Unit 6: Rotational Motion

Introduction;

- Rotational motion is an important part of everyday life. The rotation of the Earth creates the cycle of day and night, the rotation of wheels enables easy vehicular motion, and modern technology depends on circular motion in a variety of contexts, from the tiny gears in a Swiss watch to the operation of lathes and other machinery. The concepts of angular velocity, angular acceleration, and centripetal acceleration are central to understanding the motions of a diverse range of phenomena, from a car moving around a circular racetrack to clusters of galaxies orbiting a common center.

Rotational motion, when combined with Newton’s law of universal gravitation and his laws of motion, can also explain certain facts about space travel and satellite motion, such as where to place a satellite so it will remain fixed in position over the same spot on the Earth. The generalization of gravitational potential energy and energy conservation offers an easy route to such results as planetary escape speed.

6.1 Rotation about a fixed axis

Axis of rotation: is the axis about which a body is rotated.

Every rotating body has an ***axis of rotation***; ex. the Earth has

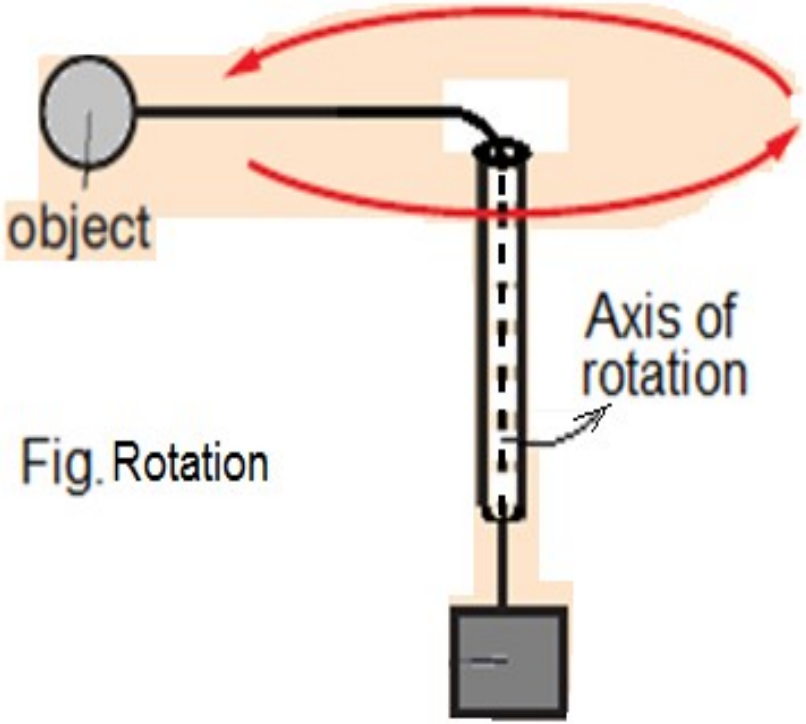
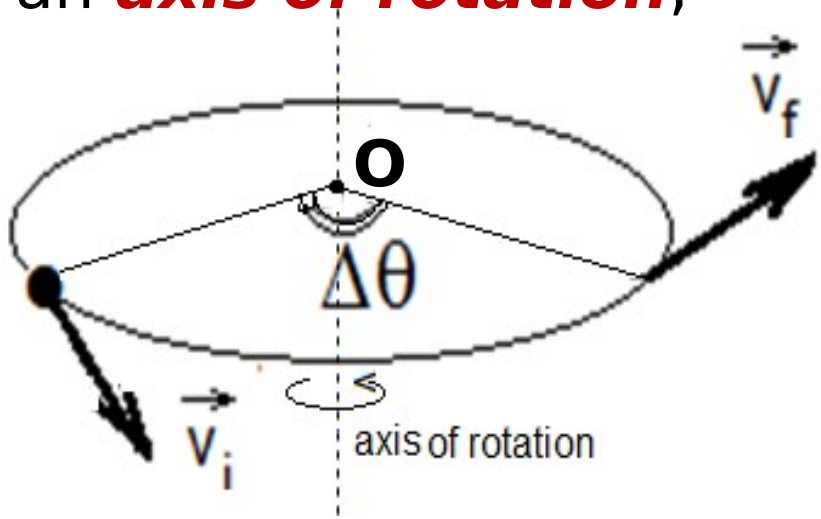


Fig. Rotation

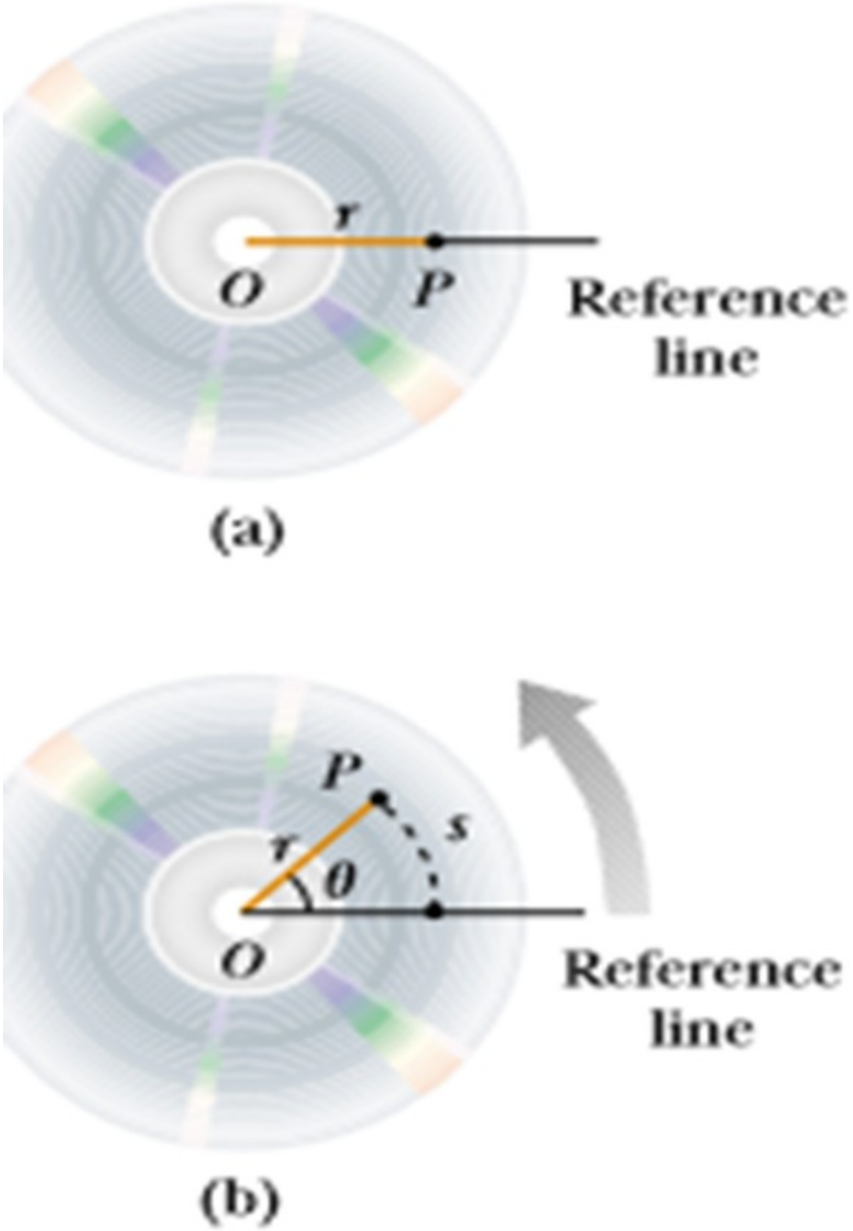


N.B.: All $\vec{\theta}$, $\vec{\omega}$, $\vec{\alpha}$, $\vec{\tau}$ and L are drawn at 'O' using ***R.H.R. of***

Angular displacement: is the angular position of a compact disc rotating about a fixed axis through O perpendicular to the plane of the figure as shown.

(a) In order to define angular position for the disc, a fixed reference line is chosen. A particle at P is located at a distance r from the rotation axis at O.

(b) As the disc rotates, point P moves through an arc length **s** on a circular path of radius r.



As the particle moves along the circle from the reference line (Θ= 0), it moves through an arc of length **s**, as shown in Figure b . The arc length **s** is related to the angle **Θ** through the relationship

angular displacement(Θ)= arc length(S)/radius(r)

$$\theta = \frac{s}{r}$$

the SI unit of angular displacement is radian (rad).

one radian is the angle subtended by an arc le

1

$$\theta(\text{rad}) = \frac{\pi}{180^\circ}\theta(\text{deg})$$

For example, 60° equals $\pi/3$ rad and 45° equals $\pi/4$ rad.

1 revolution = $360^o = 2\pi$ radians

Angular velocity (ω); it is the time rate of angular change of angular displacement.

$$\omega = \Delta\theta/t$$

It is vector quantity measured is rad/sec.

Linear velocity (V): the velocity of rotating body tangent to the curve every point along the curved path.

$$V=\omega \times r= \omega r$$

the SI unit of V is m/s=rad/s x m

Example: calculate the angular velocity of a flywheel having radius of 2m rotating at a velocity of 16m/s.

sol/n:

$\omega=V/r =$

Exercise 1: **A flywheel spins 250 times per minute. What is its angular velocity?**

ANS: 26.17 rad/s

Exercise 2: **A c.d. is rotating at 4800 rev/min. What is the linear speed of a point at the rim of the c.d. if the c.d. has a radius of 40 mm? ANS:**

Exercise 3: **A car is traveling at 16m/s; wheel: diam. = 70cm. a) What is the angular speed of a wheel? b) What is the tan. speed of a point 30cm from the axis on a wheel?**

Worked example 6.2

A CD is rotating at 4800 revolutions per minute. What is the linear velocity of a point on the CD, 40 mm from the axis of rotation?

First find the angular velocity of the CD.

Angle turned through in 60 s = $4800 \times 2\pi$ radians

Use the equation: $\omega = \theta/t$

Sustituting in the values $\omega = 4800 \times 2\pi/60 = 502.7$ rad/s

Velocity of point = $r\omega$

Sustituting in the values $v = 0.04$ m $\times$ 502.7 rad/s = 20.1 m/s

Review questions

1. A car engine rotates at 3000 revolutions per minute. What is its angular velocity in rad/s?
2. The linear velocity of a point on a music CD is a constant 1.2 m/s when it is being played.

   At the start of a CD, this point is 23 mm from the axis of rotation. At the end it is 58 mm from the axis of rotation. What is the angular velocity of the point at the start and the end?
3. A car is travelling at 16 m/s. The car wheel has a diameter of 0.7 m.

   What is the angular velocity of the wheel? Give your answer in rad/s.
4. A turbine is rotating at 3000 rev/min.

   a) What is the angular velocity of the turbine? Give your answer in rad/s.

   b) A turbine blade is 0.4 m long.

   Calculate the linear velocity of a point on the end of a turbine blade and halfway down a blade.

Angular acceleration: is the time rate of change of angular velocity.

Angular acceleration (α) :

$\alpha = \Delta\omega/t$,

The si unit of α is = rad/s², but other unit rev/min², …

Example. A net tangential force accelerates a flywheel from 15.1 rad/s to 23.1 rad/s in 4s. Find

a). the angle it turns through:

b). Angular acceleration

Solutions

a. Θ=(= x4s=76.4rad

b. α = (= = 2 rad/s²

Exercise

1. **Calculate the angular acceleration when the angular velocity of a wheel increases from 3rad/s to 23rad/s in 4sec.**
2. **A flywheel rotates at 3rad/s² from rest for 5se. Find its final angular velocity.**

Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

For rotational motion about a fixed axis, the simplest accelerated motion to analyze is motion under constant angular acceleration.

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

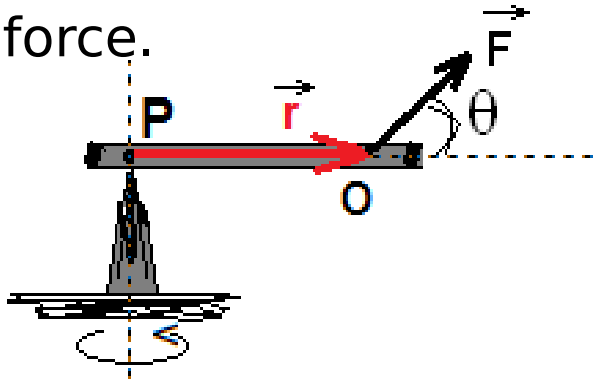
exercise

1. The angular velocity of the rear wheel of a stationary exercise bike is 4rad/s at time t=0 and its angular acceleration is constant and equal to 2rad/. A particular spoke coincides with the x axis at time t=0 .
   a. What angle does this spoke make with the axis at time t=3sec?
   b. What is the wheel's angular velocity at this time?
2. A disk rotates about an axis perpendicular to its plane. Point B on the disk is three
   times farther from the axis than point A. If the speed of point B is V. what is the speed
   of point A? A. 1/3V B. 1/2V C. 2V D. 3V
3. A disk spins with increasing angular velocity. Point B on the disk is twice as far from the
   axis of rotation as point A is. Which of the following statements is or are correct?
   A. The angular acceleration of A is greater than that of B.
   B. The radial acceleration of A is greater than that of B.
   C. A and B have the same angular acceleration.
   D. A and B have the same radial acceleration
4. A discus thrower turns with angular acceleration 50rad/moving the discus in a circle
   of radius 0.80 m. Find the radial and tangential components of acceleration of the

6.2 Torque and angular acceleration

- **Torque ($\vec{\tau}$):**

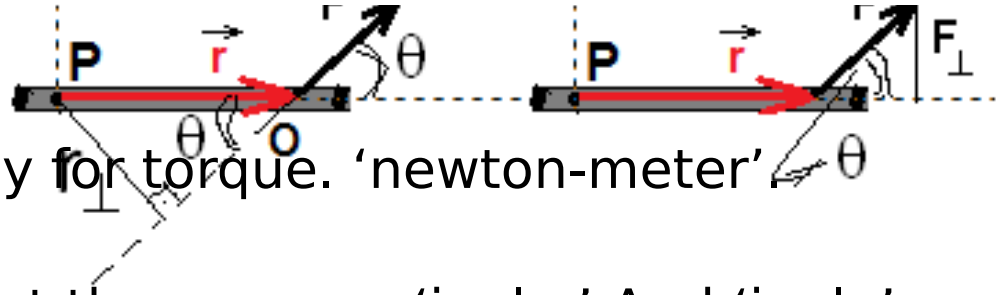
  Torque is the rotational effect of a force.



Definition: torque $\vec{\tau} = \vec{r} \times \vec{F}$

and $\tau = r.F.\sin\theta = F.r.\sin\theta$

$= F_{\perp}.r = F.r_{\perp}$



$\therefore \ [\vec{\tau}] = N \cdot m$, ...

N.B. : . 'N.m' is reserved only for torque. 'newton-meter'.

Thus

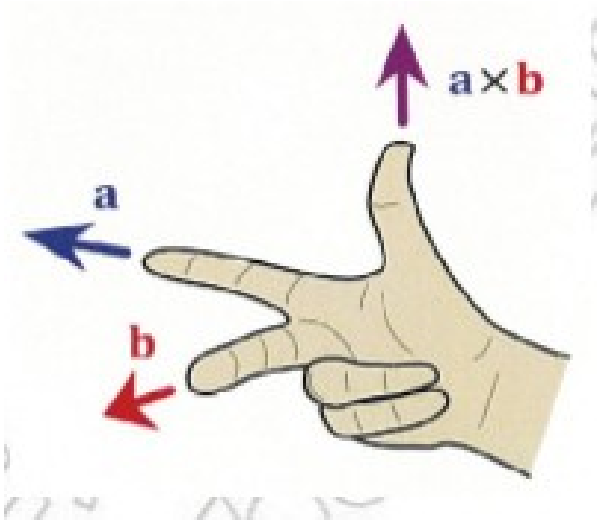
'newton-meter' is not the same as 'joule.' And 'joule' is

reserved only for 'work' and 'energy.'

If the force and arm length are given as a vector form in space

In Unit 2, you learnt about the vector product of two vectors which is:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \sin\theta\, \hat{\mathbf{n}}$$



From above, you can see that

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = |\mathbf{r}|\,|\mathbf{F}| \sin\theta$$

From Unit 2, you should also remember that we can also express the vector product as:

$$\boldsymbol{\tau} = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

Exercise 1: A force of $(3\hat{i} + 4\hat{j})$ N acts at a position of $(120\hat{i} + 50\hat{j})$ cm from the **pivot** of a rotable bar. What is the ***torque*** produced by the force? (Find the ***angle*** between $\vec{r}$ and $\vec{F}$.)

Exercise 2: A mechanic turns a nut with a spanner by a force of 270N perpendicular to the position vector of the point of force application which is 50cm from the center of the nut (**pivot** point). What are the torque and the work if he rotates the spanner through a half turn?

Exercise 3: (R.Q. 3, p.124) A force of (2N, - 5N) acts at a position of (3m, 1m) from a **pivot**. What is the torque exerted by the force?

→

6.3 Rotational Kinetic Energy and Rotationa lnertia

- **Rotational inertia**:

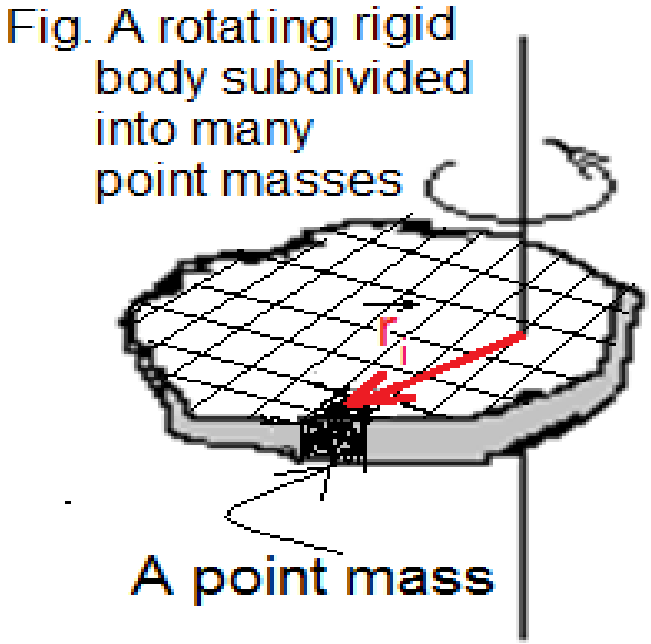
Rotational inertia, or **m**oment **o**f **i**nertia (I), is the rotational equivalent of 'mass' in linear motion. M.o.i. is a measure of inertia in rotational motion.

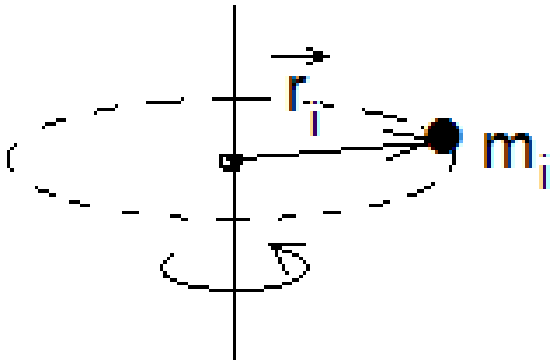
⇒ **torque** $\tau = I.\alpha$

The **moment of inertia (m.o.i.)** of a **rotating body** depends on its '**mass**' as well as its **mass distribution relative to an axis**.

For a point mass, m.o.i. is just $\mathbf{I = mr^2}$ **,** where m=mass…, r=arm length…

For a rigid body, we add the m.o.i. of its point masses.





For a point mass: $I_i = m_i r_i^2$

For a rigid body: $I = \Sigma I_i = \Sigma m_i r_i^2$

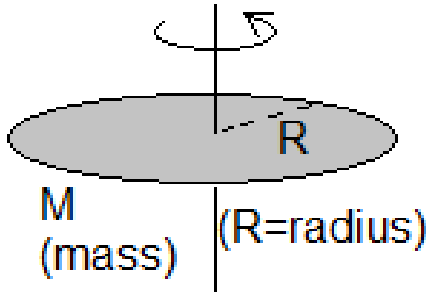
The sum depends on the

- position of the axis of rotation.
- Mass of a body
- Shape of a body
- Size of a body

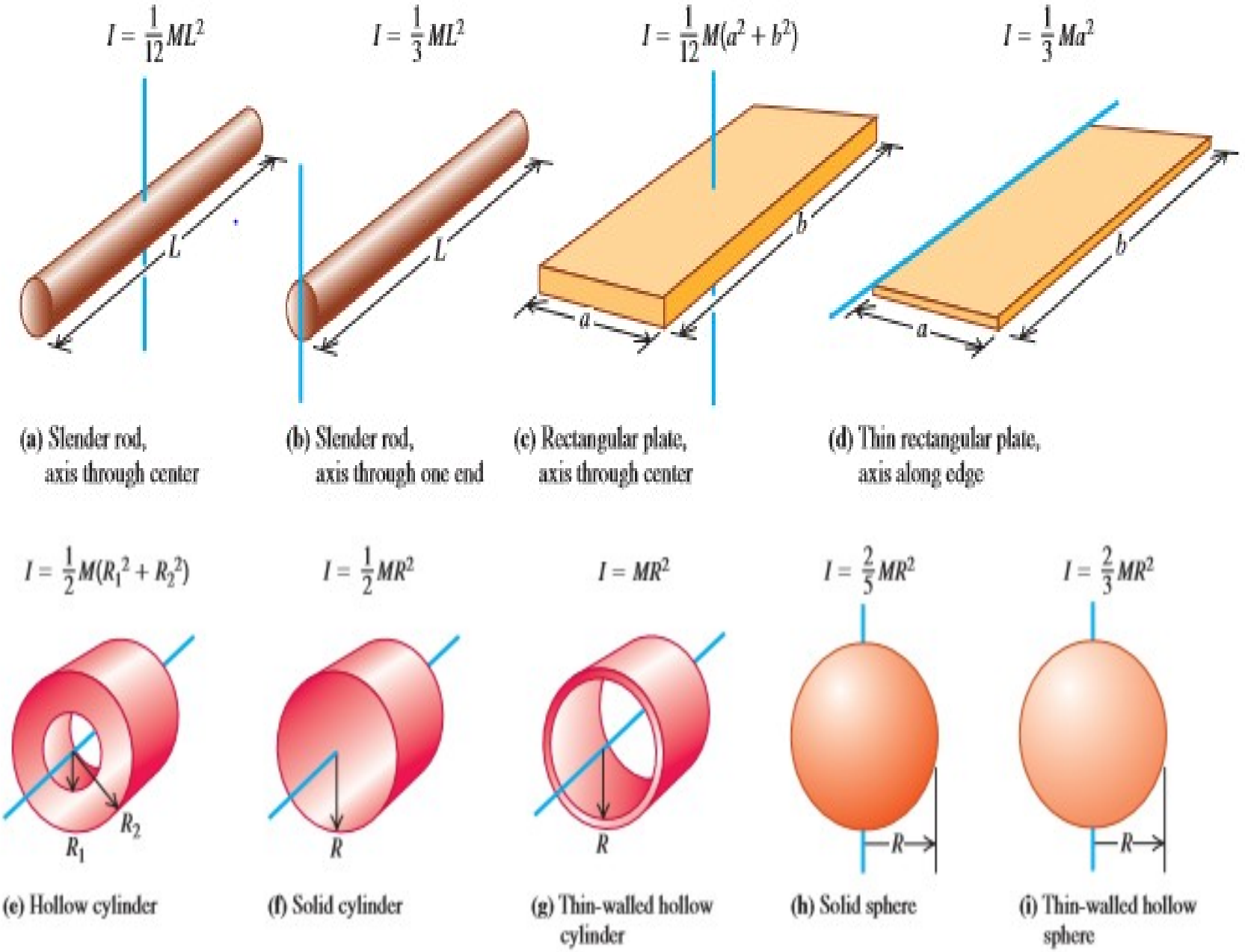
Examples:

1.) Thin disk :

$I = \frac{1}{2} M.R^2$



2. For a point mass: $I = m.R^2$


$I = \frac{1}{12}ML^2$
$I = \frac{1}{3}ML^2$
$I = \frac{1}{12}M(a^2 + b^2)$
$I = \frac{1}{3}Ma^2$
L
L
a
b
a
b
(a) Slender rod, axis through center
(b) Slender rod, axis through one end
(c) Rectangular plate, axis through center
(d) Thin rectangular plate, axis along edge
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
$I = \frac{1}{2}MR^2$
$I = MR^2$
$I = \frac{2}{5}MR^2$
$I = \frac{2}{3}MR^2$
R_1
R_2
R
R
R
R
(e) Hollow cylinder
(f) Solid cylinder
(g) Thin-walled hollow cylinder
(h) Solid sphere
(i) Thin-walled hollow sphere

→

- **Rotational kinetic energy**:

In linear motion: – Translational kinetic energy is $E_K = \frac{1}{2} m.v^2$

→ For rotation: $v=\omega.r$, and 'm' is replaced by 'I' (WHY?).

In **rotational motion**: – We have ***rotational kinetic energy***,

and $E_{rot} = \frac{1}{2} I.\omega^2$.

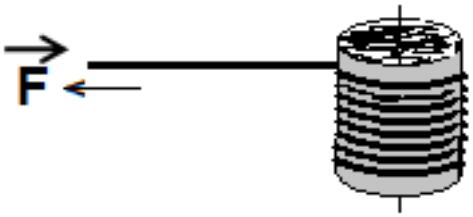
→ I depends on the position of the axis of rotation!

Exercise 1: A point mass of mass 2 kg is revolving round an axis at 5 rad/s at a distance of ½ m from the axis. Calculate its rotational kinetic energy.

→

Exercise 2: A rotating drum has a m.o.i. of 16 000kg.m^2 and an angular speed of 3rad/s. Find its rot. kin. energy.

Exercise 3: A starter cord for a generator is wound round onto a drum of diameter 32cm. A person starts the generator by pulling with a



of 100N horizontally.

a) What torque does the person apply?

b) How much work does the person do if he pulls a 1m long cord?

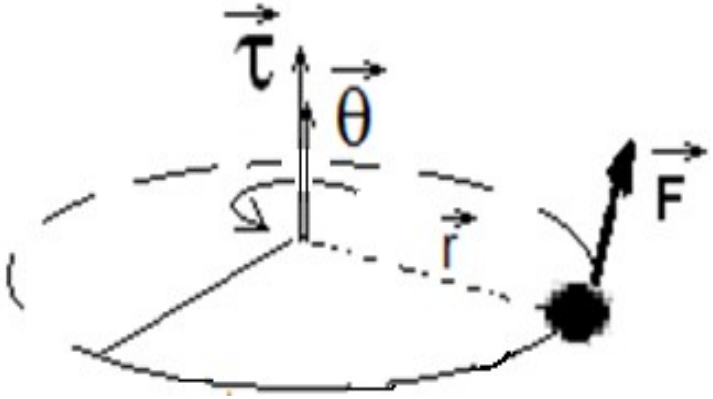
c) Through what angle does the drum turn?

Exercise 4: A string is wrapped round a full cylinder of mass 500g and radius 10cm which is lying on its sides on a flat surface. The string comes off the top of the cylinder horizontally. The string is pulled with a force of 16N. What is the acc of the cylinder if it rolls without slipping? (Use: $\mu_s = 0.4$ between the cylinder and surface.)

Exercise 5: A 5-kg ball rolls at 4m/s at some moment. If its radius is 10cm, find its kinetic energy (total). (**Assume that it rolls without slipping with the help of static friction**.)

Exercise 6: (See ACTIVITY 6.8) **A ring**, **a solid cylinder** and **a solid sphere** roll down a 24-degree inclined plane from a height of 1m simultaneously ***without slipping***. All have equal masses. They are all made from ***the same material***, and $\mu_{st} = 0.3$. **a)** Work out what order they would get to the bottom of the slope. **b)** How long will it take each shape to reach the bottom of the inclined plane ?

- **Work**: the work done by torque is given by the equations



$\cdot\ \theta \quad \Rightarrow\ W = \tau\ .\ \theta.\cos\gamma = \tau\ .$

$\therefore\ [W] = N.m.rad/s,$

- **Power**: the rotational power is the rate of rotational work done

$$\mathbf{P = W/t = \tau.\theta/t = \tau.\omega \quad \Rightarrow\ P = \tau\ \cdot\ \omega}$$

- **we know that the linear power is rate of linear work done as**

$$\mathbf{P = W/t = F.S/t = F.V}$$

Exercise 1: A mechanic applies a torque of 160N.m over three-quarters of a turn and takes 2s to do this.

6.5 Parallel Axis Theorem

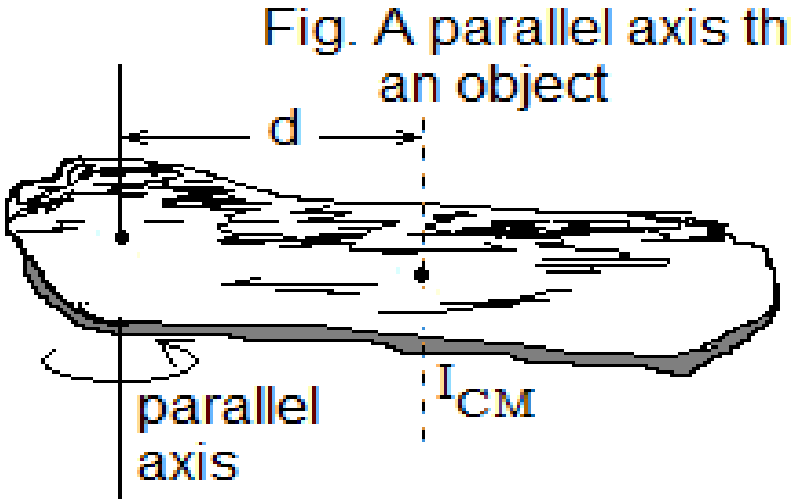


Fig. A parallel axis through an object

If the

$$I_P = I_{CM} + Md^2$$

i.

relative to the

n new

parallel axis becomes. Find the m.o.i. of 6kg rod

where: M = mass of the rigid body,

and

d = distance between the two parallel axes

Exercise 1

A body of mass 8kg has a m.o.i. of 6kg.m^2 about its CM. It is then rotated about a parallel axis which has a displacement of $0.6m\hat{i} + 0.8m\hat{j}$ from the CM.

Find the m.o.i. about the new axis.

→

Exercise 2: The moment of inertia (m.o.i.) of a uniform solid sphere is $(2/5)mR^2$ about an axis through its CM. Show that the m.o.i. about a parallel axis at the edge of its surface is $(7/5)mR^2$.

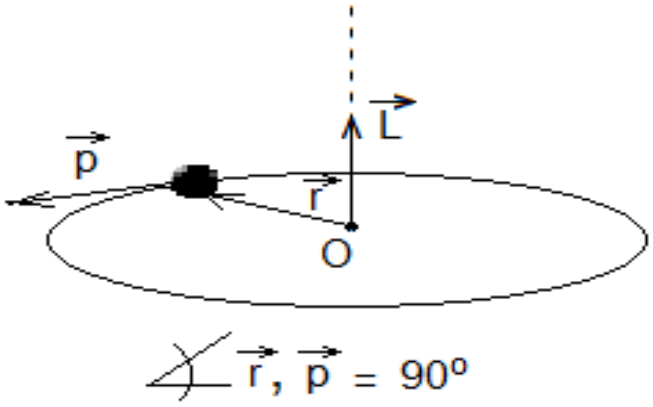
Exercise 3 find the moment of inertia of **solid cylinder**, hallow cylinder, hollow sphere and disk when the axis is taken to their edge taking the moment of inertia of each object about axis at its center is given in **slide no 18 and 19**.

6.6 Angular Momentum and Angular Impulse

- **Angular Momentum ($\vec{L}$):**

  linear momentum $p = m.v$

  angular momentum $L = I.\omega$



For a point mass:

$L = mr^2.\omega = r.\ m\omega r =$

and since $\vec{r} \perp \vec{p}$, we get:

ang. mom. $\vec{L} = \vec{r} \times \vec{p}$

$\therefore [\ \vec{L}\ ] = kg.m^2.rad/s,$

$\ldots = kg.m^2/s$

Next, we introduce the concept of impulse.

Definition 1: **linear impulse** is equal to the change in linear momentum thus, $\rightarrow$

linear impulse $\mathbf{J = F_{net}.\ \Delta t \Rightarrow J = \Delta p = mv - mu}$

Definition **2:Angular impulse** is thus the product of net torque

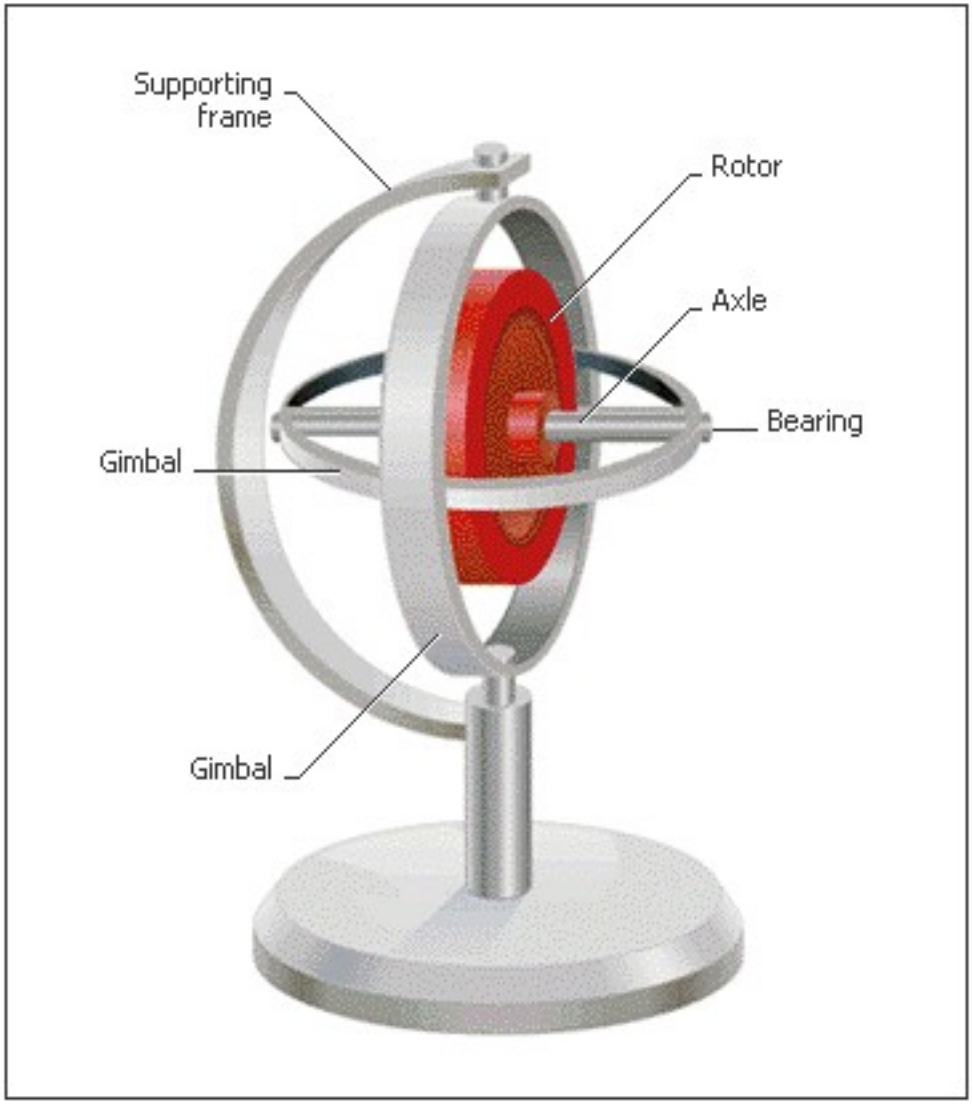
and time for which the torque acts on a rotating body.

- **it is equal to the change in angular momentum**

angular impulse $\mathbf{J = \tau_{net}\ .\Delta t \Rightarrow J = \Delta L \Rightarrow J = I.\Delta\omega}$

the SI unit of angular impulse is **newton-meter-second**

$[\ J\ ] = N.m.s =$

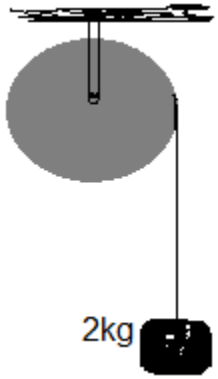


Gyroscope

This gyroscope is designed so that the flywheel and axle are free to point in any direction. Gyroscopes are useful in navigation because they are "rigid in space;" a spinning gyroscope mounted within a vehicle always points in the same direction. Thus a gyroscope provides a means to determine a vehicle's orientation, without relying on visual cues that may not always be available (in fog or at night, for example).

Exercise 18: A flywheel is rotating at 1000rev/min and has a m.o.i. of 50kg.m^2. What is its angular momentum ?

Exercise 19: A flywheel of mass 12kg and radius 20cm is rotated by a 2-kg mass, as shown in the diagram, starting from rest. Find the angular speed of the flywheel after 2.5s.



6.7 Conservation of angular momentum

" When the net torque acting on a rotating body is zero, its angular momentum remains constant."

$\vec{\tau}_{net} = 0 \Rightarrow \vec{L} = \text{const}$, (because $\vec{\tau}_{net} = \Delta L/\Delta t$)

Application: skaters, and divers, …

Exercise 20: A skater that spins with a moment of inertia of $1.2 kg.m^2$ and an angular velocity of 5rad/s, puts her arms out and begins to spin with an angular speed of 1.8rad/s. What is her new m.o.i.?

6.8 Center of Mass of a rigid body

A rigid body balances freely when it is supported at its C.M.

- Uniform rigid body:



C.M. at geometric center; ex. a disc

- Non-uniform rigid body:



ted at an end where there is more

Fig. An irregular body

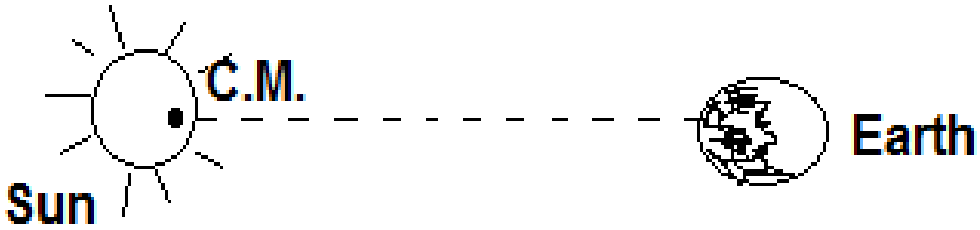
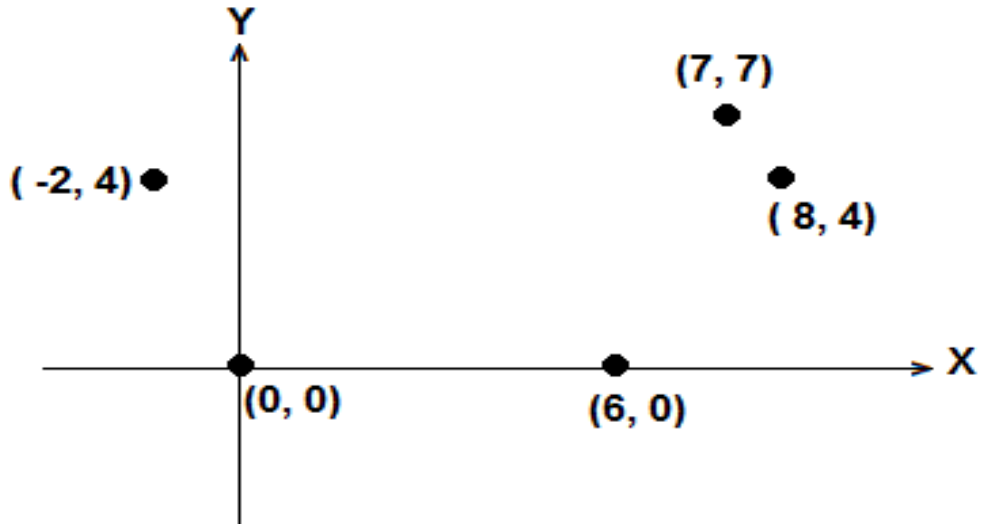


Fig.The Sun-

Exercise 21: Determine the C.M. of the system of



Solved problems for the whole unit 6

PROBLEM 1. The rotor on a helicopter turns at an angular velocity of 3.20 x revolutions per
minute.

(a) Express this angular velocity in radians per second.

(b) (b) If the rotor has a radius of 2.00 m, what arc length does the tip of the blade trace out in 3 x s?

(c) (c) The pilot opens the throttle, and the angular velocity of the blade increase nes in 3.60 s. Calculate the average angular

solut

$$\omega = 3.20 \times 10^2 \frac{\text{rev}}{\text{min}}$$

$$= 3.20 \times 10^2 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1.00 \text{ min}}{60.0 \text{ s}}\right)$$

$$= 33.5 \text{ rad/s}$$

$$\Delta\theta = \omega t = (33.5 \text{ rad/s})(3.00 \times 10^2 \text{ s}) = 1.01 \times 10^4 \text{ rad}$$

$$\Delta s = r\,\Delta\theta = (2.00 \text{ m})(1.01 \times 10^4 \text{ rad}) = 2.02 \times 10^4 \text{ m}$$

c. $\Delta\theta = (26 \text{ rev})(2\pi \text{ rad/rev}) = 52\pi \text{ rad}$:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{52\pi \text{ rad}}{3.60 \text{ s}} = 45 \text{ rad/s}$$

Promlem 2. If the angular velocity goes from 15 rad/s to 9.0 rad/s in 3.0 s, What is the average angular acceleration during that time interval ?

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{9.0 \text{ rad/s} - 15 \text{ rad/s}}{3.0 \text{ s}} = -2.0 \text{ rad/s}^2$$

solution

Problem 3.

A wheel rotates with a constant angular acceleration of 3.50rad/. If the angular velocity of the wheel is 2.00 rad/s at t = 0,

(a) through what angle does the wheel rotate between t =0 and t = 2.00 s?

Give your answer in radians and in revolutions.

(b) What is the angular velocity of the wheel at t = 2.00 s?

(c) What angular displacement (in revolutions) results while the angular

fou

Solutio

$$\Delta\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$= (2.00 \text{ rad/s})(2.00 \text{ s}) + \tfrac{1}{2}(3.50 \text{ rad/s}^2)(2.00 \text{ s})^2$$
$$= 11.0 \text{ rad}$$

a. $\Delta\theta = (11.0 \text{ rad})(1.00 \text{ rev}/2\pi \text{ rad}) = 1.75 \text{ rev}$

b. $\omega = \omega_i + \alpha t = 2.00\ \text{rad/s} + (3.50\ \text{rad/s}^2)(2.00\ \text{s})$

$= 9.00\ \text{rad/s}$

C $\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

$(2 \times 9.00\ \text{rad/s})^2 - (9.00\ \text{rad/s})^2 = 2(3.50\ \text{rad/s}^2)\Delta\theta$

$\Delta\theta = (34.7\ \text{rad})(1\ \text{rev}/2\pi\ \text{rad}) = 5.52\ \text{rev}$

PROBLEM 4 In a compact disc player, as the read head moves out from the center of the disc, the angular speed of the disc changes so that the linear speed at the position of the head remains at a constant value of about 1.3 m/s.

(a) Find the angular speed of a CD of radius 6.00 cm when the read head is at r = 2.0 cm and again at r = 5.6 cm.

(b) An old-fashioned record player rotates at a constant angular speed, so the linear speed of the record groove moving under the detector (the stylus) changes. Find the linear speed of a 45.0-rpm record at points 2.0 cm and 5.6 cm from the center.

(c) In both the CDs and phonograph records, information is recorded in a

Solutions

a. $\omega = \frac{v_t}{r} = \frac{1.3\ \text{m/s}}{2.0 \times 10^{-2}\ \text{m}} = 65\ \text{rad/s}$

$\omega = \frac{v_t}{r} = \frac{1.3\ \text{m/s}}{5.6 \times 10^{-2}\ \text{m}} = 23\ \text{rad/s}$

$45.0\ \frac{\text{rev}}{\text{min}} = 45.0\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1.00\ \text{min}}{60.0\ \text{s}}\right) = 4.71\ \frac{\text{rad}}{\text{s}}$

$v_t = r\omega = (2.0 \times 10^{-2}\ \text{m})(4.71\ \text{rad/s}) = 0.094\ \text{m/s}$

$v_t = r\omega = (5.6 \times 10^{-2}\ \text{m})(4.71\ \text{rad/s}) = 0.26\ \text{m/s}$

c. $d = v_t t = (1.3\ \text{m/s})(3\,600\ \text{s}) = 4\,700\ \text{m}$

PROBLEM 5. A race car accelerates uniformly from a speed of 40m/s to a speed of 60m/s in 5s while traveling counterclockwise around a circular track of radius 4 x m. When the car reaches a speed of 50m/s, calculate

(a) the magnitude of the car's centripetal acceleration,

(b) the angular velocity,

(c) the magnitude of the tangential acceleration, and

(d) the magnitude of the total acceleration.

Solutions

a. $a_c = \frac{v^2}{r} = \frac{(50.0\ \text{m/s})^2}{4.00 \times 10^2\ \text{m}} = 6.25\ \text{m/s}^2$

$\omega = \frac{v}{r} = \frac{50.0\ \text{m/s}}{4.00 \times 10^2\ \text{m}} = 0.125\ \text{rad/s}$

c. $a_t = \frac{v_f - v_i}{\Delta t} = \frac{60.0\ \text{m/s} - 40.0\ \text{m/s}}{5.00\ \text{s}} = 4.00\ \text{m/s}^2$

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4.00\ \text{m/s}^2)^2 + (6.25\ \text{m/s}^2)^2}$

$a = 7.42\ \text{m/s}^2$

PROBLEM 6. A car travels at a constant speed of 30.0 mi/h or (13.4 m/s) on a level circular turn of radius 50 m, as shown in the bird's-eye view in Figure below. What minimum coefficient of static friction, between the tires and roadway will allow the car to make the circular t



Solutions

$$-m\frac{v^2}{r} = -f_{s,\max} = -\mu_s n$$

$$n - mg = 0 \quad \rightarrow \quad n = mg$$

$$-m\frac{v^2}{r} = -\mu_s mg$$

$$\mu_s = \frac{v^2}{rg} = \frac{(13.4\ \text{m/s})^2}{(50.0\ \text{m})(9.80\ \text{m/s}^2)} = 0.366$$

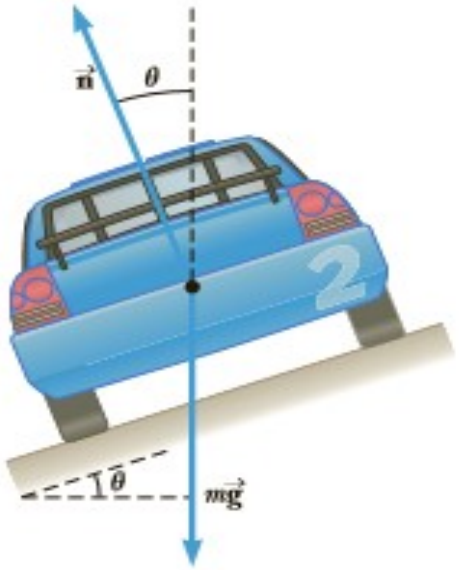
PROBLEM 7 The Daytona International Speedway in Daytona Beach, Florida, is famous for its races, especially the Daytona 500, held every February. Both of its courses feature four-story, 31.0° banked curves, with maximum radius of 316 m. If a car negotiates the curve too slowly, it tends to slip down the incline of the turn, whereas if it's going too fast, it may begin to slide up the incline.

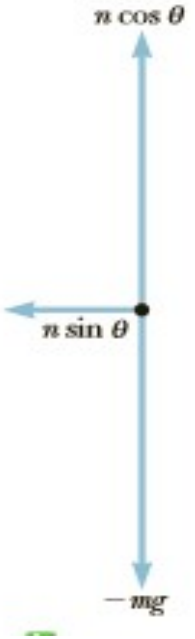
(a) Find the necessary centripetal acceleration on this banked curve so the

car won't tend to slip down or slide up the incline. (Neglect friction.)

(b) Calculate the speed of the race car.

Solutions





a. $m\vec{a} = \sum \vec{F} = \vec{n} + m\vec{g}$

$$n\cos\theta - mg = 0$$

$$n = \frac{mg}{\cos\theta}$$

$$F_c = -n\sin\theta = -\frac{mg\sin\theta}{\cos\theta} = -mg\tan\theta$$

$$ma_c = F_c$$

$$a_c = \frac{F_c}{m} = -\frac{mg\tan\theta}{m} = -g\tan\theta$$

$$a_c = -(9.80\ \text{m/s}^2)(\tan 31.0^\circ) = -5.89\ \text{m/s}^2$$

b. $-\frac{v^2}{r} = a_c$

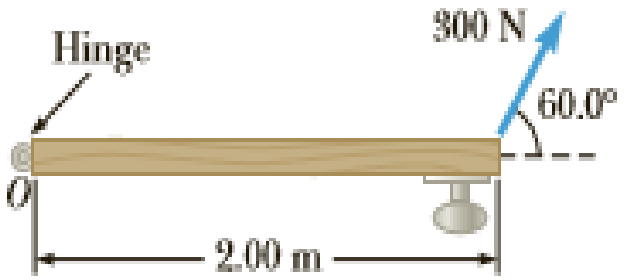
$$v = \sqrt{-ra_c} = \sqrt{-(316\ \text{m})(-5.89\ \text{m/s}^2)} = 43.1\ \text{m/s}^2$$

PROBLEM 8 (a) A man applies a force of F= 3 x N at an angle of 60° to the door of Figure as shown below, 2.00 m from well-oiled hinges. Find the torque on the door, choosing the position of the hinges as the axis of rotation.

solutions

$$\tau_P = rF\sin\theta = (2.00\text{ m})(3.00 \times 10^2\text{ N})\sin 60.0°$$

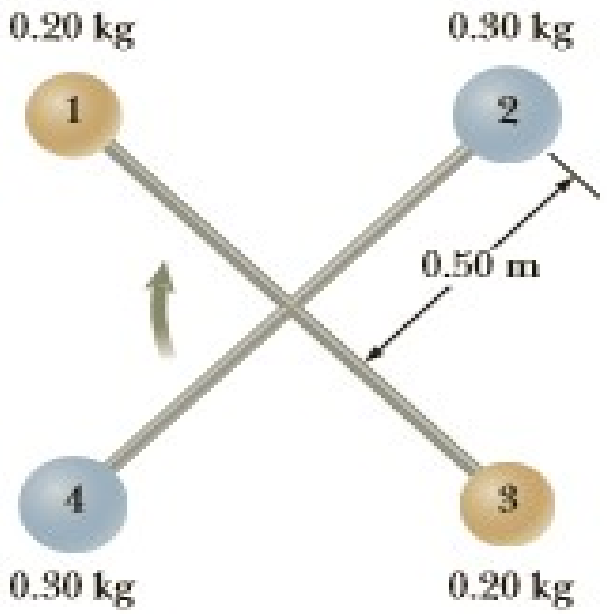
$$= (2.00\text{ m})(2.60 \times 10^2\text{ N}) = 5.20 \times 10^2\text{ N}\cdot\text{m}$$



PROBLEM 9 In an effort to be the star of the halftime show, a majorette twirls an unusual baton made up of four balls fastened to the ends of very light rods as shown below. Each rod is 1.0 m long. Find the moment of inertia of the baton about an axis perpendicular to the page and passing through the point where the rods cross.

Solutions

$$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$
$$= (0.20\ \text{kg})(0.50\ \text{m})^2 + (0.30\ \text{kg})(0.50\ \text{m})^2$$
$$+ (0.20\ \text{kg})(0.50\ \text{m})^2 + (0.30\ \text{kg})(0.50\ \text{m})^2$$
$$I = 0.25\ \text{kg} \cdot \text{m}^2$$



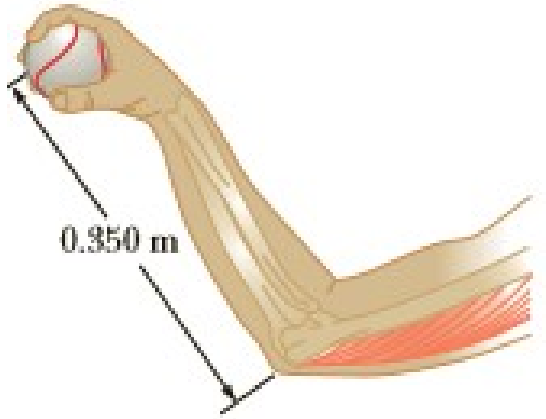
PROBLEM 10. A baseball player loosening up his arm before a game tosses a 0.150-kg baseball, using only the rotation of his forearm to accelerate the ball. The forearm has a mass of 1.50 kg and the length from the elbow to the ball's center is 0.350 m. The ball starts at rest and is released with a speed of 30.0 m/s in 0.300 s.

(a) Find the constant angular acceleration of the arm and ball.

(b) Calculate the moment of inertia of the system consisting of the forearm and ball.

(c) Find the torque exerted on the system that results in the angular acceleration found in part (a).

Solutions



a. $\omega = \omega_i + \alpha t \quad \rightarrow \quad \alpha = \dfrac{\omega}{t}$

$$\alpha = \frac{\omega}{t} = \frac{v}{rt} = \frac{30.0\ \text{m/s}}{(0.350\ \text{m})(0.300\ \text{s})} = \boxed{286\ \text{rad/s}^2}$$

b. $I_{\text{ball}} = mr^2 = (0.150\ \text{kg})(0.350\ \text{m})^2 = 1.84 \times 10^{-2}\ \text{kg} \cdot \text{m}^2$

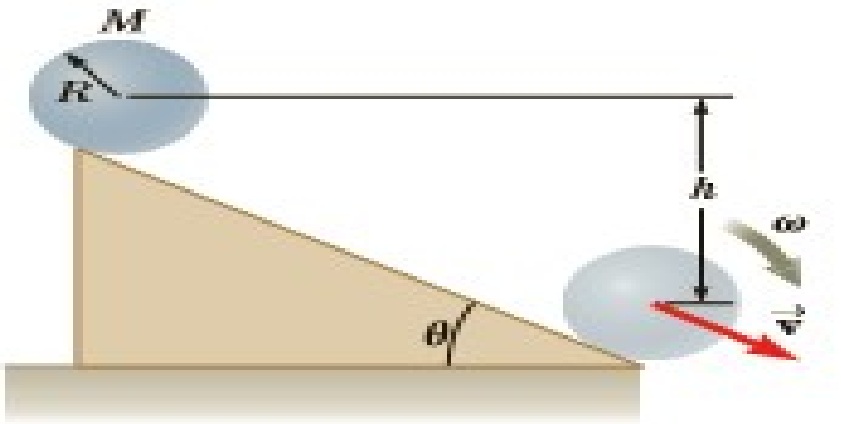
$$I_{\text{forearm}} = \tfrac{1}{3} ML^2 = \tfrac{1}{3}(1.50\ \text{kg})(0.350\ \text{m})^2$$
$$= 6.13 \times 10^{-2}\ \text{kg} \cdot \text{m}^2$$

$$I_{\text{system}} = I_{\text{ball}} + I_{\text{forearm}} = \boxed{7.97 \times 10^{-2}\ \text{kg} \cdot \text{m}^2}$$

c.

$$\tau = I_{\text{system}}\alpha = (7.97 \times 10^{-2}\ \text{kg} \cdot \text{m}^2)(286\ \text{rad/s}^2)$$
$$= \boxed{22.8\ \text{N} \cdot \text{m}}$$

PROBLEM 11. A uniform, solid ball of mass M and radius R starts from rest at a height of h=2.00 m and rolls down a Θ=30.0° slope, as in Figure below. What is the linear speed of the ball when it leaves the incline at the bottom without slipping.



solutions

$$(KE_t + KE_r + PE_g)_i = (KE_t + KE_r + PE_g)_f$$

$$0 + 0 + Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}(\tfrac{2}{5}MR^2)\omega^2 + 0$$

$$Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{5}Mv^2 = \tfrac{7}{10}Mv^2$$

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10(9.80\ \text{m/s}^2)(2.00\ \text{m})}{7}} = 5.29\ \text{m/s}$$

PROBLEM 12 A student sits on a pivoted stool while holding a pair of weights as shown below. The stool is free to rotate about a vertical axis with negligible friction. The moment of inertia of student, weights, and stool is **2.25**. The student is set in rotation with arms outstretched, making one complete turn every **1.26 s**, arms outstretched.

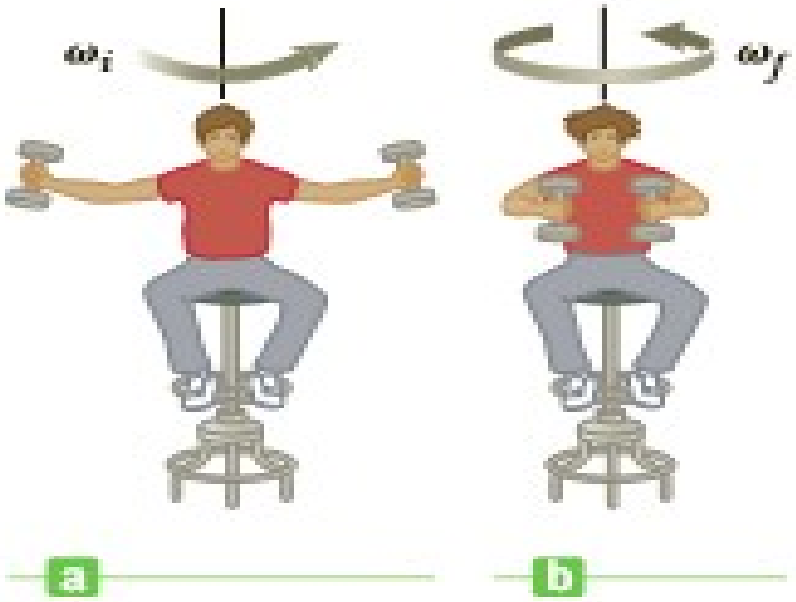
(a) What is the initial angular speed of the system?

(b) As he rotates, he pulls the weights inward so that the new moment of inertia of the system (student, objects, and stool) becomes **1.80**. What is the new angular speed of the system?

(c) Find the work done by the student on the system while pulling in the weights. (Ignore energy lost through dissipation in his muscles.)

Solutions.

a. $\omega_i = 2\pi f = 2\pi/T =$ **4.99 rad/s**



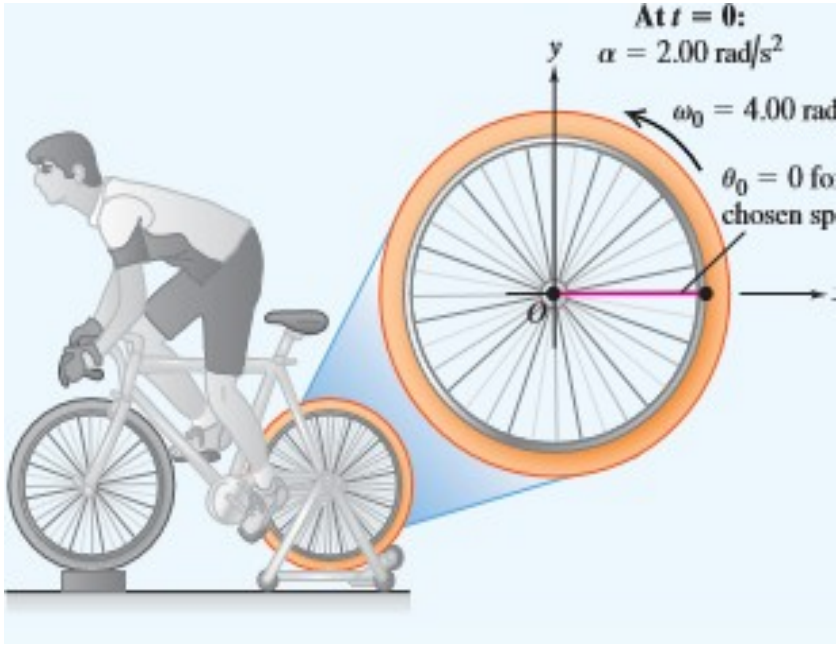
b. (1) $L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f$

(2) $(2.25\ \text{kg}\cdot\text{m}^2)(4.99\ \text{rad/s}) = (1.80\ \text{kg}\cdot\text{m}^2)\omega_f$

$\omega_f =$ **6.24 rad/s**

c. $$W_{\text{student}} = \Delta K_r = \tfrac{1}{2}I_f\omega_f^2 - \tfrac{1}{2}I_i\omega_i^2$$
$$= \tfrac{1}{2}(1.80\ \text{kg}\cdot\text{m}^2)(6.24\ \text{rad/s})^2$$
$$- \tfrac{1}{2}(2.25\ \text{kg}\cdot\text{m}^2)(4.99\ \text{rad/s})^2$$
$$W_{\text{student}} = \textbf{7.03 J}$$

Problem 13 The angular velocity of the rear wheel of a stationary exercise bike is 4ad/s at time t=0 and its angular acceleration is constant and equal to 2rad/A particular spoke coincides with the x- axis at time t=0 as shown in fig below. What angle does this spoke make with the +x- axis at time t=o? What is the wheel's angular velocity at this



$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$= 0 + (4.00\ \text{rad/s})(3.00\ \text{s}) + \tfrac{1}{2}(2.00\ \text{rad/s}^2)(3.00\ \text{s})^2$$
$$= 21.0\ \text{rad} = 21.0\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 3.34\ \text{rev}.$$

The wheel has turned through three complete revolutions plus an additional 0.34 rev, or $(0.34\ \text{rev})(2\pi\ \text{rad/rev}) = 2.14\ \text{rad} = 123°$. Thus, the chosen spoke is at an angle of 123° with the $+x$ axis.

$\omega = \omega_0 + \alpha t$. At time $t = 3.0$ s,

$$\omega = 4.00 \text{ rad/s} + (2.00 \text{ rad/s}^2)(3.00 \text{ s}) = 10.0 \text{ rad/s}.$$

Problem 14 An electric motor exerts a constant torque of 10Nm on a grindstone mounted on its shaft; the moment of inertia of the grindstone 2 is If the system starts from rest, find the work done by the motor in 8.0 s and the kinetic energy at the end of this time. What was the average power delivered by the motor?

sol

$$\alpha = \frac{\tau}{I} = \frac{10 \text{ N} \cdot \text{m}}{2.0 \text{ kg} \cdot \text{m}^2} = 5.0 \text{ rad/s}^2.$$

For constant angular acceleration,

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 = 0 + 0 + \tfrac{1}{2}(5.0 \text{ rad/s}^2)(8 \text{ s})^2 = 160 \text{ rad}.$$

The total work is

$$W = \tau \, \Delta\theta = (10 \text{ N} \cdot \text{m})(160 \text{ rad}) = 1600 \text{ J}.$$

Finally, the average power is the total work divided by the time interval:

$$P_{av} = \frac{W}{\Delta t} = \frac{1600 \text{ J}}{8.0 \text{ s}} = 200 \text{ J/s} = 200 \text{ W}.$$