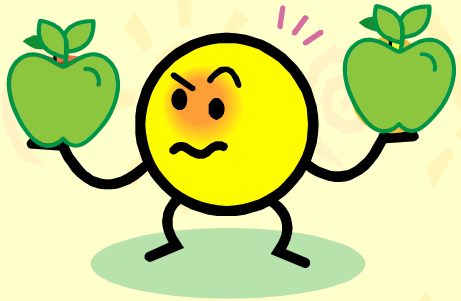
Ratios, Rates, and Proportions

Section 1.8



RATIOS



- A ratio is the comparison of two quantities with the same unit.
- A ratio can be written in three ways:
  - As a quotient (fraction in simplest form)
  - As two numbers separated by a colon (:)
  - As two numbers separated by the word “to”
- Note: ratios are “unitless” (no units)

Ex: Write the ratio of 25 miles to 40 miles in simplest form.

What are we comparing?

miles 25 miles to 40 miles

$$\frac{25\text{mile}}{40\text{mile}} = \frac{25}{40} = \frac{5}{8}$$

Units, like factors, simplify (divide common units out)

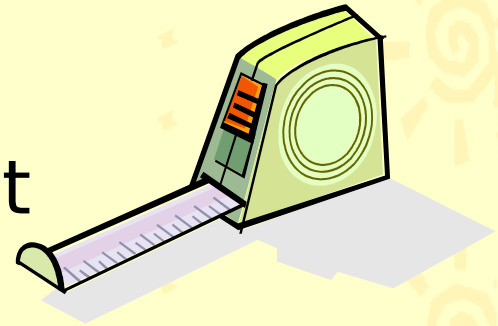
Simplify

The ratio is 5/8 or 5:8 or 5 to 8.

Ex: Write the ratio of 12 feet to 20 feet in simplest form.

What are we comparing?

feet 12 feet to 20 feet



$$\frac{12\,\text{feet}}{20\,\text{feet}} = \frac{12}{20} = \frac{3}{5}$$

Units, like factors, simplify (divide common units out)

Simplify

The ratio is 3/5 or 3:5 or 3 to 5.

Ex: Write the ratio of 21 pounds to 7 pounds in simplest form.



What are we comparing?

pounds 21 pounds to 7 pounds

$$\frac{21\text{lbs}}{7\text{lbs}} = \frac{21}{7} = \frac{3}{1}$$

Units, like factors, simplify (divide common units out)

Simplify

The ratio is 3/1 or 3:1 or 3 to 1.

What is the ratio of cats to mice?



Number of Cats: 3

Number of Mice: 6

Express the ratio as a fraction: $\frac{1}{2}$

Express the ratio in words: 1 to 2

Express the ratio with a colon: 1:2

What is a ratio?

Example: There are 300 computers and 1200 students in our school. What is the ratio of computers to students?

A **ratio** is a comparison of two quantities.

Express the ratio in words: **1 to 4**

Express the ratio with a colon: **1 : 4**

Express the ratio as a fraction: $\frac{1}{4}$



How many students are there for one computer?

Practice With Equivalent Ratios

Find an equivalent ratio by dividing:

 # 1

$$\frac{30}{90} = \frac{30 \div 30}{90 \div 30} = \frac{1}{3}$$

→ Divide by 30

 # 2

$$\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = \frac{5}{4}$$

→ Divide by 3

 # 3

$$\frac{125}{300} = \frac{125 \div 25}{300 \div 25} = \frac{5}{12}$$

→ Divide by 25



John and Mary make strawberry punch. Whose punch has a stronger strawberry taste?



John:
2 parts concentrate
4 parts water

Mary:
3 parts concentrate
5 parts water

→ **Write the ratio**

$$\frac{2}{4}$$

→ **Divide 2 by 4**

$$\frac{2}{4} = 0.5$$

→ **Write as a percentage**

0.5x100 = 50 % concentrate

→ **Write the ratio**

$$\frac{3}{5}$$

→ **Divide 3 by 5**

$$\frac{3}{5} = 0.6$$

→ **Write as a percentage**

0.6x100 = 60 % concentrate

stronger strawberry taste

Ex: The ratio of games won to games lost for a baseball team is 3:2. The team won 18 games. How many games did the team lose?

Using ratios



The ratio of faculty members to students in one school is 1:15. There are 675 students. How many faculty members are there?

$$\frac{\text{faculty}}{\text{students}} \rightarrow \frac{1}{15}$$

$$\frac{1}{15} = \frac{x}{675}$$

$$15x = 675$$

x = 45 faculty

Rates

A ***rate*** is a ratio that is measured using two different units.

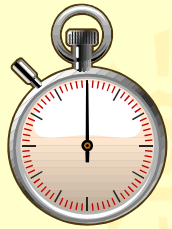
A ***unit rate*** is a rate per one given unit, like 6 miles per 1 hour.

Ex: You can travel 120 miles on 6 gallons of gas. What is your fuel efficiency in miles per gallon?

$$\text{Rate} = \frac{120 \text{ miles}}{6 \text{ gallons}} = \frac{20 \text{ miles}}{1 \text{ gallon}}$$

Your fuel efficiency is 20 miles per gallon.

Ex: Write the rate of 25 yards to 30 seconds in simplest form.



What are we comparing?

yards & seconds

25 yards to 30 seconds

$$\frac{25\,\text{yard}}{30\,\text{sec}} = \frac{5\,\text{yard}}{6\,\text{sec}}$$

Units can't simplify since they are different.

Simplify

The rate is 5 yards/6 seconds.

Ex: Write the rate of 140 miles in 2 hours in simplest form.

What are we comparing?

miles & hours 140 miles to 2 hours

$$\frac{140\,\text{mile}}{2\,\text{hours}} = \frac{70\,\text{mile}}{1\,\text{hour}}$$



Units can't simplify since they are different.

Simplify

The rate is 70 miles/1 hour (70 miles per hour, mph).

Notice the denominator is 1 after simplifying.

Ex: Write as a unit rate

20 patients in 5 rooms

What are we comparing?

patients & rooms

20 patients in 5 rooms

$$\frac{20\,\text{patien}}{5\,\text{rooms}} = \frac{4\,\text{patien}}{1\,\text{room}}$$

Units can't simplify since they are different.

Simplify

The rate is 4 patients/1room →

Four patients per room



Examples

You are shopping for t-shirts. Which store offers the better deal?

Store A:$25 for 2 shirts Store B: $45 for 4 shirts Store C: $30 for 3 shirts

Write each ratio as a unit rate.

Store A: $25/2 shirts = $12.50

Store B: $45/4 shirts = $11.25

Store C: $30/3 shirts = $10

Examples

Find each unit rate.

1. 300 miles in 5 hrs

2. $6.75 for 3 coloring books

3. 60 miles using 3 gal of gas

Example 2

A floral design uses two red roses for every three yellow roses. How many red roses will be in a garden that contains 500 roses in total?

 # 1

Let r be the number of red roses.
Let y be the number of yellow roses.

 # 2

Write the ratio: $r : y = 2 : 3$

 # 3

One design requires 2 + 3 = 5 roses in total

 # 4

How many designs are there in the garden?

 # 5

500 ÷ 5 = 100 designs
How many red roses are in the garden?

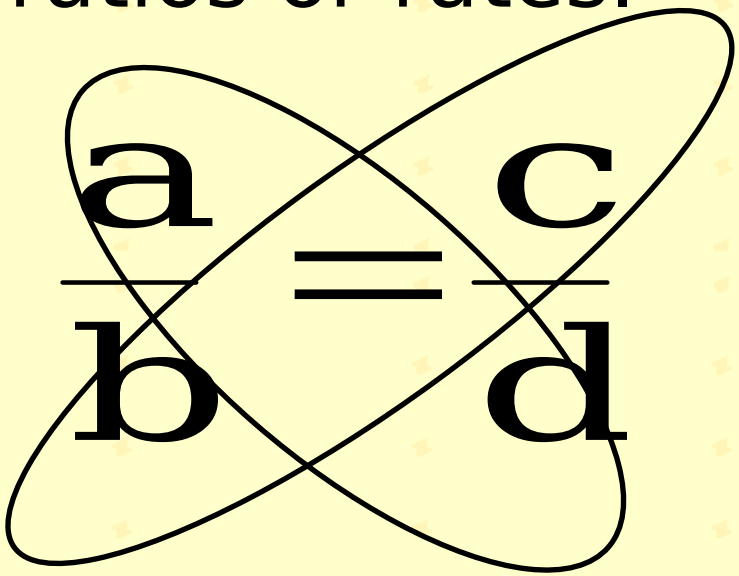
100 designs x 2 red roses per design

= 200 red roses



PROPORTIONS

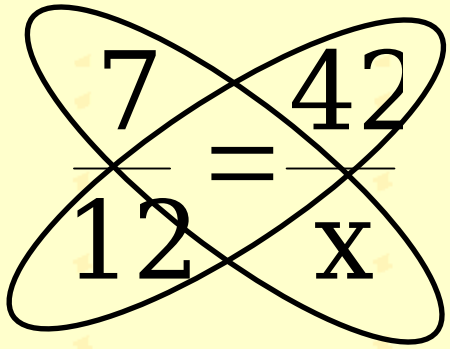
- A proportion is the equality of two ratios or rates.



→ Cross products are equal!

Ex: Solve the proportion

If the proportion is to be true, the cross products must be equal → find the cross product equation:



→ $7x = (12)(42)$

→ $x = \frac{504}{72}$

Ex: Solve the proportion $\frac{4}{3} = \frac{n-2}{6}$

If the proportion is to be true, the cross products must be equal → find the cross product equation:

$\frac{4}{3} = \frac{n-2}{6}$

→ $24 = 3(n - 2)$

→ $24 = 3n - 6$

→ $30 = 3n$

→ $10 = n$

Check: $\frac{4}{3} = \frac{10-2}{6} \rightarrow \frac{4}{3} = \frac{8}{6}$

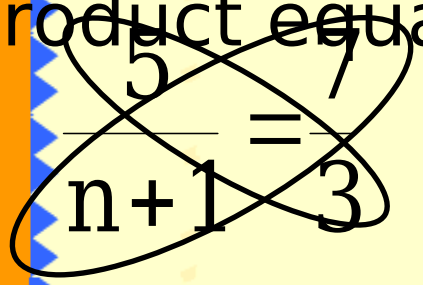
x 2

x 2



Ex: Solve the proportion $\frac{5}{n+1} = \frac{7}{3}$

If the proportion is to be true, the cross products must be equal → find the cross product equation:



→ (5)(3) = 7(n + 1)

→ 15 = 7n + 7

→ 8 = 7n

→ 8/7 = n

Check: $\frac{5}{\left(\frac{8}{7}\right)+1} = \frac{7}{3}$ $\frac{5}{\frac{15}{7}} = \frac{7}{3}$ $(5)(3) = \left(\frac{15}{7}\right)(7)$

Solve each Proportion

$$\frac{5}{9} = \frac{3}{w}$$

$$\frac{8}{x+10} = \frac{1}{12}$$

$$\frac{g+3}{5} = \frac{7}{4}$$