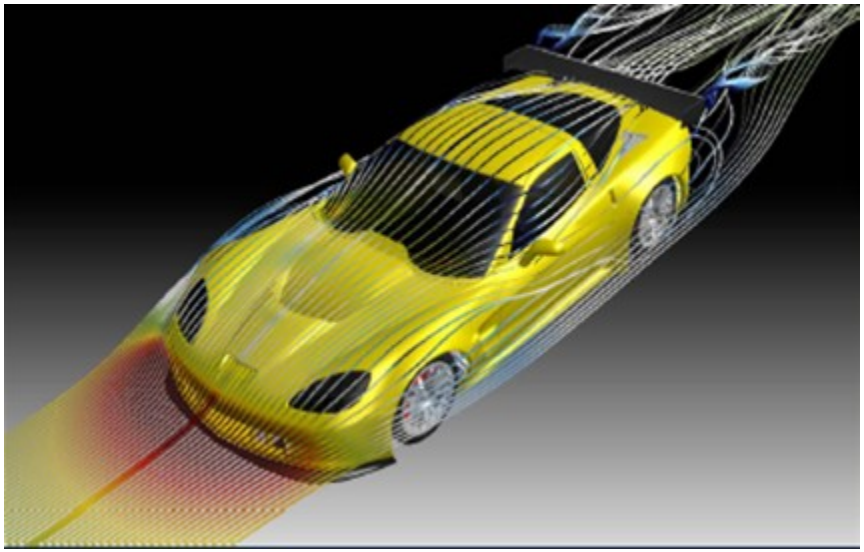


Non-viscous flow – no internal friction (water OK, honey not)

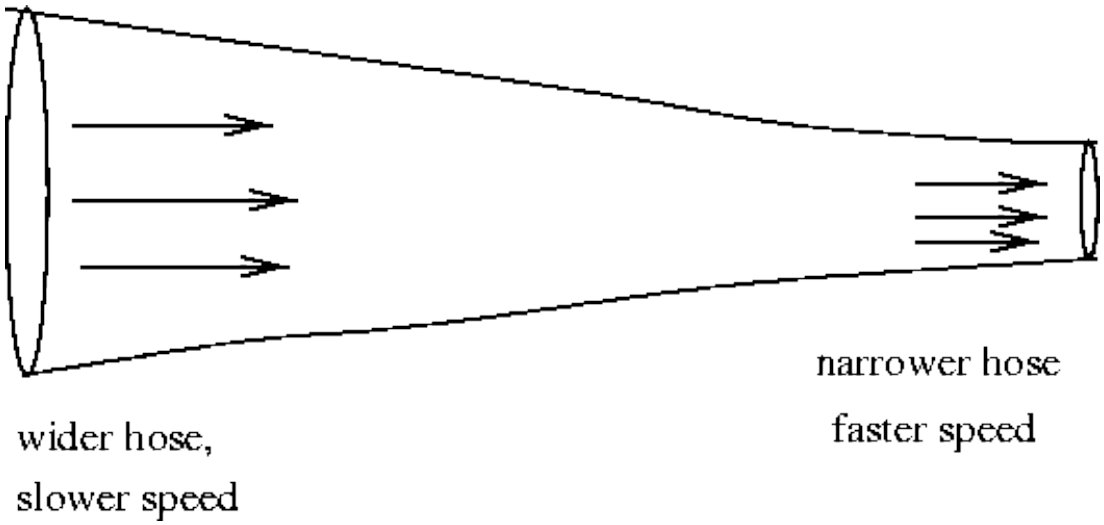
Turbulent flow above a critical speed. Whirlpools appear, paths cross, and become irregular and chaotic.

LAMINAR AT THE FRONT AND TURBULENT AT THE BACK

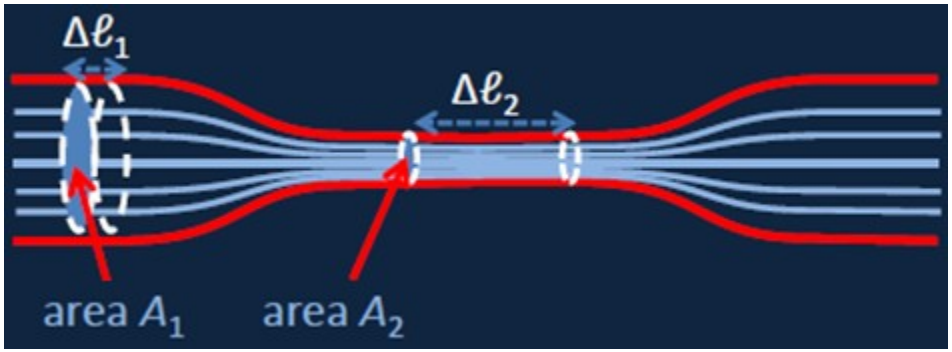
Air flow over this Corvette is laminar until the end: the air cannot curve in completely at the back, it breaks away forming a turbulent wake.



Conservation of Mass: The Continuity Eqn.

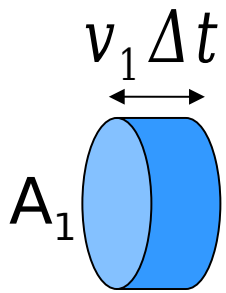


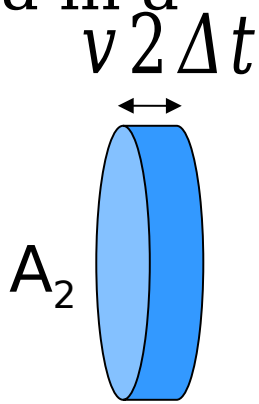
The rate of flow is the **same** at all cross-sections.
There can be **no sources or sinks** of fluid.



Conservation of Mass: The Continuity Eqn.

Q. How much fluid flows across each area in a time :





$$\Delta m = \rho_1 V_1 = \rho_1 A_1 v_1 \Delta t$$

$$\Delta m = \rho_2 V_2 = \rho_2 A_2 v_2 \Delta t$$

$$flow\,rate: \frac{\Delta m}{\Delta t} = \rho A v$$

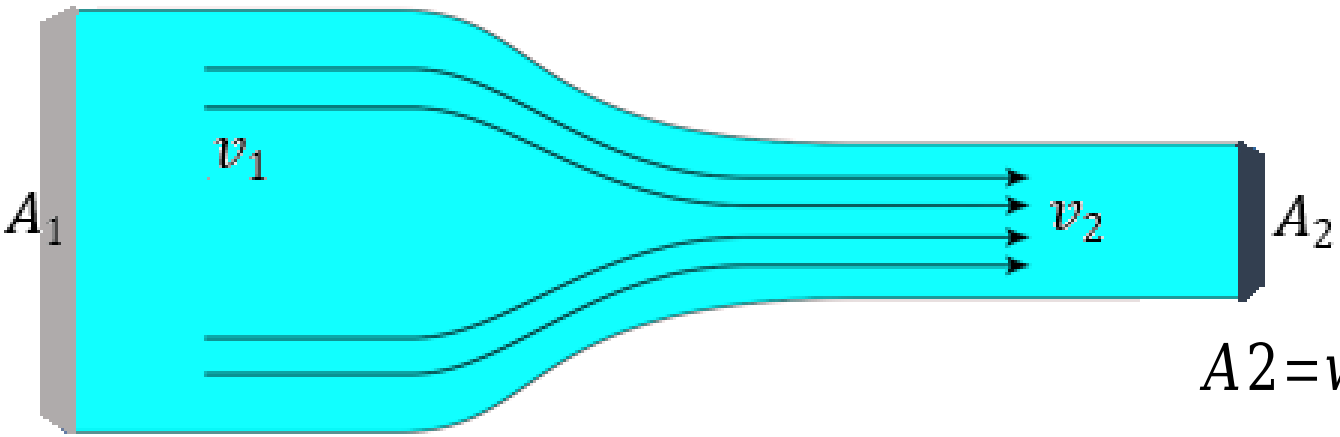
C

Continuity equation for incompressible fluid

- Density is constant for incompressible fluid.
- The continuity equation then becomes

Conservation of Mass: The Continuity Eqn.

Q. A river is 40m wide, 2.2m deep and flows at 4.5 m/s. It passes through a 3.7-m wide gorge, where the flow rate increases to 6.0 m/s. How deep is the gorge?



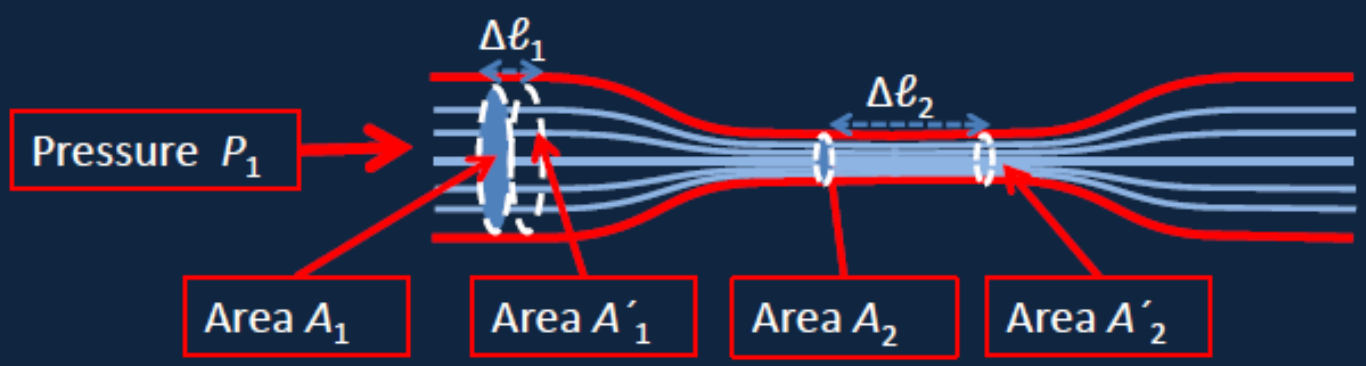
$A2 = w2\,d2$

$A1 = w1\,d1$

$$Continuity\ equation: A1\,v1 = A2\,v2 \rightarrow w1\,d1\,v1 = w2\,d2\,v2$$

$$d2 = \frac{w1\,d1\,v1}{w2\,v2} = \frac{40 \times 2.2 \times 4.5}{3.7 \times 6.0} = 18\,m$$

Bernoulli's Equation



For constant density, $A_1\Delta\ell_1 = A_2\Delta\ell_2$

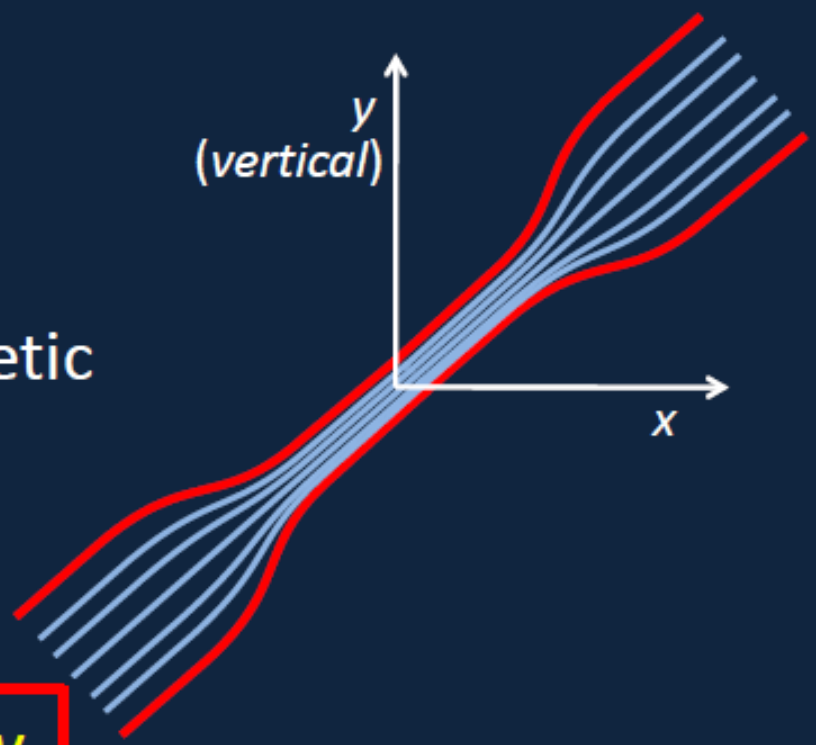
- In the time Δt, there is a KE gain of $\frac{1}{2}\rho A_1\Delta\ell_1(v_2^2 - v_1^2)$.
- Where did that energy come from?
- In the time Δt, the pressure P_1 on the area A_1 does work:

  force x distance $= P_1A_1\Delta\ell_1$
- BUT at the same time, our block of fluid did some work itself: it pushed the fluid in front of it, doing work $= P_2A_2\Delta\ell_2$.
- SO net work done $= (P_1 - P_2)\, A_1\Delta\ell_1$ =KE gain $\frac{1}{2}\rho A_1\Delta\ell_1(v_2^2 - v_1^2)$
- That is, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Uphill Work…

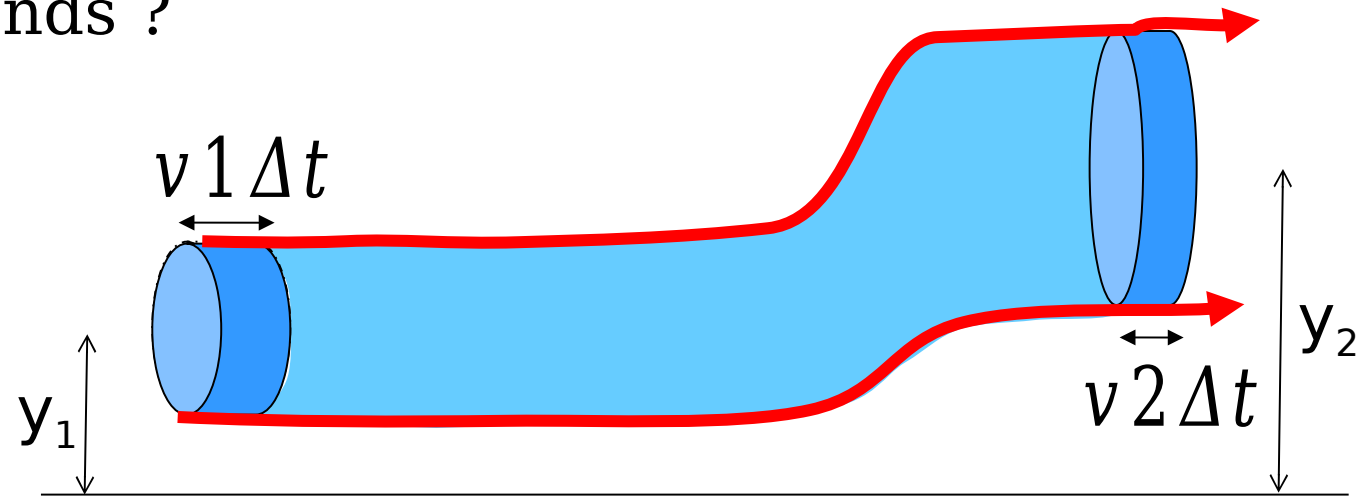
- What if the pipe is *tilted upwards*?
- Now the pressure speeding the fluid along has to lift it as well!
- So the pressure adds potential energy corresponding to how much it was lifted as well as kinetic energy from speeding it up.
- This gives the full Bernoulli's equation:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$



Conservation of Energy: Bernoulli's Eqn.

What happens to the energy density of the fluid if I raise the ends ?



Energy per unit volume

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = cons$$

Total energy per unit volume is constant at **any** point in fluid.

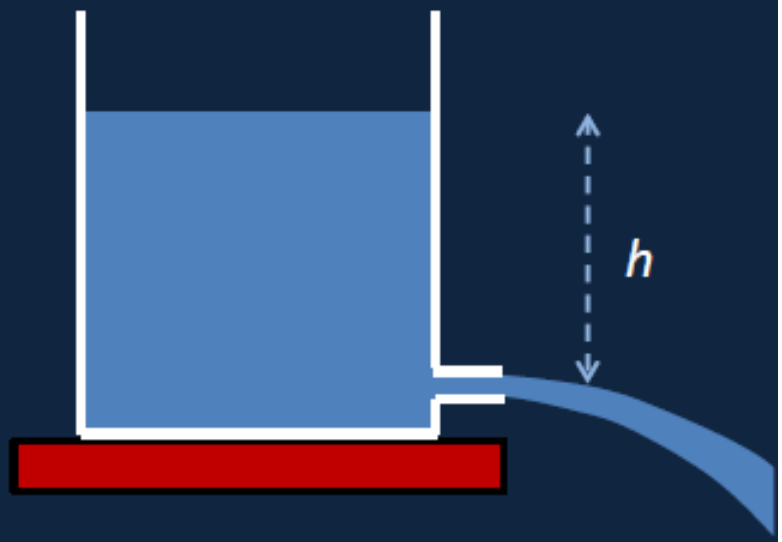
$$p + \frac{1}{2}\rho v^2 + \rho g y = cons$$

Torricelli's Theorem

- Water coming from a small spigot in a large tank has a speed given by

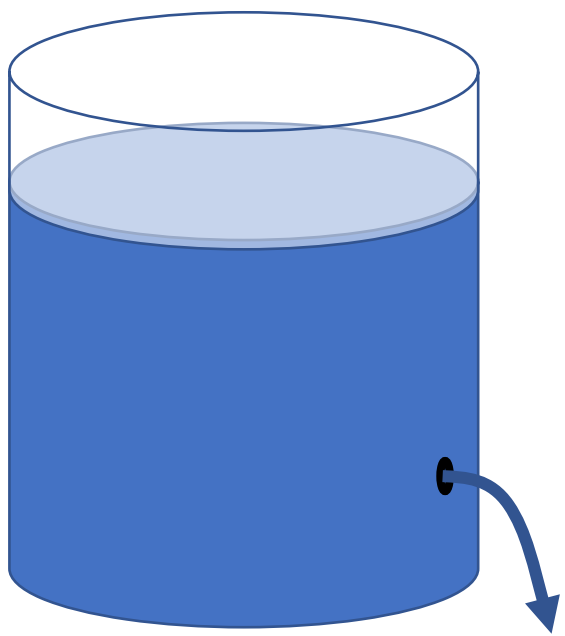
$$\boxed{v^2 = 2gh}$$

- This is a special case of Bernoulli's equation, because the outside pressure at the spigot is the same as that at the top of the fluid, and fluid velocity at the top is negligible.



Conservation of Energy: Bernoulli's Eqn.

Q. Find the velocity of water leaving a tank through a hole in the side 1 metre below the water level.



$$P+\frac{1}{2}\rho v^2+\rho g y=constant$$

$$At\ the\ top: P=1\,atm,\ v=0,\ y=1\,m$$

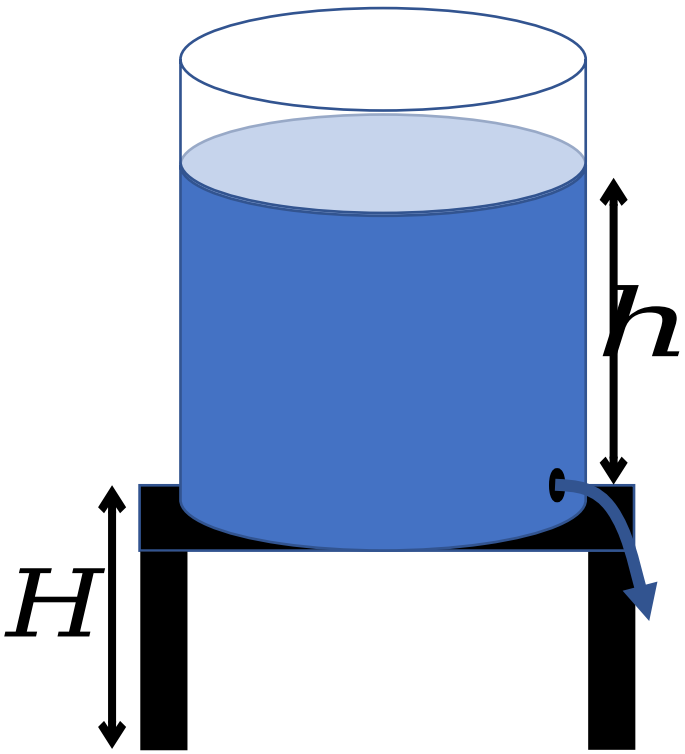
$$At\ the\ bottom: P=1\,atm,\ v=?,\ y=0\,m$$

$$P+\rho g y=P+\frac{1}{2}\rho v^2$$

$$v=\sqrt{2gy}=\sqrt{2\times 9.8\times 1}=4.4\,m/s$$

Which of the following can be done to increase the **flow rate** out of the water tank ?

1. **Raise the tank**
2. **Reduce the hole size**
3. **Lower the water level**
4. **Raise the water level**
5. **None of the above**



Summary: fluid dynamics

Continuity equation: mass is conserved!

$$\rho v A = constant$$

For liquids:

$$\rho = constant \rightarrow vA = constant$$

$$(Density\ \rho, velocity\ v, pipe\ area\ A)$$

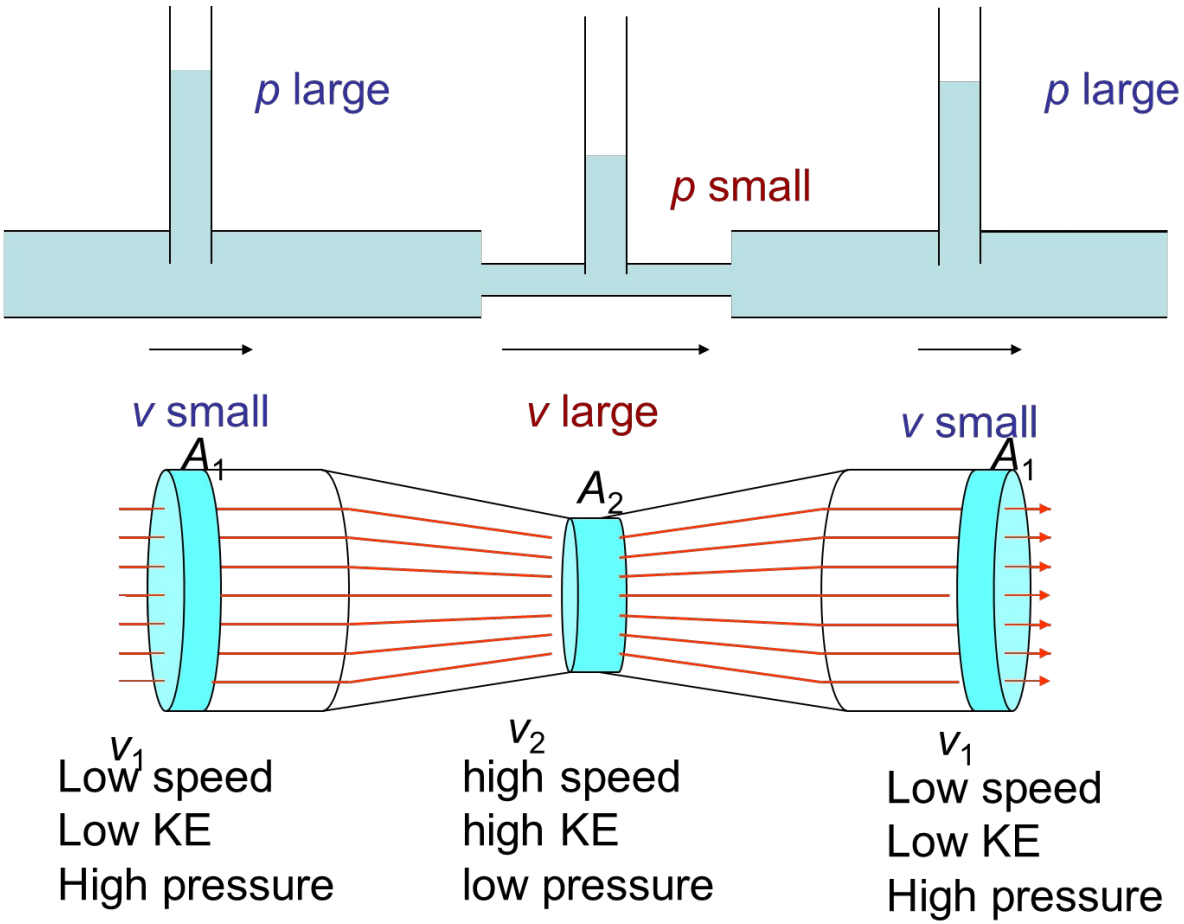
Bernoulli's equation: energy is conserved!

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

$$(Pressure\ P, density\ \rho, velocity\ v, height\ y)$$

Applications of Bernoulli's Principle

- Venturi and pitot tubes

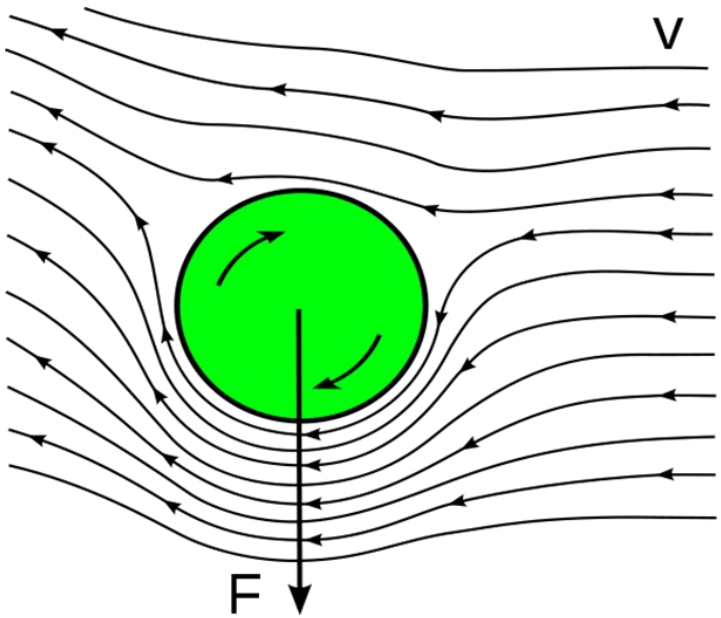


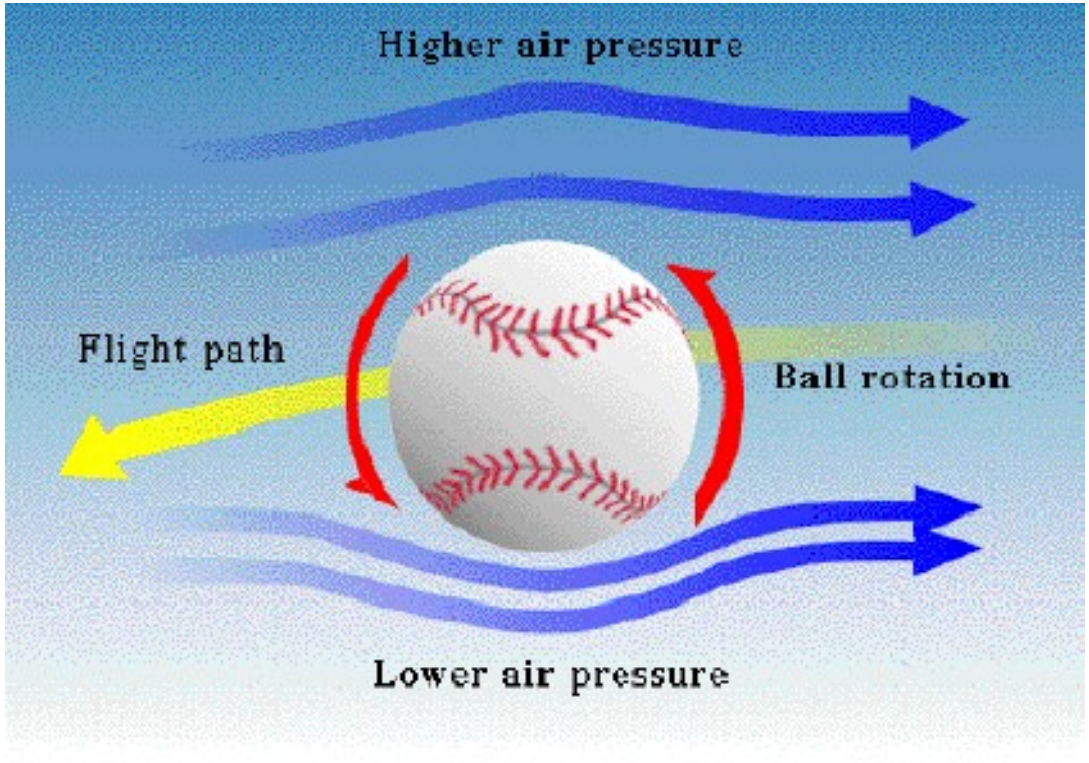
Applications of Bernoulli’s Principle

- Scent pray

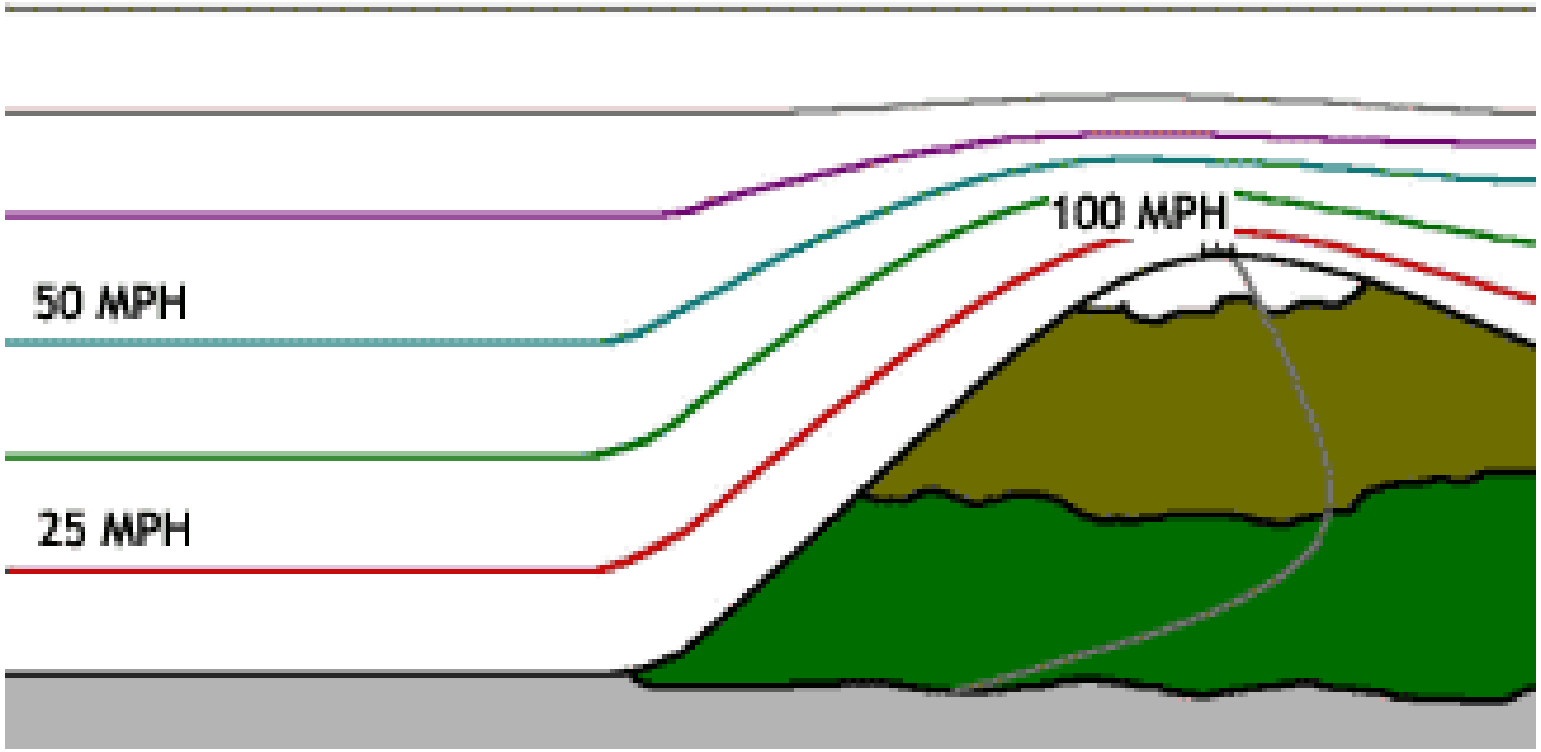
Applications of Bernoulli's Principle

- Swerving ball

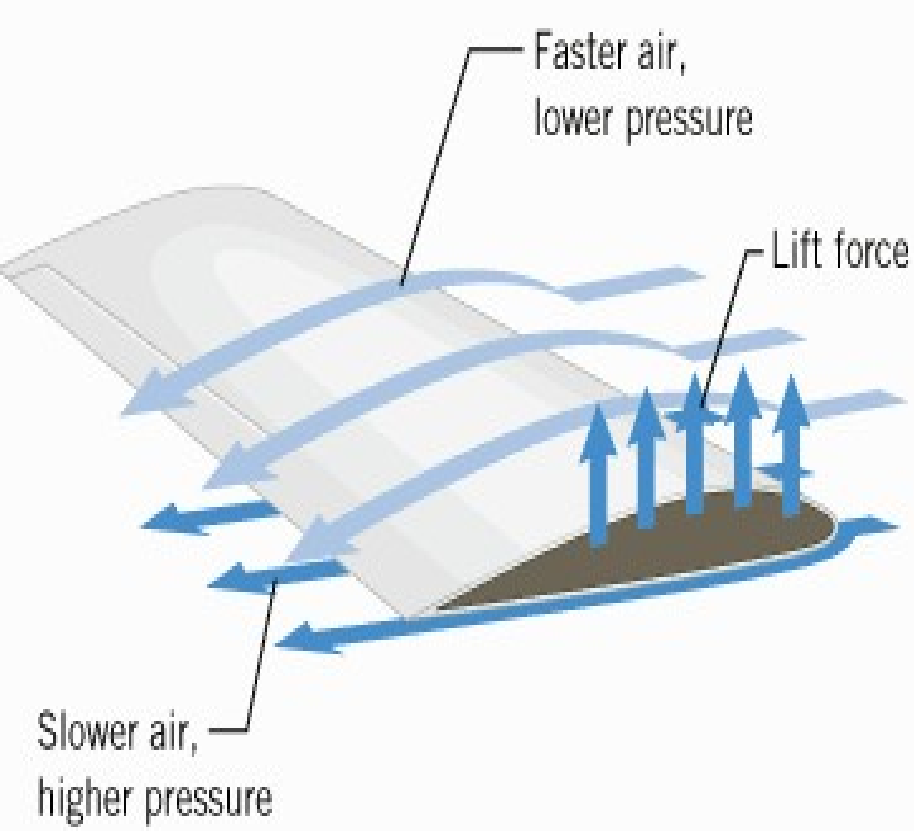




Applications of Bernoulli’s Principle



Applications of Bernoulli's Principle



(a)

(b)

Applications of Bernoulli’s Principle

Properties of Bulk Matter

Reading assignment