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PART ONE

INTRODUCTION

Operations Research (or) OR management Science (MS) provides methodology for solving managerial problems. Management OR/MS methodology is appropriate to some but very important problems, which managers encounter in various functional areas such as in finance, operations or production, human resource management, marketing, and in general application areas such as in investment planning, project management, and decision analysis. The course deals with mainly decision processes by way of model building and OR techniques, linear programming models using Simplex procedures, sensitivity analysis and stochastic models that are decision theories and network analysis.

During the 1950s, there was substantial progress in the application of OR techniques for civilian activities along with a great interest in the professional development and education in OR. Many colleges and universities introduced OR in their curricula. They were generally schools of engineering, public administration, business management, applied mathematics, economics, computer science, etc. Today, however, service organizations such as banks, hospitals, libraries, airlines, railways, etc., recognize the usefulness of OR in improving efficiency. In 1948, an OR club was formed in England which later changed its name to the operational Research Society of UK. Its journal, OR Quarterly, first appeared in 1950. The Operations Research Society of America (ORSA) was founded in 1952 and its journal, Operations Research was first published in 1953. In the same year, the institute of Management Sciences (TIMS) was founded as an international society to identify, extend and unify scientific knowledge pertaining to management.

Course objectives :

Upon completion of this course, students are expected to

- a. identify and describe basic concepts in modeling and model use as appropriate to management problems
- b. appreciate representative problems which managers encounter in managing their business firms
- c. appreciate the logical approach to the analysis of problems; and
- d. apply to a limited degree, the concepts and techniques they have learned to sample exercise problems

CHAPTER ONE

INTRODUCTION

Chapter Overview

Operations research attempts to provide a systematic and rational approach to the fundamental problems involved in the control of systems by making decisions which, in a sense, achieve the best results considering all the information that can be profitably used. A classical definition of OR is given by Chruchman et al, ". . . Operations Research is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those in control of operations with optimum solutions to the problems." * Thus, it may be regarded as the scientific method employed for problem solving and decision-making by the management.

Objectives of the chapter

After completing this chapter students are expected to :

- a. Identify the basic tenets of operations research
- b. Grasp the significant features of management science
- c. The development of operations research as a science through its various stages
- d. Application of management science to solve managerial problems

Topic 1- Introduction

It is generally agreed that operations research came into existence as a discipline during World War II when there was a critical need to manage scarce resources. However, a particular model and technique of OR can be traced back to much earlier times. The term 'operations research' was coined as a result of research on military operations during this war. Since the war involved strategic and tactical problems which were greatly complicated, to expect adequate solutions from individuals or specialists in a single discipline was unrealistic. Therefore, groups of individuals who collectively were considered specialists in mathematics, economics, statistics and probability theory, engineering, behavioral, and physical science were formed as special units within the armed forces to deal with strategic and tactical problems of various military operations.

Such groups were first formed by the British Air Force and later, the American armed forces formed similar groups. One of the groups in Britain came to be known as Blackett's Circus. This group, under the leadership of Prof. PMS Blackett was attached to the Radar Operational Research unit and was assigned the problem of analysis the coordination of radar equipment at gun sties. The efforts of such groups, especially in the area of radar detection are considered vital in Britain winning the air battle. Following the success of this group, such a mixed-team approach was also adopted in other allied nations.

After the war ended, scientists who had been active in the military OR groups made efforts to apply the operations research approach to civilian problems, related to business, industry, research and development, etc. There are three important factors behind the rapid development in the use of operations research approach.

- (i) The economic and industrial boom after World War II resulted in continuous mechanization, automation, decentralization of operations and division of management functions. This industrialization also resulted in complex managerial problems, and therefore application of operations research to managerial decision-making became popular.
- (ii) Many operations researchers continued their research after war. Consequently, some important advancements were made in various operations research techniques. A key person in the post war development of OR was George B Dantizing. In 1947, he developed the concept of linear programming and its solution by a method known as simplex method. Besides linear programming, many other techniques of OR, such as statistical quality control, dynamic programming, queuing theory and inventory theory were well developed before the end of the 1950.
- (iii) Analytic power was made available by high-speed computers, The use of computers made it possible to apply many OR techniques for practical decision analysis.

1.2 Definition

An operations research approach seeks to obtain an optimal solution to the problem under analysis. For this, a measure of desirability (or effectiveness) is defined, based on the objective(s) of the organization. A measure of desirability so defined is then used to compare alternative courses of action with respect to their outcomes.

- Operations research is the application of the methods of science to complex problems in the direction and management of large systems of men, machines, materials and money in industry, business, government and defense. The distinctive approach is to develop a scientific model of the system incorporating measurements of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management in determining its policy and actions scientifically.

Operations Research Society, UK

- Operations research is concerned with scientifically deciding how to best design and operate man-machine systems usually requiring the allocation of scarce resources.

Operations Research Society, America

Apart from being lengthy, the definition given by operational Research Society of UK, has been criticized because it emphasizes complex problems and large systems, leaving the reader with the impression that it is a highly technical approach suitable only to large organizations. The definition of OR Society of America contains an important reference to the allocation of scarce resources. The keywords used in the above definitions are scientific approach, scarce resources, system and model. The British definition contains no reference to optimization, while the American definition has no

reference to the word, best. A few other definitions which are commonly used and are widely acceptable are as follows:

- Operations research is the systematic application of quantitative methods, techniques and tools to the analysis of problems involving the operation of systems.
- Operations research is essentially a collection of mathematical techniques and tools which in conjunction with a systems approach, are applied to solve practical decision problems of an economic or engineering nature.

These two definitions imply another view of OR as being the collection of models and methods which have developed largely independent of one another.

- Operations research utilizes the planned approach (updated scientific method) and an interdisciplinary team in order to represent complex functional relationships as mathematical models for the purpose of providing a quantitative basis for decision-making and uncovering new problems for quantitative analysis.
- This new decision-making field has been characterized by the use of scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources.

These two definitions refer to the interdisciplinary nature of OR. However, there is nothing which can stop one person from considering several aspects of the problem under consideration. One of the best definitions is as follows.

- Operations research, in the most general sense, can be characterized as the application of scientific methods, techniques and tools, to problems involving the operations of a system so as to provide those in control of the operations with optimum solutions to the problems.
- Operations research is the art of winning wars without actually fighting them.

This definition refers to the military origin of the subject where team of experts were not actually participating in military operations for winning the war but providing advisor and intellectual support for initiating strategic military actions.

- Operations research is the art of finding bad answers to problems to which otherwise worse answers are given.

This definition refers operations research as a technique for selecting the best course of action out of several courses of action to reach a desirable solution of the problem.

A few other definitions of OR are as follows:

- Operations research has been described as a method, an approach, as set of techniques, a team activity, a combination of many disciplines, an extension of particular disciplines (mathematics, engineering, economic), a new discipline, a vocation, even a religion. It is perhaps some of all these things.
- Operations research may be described as a scientific approach to decision-making that involves the operations of organizational system.
- Operations research is a scientific method of providing executive departments with a quantitative basis for decisions under their control.

- Operations research is applied decision theory. It uses scientific, mathematical, or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem. DW Miller and MK Stan through-going rationality in dealing with his decision problems.
- Operations research is a scientific approach to problem-solving for executive management.

As the discipline of operations research grew, numerous names were given to it, such as Operations Analysis, Systems Analysis, Decision Analysis, Management Science, Quantitative Analysis, Decision Science. However, each of these emphasizes the quantitative approach to the analysis and solution of management problems.

The basic and dominant characteristic feature of operations research is that it employs mathematical representations or models to analyse problems. This distinctive approach represents an adaptation of the scientific methodology used by other physical sciences. The scientific method translates a real given problem into a mathematical representation, which is solved and re-transformed into the original context. The OR approach to problem solving consists of the following steps.

1. Formulate the problem.
2. Determine the assumptions (model building) and formulate the problem in a mathematical framework.
3. Acquire the input data.
4. Solve the model formulated and interpret the results.
5. Validate the model.
6. Implement the solution obtained.

However, one step need not be completed fully before the next is started. In fact, one or more of the steps may be modified to some extent before final results are implemented. This would of course, necessitate the subsequent steps being modified.

Activity 1.1.

What is Operations Research?

What are the main applications of management science?

Why is operations research named management science?

What types of managerial problems are addressed by OR?

1.1. Decision processes and relevance of OR

The significant features of operations research are

1. **Decision-making** Primarily, OR is addressed to managerial decision - making or problem solving. A major premise of OR is that decision-making, irrespective of the situation involved, can be considered as a general systematic process that consists of the following steps:
 - (a) Define the problem and establish the criterion, which will be used. The criterion may be the maximization of profits, utility and minimization of costs and so on.
 - (b) Select the alternative courses of action for consideration.
 - (c) Determine the model to be used and the values of the parameters of the process.

- (d) Evaluate the alternatives and choose the one that is optimal
2. **Scientific approach** OR employs scientific methods for the purpose of solving problems, and there is no place of whims and guesswork in it. It is a formalized process of reasoning and consists of the following steps:
 - (a) The problem to be analysed is defined clearly and the conditions for observations are determined.
 - (b) Observations are made under varying conditions to determine the behavior of the system.
 - (c) On the basis of the observations, a hypothesis describing how the various factors involved are believed to interact and the best solution to the problem is formulated.
 - (d) To test the hypothesis, an experiment is designed and executed. Observations are made and measurements are recorded.
 - (e) Finally, the results of the experiments are analysed and the hypothesis is either accepted or rejected. If the hypothesis is accepted, the best solution to the problem is obtained.
 3. **Objective:** OR attempts to locate the best or optimal solution to the problem under consideration. For this purpose, it is necessary that a measure of effectiveness be defined which is based on the goals of the organization. This measure is then used as the basis to compare the alternative courses of action.
 4. **Inter-disciplinary team approach:** OR is inter-disciplinary in nature and requires a team approach to a solution to the problem. No single individual can have a through knowledge of the myriad aspects of operation research and how the problems may be addressed. Managerial problems have economic, physical, psychological, biological, sociological and engineering aspects. This requires a blend of people with expertise in the areas of mathematics, statistics, engineering, economics, management, computer science and so on. Of course, it is not always so. Some problem situations may be adequately handled even by one individual.
 5. **Digital Computer** Use of a digital computer has become an integral part of the operations research approach to decision-making. The computer may be required due to the complexity of the model, volume of data required or the computations to be made. Many quantitative techniques are available in the form of 'canned' programmes.
1. **Model building:** Once the problem is defined, the next step is to build a suitable model. As already mentioned, the concepts of models and model-building lie at the very heart of the operations research approach to problem solving. A model is a theoretical abstraction of a real-life problem. In fact, many real-life situations tend to be very complex because there are literally innumerable inherent factors in any given situation. Thus, the decision-maker has to abstract from the empirical situation those factors which are most relevant to the problem. Having selected the critical factors, he combines them in some logical manner so that they form a counterpart or a model of the actual problem.

Thus, a model is a simplified representation of a real-world situation that, ideally, strips a natural phenomenon of its bewildering complexity and

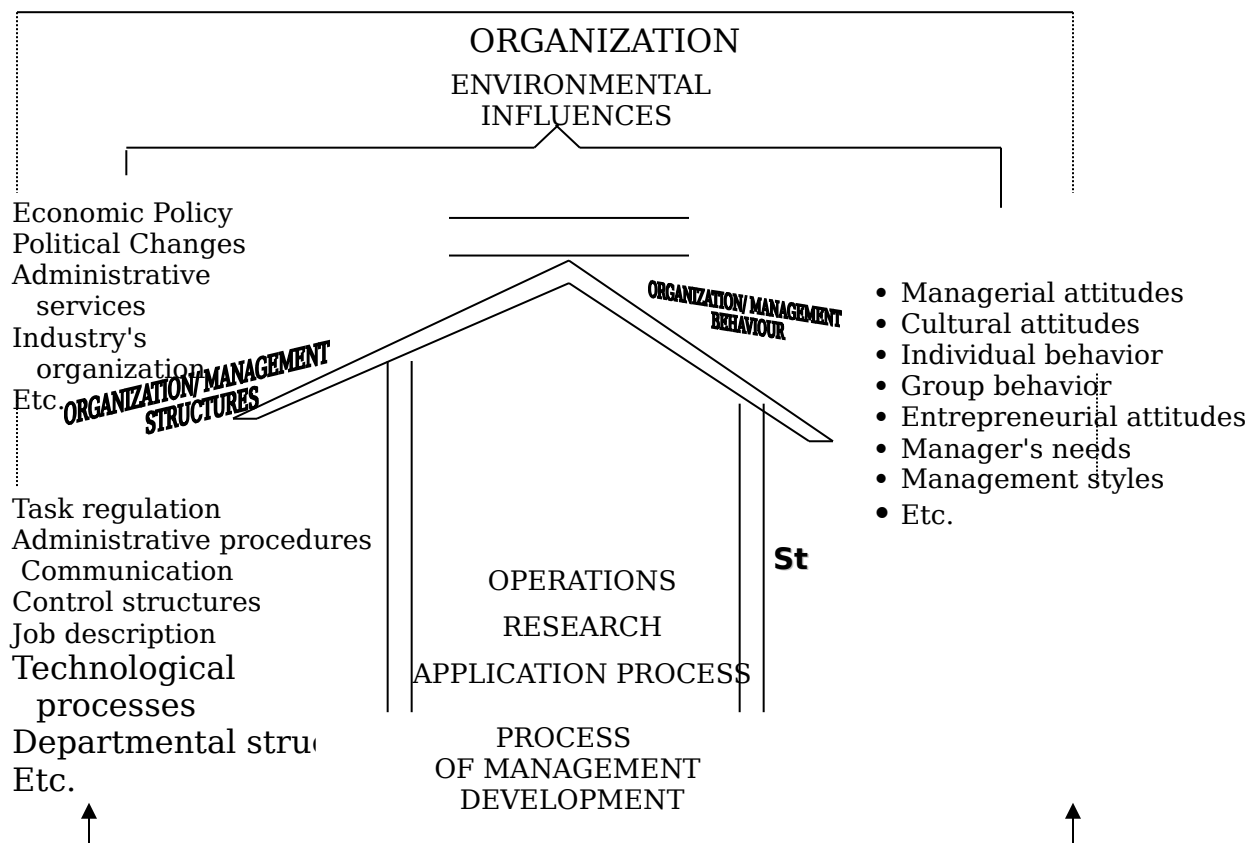
replicates its essential behavioral. Models may be represented in a variety of ways. They can be classified as physical and symbolic models.

(a) Physical models: A physical model is a physical or schematic representation of the real thing. There are two types of physical models: iconic and analogue.

(i) Iconic models-They are essentially the scaled-up/down versions of the particular thing they represent. A model aeroplane in a wind tunnel, a model of a proposed building provided by an architect, models of the sun and its planets housed in a planetarium, a model of a particular molecular structure of a chemical are examples of iconic models, because they look like what they represent (except size). Maps, pictures or drawings may also be categorized as iconic models since they represent essentially the images of certain things.

The chief merit of an iconic model is that it is concrete and specific. It resembles visually the thing it represents and, therefore, there are likely to be fewer problems in translating any 'findings' from the model into the real-life situation. However, the disadvantage of such models is that they often do not lend themselves to manipulation for experimental purposes.

(ii) Analogue models. The analogue models use one set of properties to represent another set. To illustrate, an electrical network used as an analogue model to study the flows in a transportation system. Similarly, a barometer that indicates changes in atmospheric pressure through movements of a needle is an example of an analogue model. Contour lines on a map are analogues of elevation. Analogue models are less specific and concrete but they are easier to manipulate as compared to the iconic models.



Structural Analysis of OR Application Process in an organization

Because of OR's multi-disciplinary character and application in varied fields, it has a bright future, provided people devoted to OR study can help meet the needs of society. Some of the problems in the area of hospital management, energy conservation, environmental pollution, etc., have been solved by OR specialists and this is an indication that OR can also contribute towards the

improvement in the social life and areas of global need. However, in order to make the future of OR brighter, its specialists have to make good use of the opportunities available to them.

1.3 Models and model building

Both simple and complex systems can easily be studied by concentrating on some portion or key features instead of concentrating on every detail of it. This approximation or abstraction, maintaining only the essential elements of the system, which may be constructed in various forms by establishing relationships among specified variables and parameters of the system, is called a model. In general, models attempt to describe the essence of a situation or activity by abstracting from reality so the decision-maker can study the relationship among relevant variables in isolation. Models do not attempt to duplicate reality in all aspects, but for models, that do reveal nothing. Hence, models do not, and cannot, represent every aspect of reality because of the innumerable and changing characteristics of the real life problems to be represented. Instead, they are limited approximation of reality. For example, to study the flow of material through a factory, a scaled diagram on paper showing the factory floor, position of equipment, tools, and workers can be constructed. It would not be necessary to give such details as the color of machines, the heights of the words, or the temperature of the building. In other words, for a model to be effective, it must be representative of those aspects of reality that are being investigated and have a major impact on the decisions situation.

From the above discussion, it appears that a model is constructed to analyse and understand the given system for the purpose of improving its performance. The reliability of the solution obtained from a model depends on the validity of the model in representing the system under study. A model, however, allows the opportunity to examine the behavioral changes of a system without disturbing the on-going operations.

The key to model-building lies in abstracting only the relevant variables that affect the criteria of the measures-of-performance of the given system and expressing the relationship in a suitable form. However, a model should be as simple as possible so as to give the desired result. But oversimplifying the problem can lead to a poor decision. Model enrichment is accomplished through the process of changing constants into variables, adding variables, relaxing linear and other assumptions, and including randomness.

There are many ways to classify models. Classification schemes can also provide a useful frame of reference for models. There are eight classification schemes for models as shown in Table 1.1.

Table 1.1 Model classification Scheme

1. Function	4. Degree of certainty	7. Degree of closure
• Descriptive • Predictive • Normative	• Certainty • Conflict • Risk • Uncertainty	• Closed • Open
2. Structure	5. Time reference	8. Degree of Quantification
• Iconic • Analog • Symbolic	• Static • Dynamic	• Qualitative • Mental • Verbal • Quantitative
3. Dimensionality	6. Degree of generality	
• Two-dimensional • Multi-dimensional	• Specialized • General	• Statistical • Heuristic • Simulation

A summary classification scheme of OR models based on different criteria is given in Fig. 1.2 and few of these schemes are discussed below:

Table 1.1. Classification Based on Structure

➤ **Physical Models**

These models provide a psychical appearance of the real object under study either reduced in size or scaled up. Physical models are useful only in design problems because they are easy to observe, build and describe. For example, in the aircraft industry, scale models of the proposed new aircraft are built and tested in wind tunnels to record the stresses experienced by the air frame. Since these models cannot be manipulated and are not very useful for prediction, problems such as portfolio selection, media selection, production scheduling, etc., cannot be analyzed with a physical model. Physical models are classified into the following two categories.

- (i) **Iconic Models.** Iconic models retain some of the physical properties and characteristics of the system they represent. An iconic model is either in an idealized form or is a scaled version of the system. In other words, such models represent the system as it is by scaling it up or down (i.e. enlarging or reducing the size).

Examples of iconic models are, blueprints of a home, maps, globes, photography, drawings, air planes, trains, etc.

Iconic models are simple to conceive, specific and concrete. An iconic model is used to describe the characteristics of the system rather than being explanatory. This means that such models are used to represent a static event and characteristics which are not used in determining or predicting effects due to certain changes in the actual system. For example, color of an atom does not play any vital role in the scientific study of its structure. Similarly, type of engine in a car has no role to play in the study of parking problem.

- (ii) **Analogue Models.** These models represent a system by a set of properties different from those of the original system and does not

resemble it physically. After the problem is solved, the solution is re-interpreted in terms of the original system.

For example, the organizational chart represents the state of formal relationships existing between members of the organization. Maps in different colours may represent water, desert and other geographical features. Graphs of time series, stock-market changes, frequency curves, etc., may be used to represent quantitative relationships between any two properties and predict how a change in one property effects the other. These models are less specific and concrete but are easier to manipulate and are more general than iconic models.

➤ **Symbolic models**

These models use symbols (letters, numbers) and functions to represent variables and their relationships to describe the properties of the system. These models are also used to represent relationships which can be represented in a physical form. Symbolic models can be classified into two categories.

(i) **Verbal Models.** These models describe a situation in written or spoken language. Written sentences, books, etc., are examples of a verbal model.

(ii) **Mathematical Models.** These models involve the use of mathematical symbols, letters, numbers and mathematical operators (+, -, ÷, x) to represent relationships among various variables of the system to describe its properties or behaviors. The solution to such models is then obtained by applying suitable mathematical technique.

The relationship among velocity, distance and acceleration is an example of a mathematical model. In accounting, the cost-volume-profit model is also an example of a mathematical model. Symbolic models are precise and abstract and can be analysed and manipulated by using laws of mathematics. The models are more explanatory rather than descriptive.

Classification Based on Function or Purpose

Models based on the purpose of their utility include the following types.

Descriptive models

Descriptive models simply describe some aspects of a situation, based on observation, survey, questionnaire results or other available data of a situation and do not predict or recommend anything.

Examples of such models are: Organization chart, plant layout diagram, block diagram representing an algorithm or method for solving a problem, etc.

Predictive models

These models indicate 'if this occurs, then that will follow.' They relate dependent and independent variables and permit trying out, 'what if' questions. In other words, these models are used to predict the outcomes due to a given set of alternatives for the problem. These models do not have an objective function as a part of the model to evaluate decision alternatives.

For example, $S = a + bA + cI$ is a model that describes how the sale (S) of a product changes with a change in advertising expenditure (A) and disposable personal income (I). Here, a, b and c are parameters whose values must be estimated. Thus, having estimated the values of a, b and c, the value of advertising expenditure (A) can be adjusted for a given value of I to study impact of advertising on sales. In these models, however, one does not attempt to choose the best decisions alternative, but can only have an idea about each alternative available to him.

1.4 OR Techniques

As the term implies, OR involves research on military operations. This indicates the approach as well as the area of its applications. The operations research approach is particularly useful in balancing conflicting the area of its applications. The operations research approach is particularly useful in balancing conflicting objective (goals or interests) where there are many alternative courses of action available to the decision-makers. In a theoretical sense, the optimum decision must be one that is best for the organization as a whole. It is often called the global optimum. A decision that is best for one or more sections of the organization is usually called sub-optimum decision. The OR approach attempts to find global optimum by analyzing inter-relationships among the system components involved in the problem. One such situation is described below.

Consider a large organization with a number of management specialists but not necessarily well coordinated. For example, consider the basic problem of maintaining stocks of finished goods. To the marketing manger, stocks of a large variety of products are a means of supplying the company's customers with what they want and when they want it. Clearly, according to a marketing manager, a fully stocked warehouse is of prime importance to the company. But the production manger argues for long production runs preferably on a smaller product range, particularly if significant time is lost when projection is switched from one variety to another. The result would again be a tendency to increase the amount of stock carried but it is, of course, vital that the plant should be kept running. On the other hand, the finance manger sees stocks in terms of capital ties up unproductively and argues strongly for their reduction. Finally, there appears the personnel manager for whom a steady level of production is advantageous for having better labour relations. Thus, all these people would claim to uphold the interests of the organization, but they do so only from their specialized points of view. They may come up with contradictory solutions and obviously, they cannot all be right.

In view of the above involving the whole system, the decision-maker, whatever his specialization, will need help and it is in the attempt to provide this assistance that OR has been developed. Operations research attempts to resolve the conflicts of interest among various sections of the organization and seeks the optimal solution which may not be acceptable to one department but is in the interest of the organization as a whole. Further, OR is concerned with providing the decision-maker with decision aids (or rules) derived from:

- (i) A total system orientation

- (ii) Scientific methods of investigation, and
- (iii) Models of reality, generally based on quantitative measurement and techniques.

Thus, successful application of OR techniques for solving a problem must involve the following steps.

1. Constructing mathematical, economics or statistical model of the problem under study to treat situations of complexity and uncertainty. This helps to view the problem in its entirety
2. Analysing the relationships among different variables and/or parameters associated with the problem, so as to determine consequences of decision alternatives.
3. Suggesting suitable measures of desirability (effectiveness or objective function) in order to evaluate the relative merit of decisions alternatives (courses of action, acts or strategies).

The techniques can be categorized thus:

1.4.1. A Quantitative Approach to Decision-making

Decision-making in today's social and business environment has become a complex task. The uncertainty of the future and the nature of completion and social interaction greatly increase the difficulty of managerial decision-making. Knowledge and technology are changing rapidly, the new problem with little or no precedents continually arise. Well-structured problems are routinely optimized by computerized systems at the operational level of organizations, and increased attention is now focused on broader tactical and strategic issues. Hence, in order to effectively address these problems and provide leadership in the advancing global age, decision-makers cannot afford to make decisions by simply applying their personal experiences, guesswork or intuition, because the consequences of wrong decisions are serious and costly. Fore example entering the wrong markets, producing the wrong products, providing inappropriate services, etc., will have major, often disastrous consequences for organizations.

The practice of operations research (OR) must maintain stride with above said trends in the global age. Criticism has periodically been leveled at the failure of OR to adequately meet the needs of business and industry. Often the reasons for failure are behavioral in nature, and significant research on implementation process has been carried out. Focusing on implantation, however, presumes that the definition, analysis, modeling, and solution phases of an OR project have been adequately performed. One can easily hypothesize that among the reasons for implantation failure is the lack of creative problem solving abilities by the analyst. This module introduces the set of operations research techniques for decision-makers to help them make rational and effective decisions by having a basic knowledge of mathematics and statistics, as well as the use of computer software needed for computational purposes.

Operations research as one of the quantitative aid to decision-making offers the decision-maker a method of evaluating every possible alternative (act or course of action) by using various techniques to know the potential

outcomes. This is not to say, however, that management decision-making is simply about the application of operations research techniques.

In general, while solving a real-life problem, the decision-maker must examine it both from quantitative as well as qualitative perspective. Information about the problem from both these perspectives needs to be brought together and assessed in the context of the problem. Based on some mix of the two sources of information, a decision should be taken by the decision-maker. For example, consider a problem of an investor considering investments in three alternatives: Stock market, Real Estate and Bank Deposit. To suggest an acceptable solution, we need to consider certain quantitative factors and information to be examined in the light of the problem. For instance, factors like financial ratios from the balance sheets of several companies whose stocks are under consideration; real estate companies' cash flows and rates of return for investment in property; and how much the investment will be worth in future when deposited at a bank at a given interest rate for a certain number of years, need to be examined. However, before reaching a conclusion, certain other qualitative factors, such as weather, state and central policies, new technology, the political situation, etc., need to be considered. These factors may be difficult to quantify.

The evaluation of each alternative is extremely difficult or time consuming for two reasons: First, the amount and complexity of information that must be processed, second the number of alternative solutions could be so large that a decision-maker simply cannot evaluate all of them to select an appropriate one. For these reasons when there is a lack of qualitative factors, decision-makers increasingly turn to quantitative methods and use computers to arrive at the optimal solution to problems involving a large number of alternatives. The study of these methods and how decision-makers use them in the decision process is the essence of operations research approach.

However, still there is an imperative need for an analytical research in this subject to present a structural analysis by critically examining the levels of interaction between the application process of operations research and the systems and organizations. Figure 1.1 summarizes the operations research approach by sketching the conceptual framework of the main elements of the analysis.

The steps are shown in Figure 1.1 and we now discuss each of these one by one.

- **Problem formulation:** The first step in quantitative analysis is to develop a clear and concise statements of the problem. In many cases, defining the problem proves to be the most important and difficult step. It is essential, therefore, that the root problem should be identified and understood in the first place. Logically speaking, we cannot expect to get the right answer if the problem is identified incorrectly. In that case, the solution derived from it is apt to be useless and all the efforts in that direction shall be a waste. The problem should be identified properly because often what is described as a problem may only be its symptom. For example excessive costs per se do not constitute a problem. They are only

an indication of some problem that may, for instance, be improper inventory levels, excessive wastage and the like. Often the symptoms of a problem may extend beyond a single manager's control to other personnel and other departments in an organization. It is essential, therefore, to go beyond symptoms of the problem and identify their actual causes. Also, one problem may be related to other problems and solving one problem without having regard to the others may make the matters worse. It is essential, therefore, that an analysis be made as to how the solution to one problem is likely to affect other problems or the situation in general. Further, it is likely for an organization to have several problems. However, an analyst usually cannot deal with all the problems and has to focus only on a few of them. For most companies it translates in to selecting those problems whose solutions are likely to result in greatest profit increases or cost reductions. Thus, it is imperative that right problems be selected for solution. In sum, it is necessary for an analyst to understand that the formulation of a problem develops from a complicated interaction that involves the selection and interpretation of data between him and the management.

Once the problem has been identified, it is categorized as being standard or special. The standard

Problems are also known as programmed problems. As has already been motioned, they are the well-structured problems characterized by routine, repetitive decisions that utilize specific decision-making techniques in their solution strategy. Standard solution procedures have been developed to handle such prototype problems. Examples of these problems include the assignments of workers to jobs, fixing the product-mix for a month and determination of the quantity of materials to be bought. On the other hand, there are special or non-programmed problems. They are unique and non-recurrent in nature and, therefore, ill-structured. Undertaking of a research and development project and the merger and consolidation decisions illustrate such type of decision situations.

➤ **Interdisciplinary Approach**

Interdisciplinary teamwork is essential because while attempting to solve a complex management problem, one person may not have the complete knowledge of all its aspects (such as economic, social, political, psychological, engineering, etc). This means we should not expect a desirable solution to managerial problems. Therefore, a team of individuals specializing in mathematics, statistics, economics, engineering, computer science, psychology, etc., can be organized so that each aspect of the problem could be analysed by a particular specialist in that field in order to arrive at an appropriate and desirable solution of the problem. However, there are certain problem situations which may be analysed even by one individual.

➤ **Methodological Approach**

Operations research is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide those

in control of operations with optimum solutions to the problems (Churchman et al.). The scientific method consists of observing and defining the problem; formulating and testing the hypothesis; and analyzing the results of the test. The data so obtained is then used to decide whether the hypothesis should be accepted or not. If the hypothesis is accepted, the results should be implemented. Otherwise an alternative hypothesis has to be formulated.

➤ **Objectivistic Approach**

1.4.2. Scientific Method In Operations Research

The most important feature of operations research is the use of the scientific method and building of decision models. There are three phases of the scientific method on which OR approach is based.

➤ **Judgment Phase**

This phase includes: (i) identification of the real-life problem, (ii) selection of an appropriate objective and the values of various variables related to this objective, (iii) application of the appropriate scale of measurement, i.e. deciding the measures of effectiveness (desirability) and (iv) formulation of an appropriate model of the problem, abstracting the essential information, so that a solution to the decision-maker's goals can be obtained.

➤ **Research Phase**

This phase is the largest and longest among other two phases. However, the remaining two are also equally important as they provide the basis for a scientific method. This phase utilizes: (i) Observations and data collection for a better understanding of the problem, (ii) formulation of hypothesis and mode, (iii) observation and experimentation to test the hypothesis on the basis of additional data, (iv) analysis of the available information and verification of the hypothesis using pre-established measures of desirability, (v) prediction of various results from the hypothesis, and (iv) generation of the result and consideration of alternative methods.

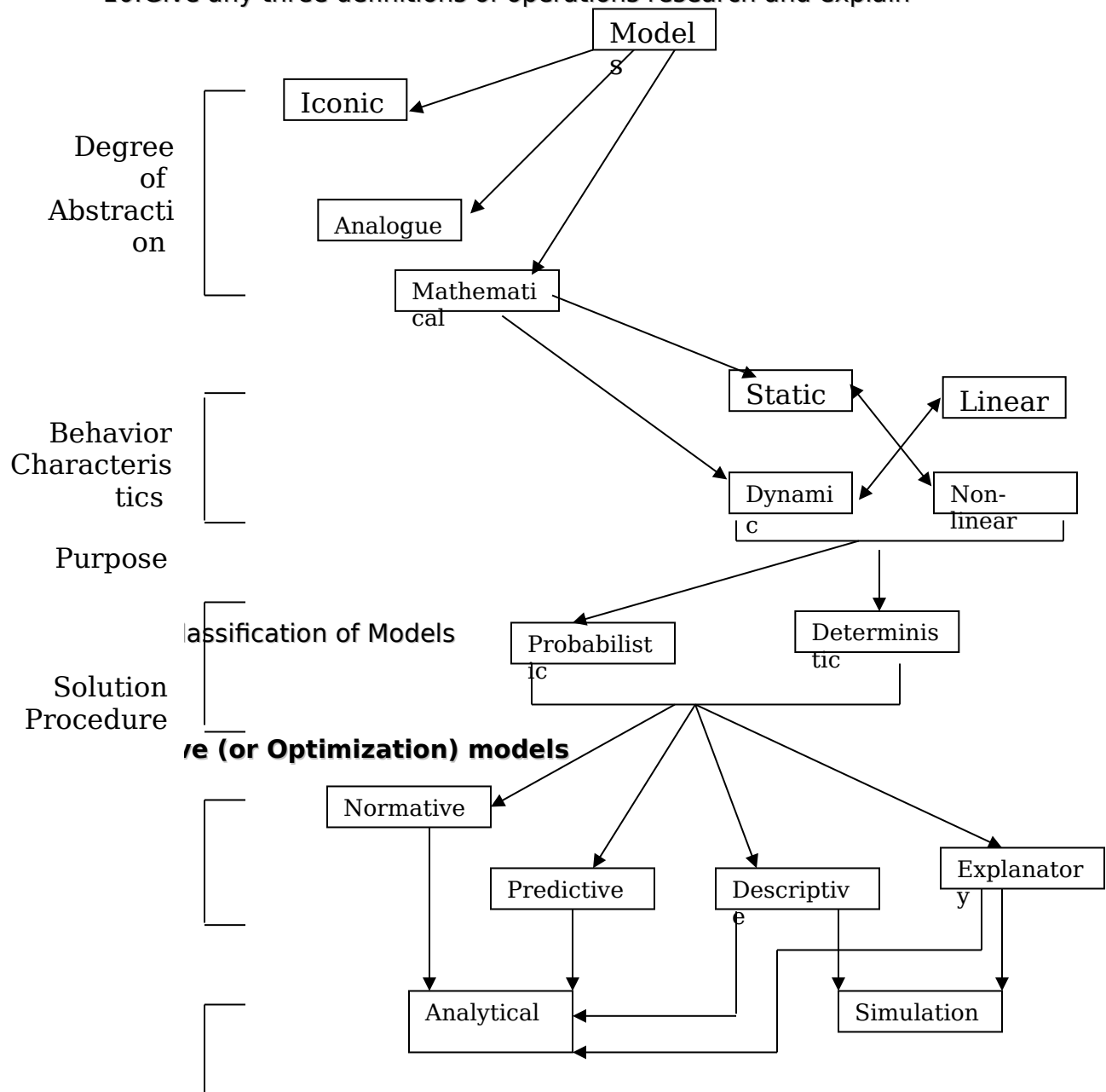
➤ **Action Phase**

This phase consists of making recommendations for implementing the decision by an individual who is in the position to implement results: This individual must be aware of the environment in which the problem occurred, objective, assumptions and omissions of the model of the problem.

Review Questions 1.1

1. Briefly trace the history of operations research. What happened to the development of operations research after World War II?
2. Is operations research a discipline or a profession or a set of techniques or a philosophy or a new name for old things? Discuss.
3. What are the situation (areas) where OR techniques will be applicable?
4. Discuss the following:
(a) OR is an inter-disciplinary approach. (b) Scientific method in OR.
(c) OR is more than a quantitative analysis of the problem.
5. What are the essential characteristics of operations research? Mention different phases in an operations research study. Point out its limitations, if any.

6. Define operations research as a decision-making science.
 - (a) Give the main characteristics of OR.
 - (b) Discuss the scope of OR.
7. (a) Outline the broad features of the judgment phase and the research phase of the scientific method in OR. Discuss in detail any one of these phases.
 - (b) What are various phases of the OR problem? Explain them briefly.
 - (c) State the phases of OR study and their importance in solving problems.
8. What is the role of operations research in decision making?
9. Define operations research. Explain critically the limitations of various definitions as you understand them.
10. Give any three definitions of operations research and explain



These models provide the 'best' or 'optimal' solution to problems subject to certain limitations on the use of resources. These models provide recommended course of action. For example, in mathematical programming, models are formulated for optimizing the given objective function, subject to restrictions on resources in the context of the problem under consideration and non-negativity of variables. These models are also called prescriptive models because they prescribe what the decision-maker ought to do.

1.4.3. Classification Based on Time Reference

- **Static models**

Static models represent a system at some specified time and do not account for changes over time. For example, an inventory model can be developed and solved to determine an economic order quantity for the next period assuming that the demand in planning period would remain the same as the for today.

- **Dynamic Models**

In a dynamic model, time is considered as one of the variables and allows the impact of changes due to change in time. Thus, a sequence of interrelated decisions over a period of time are made to select the optimal course of action to optimize the given objective. Dynamic programming is an example of a dynamic model.

1.4.4. Classification Based on Degree of Certainty

- **Deterministic models**

If all the parameters, constants and functional relationships are assumed to be known with certainty when the decision is made, the model is said to be deterministic. Thus, in such a case, the outcome associated with a particular course of action is known. That is, for a specific set of input values, there is a uniquely determined output which represents the solution of the mode under conditions of certainty. The results of the models assume single value. Linear programming models are examples of deterministic models.

- **Probabilistic (Stochastic) models**

Models in which at least one parameter or decision variable is a random variable are called probabilistic (or stochastic) models. Since at least one decision variable is random, therefore, an independent variable which is the function of dependent variable(s) will also be random. This means consequences or pay-off due to certain changes in the independent variable cannot be predicted with certainty. However, it is possible to predict a pattern of values of both the variables by their probability distribution.

Insurance against risk of firm, accidents, sickness, etc. are examples where the pattern of events is studied in the form of a probability distribution.

1.4.5. Classification Based on Method of Solution or Quantification

- **Heuristic Models**

These models employ some sets of rules which, though perhaps not optimal, do facilitate solutions of problems when applied in a consistent manner.

- **Analytical models**

These models have a specific mathematical structure and thus can be solved by known analytical or mathematical techniques. Any optimization model (which required maximization or minimization of an objective function) is an analytical model.

- **Simulation models**

These models also have a mathematical structure but are not solved by applying mathematical techniques to get a solution. Instead, a simulation mode is essentially a computer-assisted experimentation on a mathematical structure of a real-life problem in order to describe and evaluate its behavior under certain assumptions over a period of time.

Simulation models are more flexible than mathematical ones and therefore, can be used to represent a complex system which otherwise cannot be represented mathematically. These models do not provide general solution like those of mathematical models.

1.4.6. General Methods for solving operations research models

In general, the following three methods are used for solving OR models. In all these methods, values of decision variables are obtained that optimize the given objective function (a measure of effectiveness).

- **Analytical (or Deductive) Method**

In this method, classical optimization techniques such as calculus, finite difference and graphs are used for solving an OR model. In this case, we have a general solution specified by a symbol and we can obtain the optimal solution in a non-iterative manner. For example, in the inventory model, in order to calculate economic order quantity (optimal order size), the analytical method requires that the first derivative of the mathematical expression.

$$TC = (D/Q) C_o + (Q/2)C_h$$

where TC = total variable inventory cost

C_o = ordering cost per order

Q = size of an order

D = annual demand

C_h = carrying cost

Be taken and equated to zero as the first step towards identifying optimum value of, $EOQ(A) = \sqrt{2DC_o/C_h}$. This is because of the concept of maxima and minima for optimality. Here, it may be noted that the validity of the

conclusion depends only on the validity of the mathematical approach that is being used.

• **Numerical (or Iterative) Method**

When analytical methods fail to obtain the solution of a particular problem due to its complexity in terms of constraints or number of variables, a numerical (or iterative) method is used to get the solution. In this method, instead of solving the problem directly, a general algorithm is applied to obtain a specific numerical solution.

The numerical method starts with a solution obtained by trial and error, and a set of rules for improving it towards optimality. The solution so obtained is then replaced by the improved solution and the process of getting an improved solution is repeated until such improvement is not possible or the cost of further calculation cannot be justified.

• **Monte Carlo Method**

This method is based upon the idea of experimenting on a mathematical model by inserting into the model specific values of decision variables at different points of time and under different conditions and then observing their effect on the criterion chosen for variables. In this method, random samples of specified random variables are drawn to be known what is happening to the system for a selected period of time under different conditions. The random samples form a probability distribution that represents the real-life system and from this probability distribution, the value of the desired random variable can be estimated.

1.4.7. Advantages of Models

Models in general are used as an aid for analyzing complex problems. However, a model can also serve other purposes as given below:

1. A model provides economy in representation of the realities of the system. That is, models help the decision-maker to visualize a system so that he can understand the system's structure or operation in a better way. For example, it is easier to represent a factory layout on paper than to construct it. It is cheaper to try out modifications of such systems by rearrangement on paper.
2. The problem viewed in its entirety, with all the components being considered simultaneously.
3. Models serve as aids to transmit ideas and visualization among people in the organization. For example, a process chart can help the management to communicate about better work methods to workers.
4. A model allows us to analyse and experiment in a complex situation to a degree that would be impossible in the actual system and its environment. For example, the experimental firing of INSAT satellite may cost millions of birr and required years of preparation.
5. Models simplify the investigation considerably and provide a powerful and flexible tool for predicting the future state of the process or system.

1.4.8. Methodology of Operations Research

Every OR specialist may have his/her own way of solving problems. However, for effective use of OR techniques, it is essential to follow some steps that are helpful for decision-makers to make better decisions. The flow diagram representing the methodology of operations research is also shown in Fig. 1.3.

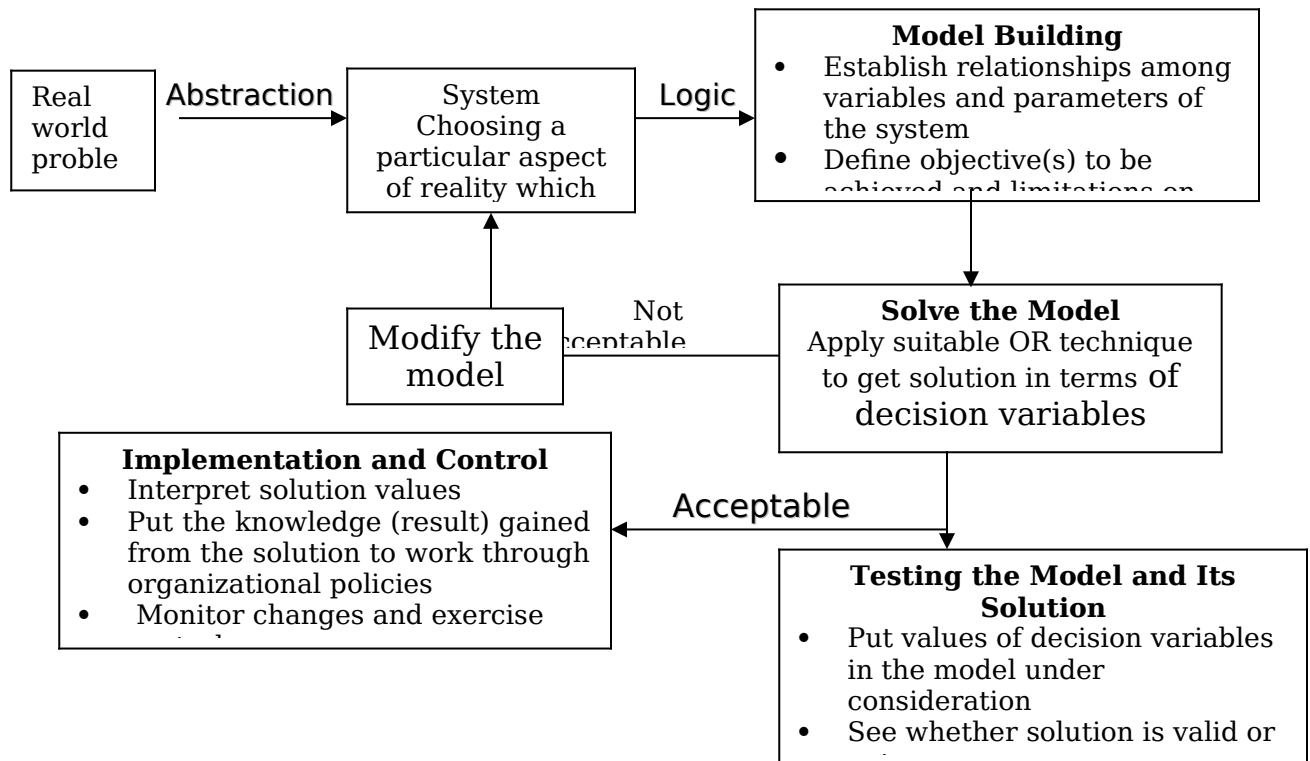


Fig. 1.3 Methodology of Operations Research

Step 1 Formulating the Problem

Problem formulation involves an analysis of the system under study, the objectives of the decision-maker, and alternative courses of action, etc., so as to understand and describe, in precise terms, the problem that an organization faces.

The analysis begins by detailed observation of the organizational structure, climate, communication and control system, its objectives and expectations. Such information will help in assessing the difficulty of the study in terms of costs, time requirement, resource requirement, probability of success of the study, etc.

The major steps which have to be taken into consideration for formulating the problems are as follows:

The first component of the problem to be defined is the decision-maker who is not satisfied with the existing state of affairs. The interaction with the decision-maker will help the OR specialist in knowing his objectives. That is,

either he has already obtained some solution of the problem and wants to retain it, or he wants to improve it to a higher degree. If the decision-maker has conflicting multiple objectives, he may be advised to rank his objectives in order of preference; overlapping among several objectives may be eliminated.

Decision environment It is desirable to know about the resources such as manager, employees, equipments, etc., which are required to carry out the policies of the organization considering the social and ecological environment in which the organization functions. Knowledge of such factors will help in modifying the initial set of the decision-maker's objectives.

Alternative courses of action The problem arises only when there are several courses of action can be prepared in the process of going through the above steps of formulating the problem. Course of action which are not feasible with respect to objectives and resources may be ruled out.

Measure of effectiveness A certain measure of effectiveness or performance is required in order to evaluate the merit of the several courses of action listed above. The performance or effectiveness can be measured in different units such as birr (net profits), (share of market desired), time dimension (service or waiting time). In order to bring uniformity in the measurement, one of the performance criteria must be chosen to express the value in terms of the value of the criterion chosen earlier.

Qualitative objectives which cannot be quantified must be assigned weightage and probability measures to their attainability.

Step 2 Collecting Data and Constructing a Mathematical Model

After the problem is clearly defined and understood, the next step is to collect required data and then formulate a mathematical model. Model construction consist of hypothesizing relationships between variables subject to and not subject to control by decision-maker. Certain basic components required in every decision problem model are:

Controllable (decision) variables These are the issues or factors in the problem whose values are to be determined (in the form of numerical values) by solving the model. The possible values assigned to these variables are called decision alternatives (strategies or courses of action). For example, in queuing theory, the number of service facilities is the decision variable.

Uncontrollable variables These are the factors whose numerical value depends upon the external environment prevailing in the organization. The values of these variables are not under the control of the decision-maker and are also termed as state of nature.

Objective function It is a representation of (i) the criterion that expresses the decision-maker's manner of evaluating the desirability of alternative values of the decision variables, and (ii) how that criterion is to be optimized (minimized or maximized). For example, in queuing theory the decision-maker may consider several criteria such as minimizing the average waiting

time of customers, or the average number of customers in the system at any time.

Constraints (or Limitations) These are the restrictions on the values of the decision variables. These restrictions can arise due to limited resources such as space, money, manpower, material, etc. The constraints may be in the form of equations or inequalities.

Functional relationships In a decision problem, the decision variables in the objective function and in the constraints are connected by a specific functional relationship. A general decision problem model might take the form:

Optimize (Max or Min) $Z = f(x)$
subject to the constraints
 $g_i(x) \{ \leq, =, \geq, \} b_i; i = 1, 2, \dots, m$
and $x \geq 0$
where, x = a vector of decision variables ($x_1, x_2, \dots, x_n$)
 $f(x)$ = criterion or objective function to be optimized
 $g_i(x)$ = the i th constraint
 b_i = fixed amount of the i th resource

A model is referred to as a linear model if all functional relationships among decision variables $x_1, x_2, \dots, x_n$ in $f(x)$ and $g(x)$ are of a linear form. But if one or more of the relationships are non-linear, the model is said to be a non-linear model.

Parameters These are constants in the functional relationships. Parameters can be deterministic or probabilistic in nature. A deterministic parameter is one whose value is assumed to occur with certainty. However, if constants are considered directly or explicitly as random variables, they are probabilistic parameters.

Step 3 Solving the Mathematical Model

Once a mathematical model of the problem has been formulated, the next step is to solve it, that is, to obtain numerical values of decision variables. Obtaining these values depends on the specific form or type, of mathematical model. Solving the model requires the use of various mathematical tools and numerical procedures. In general, the following two categories of methods are used for solving an OR model.

- (i) **Optimization Methods** These methods yield the best values for the decision variables both for unconstrained and constrained problems. In constrained problems, these values simultaneously satisfy all of the constraints and provide an optimal or acceptable value for the objective function or measure of effectiveness. The solution so obtained is called the optimal solution to the problem.
- (ii) **Heuristic Methods** These methods yield values of the variables that satisfy all the constraints, but not necessarily provide optimal solution. However, these values provide an acceptable value for the objective function.

Heuristic methods are sometimes described as rules of thumb which work. An example of a commonly used heuristic is 'stand in the shortest line'. Although using this rule may not work if everyone in the shortest line requires extra time, in general, it is not a bad rule to follow. These methods are used when obtaining optimal solution is either very time consuming or model is complex.

Sometimes difficulties in problem solving arise due to lack of an appropriate methodology for it and psychological perceptions on the part of the problem solver. The major difficulties in problem solving can be grouped into several categories:

- i) Failure to recognize the existence of a problem
 - Some people tend to personalize problems
 - Information is not received to signal that a problem exists
 - Problems arise in contexts with which people have had no experience.
 - There is a lack of objectives or standards
- ii) Failure to define the correct problem
 - One situation may contain many intertwined problems
 - Obvious problems are often the symptoms of much deeper problems
 - An inability to identify accurately what is going on can lead to inaccurate problem identification.
 - Incorrect inferences can lead to inaccurate problem identification.
 - Attitudes and beliefs can blind the problem solver to the real causes of an undesirable situation.
 - Problems and their causes are oversimplified
 - Fixation on either a 'World view' or 'functions' provide too narrow a scope
- iii) Failure to use all available information
 - The problem -solver fails to seek out information
 - Perceptual blocks to thinking exist.
- iv) Failure to recognize or question assumptions
 - It is assumed that there is a solution to every problem
 - Rigid thinking limits one's viewpoint
- v) Failure to consider a wide range of alternatives
 - problem definition is limited
 - Premature evaluation or judgment takes place
 - Lack of time is cited as an excuse

Step 4 Validating the Solution

After solving the mathematical model, it is important to review the solution carefully to see that the values make sense and that the resulting decisions can be implemented. Some of the reasons for validating the solution are:

- i) The mathematical model may not have enumerated all the limitations of the problem under consideration.
- ii) Certain aspects of the problem may have been overlooked, omitted or simplified.

- iii) The data may have been incorrectly estimated or recorded, perhaps when entered in to the computer.

Step 5 Implementing the Solution

The decision-maker has not only to identify good decision alternatives but also to select alternatives that are capable of being implemented. It is important to ensure that any solution implemented is continuously reviewed and updated in the light of a changing environment. The behavioral aspects of change are exceedingly important to the successful implementation of results. In any case, the decision-maker who is in the best position to implement results must be aware of the objective, assumption, omissions and limitations of the model.

Step 6. Modifying the Model

For a mathematical model to be useful, the degree to which it actually represents the system or problem being modeled must be established. If during validation, the solution cannot be implemented, one needs to identify constraint that were omitted during the original problem formulation or to find if some of the original constraints that were omitted during the original problem formulation or to find if some of the original constraints were incorrect and need to be modified. In all such cases, one must return to the problem formulation step and carefully make the appropriate modifications to represent more accurately the given problem.

A model must be applicable for a reasonable time period and should be updated from time to time, taking in to consideration the past, present and future aspects of the problem.

Step 7 Establishing Controls over the Solution

The dynamic environment and changes with in the environment can have significant implications regarding the continuing validity of models and their solutions. Thus, a control procedure has to be established for detecting significant changes in decision variables of the problem so that suitable adjustments can be made in the solution without having to build a model every time a significant change occurs.

1.4.9. Advantages of Operations Research Study

A decision-maker in business and industry comes across the problem of making decisions in his day to day activities. In addition to learning about various OR models useful in making a decision, the decision maker can also get several other advantages listed below.

Structured Approach to Problems

A substantial amount of time and effort can be saved in developing and solving OR models if one uses a logical and consistent approach. This implies that the decision-maker has to be careful while defining controllable and uncontrollable variables, availability of resources man, machine, material and money, etc.... functional relationships among variables in the objective

function and constraints. This will also reduce the chance of conceptual and computational errors. Any such error can also be detected easily and corrected early.

Critical Approach to Problem Solving

The decision-maker will come to understand various components of the problem and accordingly select a mathematical model for solving the given problem. He will be aware of the explicit and implicit assumptions and inherent limitation of such models. Problem solutions are examined critically and the effect of any words, the ability to judge applicability and limitations of solution techniques to a particular problem, as well as to critically analyse a proposed solution and data on which they are based, is bound to be a valued skill in any business organization.

Computer Software for Operation Research

Prentice- Hall distributes some of the leading software packages for quantitative approach to decision making. Features of a few such software packages are explained below.

Quantitative Systems for Business plus (QSB+) Version 3.0, by Yih-long chang and Roberts Sullivan, is a software package that contains problem solving algorithms for operations research/ management science, as well as modules on basic statistics, non-linear programming and financial analysis. QSB+ describes tips for Many real-life OR models require long and complex mathematical calculations. Thus, Computer software packages that are used to do these calculations rapidly and effectively have become part of or approach to problems solving computer facilities such spread sheets or statistical and mathematical software packages make such analysis readily available to a decision maker.

Summary

There is no unique set of problems which can be solved by using OR models or techniques. Several OR models or techniques can be grouped in to some basic categories as given below. In this module, a large number of OR models have been discussed in detail.

- **Allocation Models**

Allocation models are used to allocate resources to activities in such a way that some measure of effectiveness (objective function) is optimized. Mathematical programming is the broad term of the OR techniques used to solve allocation problems.

- **Inventory models**

Inventory models deal with the problem of determination of how much to order at a point in time and when to place an order. The main objective is to minimize the sum of three conflicting inventory costs: The cost of holding or carrying extra inventory, the cost of shortage or delay in the delivery of items when it is needed, a cost of ordering or set- up. These are also useful in dealing with quantity discounts and selective inventory control.

- **Waiting Line (or Queuing) Models**

These models have been developed to establish a trade-off between costs of providing service and the waiting time of a customer in the queuing system. Constructing a model entails describing the components of the system: Arrival process, queue structure and service process and solving for the measure performance – average length of waiting time, average time spent by the customer in the line traffic intensity, etc., of the waiting system.

- **Competitive (Game Theory) Models**

These models are used to characterize the behavior of two or more opponents (called players) who compete for the achievement of conflicting goals. These models are classified according to several factors such as number of competitors, sum of loss and gain, and the type of strategy which would yield the best or the worst outcomes.

- **Network Models**

These models are applied to the management (Planning, controlling and scheduling) of large -scale projects PERT/CPM techniques help in identifying potential trouble spots in a project through the identification of the critical path. These techniques improve project coordination and enable the efficient use of resources network methods are also used to determine time cost trade - off, resource allocation and updating of activity time.

- **Sequencing models**

The sequencing problem arises whenever there is a problem in determining the sequence (order) in which a number of tasks can be performed by a number of service facilities such as hospital, plant etc., in such a way that some measure of performance, for example, total time to process all the jobs on all the machines is optimized

- **Replacement Models**

These models are used when one must decide the optimal time to replace an equipment for one reason or the other -for instance, in the case of equipment whose efficiency deteriorates with time or fails immediately and completely. For example, in case of an automobile, the user has his own measure of effectiveness. So, there will not be a single optimal answer for everyone even if each automobile gives exactly the same service.

- **Dynamic Programming Models**

Dynamic Programming may be considered as an outgrowth of mathematical programming involving the optimization of multi-stage (sequence of inter-related decisions) decision processes. The method starts by dividing a given problem in to stages or sub-problems and then solves those sub-problems sequentially until the solution to the original problem is obtained.

- **Markov-Chain Models**

These models are used for analyzing a system which changes over a period of time among various possible outcomes or states. The model while dealing with such systems describes transitions in terms of transition probabilities of various states. These models have been used to test brand -loyalty and brand switching tendencies of consumers, where each system state is considered to be a particular brand purchase. These have been used in reliability analysis, where the states of the system are the various levels of performance of the equipment being monitored.

- **Simulation Models**

These models are used to develop a method to evaluate the merit of alternative courses of action by experimenting with a mathematical model of the problems where various variables are random. That is, these provide a means of generating representative samples of the measures of performance variables. Thus, repetition of the process by using the simulation model provides an indication of the merit of alternative course of action with respect to the decision variables.

- **Decision Analysis Models**

These models deal with the selection of an optimal course of action given the possible payoffs and their associated probabilities of occurrence. These models are broadly applied to problems involving decision making under risk and uncertainty.

Some of the industrial/ government /business problems which can be analysed by OR approach have been arranged by functional areas as follows:

- Finance and Accounting
- Marketing
- Purchasing, Procurement and Exploration
- Production Management

i) Facilities Planning

ii) Manufacturing

iii) Maintenance and project scheduling

Personnel Management

Techniques and General Management

Government

Economic Planning, natural resources, social planning and energy.
Urban and housing problems Military, police, Pollution control, etc.

PART TWO

DETERMINISTIC MODELS

Overview

Linear programming is a mathematical technique that makes use of linear algorithms to find solutions to constrained optimization problems.

In a managerial decision making process, the decision maker undertakes various analyses. These analyses are supported by the decision maker's own experience, intuition, and so on; or by mathematical (scientific) techniques. One of the mathematical techniques is LP model.

This model has wide areas of application. Some of them are: problems that involve product mix, diet, portfolio selection, job assignments, distribution, inventory planning etc.

Activity 2.1.

What is linear programming?

What are the applications of linear programming?

What is the role of linear programming to the decision making process?

CHAPTER- 2

MATHEMATICAL MODEL FORMULATION OF LINEAR PROGRAMMING (LP) PROBLEMS

To understand the model very well, we have to be familiar with its basic characteristics. We classify them into two as: components and assumptions.

a) The components

Do you remember the different groups of mathematical expressions (equations) in a linear programming model? Knowing these groups of equations can help us better understand the components of an LP model. The components are the structural units or the building blocks of an LP model. The following are the components of an LP model:

- **Objective:** Every linear programming model represents a certain problem. The focus of the decision maker is to solve that problem. This is his objective. This objective is one component which must be included in an LP model. It is only one. We can not entertain two or more objectives in a single model. Since linear programming is a constrained optimization model, our objective will be either maximization or minimization but not both. Maximization problems usually involve profit, market share, return on investment, etc. Whereas minimization problems involve cost, time, distance, etc. For example, if our problem is to get an optimal level of profit, the objective is maximization. This objective is expressed mathematically using a function called *Objective Function*. The objective function incorporates the profit per unit of each variable to the objective.
- **Decision variables:** These are variables under the control of the decision maker. By making an appropriate choice in the level of the decision variables, the decision maker controls the level of the objective function. This means the role of the decision maker here is to determine the level of each variable involved in the model (to find the best combination) so as to know the optimal level of the objective function. For example, if a company is engaged in the production and sale of four products to generate profit, the focus of the decision maker is how to get an optimal profit. It is obvious that unless there is problem in the market (low demand for the product), we can increase profit by increasing the quantity of the products. But this is not the case in reality. We cannot increase the quantity of the products without limit. Of course the quantities for each variable vary between zero and the maximum possible quantity. From this wide variation in quantity, the manager decides on the best quantity combination that gives the optimal level of the objective function. That is why we say these variables decision variables.

The decision variables will be defined in terms of a quantity of an amount which is usually represented by a variable and a subscript number like $x_1, x_2, x_3, \dots$. If for example our case is to maximize profit through the production and sell of two products, x_1 represents the

quantity of the first product and x_2 the quantity of the second product. It is this way we express decision variables in the model.

- **Constraints:** In the previous paragraph we have seen some role of the decision maker to choose the level of decision variables. His ability to do so is subject to certain limiting factors or restrictions. These restrictions are called *Constraints*. Constraints may reflect availability of resources like material, financial, time, space etc. All the constraints that are called for in the model should be expressed mathematically. The solution to the model must satisfy all these constraints (restrictions). There is a constraint common to all LP models -the non-negativity constraint.
Constraints may be in terms of an upper limit, lower limit or equality. So when you formulate the model, carefully differentiate which one is called for. If for example one constraint says “at least 20 unit of the first product must be made”, the constraint will be expressed as: $x_1 \geq 20$. If it were “to a maximum of 20 units of the first product must be made”, the constraint will be: $x_1 \leq 20$. And had the constraint been explained like “exactly 20 units of the first product is needed”, the constraint could have been $x_1 = 20$. The usual way to write constraints is by putting the variables in the left side of the expression while a constant alone is placed on the right side. This constant is usually referred to as the *Right hand side value* (RHS)
- **Parameters:** The objective and the constraints in the model are mathematical expressions (equations). Each equation consists of numerical values and symbols. The symbols represent decision variables, whereas the numerical values are called *Parameters*. Parameters specify the impact that a unit of each decision variable will have on the level of the objective function, and on the numerical value (RHS) of each constraint.

b) The Assumptions

The assumptions represent the framework within which the model will be valid. The major assumptions are: linearity, divisibility, certainty, and non-negativity.

- **Linearity:** Linearity assumption in LP models has two aspects: the first is that every decision variable has a linear impact on the objective function and the constraint in which it appears. The second aspect of this assumption is that all the constraint functions in the model must be linear functions. (i.e. functions to the first degree).
- For example, look at the following constraint functions:
 - $2x_1 + 3x_2 = 20$
 - $x_1 + 2x_1x_2 = 40$
 - $2x_1^2 + 3x_2 = 10$

Only the first one is a linear function hence can be a constraint function in a linear programming model.

- **Divisibility:** According to this assumption, non-integer values for decision variables are acceptable. For example an optimal solution to a decision variable can take values like 3.5, 1.2, 2.5

etc... regardless of the nature of the item represented by the decision variable.

- **Certainty:** Certainty has two aspects in LP models. The first relates to the parameters in the model. It means that we know for certain these parameters (numerical values). If for example we do have an objective function: Maximize profit $Z = 2x_1 + 3x_2$, the parameters (coefficients) 2 and 3 represent the per unit contribution of x_1 and x_2 and we now these unit contributions for certain.
- **Non-negativity:** It customary to see one common constraint at the end of every linear programming model. This constraint is about the exclusion of negative values from our solution to the decision variables. This is because decision variables represent quantities and there is no negative quantity. Only positive values and zero will be allowed.

Distance learner; make sure that you are very clear about the characteristics of LP models.

Formulating LP Models

It is worth acquiring knowledge of formulating LP models since the model is the representative (abstraction) of the real problem at hand. That is, if the model is ill-formulated, it will fail to represent the problem properly. Therefore, solving a model which is ill-formulated means ending up with a wrong conclusion about the problem. Take care when you formulate a model!

Formulation of the model is the same as assembling the components that are mentioned above. Each component will be depicted as a mathematical expression or equation in the linear programming model. You are assumed to have the appropriate skill to express these components mathematically. Just to refresh your memory, the following are steps involved in the formulation of linear programming models:

- Identifying the decision variables
- Determining the objective functions
- Identifying the constraints
- Determining appropriate values for parameters (i.e. determining whether an upper limit, lower limit, or equality is involved).
- Building the model using the information
- Validating the model. (Checking if the model is in conformity with the assumptions we have seen before)

CHAPTER -3

GRAPHICAL AND ANALYTICAL SOLUTION OF (LP) PROBLEMS

There are two approaches to solving linear programming models. The first one is the *Algebraic approach which is also called the Graphical approach* and the second approach is *the simplex approach*. We are going to have a discussion on these two approaches.

3.1. Graphical approach to solving LP problems

This is a simple and straightforward approach for determining the optimal solution to certain linear programming problems. This is done by plotting the constraints and determining the solution space (which is equivalent to the feasible solution) and pointing out the optimal solution from the solution. The graphical approach is only applicable to problems that involve two decision variables. This is because we can't plot an equation having more than two variables in a two-dimensional coordinate plane.

There are about three basic steps in finding the optimal solution using the graphical approach:

1. Plotting each of the constraints
2. Determining the region or area that contains all of the points that satisfy the entire set of constraints (the solution space)
3. Determining the optimal solution

Illustration:

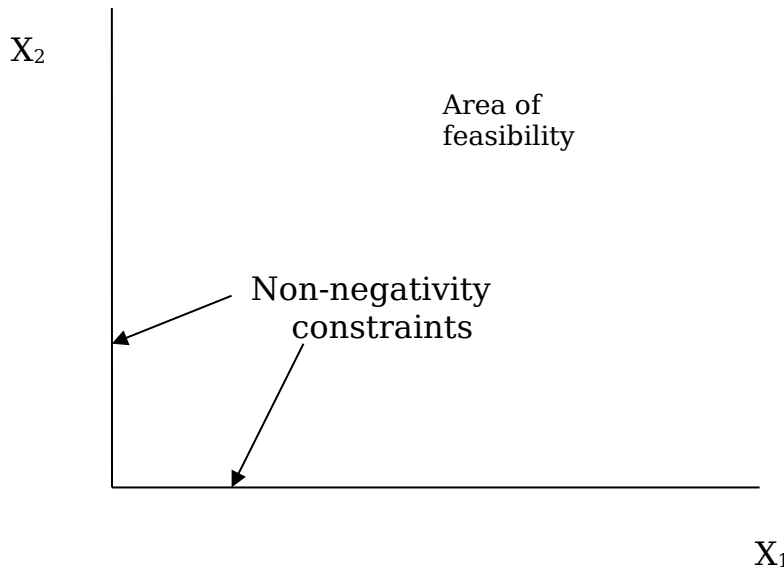
Solve the following linear programming model using graphical approach:

$$\begin{array}{ll}\text{Maximize} & 10x_1 + 16x_2 \text{ (profit)} \\ \text{Subject to} & \\ & 8x_1 + 20x_2 \leq 120 \\ & 25x_1 + 20x_2 \leq 200 \\ & x_1, \quad x_2 \geq 0\end{array}$$

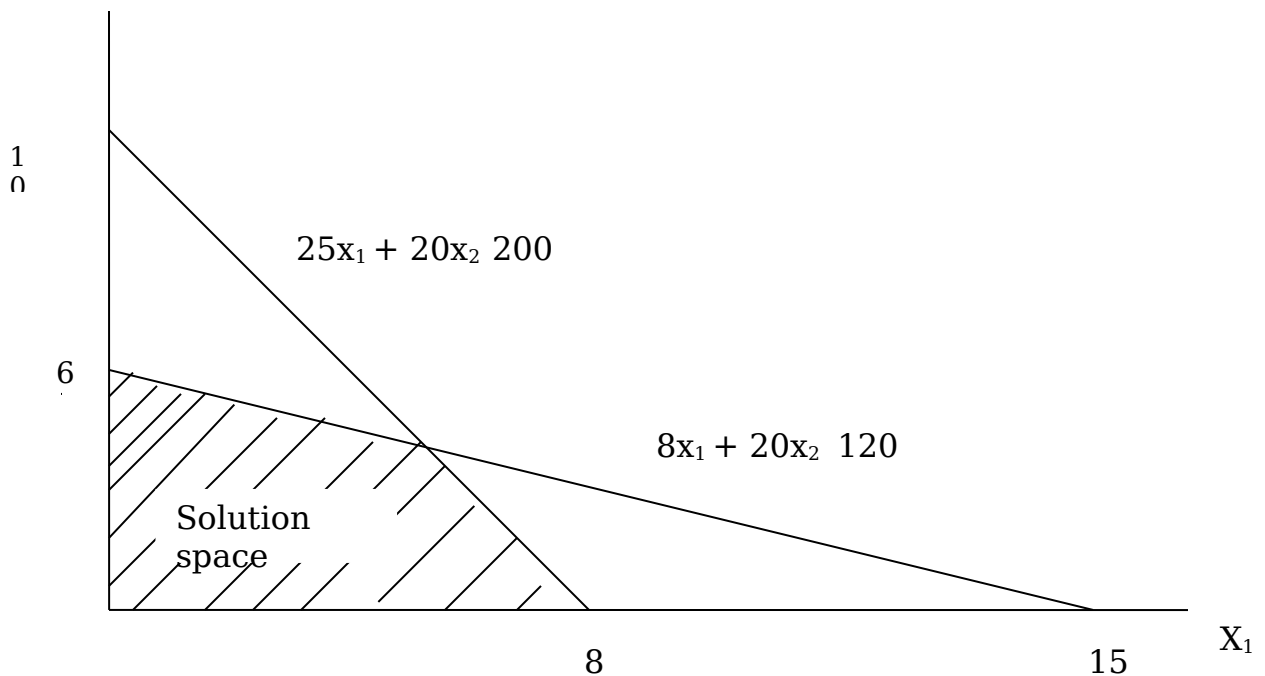
Now we can solve the above model following the above mentioned steps.

Plotting the constraints

Plotting the constraint functions starts by plotting the non-negativity constraints. This is the same as taking only one quadrant of the coordinate plane (where both X and Y have positive values).



The other two constraints can be plotted usually by determining the vertical and horizontal intercepts of them and then by connecting the two points to draw (plot) the straight line representing the constraint function. The first constraint is plotted by connecting two points (0, 6) and (15, 0) which are the vertical and horizontal intercepts respectively. And the second constraint is plotted by connecting points (0, 10) and (8, 0) again the vertical and horizontal intercepts. Look at the following figure.



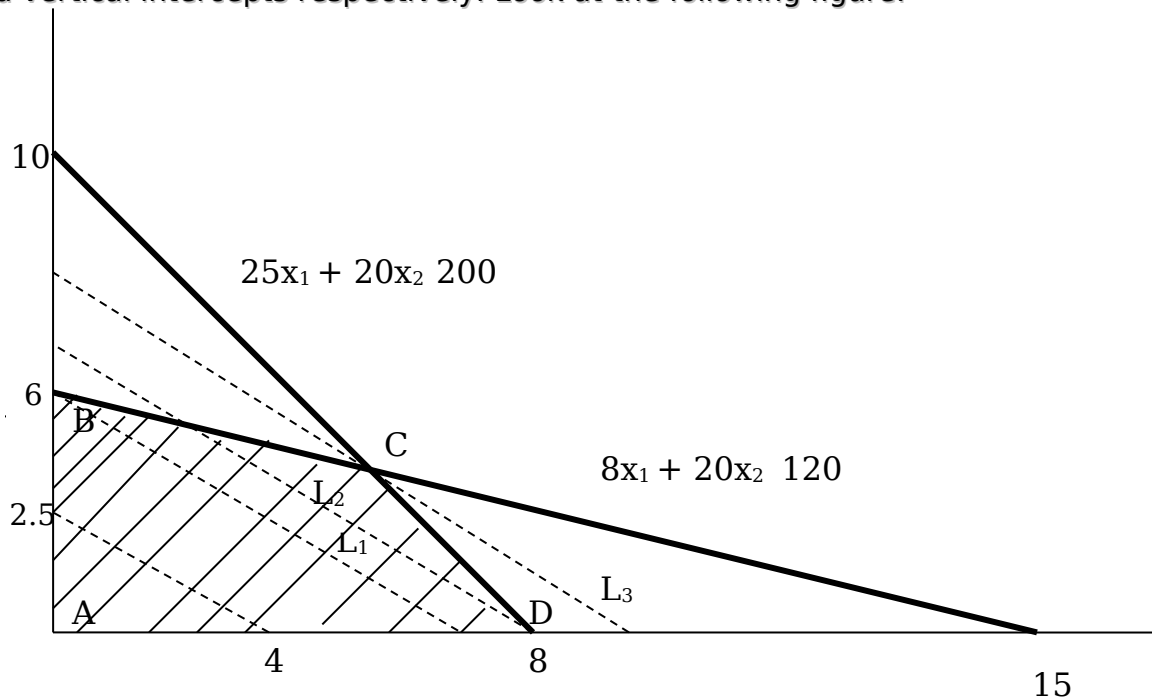
As can be seen from the above figure (shaded region), the solution space is the region that represents all points that satisfy all the constraints (these points are called feasible solutions to the model).

The last step is the determination of the optimal solution. The optimal solution is represented by a point which is in the solution space. There are infinitely many points in the solution space from which we are going to

determine the point(s) representing the optimal solution to the problem. There are different approaches to determine the optimal point.

We shall discuss two approaches, one involves graphing the objective function (objective function approach) and the other involves examining the points at the edges of the feasible solution space (corner point approach).

3.1.1. Objective function approach: This approach is done by plotting the objective function like we did in the case of constraint functions. But the objective function is not an equation like the constraint functions. Therefore, we have to make it first an equation by assigning any number arbitrarily to the right side. In our example the objective function $10x_1 + 16x_2$ is not an equation because it does not include an equality sign. We make it by simply setting it equal to any quantity. Suppose we decide to set the objective function equal to 40. That is, $10x_1 + 16x_2 = 40$, we can now plot the function by just following the same procedure we used to plot constraint functions. Connecting two points (4, 0) and (0, 2.5) which are the horizontal and vertical intercepts respectively. Look at the following figure.



Then we can identify the optimal point by plotting parallel lines with the objective function. Look at L_1 , L_2 , and L_3 above. These parallel lines are drawn to pass through the corner points of the solution space. If we take L_1 it passes through point (0, 6), our intention here is to check if the point is the optimal point. This point is not the optimal point because there are points above the line which are in the solution space and can still maximize the objective function. The same is true for L_2 . But look at L_3 , there is no point which is above the line and in the solution space, the line passes only through one point which is in the solution space. This indicates that this point is the optimal solution.

To determine the optimal solution to the model we will just determine the coordinate values of the point we have identified to be the optimal. This could be done by finding out the intersection point of the two constraint functions.

The intersection is when $x_1 = \frac{80}{17}$ and $x_2 = \frac{70}{17}$. This means the optimal solution to the model. The optimal profit is approximately 113. Computed by substituting the optimal solution to the objective function

$$10x_1 + 16x_2 \Rightarrow 10\left(\frac{80}{17}\right) + 16\left(\frac{70}{17}\right) = \underline{\underline{112.94}}$$

Exercise

Solve the following linear programming model graphically.

Maximize $60x_1 + 50x_2$

Subject to

$$4x_1 + 10x_2 \leq 100$$

$$2x_1 + 1x_2 \leq 22$$

$$3x_1 + 3x_2 \leq 39$$

Solution: $x_1 = 9$ and $x_2 = 4$

3.2 The extreme point approach:

This approach is less efficient as compared with the previous one (plotting the objective function). However, it better reflects the nature of non graphical approaches to solve linear programming models.

This approach states that the optimal solution to the model will occur at an extreme or corner points of the feasible region. Thus, we only consider corner points in searching for an optimal solution. This is done by determining the value of the objective function at each extreme point. For the example above, the corner point approach is summarized by the following table.

Points (Coordinates)		Value of the objective function
A	(0, 0)	$10(0) + 16(0) = 0$
B	(0, 6)	$10(0) + 16(6) = 96$
C	$\left(\frac{80}{17}, \frac{70}{17}\right)$	$10\left(\frac{80}{17}\right) + 16\left(\frac{70}{17}\right) = 113$
D	(8, 0)	$10(8) + 16(0) = 80$

Since our objective is maximization, we will take the point which gives us the largest value of the objective function. That is point C.

If our problem is a minimization problem our selection will be the least value to the objective function. Otherwise all the procedures we used in the above example (a maximization problem) are the same.

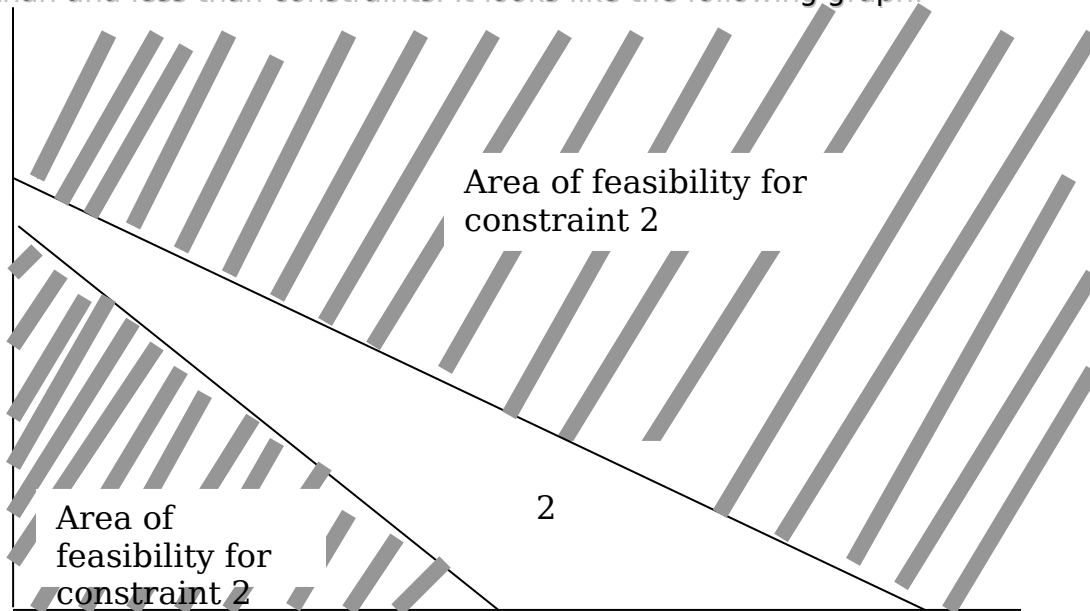
3.3. Some Special Issues:

The following are some of the special issues that may arise during the formulation of linear programming problems:

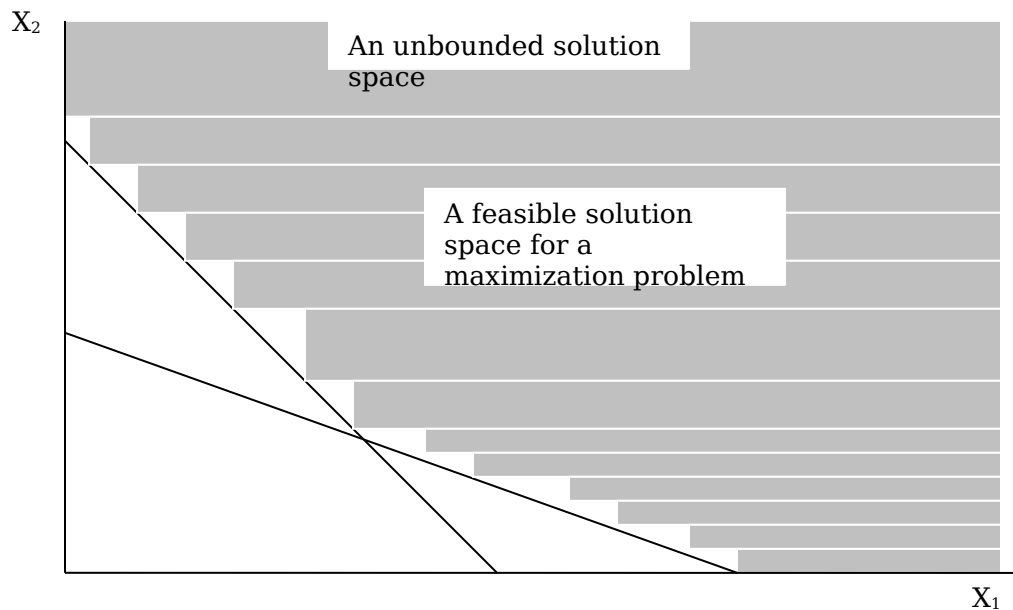
- No feasible solutions
- Unbounded problems
- Redundant constraints
- Multiple optimum solutions

No feasible solutions: These are cases when it is not possible to formulate the problem because it is impossible to satisfy all the constraints that are called

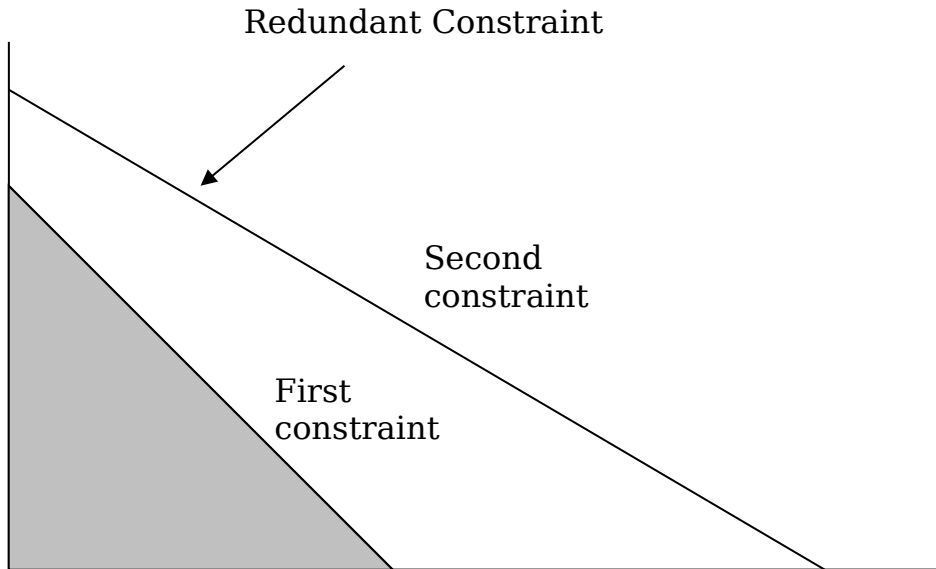
for in the model. It may happen for example when there is mix of greater than and less than constraints. It looks like the following graph.



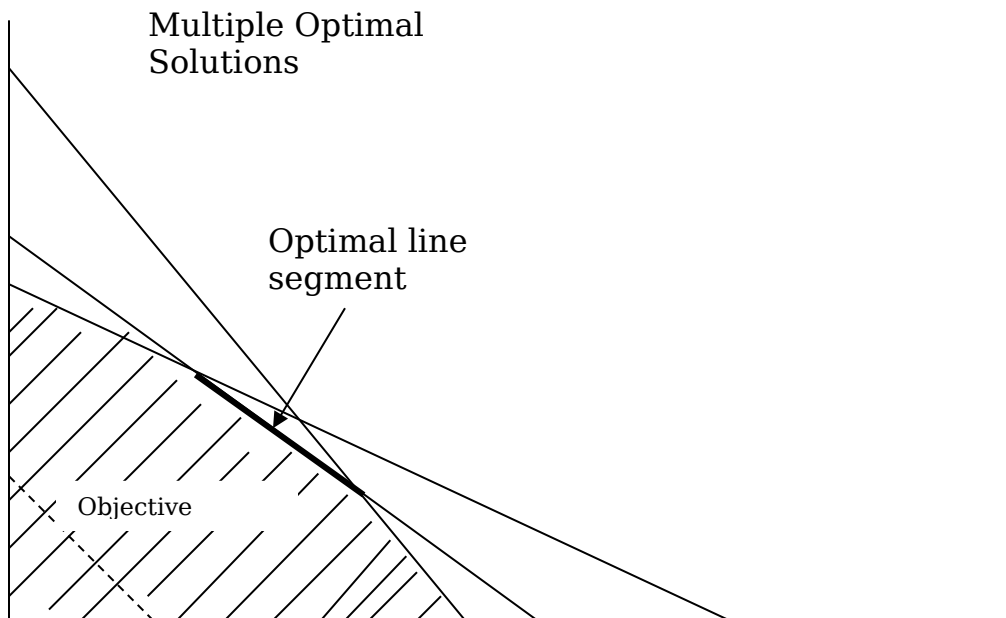
Unbounded problems: This is the case when we can increase the value of the objective function without limit. This case happens if the objective is maximizing while all the constraints are greater than constraints.



Redundant Constraints: When a constraint does not form a boundary to the feasible solution region, this constraint is a redundant constraint. Its absence (removal) does not make change to the feasible solution space.



Multiple Optimal Solutions: Linear programming problems sometimes may have multiple optimal solutions, where different combination of values of the decision variable can yield the same optimal value for the objective function. This will happen when any of the constraint functions is parallel to the objective function. The entire portion of this line (parallel to the objective function) which makes the boundary of the feasible solution space will touch the objective function. All points in this segment will yield the same optimal value to the objective function (i.e. Points on the segment are solutions to the problem). The following graphs indicate the above cases.



CHAPTER- 4

THE SIMPLEX METHOD OF SOLVING LP PROBLEMS

The graphical approach is useful in solving linear programming models having only two variables. When the model has more than two variables, the appropriate approach is the Simplex procedure.

The simplex procedure is a general purpose approach that can be used in any LP model regardless of the number of decision variables. This procedure is discussed in different conditions: Maximization problems having only $\leq$ constraints, Minimizations and Maximizations with mixed constraints (having some constraints that are not $\leq$ type).

4.1. Overview of the simplex Method

The simplex approach is an algebraic technique to solve LP problems. It begins with a feasible solution which is not optimal, and the solution is improved through continuous algebraic manipulations (iterations) until the optimal solution is determined.

In our previous discussion (graphical approach), we learned that to identify the optimal solution we have to first identify the feasible solution space. This will help us to know where to concentrate our attention in the process of finding the optimal solution. Even from the feasible solution space the corner points (extreme points) along the boundaries of the feasible solution space are the only solutions that are worthwhile to examine because, optimal solution is one of these points. The simplex method has a very similar approach that enables us to do just the same!

Like we identified the feasible solution space first in the graphical approach, here also we identify the initial feasible solution first. Then this solution will be tested for optimality using a certain approach, if not optimal, another feasible solution (called basic feasible solution) will be developed. This procedure will continue until the optimal solution is identified. This will be achieved through the help of a table called the simplex tableau.

4.2. Steps in the simplex procedure

(for a Maximization LP Problem)

The following are the steps in the simplex procedure for a maximization linear programming model with all the constraints with $\leq$ signs:

1) Initial simplex tableau

- (1) **Write the LP model in a standard form:** This is writing each constraint in a way that all the variables are on the left side of the constraint function and the non-negative constant in the constraint is on the right side. Then add (introduce) a slack variable to the left side of the constraint, to make it an equality.

Slack variables are variables we introduce to the left side of $\leq$ constraints to make them an equality. These variables represent the amount of unused scarce resources or capacity. A slack variable is denoted by letter 'S' with a

subscript number to indicate the constraint to which the variable is introduced. For example S_1 indicates the value of a slack variable in the first constraint.

Given the following LP problem,

$$\begin{array}{ll}\text{Maximize} & 10x_1 + 6x_2 + 7x_3 \\ \text{Subject to} & \\ & \text{Material } 4x_1 + 8x_2 + 5x_3 \leq 860 \\ & \text{Labor } 2x_1 + 3x_2 + 1x_3 \leq 400 \\ & x_1, x_2, x_3 \geq 0\end{array}$$

The standard form will be:

$$\begin{array}{ll}\text{Maximize} & 10x_1 + 6x_2 + 7x_3 + 0S_1 + 0S_2 \\ \text{Subject to} & \\ & \text{Material } 4x_1 + 8x_2 + 5x_3 + S_1 = 860 \\ & \text{Labor } 2x_1 + 3x_2 + 1x_3 + S_2 = 400 \\ & x_1, x_2, x_3, S_1, S_2 \geq 0\end{array}$$

Do you see the coefficients of variables in the objective function of the standard form? Slack variables are assigned coefficients of zero because they have no any real contribution to the objective.

(2) Develop the initial Simplex tableau

- List the variables across the top of the table and write the objective function coefficient just above it.
- Provide one row for each constraint in the body of the tableau. List each slack variable in the basis column, one per row.
- Enter the objective function coefficient of 0 in the C column for each slack variable.
- Compute values for row Z
- Compute values for row C - Z

By using the following notations for a general reference purpose,

c_j = Coefficient of variable j in the objective function

a_{ij} = Coefficient of variable j in constraint i

b_i = Right hand side value of constraint i

we can have the following general format for a linear programming model:

$$\begin{array}{ll}\text{Maximize} & c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_mx_n + 0S_1 + 0S_2 + 0S_3 + \dots + 0S_n \\ \text{Subject to} & \end{array}$$

$$\begin{array}{ll}a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1m}x_n + S_1 & = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2m}x_n + S_2 & = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3m}x_n + S_3 & = b_3 \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n + S_n & = b_n\end{array}$$

$$x_1, x_2, x_3 \dots x_n, S_1, S_2, S_3 \dots S_n \geq 0$$

The initial simplex tableau for the above general format looks like:

Basis		C_1 X_1	C_2 X_2	C_3 X_3	...	0 X_n	0 S_1	0 S_2	0 S_3	...	0 S_n	RHS values
S_1	0	a_{11}	a_{12}	a_{13}	...	a_{1n}	1					b_1
S_2	0	a_{21}	a_{22}	a_{23}	...	a_{2n}		1				b_2
S_3	0	a_{31}	a_{32}	a_{33}	...	a_{3n}			1	...		b_3
S_n	0	a_{n1}	a_{n2}	a_{n3}	...	a_{nn}					1	b_n
Z												Optima
C-Z												Level

Example: Develop the initial simplex tableau for the following linear programming model

$$\begin{aligned}
 &\text{Maximize} && 10x_1 + 6x_2 + 7x_3 \text{ (Profit)} \\
 &\text{Subject to} && \\
 &\quad \text{Material} && 4x_1 + 8x_2 + 5x_3 \leq 860 \\
 &\quad \text{Labor} && 2x_1 + 3x_2 + 1x_3 \leq 400 \\
 &&& x_1, x_2, x_3 \geq 0
 \end{aligned}$$

The standard form for the model is as developed above. Therefore we can just go to the representation of the standard form using the simplex tableau.

Before developing the initial simplex tableau, we have to be able to understand the meaning of each bits and pieces of information depicted in it. Before coming to this discussion it is worth saying something about the relationship between the number of variables in a standard form linear programming model and the number of constraint functions. If you look at the number of variables, we do have a total of three decision variables, and two slack variables. This indicates that the number of constraints is less than the number of variables. The number of slack variables is equal to the number of constraints.

The simplex tableau in the simplex procedure is equivalent to the solution space and the checking process is done for each corner points. The corner points represent the intersection of two constraints. (i.e. the coordinate value for decision variables at the intersection point). The variables in the basis of a simplex tableau are like variables at the intersection point. The value for these variables at this point is found from the RHS column of the tableau. All constraints must also be represented in the simplex tableau if the simplex procedure is to yield a solution that takes all the constraints into account.

Having this in mind, in a maximization problem to get the optimal corner point using the graphical approach obviously we don't test the origin where the value of the decision variables is zero. But the simplex procedure starts checking from this point, where the value of all the decision variables is zero. If we start from the origin by assuming a value of zero for all the decision variables, and if all the constraints are to be represented in the tableau; how can we develop the initial simplex tableau?

The initial simplex tableau represents all the constraints in the model without violating the assumption of starting from the origin through the help of other variable(s). These variables are slack variables. Look at the basis in following simplex tableau.

C → ↓		10	6	7	0	0	RHS
Basis		X_1	X_2	X_3	S_1	S_2	values
S_1	0	4	8	5	1	0	860
S_2	0	2	3	1	0	1	400
Z		0	0	0	0	0	0
C-Z		10	6	7	0	0	

The meaning of the above simplex tableau is that the solution to the model is $S_1 = 860$, $S_2 = 400$, and all the decision variables are equal to zero. And the level of the objective function at this point is 0.

Then our next task is to check if the solution so determined is the optimal solution to the model. This will be proved by looking at the net evaluation row (C-Z) row in the tableau. The rule is check if there is any positive value in this row. If there is one, the solution is not an optimal solution to the model. That is the level of the objective function can be improved by searching for other combination(s) of variables. This demands the development of subsequent simplex tableau(s). In our example, there are three positive numbers. This indicates that the initial simplex tableau doesn't give us the optimal solution to the model.

2) Subsequent tableaus

- (1) Identify the variable that has the largest positive value in row C-Z (the net evaluation row). This is the next variable to come into the solution mix (basis). The largest positive value in the above example is 10. The column containing this largest positive value is called the pivot column; and the variable in the pivot column is the incoming variable to bring about the improvement in the level of the objective function.
- (2) Using the constraint coefficients in the entering variable's column (called substitution rates), divide each one into the corresponding quantity column value. However, do not divide by a 0 or a negative value. The smallest non negative ratio indicates which variable will leave the solution mix.

This resulting values (ratios) so computed indicates the quantity or units of the incoming variable that we can get by replacing it with another variable which was previously basic. In our example, the column containing the entering variable (pivot column) as indicated below contains the substitution rates that indicate the amount of slack variable to be given up so as to increase the incoming variable just by one unit. That is in order to increase x_1 by one unit; we need to give up 2 units of S_1 and 4 units of S_2 . And to know the total number of x_1 that we can get by giving up any of the basic variables

is determined by dividing the RHS values with corresponding substitution rates. In our example, we will take two ratios: $\frac{860}{4}$ and $\frac{400}{2}$ which is 215 and 200. This means if we give up S_1 and bring x_1 into the basis we will have an additional 215 units of x_1 whereas additional 200 units of x_1 by giving up S_2 .

Our objective is maximization; per unit contribution of x_1 to our objective is 10. This shows that we have to have as much x_1 as possible. Therefore, the largest ratio is the largest contribution to profit; and the least ratio means least contribution to profit. That is why the variable with the least positive ratio is the leaving variable. S_2 is therefore the leaving variable.

		<i>Pivot column</i>					RHS	
		<i>C</i> →	<i>X</i> ₁	<i>X</i> ₂	<i>X</i> ₃	<i>S</i> ₁	<i>S</i> ₂	values
Basis								
<i>S</i> ₁	0		4	8	5	1	0	860 $\frac{860}{4} = 215$
<i>S</i> ₂	0		2	3	1	0	1	400 $\frac{400}{2} = 200$
<i>Z</i>			0	0	0	0	0	0
<i>C-Z</i>			10	6	7	0	0	

Leaving variable →

Entering variable *Pivot*

- (3) Compute replacement values for the leaving variable: Divide each element in the pivot row by the pivot element. These resulting values will be the values in the same row (major row) of the new tableau. This row will also contain the new variable (entered the solution mix) together with its objective function coefficient next to it in column C. Look at the following tableau.

		<i>C</i> →	10	6	7	0	0	RHS
		↓	<i>X</i> ₁	<i>X</i> ₂	<i>X</i> ₃	<i>S</i> ₁	<i>S</i> ₂	values
Basis								
<i>S</i> ₁								
<i>X</i> ₁	10		1	3/2	1/2	0	1/2	200
<i>Z</i>								

- (4) Compute values for each of the other constraint equations: This will be done in such a way that all the entries in the pivot column of the previous tableau will be converted from top to down with zero except the one which is the pivot element by the application of elementary row operations. Every other entry in the heart of the former tableau will also be converted as a result of the row operations to make the entries for the new tableau.

C →		10	6	7	0	0	RHS
Basis	↓	X ₁	X ₂	X ₃	S ₁	S ₂	values
		0					
S ₁		0					
X ₁	10	1	3/2	1/2	0	1/2	200
Z							
C-Z							

The entries in the pivot column are now converted into zero except the pivot element. By the way, would you go back to the initial simplex tableau and see what is common to all the columns containing basic variables?

As you might have successfully realized, these columns are unit columns (having entries of all zeros and only a single one at the intersection point of the column and the row containing that same variable). For example look the column containing S₁ one of the basis, it has a one in the first row where it is in; and a zero in the other row. To develop our new tableau in the same fashion, we will identify appropriate row operations that can yield the desired outcome.

Elementary row operations may be:

- Multiplying (dividing) all of the elements in a row by a constant.
- Adding or subtracting the multiple of a row to or from another row.

The desired value in the other entry for the pivot column is zero. But this value was 4 in the previous tableau. So we have to search for the row operation that helped to convert 4 into 0. This can be achieved by multiplying the second row of the new tableau by -4 and adding it to the entries in the first row of the former tableau. This way, entries in the second tableau are constructed.

2nd Simplex tableau

C →		10	6	7	0	0	RHS
Basis	↓	X ₁	X ₂	X ₃	S ₁	S ₂	values
		0					
S ₁	0	0	2	3	1	-2	60
X ₁	10	1	3/2	1/2	0	1/2	200
Z		10	15	5	0	10	2000 = Profit
C-Z		0	-9	2	0	-10	

- (5) Compute values for row Z: For each column take the sum of the products of the row coefficients (constraint coefficients) and

- coefficients of basic variables (row values in column C). For example the first entry is determined as: $0(0) + 1(10) = 10$
- (6) Compute values for row C - Z (the net evaluation row): For each column, subtract the value in row Z from the objective function coefficient values in row C at the top of the tableau. For example the first entry is determined as: $10 - 10 = 0$
 - (7) Test the solution for optimality by looking if there is any negative value in the C - Z row. If there is a positive value, the solution is not optimal and hence we have to develop other tableau. This is done by repeating step **B**. This process will continue until you get an optimal solution (all the entries in the last row are converted into zeros or negatives but no positives).

The pivot column is the one with the largest positive value in the net evaluation row. The only positive value in the row is 2. The column containing this value is therefore the pivot column as indicated below.

		C →	10	6	7	0	0	RHS
		↓						
Basis			X ₁	X ₂	X ₃	S ₁	S ₂	values
S ₁	0		0	2	3	1	-2	60
X ₁	10		1	3/2	1/2	0	1/2	200
Z			10	15	5	0	10	2000
C-Z			0	-9	2	0	-10	

This will lead us to the development of the fourth tableau. The major row in the fourth tableau is the first row (the pivot row in the third tableau). Then by dividing every element in the pivot row by the pivot element, we develop the major row of the new tableau. The remaining entries are determined through the help of elementary row operations as usual. Look at the following tableau:

4th Tableau

		C →	10	6	7	0	0	RHS
		↓						
Basis			X ₁	X ₂	X ₃	S ₁	S ₂	values
X ₃	7		0	2/3	1	1/3	-2/3	20
X ₁	10		1	7/6	0	-1/6	5/6	190
Z			10	70/6	7	4/9	11/3	2040 = Optimal profit
C-Z			0	-5.67	0	-4/9	-11/3	

The fourth tableau is the optimal tableau because there is no any positive value in the net evaluation row. This indicates that the optimal level of profit is achieved when: $x_1 = 20$, $x_2 = 0$, $x_3 = 190$, $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$. The maximum profit is 2,040.

4.3. Some special issues

Unbounded ness

When the problem was not correctly formulated, constraints don't properly bind the solution, or a minimization problem incorrectly stated as a maximization problem; unbounded ness can be the case. This condition can easily be identified in the simplex procedure. When we try to determine the leaving variable in the construction of a new tableau, we have said that we identify the least positive ratio of values in the RHS column and values in the

pivot column. But in this special case all the ratios will be either negative or zero. Therefore we can't identify the leaving variable.

Example: Solve the following linear programming model using the simplex procedure.

$$\begin{aligned} \text{Maximize} \quad & 2x_1 + 3x_2 \\ \text{Subject to} \quad & -x_1 + x_2 \leq 1 \\ & -1/2x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The final tableau for the problem is:

C →		10	6	7	0	RHS
Basis		X ₁	S ₁	S ₁	S ₂	values
X ₂	0	0	1	-1	2	5
X ₁	10	1	0	-2	2	4
Z		2	3	-7	10	23
C-Z		0	0	7	-10	

Even though there is a positive value in the net evaluation row indicating that the solution is not optimal (can be improved by bringing the variable in the pivot column into the solution mix), we can't determine the variable to be replaced because there is no positive ratio in the pivot column. This special condition is referred to as *unboundedness*.

Degeneracy

Student, think about what will you do to develop the next simplex tableau after recognizing that a tableau is not optimal.

Normally you will identify the leaving and entering variables and make the appropriate replacement. In some cases however, when you are determining the leaving variable, there may be a tie for the lowest positive ratio. Such a case is known to be the case of *degeneracy*. If you try to generate the next tableau by just taking any one variable with the least ratio, you encounter no problem in the procedure but there will be no improvement in the solution (level of the objective function); it will rather be the same as the previous solutions.

Multiple optimal solutions

This is the condition when same maximum value of the objective function might be possible with a number of different combinations of values of the decision variables. It occurs when the objective function is parallel to any of the binding constraint function. Two functions are said to be parallel if the ratio of coefficients of variables is the same.

When using the simplex approach, we can recognize alternate optima if the C-Z row contains a zero value for one or more of the non-basic variables in the final simplex tableau.

4.4. Simplex maximization with mixed constraints

As we have discussed earlier, the simplex technique requires that the LP model should be in its standard form first. To put an LP model with all constraints having $\leq$ in standard form, we added a slack variable to the left sides of the constraint functions. But constraints with $\geq$ and $=$ signs will be

handled differently. Our attention in this section is to solve LP models with all the $=$, $\geq$, and $\leq$ signs. This is what is meant by mixed constraints.

In the case of equality constraints slacks are not applicable. These constraints require an exact (precise) amount. But to use the simplex technique in solving a maximization problem we assume to start from the origin by making the values of decision variables zero (excluding them from the basis of the initial simplex tableau). In the simplex procedure we need to represent each constraint function through the help of a variable. This implies that having a variable that will serve this end in the simplex tableau is a must. Therefore, we will introduce a variable to the equality constraint called an *artificial variable*. Artificial variables have no physical interpretations, they merely serve as a device to enable us use the simplex procedure. Because of this fact, after serving their purpose, artificial variables should leave the simplex process (solution mix) as quickly as possible. The optimal solution should never contain an artificial variable with non-zero value (artificial variable should not appear in the basis of the final simplex tableau).

Example: Let us assume we do have the following constraint in an LP model under consideration

$$2x_1 + 10x_2 = 30,$$

Its standard form will be: $2x_1 + 1x_2 + A_1 = 30$

A_1 does mean an artificial variable in the first constraint of an LP model.

What will happen if our constraint is with $\geq$ sign? In a $\geq$ constraint, the restriction is to meet a specific lower limit. The constraint is never allowed to have a value which is less than the minimum amount designated by the constraint. But it can have a value which exceeds the minimum designated amount. This excess amount is referred to as *Surplus*. This surplus amount is subtracted from the left side of the constraint function to put it in standard form.

Example: $4x_1 + 11x_2 \geq 60$ will be look like:

$4x_1 + 11x_2 - S_1 = 60$, where S_1 = the surplus amount for the first constraint.

By subtracting the surplus, the constraint becomes equality. Therefore, to allow for the initial solution that will be less than that amount, an artificial variable must be added. Thus the above constraint will be put in its standard form as:

$$4x_1 + 11x_2 - S_1 + A_1 = 60$$

Our next vital issue is how to express these surplus and artificial variables in the objective function.

Surplus variables are expressed in exactly the same way as slack variables. They are assigned coefficients of zero.

Assigning coefficients for artificial variables depends on whether the problem is a maximization or minimization problem. Moreover, we don't want the artificial variables to appear in the final solution because they are not real variables.

In a maximization problem, we assign a large negative number denoted by M to facilitate the solution. Which means, the contribution of the variables will

be negative (reduction) to the level of the objective function for a maximization problem (opposing the objective of the model). The simplex procedure does exclude a basic variable that has least contribution to the objective function and brings another variable (previously non basic) with the most desirable contribution to the basis. This being the procedure, assigning large negative number to be the coefficient of the artificial variable in the objective function for a maximization problem will facilitate the exclusion of the artificial variable from the basis.

Example:

Maximize $2x_1 + 3x_2 + 0S_1 + 0S_2 - MA_2$ indicates that the first constraint in the model is a $\leq$ constraint and the second is a $\geq$ constraint.

In a minimization problem, we assign a large positive number to be the coefficient of the artificial variable. This will have the same effect on the variable under consideration. Here also the large positive number is denoted by M .

Solving Maximization problems that have mixed constraints

The simplex procedure can also be applied here. All the logics we have seen so far holds true again. The only difference is the way we will be treating the new variables we introduced to the model (i.e. the artificial variable A and its coefficient M). Let's look at the following example:

Example Maximize $6x_1 + 8x_2$
 Subject to

$$\begin{aligned} x_2 &\leq 4 \\ x_1 + x_2 &= 9 \\ 6x_1 + 2x_2 &\geq 24 \\ x_1, x_2 &\geq 0 \end{aligned}$$

As usual, the simplex procedure starts solving LP models first by putting them into their standard form. The standard form to the above problem is:

Maximize $6x_1 + 8x_2 + 0S_1 + 0S_3 - MA_2 - MA_3$
 Subject to

$$\begin{aligned} x_2 + S_1 &= 4 \\ x_1 + x_2 + A_2 &= 9 \\ 6x_1 + 2x_2 - S_3 + A_3 &= 24 \\ x_1, x_2, S_1, S_3, A_1, A_3 &\geq 0 \end{aligned}$$

Then we will develop the initial simplex tableau.

The initial simplex tableau

C → ↓		6	8	0	0	-M	-M	RHS
Basis		X_1	X_2	S_1	S_3	A_2	A_3	values
S_1	0	0	1	1	0	0	0	4
A_2	-M	1	1	0	0	1	0	9
A_3	-M	6	2	0	-1	0	1	24
Z		-7M	-3M	0	M	-M	-M	-33M
C-Z		$6 + 7M$	$8 + 3M$	0	-M	0	0	

The next step is testing the initial simplex tableau for optimality.

Remember that the test is done by checking whether there is a positive number in the net evaluation row, and taking the largest one if there are

more than one positive numbers in that row. So, what is the largest positive number?

The variable M is just a very large number, therefore when we try to identify the largest positive number, what we do is just to take the positive M with the largest coefficient regardless of the constant number added with it. In the above tableau, we do have two positive numbers, $6 + 7M$ and $8 + 3M$. From these two, the first one is the larger even though 8 is greater than 6. This is because M is very large as compared with the difference between 8 and 6. This indicates that the next variable to enter the solution mix is X_1 .

The leaving variable is determined in the same way as we did in our previous discussions. The one with the least positive ratio will be the leaving variable.

		C → ↓	6	8	0	0	-M	-M	RHS
Basis			X_1	X_2	S_1	S_3	A_2	A_3	value s
S_1	0		0	1	1	0	0	0	4
A_2	-M		1	1	0	0	1	0	9
A_3	-M		6	2	0	-1	0	1	24
Z			-7M	-3M	0	M	-M	-M	-33M
C-Z			$6 + 7M$	$8 + 3M$	0	-M	0	0	

Leaving variable

↑
Entering variable

Don't forget to exclude the third artificial variable from the simplex tableau when you develop the second tableau. The second tableau will look like:

Second Tableau

		C → ↓	6	8	0	0	-M	RHS
Basis			X_1	X_2	S_1	S_3	A_2	values
S_1	0		0	1	1	0	0	4
A_2	-M		0	$2/3$	0	$1/6$	1	5
X_1	6		1	$1/3$	0	$-1/6$	0	4
Z			6	$2 - 2/3M$	0	$-1 - 1/6M$	-M	$24 - 5M$
C-Z			0	$6 + 2/3M$	0	$1 + 1/6M$	0	

↑
 M

The largest positive number in the net evaluation row is $6 + 2/3M$. This shows that the next variable to enter the basis is X_2 . And the leaving variable

is S_1 . Therefore, we develop the next tableau by taking the above facts into consideration.

Third Tableau

C → ↓		6	8	0	0	-M	RHS
Basis		X_1	X_2	S_1	S_3	A_2	values
X_2	8	0	1	1	0	0	4
A_2	-M	0	0	$-2/3$	$1/6$	1	$7/3$
X_1	6	1	0	$-1/3$	$-1/6$	0	$8/3$
Z		6	8	$6 + 2/3M$	$-1 - M/6$	-M	$48 - 7/3M$
C-Z		0	0	$-6 - 2/3M$	$1 + M/6$	0	

Still we do have a positive number in the net evaluation row. Therefore, we have to search for a better solution. We need to determine the next variable to enter the solution mix and the one which must leave the basis. Look at the following table.

C → ↓		6	8	0	0	-M	RHS
Basis		X_1	X_2	S_1	S_3	A_2	values
X_2	8	0	1	1	0	0	4
A_2	-M	0	0	$-2/3$	$1/6$	1	$7/3$
X_1	6	1	0	$-1/3$	$-1/6$	0	$8/3$
Z		6	8	$6 + 2/3M$	$-1 - M/6$	-M	$48 - 7/3M$
C-Z		0	0	$-6 - 2/3M$	$1 + M/6$	0	

The next tableau will be developed by excluding A_2 from the basis and bringing S_3 to the basis.

Fourth Tableau

C → ↓		6	8	0	0	RHS
Basis		X_1	X_2	S_1	S_3	values
X_2	8	0	1	1	0	4
S_3	0	0	0	-4	1	14
X_1	6	1	0	-1	0	5
Z		6	8	2	0	62
C-Z		0	0	-2	0	

Look at the above tableau. There is no positive number in the net evaluation row. This means, we cannot get an improved solution by introducing another variable (developing the next tableau). Therefore, the fourth tableau is the final tableau.

4.5. Solving Minimization Problems

The simplex procedure to solving minimization problems is almost the same as the procedure for maximization with mixed constraints except the following differences:

- The M coefficients in the objective function are given positive signs instead of negative signs.

- The selection of the variable to enter the solution is based on the largest negative value in the C - Z row.

Note also that minimization problems always require at least one artificial variable.

What do the M variables and the values in the C - Z row indicate? If you know the answer to this question, you will also know the reason behind the above differences between maximization and minimization problems.

M, the coefficient of artificial variables when we indicate them in the objective function, has same meaning as other coefficients of variables in the objective function. It indicates the extent of the artificial variable's per unit contribution to the level of the objective function.

The values in the net evaluation row indicate the extent of possible increase to the level of the objective function as a result of bringing an additional unit of the variable in that column to the solution mix.

Therefore, in a minimization problem, a large positive coefficient to a variable in the objective function means a large undesirable impact towards achieving the objective of the model (minimizing). And a large negative number in the net evaluation row means large desirable contribution to the level of the objective function.

The first one, creating an adverse impact towards achieving the objective seems paradox. The reason is to exclude the artificial variable (a variable with no real meaning) from the basis and the tableau as a whole as quickly as possible. Assigning a large positive coefficient is the appropriate measure since we are solving the model using the simplex procedure. If you consider the simplex procedure; each iteration in the procedure is about excluding a variable with least contribution to the achievement of the objective, and replacing it with another variable which is the most desirable contributor of those which previously were not in the basis. Therefore, assigning a large positive number to be the coefficient of an artificial variable will force the variable to be the first to leave the solution mix.

Selecting the largest negative number from the net evaluation row is a straight forward action in a minimization problem. Large negative number in this row means large deduction to the level of the objective function. This will facilitate the achievement of the objective of the model (minimization).

To sum up; in minimization problems, while the variable with the largest negative value in the net evaluation row enters the next tableau, the variable with the least positive ratio will leave the solution mix like the maximization case.

CHAPTER- 5

SENSITIVITY ANALYSIS AND DUALITY

Post optimality is concerned about the study of possible changes to a linear programming model and the associated impacts to the solutions to the model and to the level of the objective function. Such a kind of analysis is done after the determination of the optimum solution to the model. That is why we refer it as “post optimality analysis.” It is also called “sensitivity analysis” because we are observing the sensitivity of the model to possible changes.

Activity 5.1

What is post optimality Analysis?

How do you perform sensitivity analysis?

What is the physical interpretation of sensitivity analysis?

When we were developing the LP models, we assumed that we know all the parameter values for certain (Certainty assumption). However, in reality, the parameter values are just educated guesses. Therefore, the optimum solutions computed under this assumption may not be optimal depending on how sensitive that solution is to alternate values of parameters. That is why the decision maker needs to perform sensitivity analysis before implementing the solution.

Changes to a model might happen due to changes in any of the parameters (coefficients of the objective function, to the coefficients of the constraint functions, or to the RHS values of the constraints). In our discussion in this section, the possible changes to the model will be seen being classified into two categories. The first one is a change to the coefficients of the objective function, and the second is a change to the RHS values of the constraints. But we will not discuss the change to the coefficients of constraints.

Changes can also be resulted due to changes in the product lines especially when introducing a new product. To deal with these kinds of changes, we use a technique called “duality.” Duality is another way of formulating (representing) linear programming models.

We shall analyze the above changes by using the final simplex tableau (optimal solution) as a starting point.

5.1. Sensitivity Analysis

Here, we will be concentrating on the sensitiveness of the solutions of the model to changes in the parameter values of the model. Special attention is given to changes in RHS values of Constraints and Changes in the coefficients of the objective function.

5.1.1. A change in the RHS of a constraint

Our analysis will be made by looking at the impact one unit change in the constraint would have on the value of the objective function. If we know this,

we will be able to determine the total impact of change in the RHS of a constraint to the level of the objective function.

But this kind of analysis has some other associated questions:

- How can we determine the impact of a per unit change in the RHS value of a constraint?
- To what extent can the RHS value of a constraint be changed and still the per unit impact (marginal value) is the same?

If we get the answer to these questions, we can make our decision without being persecuted by possible changes to the RHS values of constraints. Therefore, our discussion in this section will focus on getting answers to the above questions.

As indicated in the beginning, our analysis is based on the optimal solution of a linear programming problem. Therefore, all our discussions right now are based on final simplex tableaux. The following simplex tableau is a final tableau for an LP model representing an LP problem presented below for simplicity purpose.

ABC Company is to start a production of two spare parts. The production manager would like to produce so many parts to earn per unit profit of Birr 60 from the sale of type I and Birr 50 from the sale of type II parts. However, the total available time to assemble the items, to inspect them and the space for their storage is limited to 100 hours, 22 hours and 39 square feet respectively. Four hours of assembly time, two hours of inspection time and three square feet of storage space is required to produce each unit of type I spare part. On the other hand 10 hours of assembly time, 1 hour of inspection time and 3 square feet of storage space is needed to produce each unit of type II spare parts. How many of each item should the Company produce and sale to maximize its profit?

The following is an LP model representing the above problem and its final tableau.

Maximize		$60x_1 + 50x_2$					
Subject to							
		$4x_1 + 10x_2 \leq 100$					
		$2x_1 + x_2 \leq 22$					
		$3x_1 + 3x_2 \leq 39$					
		$x_1, x_2 \geq 0$					
Basis	C →	60	50	0	0	0	RHS
	↓	x_1	x_2	s_1	s_2	s_3	values
s_1	0	0	0	1	6	-16/3	24
x_1	60	1	0	0	1	-1/3	9
x_2	50	0	1	0	-1	2/3	4
Z		60	50	0	10	40/3	740
C-Z		0	0	0	-10	-40/3	

Now let's go back to the above two questions. To get an answer to the first question (determining the per unit impact of a change in the RHS value of constraints to the objective function), look at the slack column values in the Z row of the above tableau. These values are called "shadow prices." That is 0,

10 and $40/3$. They indicate the impact that one unit change in the constraint would have on the value of the objective function.

To be specific, if the value of the second constraint in the above problem increases just by one unit (i.e. inspection time rose from 22 hrs to 23 hrs), profit will be increased by 10. This value (10) also shows the amount by which the value of the objective function will be decreased due to a unit decrease in the level of the constraint. If the value of the third constraint (storage space) increases by one unit, the level of the objective function will also be increased by $40/3$. But a unit increase or decrease in the first constraint has no effect to the level of the objective function.

However, knowing the impact of a unit change is not sufficient; because, we cannot make changes to the values of the constraints without limit. For example, we have said that the unit increase in inspection time will result in an increase to the level of the objective function by 10. This means, if we increase inspection time by 2 units, the level of the objective function will be increased by 20. But, we can not increase time (a precious resource) unlimitedly. And also we can not decrease it without limit, because we need time to produce our product. Therefore, we should be informed that we can have a change to the RHS value of constraints only within a certain range. This indicates that, our effort to analyze the impact of changes to the RHS values of constraints should also include knowing the range within which the possible change to the RHS values of constraints can happen and still have the same shadow price. This range is called the “*range of feasibility*” or “*the RHS range*.” This leads us to the second question we raised at the beginning of our discussion.

We can determine the upper and lower limits of the range of feasibility from the final simplex tableau. To do so, take the ratios of the entries in the RHS column to the corresponding entries in each slack column. This will give you the amount by which the level of the constraint function can be increased or decreased. This is summarized as follows:

- Allowable decrease: The smallest positive ratio
- Allowable increase: The smallest negative ratio

This means, we add the smallest negative ratio to the original RHS value to determine the upper limit of the range of feasibility and we subtract the smallest positive ratio from the original RHS value to determine the lower limit of the range of feasibility. Look at the following table.

Slack variables			Assembly	Inspection	Storage
S_1	S_2	S_3			
1	6	-16/3	$24/1=1$	$24/6 = 4$	$\frac{24}{-16/3} = -4.5$
0	1	-1/3	$9/0 = \text{Undif.}$	$9/1 = 9$	$\frac{9}{-1/3} = -27$
0	-1	$2/3$	$4/0 = \text{Undif.}$	$4/-1 = -4$	$\frac{4}{2/3} = 6$
Original Amount			100	22	39
Upper Limit			None	$22 + 4 = 26$	$39 + 4.5 = 43.5$
Lower Limit			$100 - 24 = 76$	$22 - 4 = 18$	$39 - 6 = 33$

As can be clearly observed from the above table, the range of feasibility for each constraint is:

Assembly time, from 76 hrs to no upper limit

Inspection time, from 18 hrs to 26 hrs

Storage space, from 33 cubic feet to 43.5 cubic feet

To this point, the decision maker knows the extent to which the right hand side value of constraints can be changed. But what would be the impact of this possible change to the level of constraints to the optimal solutions of the model and to the level of the objective function? The answer to this question will increase the level of confidence of the decision maker. To determine this, you just multiply the proposed amount of increase or decrease in the level of the constraint and multiply it by the corresponding substitution rates in the slack column.

For example, let us assume that inspection time is increased by 4 hours. The impact is calculated as:

Basis	S_2	Current Solution	Change	Revised solution
S_1	6	24	$+4(6) = +24$	48
X_1	1	9	$+4(1) = +4$	13
X_2	-1	4	$+4(-1) = -4$	0
Z	10	740	$+4(10) = 40$	780

NB. If we try to increase or decrease inspection time by more than 4, it would be beyond the feasible region. However, we can calculate the impact to changes to the level of inspection time by any value within the range of feasibility by using the above approach.

5.1.2. A Change in an Objective Function Coefficient

The coefficient of variables in the objective function indicates the per unit contribution of that variable to the objective function. A change to the objective function coefficient means therefore, a change in the per unit contribution of variables to the level of the objective function.

A change in the objective function coefficient can occur because of changed costs, new pricing policies, product or process design changes, as well as other factors. Here, the direction of our analysis is geared towards knowing the extent to which the contribution of a given decision variable to the objective function can be changed without altering the optimal solution.

When we talk about a change in the coefficient of decision variables in the objective function, we should not forget the fact that the variable under consideration may or may not be a basic variable in the optimal solution mix. On the basis of whether the variable is basic or non basic, we can determine two types of ranges to the possible changes in the coefficients of variables in the objective function. These are:

- The range over which a non-basic variable's objective function coefficient can change without causing that variable to enter the solution mix (range of insignificance).
- The range over which the objective function coefficient of a variable that is in solution can change without causing that variable to leave the solution mix (range of optimality).

A non-basic variable's objective function coefficient would have to increase by an amount that exceeds the C-Z value for that variable in the final tableau in order for it to be end up as a basic variable in the optimal solution.

Example:

Given the following final tableau of a maximization linear programming model:

C → ↓		11	10	14	0	0	0	RHS
Basis		X_1	X_2	X_3	S_1	S_2	S_3	values
x_2	10	.227	1	0	.227	.04	0	3.27
x_3	14	.909	0	1	-.091	.18	0	13.09
S_3	0	1	0	0	0	0	1	13.00
Z		15	10	14	1	3	0	215.96
C-Z		-4	0	0	-1	-3	0	

Determine the range of insignificance.

The range of insignificance for a non-basic decision variable is a range over which the objective function coefficient of the variable can vary without causing that variable to enter the basis as we have said earlier. X_1 is a non-basic decision variable as we can see from the final tableau of the above problem. This variable should have a positive value in the net evaluation row to enter the solution mix. This will be achieved if its coefficient increases by more than 4. This indicates that the range of insignificance for x_1 is less than or equal to 15.

The range of insignificance for a minimization problem will be determined again by taking the step to determine the entering variable in the simplex procedure into consideration. To remind you that step, the variable with the largest negative ratio in the net evaluation row will be selected to be the next variable to enter the basis in a minimization problem. The optimum tableau therefore has no negative value in the C-Z row. This implies that the range of insignificance for a non basic variable can include any value as far as that value will not make the value in the net evaluation row will not make negative. Any value that exceeds the Z value for that variable by more than the C-Z value for the same variable will satisfy this.

To determine the range of optimality for basic variables, the values in the C-Z row will be divided by the substitution rates in the row of that variable to have a series of ratios. The rules to determine the range of optimality for both maximization and minimization problems are:

- *Allowable increase: The smallest positive ratio of C-Z value and the variable substitution rate*
- *Allowable decrease: The smallest negative ratio of C-Z value and variable substitution rate*

5.2. Duality

Linear programming problems have two forms. The original formulation of a problem is known as its **Primal** form. The other form formulated from the primal is the dual form. The dual is the “mirror image” of the primal. The solution to the primal contains the solution to the dual and vice versa. Therefore, solving either of the two is enough to determine the solutions.

Analysis of the dual can help the decision maker assess the potential impact of a new product by determining the marginal value of resources. This is generally known to be the economic interpretation of the dual. This is discussed later in this section. But first let's have a look at the technical aspects of the dual-its formulation.

5.2.1. Formulating the dual:

The dual for a primal problem of maximization with all $\leq$ constraints, is a minimization problem with all $\geq$ constraints. The constraint coefficients of the primal are constraint coefficients of the dual except that the coefficients of first “row” of the primal become the coefficients of the first “column” of the dual, and the coefficients of the second “row” of the primal become the coefficients of the second “column” of the dual; and so on.

The following procedures can be followed to formulate the dual:

- For the objective function: Develop the objective function by making it a maximization if the primal is a minimization and vice versa; and assign the right hand side (RHS) values of constraints of the primal to be coefficients of the dual.
- To construct the constraints: Make the objective function coefficients of the primal, the right hand side (RHS) values to the constraints of the dual taken in order.

Example: Formulate the dual of the following problem

$$\begin{aligned} &\text{Minimize } 5x_1 + 7x_2 + 9x_3 \\ &\text{Subject to} \\ &\quad 20x_1 + 10x_2 + 30x_3 \geq 300 \\ &\quad 40x_1 + 5x_2 + 10x_3 \geq 200 \\ &\quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

To formulate the dual, we start with the objective function. Since the primal is a minimization problem with all $\geq$ constraints, the dual will be maximization with all $\leq$ constraints. The RHS values of the primal will become objective function coefficients of the dual. Therefore, the objective function for the dual is

$$\text{Maximize } 300y_1 + 200y_2$$

We usually use y's as variables to the dual. The reason is to differentiate it from variables of the primal.

The constraints of the dual are developed in the following way:

$$\begin{aligned}
20y_1 + 40y_2 &\leq 5 \\
10y_1 + 5y_2 &\leq 7 \\
30y_1 + 10y_2 &\leq 9 \\
y_1, y_2 &\geq 0
\end{aligned}$$

Student, please examine the following facts from the above dual constraints:

- The number of primal decision variables is the same as the number of dual constraints.
(i.e. there are three decision variables in the primal and three constraints in the dual)
- The RHS values of the dual constraints are equal to the objective function coefficients of the primal taken in order.
(The RHS values in the dual; that is 5, 7, and 9 are the coefficients of decision variables in the primal objective function)
- The coefficients of the primal constraints are also the coefficients of the dual constraints but in a different pattern.

Exercise

Formulate the dual of the following linear programming model

$$\begin{aligned}
&\text{Maximize} && 60x_1 + 50x_2 \\
&\text{Subject to} && \\
&&& 4x_1 + 10x_2 \leq 100 \\
&&& 2x_1 + x_2 \leq 22 \\
&&& 3x_1 + 3x_2 \leq 39 \\
&&& x_1, x_2 \geq 0
\end{aligned}$$

The dual to the above LP model is:

$$\begin{aligned}
&\text{Minimize} && 100y_1 + 22y_2 + 39y_3 \\
&\text{Subject to} && \\
&&& 4y_1 + 2y_2 + 3y_3 \geq 60 \\
&&& 10y_1 + y_2 + 3y_3 \geq 50 \\
&&& y_1, y_2, y_3 \geq 0
\end{aligned}$$

The final simplex tableau for the above LP model is given below

Final tableau of the dual

C →		100	22	39	0	0	RHS
Basis		y ₁	y ₂	y ₃	S ₁	S ₂	values
y ₃	60	16/3	0	1	1/3	-2/3	40/3
y ₂	50	-6	1	0	-1	1	10
Z		76	22	39	-9	-4	740
C-Z		24	0	0	9	4	

Primal shadow prices

Primal solution quantities

As we have seen earlier, the final simplex tableau for primal of the above LP model is:

Final tableau of the primal

C → ↓		60	50	0	0	0	RHS
Basis		X ₁	X ₂	S ₁	S ₂	S ₃	values
S ₁	0	0	0	1	6	-16/3	24
x ₁	60	1	0	0	1	-1/3	9
x ₂	50	0	1	0	-1	2/3	4
Z		60	50	0	10	40/3	740
C-Z		0	0	0	-10	-40/3	

The following can be summarized about the solutions of the dual and the primal as can be seen from the above tableaus:

1. If the primal problem has an optimum solution, the dual will also have an optimum solution and vice versa. The optimum solution of the dual and the primal are the same
2. Given the final tableau of the dual, the optimal solutions for decision variables of the primal are given by the C - Z row values of slack variables in the dual
3. Given the final solution of the primal, the solution for the decision variables of the dual are determined to be the Z row values of slack variables (shadow prices) of the primal; and the solution for slack variables of the dual are given by the C - Z row values of decision variables of the primal

This indicates the technical importance of formulating the dual. That is to say, if you encounter problems in solving the primal problem, you can solve the dual and vice versa. Generally, we can reduce our computing time by developing the two models and solving the easier.

Beyond the above importance (technical), the dual has an economic interpretation. The following paragraph is about the economic interpretation of the dual.

5.2.2. Economic Interpretations of the dual:

The formulation of the dual can help us to analyze (evaluate) the feasibility of adding a new decision variable to the model. This may happen when a firm decides to start a new product in addition to its current products by using its current capacity.

Suppose ABC Company decided to start producing a new spare part by using available resources, the company's manager should try to get an answer to the following questions:

- How much of each item should be given up in order to consider the production of the new item?
- How much is the gain from the new item?
- Is the gain less than, equal to or greater than the gain that is forgone as a result of reduction in the quantity of the older items just to free a resource that could be used in the production of the new item?

Economically speaking, for the firm to consider the production of the new product, the amount of scarce resources that will be given up must produce a return to the company that is at least to the foregone profit.

To be more specific, for each unit of type I spare part that the company sacrifices in order to produce the new product, it will gain four hours of assembly time, two hours of inspection time and three cubic feet of storage

space. However, ABC Company will also give up a unit profit of Birr 60. Therefore, the return from the new product should at least equal to or greater than the forgone profit (Birr 60). That is, the value of 4 assembly hours + 2 inspection hours + 3 cubic feet of storage space must be greater than or equal to Birr 60. The same logic holds true for the second item also. These two analyses results became the constraints of the dual. The variables in the dual problems are the marginal values of scarce resources. Look at the constraint functions of the dual of ABC Company's problem.

CHAPTER- 6

TRANSPORTATION PROBLEMS

The transportation problem arises frequently in planning for the distribution of goods and services from several supply locations to several demand locations. Usually the quantity of goods available at each supply location (origin) is fixed or limited and there is a specified amount needed (demand) at user location (destination). With a variety of shipping routes and differing cost for the routes, the objective how many units should be shipped from each origin to each destination so that all destination demands are satisfied and total transportation costs are minimized.

Therefore, it is necessary to have the following information to develop transportation model these are.

- Supply quantity (capacity of each origin)
- Demand quantity of each destination
- Unit transportation cost for each origin - destination route

The transportation algorithm requires the assumptions that all goods be homogeneous so that any origin is capable of supplying any destination and quantity shipped over any route. We shall add one additional requirement that will simplify the problems the total quantity available for shipment is equal to the total quantity demanded.

Example: poster generators inc. is a firm that has production operation in Cleveland, (Ohio) Bedford (Ind) and York (pa). Production capacities for these plants over the next 3 months planning period for one particular type of generator are as follows.

Origin	Plant	Three month production capacity (Units)
1	Cleveland	5000
2	Bedford	6000
3	York	2500
Total		13,500

Suppose that the firm distributes its generators through four regional distribution centers located in Boston, Chicago, St. Louis, and Lexington and that the 3-month forecast of demand for the distribution centers is as follows.

Destination	Distribution centers	3-month Demand forecast (Units)
1	Boston	6000
2	Chicago	4000
3	St. Louis	2000
4	Lexington	1500
Total		13,500

Management would like to determine how much of its production should be shipped from each plant to each distribution center with identical production costs at the three plants, the only variable costs involved are transportation costs. Thus the problem becomes one of determining the distribution routes to be used and the quantity to be shipped via each route so that all distribution center demands can be met with a minimum total transportation cost. The cost for each unit shipped on each route is given in table below.

Destination

Origin	Boston	Chicago	St. Louis	Lexington
Cleveland	3	2	7	6
Bedford	7	5	2	3
York	2	5	4	5

	Boston	Chicago	Louis	Lexington	SS
Cleveland	3	2	7	6	5000
Bedford	7	5	2	3	6000
York	2	5	4	5	2500
DD	6000	4000	2000	1500	13,500

A solution to a transportation problem consists of quantities that are assigned to the various routes (i.e. cells in the table). These values can range from zero which implies that no units will be shipped over that route to a maximum that equals the smaller of two quantities, the row (supply) and column (demand) totals. The logic of the maximum quantity is simple. The quantity shipped cannot exceed the available supply in a row and it should not exceed the amount demanded (column total). Such a solution is feasible solution. The starting point of the transportation method is feasible solution.

6.1. Finding an initial Feasible Solution

The feasible solution is one in which assignments are made in a such a way that all supply and demand requirements are satisfied. In general the number of non zero (occupied) cells should equal one less than the sum of the number of rows and the number of columns in a transportation table. In the case of a table with 3 rows and 3 columns, the number of occupied cells should be $3+3-1 = 5$ in order to be able to use the transportation algorithm.

Sometimes, fewer occupied or completed cells appear in a solution. When that happens, the solution is referred to as a degenerate solution, such a solution requires modification in order to be able to determine if it is optimal.

Aside from a degenerate case, the transportation method will generate solutions that have the number of occupied cells equal to the no of rows plus no of columns minus one. Other feasible solutions that use more of the cells are imaginable but not really desirable. A number of different approaches can be used to find an initial feasible solution. Two of these are described here

1. The north west corner method
2. An intuitive approach

6.1.1. The North West Corner Method

The northwest-corner method is a systematic approach for developing an initial feasible solution. Its chief advantages are that it is simple to use and easy to understand. Its chief draw back is it does not take transportation costs in to account consequently such a solution may require much addition effort to obtain the optimal solution.

The northwest -corner method gets its name because the starting point for the allocation process is the upper left hand (north west) corner of the transportation table. For the foster Generators problem, this would be the cell that represents the route from Cleveland to Boston. The following set of principles guides the allocation.

1. Begin with the upper left-hand cell, and allocate as many units as possible to that cell. This will be the smaller of the row supply and the column demand. Adjust the row and column quantities to reflect the allocation.
2. Remain in a row or column until its supply or demand is completely exhausted or satisfied, allocating the maximum number of units to each cell in turn, until all supply has been allocated and all demand has been satisfied because we assume total supply and demand are equal.

	Boston	Chicago	Louis	Lexington	SS
Cleveland	5000 3	2	7	6	5000
Bedford	1000 7	5	1000 2	3	6000
York	2	4000 5	1000 4	1500 5	2500
DD	6000	4000	2000	1500	13,500

The sequence of distribution is as follows

1. Beginning in cell Cleveland. Boston 5000 units exhausting supply in row - 1 and leaving 1000 units of column 1.
2. The remaining 1000 unit demand of column - 1 is satisfied from the second supplier row - 2
3. The remaining 5000 units of row 2 will be distributed to 2nd column and 2nd row cell 4000 and to 3rd column 2nd row 1000 units
4. To satisfy the 3rd column 1000 units will be transported from York to Louis
5. The remaining 1500 units from York to Lexington.

In terms of minimizing total transportation cost, the solution may or may not be optimal. We shall make that determination very shortly. At this point, let's simply compute the total cost

$$\begin{aligned}\text{Total cost} &= 3 \times 5000 + 7 \times 1000 + 5 \times 4000 + 2 \times 1000 + 4 \times 1000 + 5 \times 1500 \\ &= 123,000 \text{ Birr}\end{aligned}$$

As noted before, the main drawback of the northwest corner method is that it does not consider cell (route) costs in making the allocation. Consequently if this allocation is optimal, that can be attributed to chance rather than the method used.

6.1.2. The Intuitive Approach

This approach is also known as the minimum cost method, uses lowest cell cost as the basis for selecting routes. The procedure is as follows.

1. Identify the cell that has the lowest unit cost. If there is a tie, select one arbitrarily. Allocate a quantity to this cell that is equal to the lower of the available supply for the row and the demand for the column.
2. Cross out the cells in the row or column that has been exhausted (or both, if both have been exhausted) and adjust the remaining row or column total accordingly.
3. Identify the cell with the lowest cost from the remaining cells. Allocate a quantity to this cell that is equal to the lower of the available supply of the row and the demand for the column.
4. Repeat step 2 and 3 until all supply and demand has been allocated.

Let us see how this method can be applied to the above problem

	Boston	Chicago	Louis	Lexington	SS
Cleveland	1000 3	2	7	6	5000
Bedford	2500 7	4000 5	2000 2	1500 3	6000
York	2500 2	5	4	5	2500
DD	2500	4000	2000	1500	13,500

This solution may or may not be optimal. In the next section, the procedure for testing for optimality will be described. For now let us simply compute the total cost of this solution and compare it to that of the northwest corner solution. Here we have

Cell	Cost /Unit	No of units	Transportation cost
Clev - Bost	3	1000	3000
Clev - Chic	2	4000	8000
Bed - Bost	7	2500	17500
Bed - Louis	2	2000	4000
Bed-Lexington	3	1500	4500
York - Bost	2	2500	5000
Total			42000

Compared to the cost using the north west - corner method. This one has a total cost that is \$ 81000 less. The fact that this plan is less costly than the previous one was expected. The previous did not involve the use of cost information in allocating units. Whether this plan is optimal or can be improved up on remains to be seen.

6.2. Evaluating a Solution for Optimality

The test for optimality for a feasible solution involves a cost evaluation of empty cells (i.e. routes to which no units have been allocated) to see if an improved solution is possible. We shall consider two methods for cell evaluation.

1. The Stepping stone method
2. The MODI method

The stepping stone method involves a good deal more effort than the MODI method as you will note. However it provides an intuitive understanding of the evaluation process. More over, when a solution is not optimal, the distribution plan must be revised by reallocating units in to and out of various cells and only the stepping stone method can be used for the reallocation. It is there fore necessary to be able to use the MODI method and then, the stepping stone method, if necessary.

6.2.1. Evaluation Using the Stepping Stone method

The stepping stone method involves tracing a series of closed paths in the transportation table, using one such path for each empty cell. The path represents a shift of one unit in to an empty cell, and it enables the manger or analyst to answer a what if question; what impact on total cost would there be if one unit were shifted in to a cell. If the shift would result in a cost savings, the stepping stone path also can be sued to determine the maximum number of units that can be shifted in to the empty cells, as well as modifications to other completed cells needed to compensate for the shift in to the previously un used cell.

The name stepping-stone relates to an analogy of crossing a pond or stream by moving from stone to stone; in the case of a transportation solution the stone are the completed cells. The initial feasible solution we found using the northwest corner method is reproduced. in the following table.

	Boston	Chicago	Lovis	Lexington	SS
Cleveland	5000 - 3	+ 2	7	6	5000
Bedford	1000 + 7	4000 - 5	1000 2	3	6000
York	2	5	1000 4	1500 5	2500
DD	6000	4000	2000	1500	13,500

Initial feasible solution obtained using northwest corner method we know it is not optimal because the intuitive method generated an initial solution that has a lower total cost. However it will be instructive to analyze the north west corner solution in order to see how the stepping stone method works and to gain some insight in to the process of developing an improved solution only unoccupied cells need to be evaluated because the question at this point is not how many units to allocate a particular route but only if converting a cell from zero units to non - zero (a positive integer) would decrease or increase total costs. The un occupied cells are

Clevel - Chicags
Bedfond - Lexington

Clevel - Louis
York - Boston

Clevel - Lexington
York - Chicago

They must be evaluated one at a time, but in no particular order. Let us begin with cell Clevel - Chicago. We start by placing a + in the cell being evaluated, which stands for the addition of one unit to the cell. In order to maintain the column total of 6000 we must subtract one unit from an occupied cell, cell Bedford - Chicago is the only option. This is designated by placing a - in cell Bedford - Chicago. Because we subtracted one unit from row Bedford, we must compensate for this, which we can do by adding a unit (placing a + sign) in cell Bedford - Boston. Similarly, we compensate for the addition of one unit to column Boston by subtracting a unit from cell Clevel Leland - Boston and place a - in that cell to reflect this. Because we initially added one unit to row Cleveland in cell Cleveland - Chicago, this last subtraction also compensates for that, and we have traced a completed path, which we can use to evaluate Cleveland - Chicago. Before doing that let's consider rules that guide tracking these paths.

Rules for tracing Stepping - Stone path

1. All unoccupied cells must be evaluated, Evaluate cells one at a time
2. Except for the cell being evaluated, only add or subtract in occupied cells (It is permissible to skip over occupied cells to find an occupied cell from which the path can continue).
3. A path will consist of only horizontal and vertical moves, starting and ending with the empty cell that is being evaluated.
4. Alternate + and - signs, beginning with a + sign in the cell being evaluated.

Note that it is not necessary to actually alter the quantities in the various cells to reflect the one unit change, the + and - signs suffice.

The general implication of the plus and minus signs is that cells with + signs mean one unit would be added. Cells with a - sign indicate one unit would be subtracted. The net impact of such a one unit shift can be determined by adding the cell costs with signs attached and noting the resulting value. Thus for cell Cleveland - Chicago we have a net change of +1

$$\begin{array}{r} + \quad - \\ 2 \quad 5 \\ 7 \quad 3 \end{array}$$

$$9 - 8 = + 1$$

$$\begin{array}{r} 19 \quad -8 \end{array}$$

This means that for each unit shifted into cell Cleveland-Chicago in this way (which is the only way a shift could be made), the total cost would increase by \$1. Consequently such a shift would not be desirable.

Turning our attention to cell Cleveland - Louis, we begin its evaluation by placing a + sign in that cell. We can move horizontally or vertically to occupied cells. Suppose we move to cell Cleveland- Boston and place - sign, then + Bedford - Boston and place + and then to Bedford - Louis and place - sign.

Evaluation of Clev-Louis = + 9

$$\begin{array}{r} + \quad - \\ 7 \quad 3 \end{array}$$

$$\begin{array}{r} 7 \\ 2 \end{array}$$

$$14 - 5 = +9$$

$$\begin{array}{r} +14 \\ -5 \end{array}$$

Hence, using this path for reallocation would increase total cost by \$ 9 per unit. Again this path would not be desirable.

Evaluation

Clev - Lengidgton Bedford - Lexigton York - Boston York - Chicago

+	-	+	-	+	-	+	-
6	5	3	5	2	7	5	5
4	2	4	2	2	4	2	4
7	3	7	7 = 0	+4	11 = -7	+7	-9 = -2

$$17 - 10 = +7$$

Not desirable

Not desirable

desirable

desirable

The negative value of cell York-Boston and York - Chicaogs indicate an improved solution is possible. For each unit we can shift in to those cells, the total cost will decrease by \$ 7 and \$ 2 respectively. The next question is to which cell should reallocation be made? obviously to the highest negative value i.e. to York - Boston. How many units can be reallocated in to that cell? Before addressing that question, let's consider an alternative method for cell evaluation that avoids having to trace all of the evaluation paths.

6.2.2. Evaluation using the MODI method

The MODI (modified distribution) methods of evaluation a transportation solution for optimality involves the use of index numbers that are established for the rows and columns. These are based on the unit costs of the occupied cells. The index numbers can be sued to obtain the cell evaluations for empty cells with out the use of stepping-stone paths. There is one index number for each column and one for each row. These can be conveniently displayed along the top and right edges of a matrix. The matrix numbers are determined in such a way that for any occupied cells, the sum of the row index and the column index equal the cells unit transportation cost.

$$\begin{array}{rcl} \text{Row index} & + & \text{Column index} \\ r_1 & + & k_1 \end{array} = \text{Cell cost} = C_{ij}$$

The index numbers are determined sequentially in a manner dictated by the position of occupied cells. The process always begins by assigning a value of zero as the index number of row 1.

This method will be illustrated by developing index numbers for the initial feasible solution for the above problem generated by the north - west corner method which is repeated below.

	+3	+1	-2	-1		
	Boston	Chicago	Louis	Lexington	SS	
Cleveland	5000 3	+1 2	-9 7	+7 6	5000	0

Bedford	1000	7	4000	5	1000	2	0	3	6000	+4
York	-7	2	-2	5	1000	4	1500	5	2500	+6
DD	6000		4000		2000		1500		13500	

We begin by assigning a value of zero as the index for row 1. once row index has been established, it will enable us to compute column index numbers for all occupied cells in that row. Similarly once a column index number has been determined, index numbers for all rows corresponding to occupied cells in that column can be determined.

The index number for column 1 is based on the fact that the sum of its value and the row index number must equal the cell cost of \$ 3 for cell Clev - Boston. $0 + K_1 = 3$, $K_1 = +3$ similarly using occupied cell Bed. Boston index number of row 2 is $3 + R_2 = 7$. $R_2 = +4$. Using occupied cell Bed - Chicago index number for column 2 will be $r_2 + K_2 = 5$ $= K_2 = 5 - r_2 = 5 - 4 = +1$. In the same way using cell Bed -louis, the index number for column 3 is, $r_2 + K_3 = 2$, $K_3 = 2 - r_2 = 2 - 4 = -2$

Using occupied cell York - Louis, index number for r_3 is $r_3 + K_3 = 4$ $= r_3 = 4 - k_3 = 4 - (-2) = +6$

Using occupied cell York Lexington, index number for K_4 is $r_3 + k_4 = 5$, $k_4 = 5 - r_3 = 5 - 6 = -1$

The complete set of row and column index number is shown in the previous table.

We can now readily determine the cell evaluations (improvement potentials) for each of the occupied cells, the sum of the row and the column index equal the unit cell cost.

Cell evaluation = cell Cost - Row index- column index

These determinations can be made in any order.

Eg the cell evaluation for York Boston is $2-6-3 = -7$. This implies an improvement (decrease in total cost of \$ 7 per unit for units that can be shifted in to cell York-Boston. Similarly for empty cell Clev - Chicago, the improvement potential is $2-0-1=+1$ which indicates any units shifted in to this cell would increase total cost by \$ 1 each. For un occupied cell York - Chicago is $5-1-6 = -2$, for clev - louis $7-0 - (-2) = +9$, for clev - lexingta $6-0-(-1) = +7$, for Bed - Lexington, $3 - 4 - (-1) = 0$. These values are summarized below.

	Boston	Chicago	Louis	Lexington	SS
Cleveland	500 ³	+1 ²	+9 ⁷	+7 ⁶	5000
Bed - ford	1000 ⁷	4000 ⁵	1000 ²	0 ³	6000
York	-7 ²	-2 ⁵	1000 ⁴	1500	2500
DD	6000	4000	2000	1500	13,500

Note that they agree with the values we computed earlier using the stepping stone method.

When cell evaluations are positive or zero, an optimal solution has been found, If one or more is negative, the cell with the largest negative should be brought in to solution because that route has the largest potential for improvement per unit. In this case, we found that cell York - Boston had an evaluation of -7 and cell York - Chicago (-2). Hence an improvement solution is possible.

6.3. Developing an Improved solution

Developing an improved solution to a transportation problem requires focusing on the unoccupied cell that has the largest negative cell evaluation. For the above case York - Boston had the largest negative.

Improving the solution involves reallocating quantities in the transportation table. More specifically, we want to take the advantage of the improvement potential of cell York-Boston by transferring as many units as possible in to that cell. The stepping stone method for that cell is necessary for determining how many units can be reallocated while retaining the balance of supply and demand for the table. The stepping stone for cell York - Boston is reproduced below.

	Boston	Chicago	Louis	Lexington	SS
Cleveland	5000 ³	²	⁷	⁶	5000
Bedford	1000 ⁷	4000 ⁵	1000 ² +	³	6000
York	² +	⁵	1000 ⁴ -	1500 ⁵	25000
DD	6000	4000	2000	1500	13,500

The + signs in the path indicate units to be added, the - sign indicate units to be subtracted. The limit on subtraction is the smallest quantity in a negative position along the cell path. There are two quantities in negative positions (both are 1000 units). So that amount will be shifted in the following manner. Subtract 1000 units from each cell on the path with a -ve sign and add 1000 units to the quantities of each cell with a + sing in it, A quick check reveals that the sum of quantities in each row and in each column are equal to original row and column totals with each iteration, it is necessary to evaluate the empty cells to see if further improvement is possible. This requires use of either the MODI or the stepping stone method. Both will yield the same value.

	Boston	Chicago	Louis	Lexington	SS
Cleveland	5000 ³	²	⁷	⁶	5000
Bedford	0 ⁷	4000 ⁵	1000 ²	³	6000
York	1000 ²	⁵	0 ⁴	1500 ⁵	25000
DD	6000	4000	2000	1500	13,500

We will continue in the same way and check for the rest un occupied cells. If we get 0 or +ve in all un occupied cells, then the table is in its optimal state other wise we will continue for iteration.

6.4. Special Issues

A number of special issues are discussed in this section in order to round out your understanding of the transportation model. They are

1. Determining if there are alternative optimal solutions
2. Defining and handling degeneracy (too few occupied cells to permit evaluations of a solution).
3. Avoiding unacceptable or prohibited route assignments
4. Dealing with problems in which supply and demand are not equal.
5. Solving maximization problems

6.4.1. Alternate optimal solutions

Sometimes transportation problems have multiple optimal solutions. In such instances, it can be useful for a manager to be aware of alternative solutions because this gives the manager an option of bringing non quantitative considerations in to the decision.

In the case of the transportation problem, the existence of an alternative solution is signaled by an empty cell's evaluation equal to zero. We can find out what that alternate solution is by reallocating the maximum number of units possible around the stepping stone path for that cell.

6.4.2. Degeneracy

A transportation problem is a degenerate when there are too few occupied cells to enable all the empty cells to be evaluated. In the case of the stepping stone method, this means that there will be at least one empty cell for which an evaluation path cannot be constructed. For the MODI method it means that it will be impossible to determine all of the row and column index numbers. It is relatively simple to determine if a solution is degenerate. A solution is degenerate if the number of occupied cells is less than the number of row plus the number of columns minus one.

Test for degeneracy

Number of occupied cells must equal

$R + C - 1$ Where R = number of rows

C = number of Columns

A quick check of the alternate solution to the foster generator Inc. developed in the preceding section will reveal that the solution was degenerate. The number of completed cells is five while the required number of completed cell is six. It should be mentioned that this particular case is some what a typical in that usually some paths can be traced but not all of them. Similarly if you want to compute index number for the rows and columns, you will be unable to compute for some of the rows and column.

Obviously, some modification has to be made in order to determine if a given solution is optimal. The modification is to treat some of the empty cells as occupied cells. This is accomplished by placing a delta (Δ) in one of the empty cells. The delta represents an extremely small quantity (eg 0.001 units), it is so small that supply and demand for the row and column involved will be unaffected even with out modifying other quantities in the row and column, and so small that total cost will not change.

The purpose of the delta is to enable evaluation of the remaining empty cells. The choice of location for the delta can be somewhat tricky. Some empty cells may be unsuitable if they do not enable evaluations of the remaining empty cells. Moreover, the delta cannot be placed in a cell which later turns out to be in a negative position of a cell path involved in reallocation, because delta will be the smallest quantity in a negative position and shifting that minute quantity around the cell path will leave the solution virtually unchanged. Consequently, a certain amount of trial and error may be necessary before a satisfactory location can be identified for delta.

6.4.3. Unacceptable Routes

In some cases, certain origin - destination combinations may be unacceptable. This may be due to weather factors, equipment breakdowns, labor problems, or skill requirements that either prohibit, or make undesirable certain combinations.

Suppose that in Foster Generators Inc. route Cleveland Boston was suddenly unavailable because of recent flooding. In order to prevent the route from appearing in the final solution (as it originally did), the manager could assign a unit cost to that cell that was large enough to make that route uneconomical and hence prohibit its occurrence. One rule of thumb would be to assign a cost that is 10 times the largest cost in the table. Thus because the largest cost is \$ 7, a unit cost of \$ 70 could be assigned instead of the original cost of \$ 3 per unit. Then this revised problem could be solved using north - west corner, intuitive, or VAM for an initial solution and either stepping - stone or MODI to evaluate the initial solution.

6.4.4. Un equal supply and Demand

Up to this point, the example has involved cases in which supply and demand were equal. As you might guess, there are situations in which the two are not equal. When such a situation is encountered, it is necessary to modify the original problem so that supply and demand are equal. This is accomplished by adding either a dummy column, or a dummy row, a dummy row is added if supply is less than demand and a dummy column is added if demand is less than supply. The dummy is assigned unit costs of zero for each cell, and it is given a supply (if a row) or a demand (if a column) equal to the difference between supply and demand. Quantities in dummy routes in the optimal solution are not shipped. Rather they indicate which supplier will hold the excess supply, and how much and which destination will not receive its total demand and how much it will be short.

6.4.5 Maximization

Some transportation type problems concern profits or revenues rather than costs. In such cases, the objective is to maximize rather than to minimize. Such problems can be handled by adding one additional step as the start. Identify the cell with the largest profit and subtract all the other cell profit from that value. Then replace the cell profits with the resulting values. These values reflect the opportunity costs that would be incurred by using routes with unit profit that are less than the largest unit profits. Replace the original unit profits with these

opportunity costs and solve in the usual way for the minimum opportunity cost solution. This will be identical to maximizing the total profit.

6.4.6. Other Applications

There are a number of other kinds of situation in which the transportation method has proved useful. They include the transportation problem, which is a variation of a transportation problem and location analysis.

6.4.6.1. Transshipment Problems

Certain transportation problems may involve the use of intermediate destinations where goods are temporarily stored before being shipped on to their final destinations. Thus these intermediate points are both destinations and origins. The transportation method can be used to handle such problem. The goal generally is the same as with other transportation problems to minimize total transportation cost.

6.4.6.2. Location Analysis

Another use of the transportation methods is to compare transportation costs for alternative locations. For instance the company may be preparing to build a new warehouse, and there may be a number of potential locations under consideration. One aspect of the decision may be differences in transportation costs that would result from each alternative.

Suppose that currently the firm has three factories that supply four warehouses and another warehouse will be added. Suppose that two separate locations for the new warehouse are being studied. The impact of each of the two potential locations can be determined by solving two transportation problems, one for each location. In other words, one extra column would be added to the transportation table representing one of the warehouses and the problem would be solved for the minimum total cost. The column then would be replaced in the table by a column representing the other warehouse and again the problem would be solved for the minimum cost. This would give decision makers an opportunity to assess the impact of each warehouse location on the total distribution costs for the system.

Exercises

1. Consider the following transportation problem

To From	Store - 1	Store 2	Store 3	Supply
Warehouse A	12	20	15	50
Warehouse B	9	11	4	15
Warehouse C	20	14	8	55
Demand	25	50	45	120

- Develop an initial feasible solution using the north west corner method, compute the total cost for this solution.
- Evaluate the solution using the stepping stone method. Is the solution optimal? Explain
- Repeat the evaluation using MODI and compare your cell evaluations to those obtained using the stepping stone method.
- Obtain an improved solution and evaluate it using MODI. Is it optimal?
- What is the total cost for your optimal solution?

2. The Microsoft enterprise manufactures the central processing unit (CPU) for a line of personal computers. The CPUs are manufactured in Seattle, Columbus, and New York, and shipped to warehouses in Pittsburgh, Mobile, Denver, Los Angeles and Washington DC for further distribution. The transportation table below shows the number of CPUs available at each plant and the number of CPUs required by each warehouse. The shipment costs are also shown below.

	Warehouse					
	Pittsburgh	Mobile	Denver	Los Angeles	Washington	SS
Seattle	10	20	5	9	10	9000
Columbus	2	10	8	30	6	4000
New York	1	20	7	10	4	8000
DD	3000	5000	4000	6000	3000	21000

- Determine the amount that should be shipped from each plant to each warehouse in order to minimize the total shipping cost.
- The Pittsburgh warehouse has just increased its order by 1000 units and Microsoft has authorized the Columbus plant to increase productivity by 1000 units. Do you expect this development to lead to an increase or a decrease in total shipping costs? Solve for the new optimal solution.

CHATER- 7

ASSIGNMENT PROBLEMS

The assignment problem is also one of the areas of application of the linear programming model. This problem arises in different situations of decision making such as, assigning jobs to machines, to workers, etc...

The basic assumption within which we can solve assignment problems using the linear programming is “assigning one job, project, etc... to one worker, machine, etc...” In doing so, we will be having different alternatives (sets) of assignments; it is from these sets of assignments we will select the optimal one. This means, that with the least possible cost, time; the maximum possible profit, etc...

7.1 Solution procedure: the assignment Problem

Read the following assignment problem. The procedure for solving assignment problems is discussed next to the problem.

XYZ Company is to undertake market studies for its three clients. The company has to do the task of assigning project leaders to each of the three market studies. The Company's management realizes that the time required by each of the three project leaders assigned to each study is different due to the experience and ability of the project leader. The estimated project completion times are summarized by the table below.

Table: Estimated project completion times

	Project leader	Client		
		1	2	3
1	Taye	10	15	9
2	Chanie	9	18	5
3	Kebede	6	14	3

The company would like to assign project leaders so that the total number of days required to complete all the projects is minimized. If a project is to be assigned to one and only one client, what assignments should be made?

The Hungarian method makes use of the matrix reduction process to determine the optimal solution to the assignment problem. There are three steps in the matrix reduction procedure:

Step 1: Reduce the initial matrix by subtracting the smallest element in each row from every element in that row. Then, subtract the smallest element in each column from every element in that column of the row-reduced matrix.

By subtracting 9 (the smallest element in row 1) from row 1, 5 from row 2, and 3 from row 3; we will have the following row-reduced matrix:

	Project leader	Client		
		1	2	3
1	Taye	1	6	0
2	Chanie	4	13	0
3	Kebede	3	11	0

Column reduction will also be performed on the row-reduced matrix to complete step 1. This is done by subtracting 1 from the first column elements, 6 from the second column elements, and 0 from column three elements. The reduced matrix becomes

	Project leader	Client		
		1	2	3
1	Taye	0	0	0
2	Chanie	3	7	0
3	Kebede	2	5	0

To reach the optimal solution, we shall go through the following two steps.

Step 2: Determine the minimum number of straight lines that must be drawn vertically or horizontally to cover all the zeros in the reduced matrix. If the minimum number of lines is found to be the same as the number of rows or columns, an optimal assignment can be made; otherwise, go to step 3.

Let's now go back to our example. The minimum number of lines required to cover all the zeros is 2 as it is shown below.

	Project leader	Client		
		1	2	3
1	Taye	0	0	0
2	Chanie	3	7	0
3	Kebede	2	5	0

These numbers of lines is different from the number of columns or rows. Therefore, we must go to step 3.

Step 3: Subtract the value of the smallest uncovered element from every uncovered element and add the same value to every element at the intersection of two lines. Return to step 2 and continue until the minimum number of lines required to cover all the zeroes in the matrix is equal to the number of rows or columns.

The minimum uncovered element is 2. Therefore, subtract 2 from all unlined elements and add it to the element at the intersection of the two lines to get the following matrix.

	Project leader	Client		
		1	2	3
1	Taye	0	0	2
2	Chanie	1	5	0
3	Kebede	0	3	0

Check the above matrix for optimality (return to step 2). Did you find that the maximum number of straight lines required to cover the zeros is 3? Look at the following matrix.

	Project leader	Client		
		1	2	3
1	Taye	0	0	2
2	Chanie	1	5	0
3	Kebede	0	3	0

As can be seen above, three lines must be drawn to cover the zeros. Therefore, we have reached the optimal solution.

Now, we can make our assignment. This assignment is done by first locating a row or column that contains only one zero. Draw a circle or a square just to locate an assignment, and eliminate that row and column from further consideration. Proceed till you finish your assignment.

When we come to our example, column 2 and row 2 have only one zero. Thus we can start our assignment either by assigning Taye to client 2 or by assigning Chanie to client 3. Let's start by assigning Taye to client 2 first. Column 2 and row 1 will be out of further consideration. Then we go to row 2 and assign Chanie to client 3; because, there is no other zero in this row. This will force us to cancel row 2 and column 3. Therefore, only row 3 is left now.

The only zero in this row is the one in column 1. This means, Kebede will be assigned to client 1. Of course we can automatically assign Kebede to client 1 since assignments are made to client 2 and 3. The final solution for XYZ Company is shown below.

	Project leader	Client		
		1	2	3
1	<i>Taye</i>	0	0	2
2	<i>Chanie</i>	1	5	0
3	<i>Kebede</i>	0	3	0

The value of the optimum assignment is determined by summing up the times in the original assignment problem associated with the optimal assignment indicated by the rectangles. That is, $15 + 5 + 6 = 26$ days are the optimal number of days that are required to complete the study for the three clients.

Summary

This chapter was about the most famous model in management- linear programming model. To understand the model, we have to know the characteristics of the model and the assumptions that are used as frameworks in the construction of the model.

Solutions to the model can be seen being classified into two. The first one is the graphical approach. This model is applicable for models having only two constraints. If the number of constraints is greater than two, we cannot use the graphical approach. In this case, another approach – the simplex procedure will be used. This procedure can

The optimal solution is just a solution to the model. A model is an abstraction of reality. So there may be a chance failure for the model to perfectly represent the reality. In this sense, before using the optimal solution of the model as an input in the decision-making, we have to consider possible changes to the model and the sensitiveness of the solution of the model to these changes. This is what we mean by sensitivity analysis.

The other type of analysis that is covered under this chapter is duality. It is also a mechanism to analyze changes to the model that is related with the introduction of new constraints. It is widely applicable in the case of introducing new product lines. It can also technically used as an alternative approach to solving the model. When we get the primal solution relatively

more difficult to solve than the dual, we can solve the dual and save our time and effort.

PART THREE

STOCHASTIC MODELS

Chapter overview

Most complex managerial decisions are made with some uncertainty. These problems are to be made by managers. That is why they are trained to do; and that is why they hired; and that is why they get paid for doing. But the manager has to have some specific knowledge before quantitative tools can be used: specification of organizational goals; knowledge about possible actions; knowledge about possible likelihood; knowledge about expected payoffs and specification of inferior on which decision will be made.

Activity

1. What are stochastic models?
2. What is decision theory?
3. What are the applications of decision theory?
4. What are the practical applications of networks?

CHAPTER -8

DECISION THEORIES

Decision is the rational choice among alternatives. Decision making is the universal concern of every rational being. It is the heart of an organization's management. We cannot think of decision in the absence of options. i.e. if there is no alternative, there is no decision making. It is also known that decision is a means not an end by itself.

Decision making is an intellectual process which consists of a sequence of steps. Let's look at these steps using example of Muger Cement Factory considering several alternative methods for expanding its production to accommodate an increase demand for its products.

Step 1 Define the objective and identify the problem

Any decision is purposeful and hence the manager has to establish specific objectives to be achieved. A necessary condition for a decision is a problem. If there is no problem, there is no decision. Well defined problem is half solved.

Step 2 List all the viable alternatives

The second action a decision maker must take is to list all the possible options that must be considered in the decision. In the case of our Muger Cement Factory, the company planners indicate that only three viable options are open to the company.

Option one: Expand the present plant

Option two: Build a new plant

Option three: Sub-contracting extra production to other factory.

Step 3 Evaluate each alternative

In this stage, once the viable alternatives are identified the decision maker has to evaluate and compare the merit and demerit of each viable alternative. In the case our example, the company planners agree that they must add capacity. If they don't, losses will be so great that doing nothing can't be considered an alternative.

Step 4 Identify the future events that may occur

Having identified all the viable alternatives and get evaluated, the decision maker must now list the future events affecting demand that may occur. In our example, the most significant future events concerned demands for the product are listed as follows:

State of nature 1: High demand (resulting high product acceptance)

State of nature 2: Moderate demand (reasonable product acceptance but heavy competitive response)

State of nature 3: Low demand (resulting from low product acceptance)

State of nature 4: Failure (resulting from no acceptance of product)

Step 5 Construct a payoff table/Decision Tree/ Influence Diagram

This step deals with the construction of payoff tables, which show the payoffs that would result from each possible combination of the decision alternatives and state of nature.

Viable alternatives	State of nature			
	High	Moderate	Low	Failure
Expand				
Build				
Sub-contracting				

Step 6 Make a choice

Once the alternatives are identified and the different states of nature are clearly specified, the last step is selecting the best decision alternative based on the decision criteria and then it will be ready for implementation.

8.1 Decision Making Criteria

To select the best decision alternative, you should depend on the decision criteria. The decision criteria; however differs from the different conditions. Let's look at some of the decision criteria of the three decision environments.

8.1.1 Decision criteria under certainty condition

Decision under certainty condition is a decision environment in which only one state of nature exists. i.e. there is complete certainty about the future. Under this condition, it is easy to analyze the situation and make good decision. Although this environment sometimes exists, it is usually uncommon to the business world and it is impossible to guarantee complete uncertainty about the future.

8.1.2 Decision criteria under risk condition

The decision environment in which the decision maker has information supporting the assignment of probability values to each of the possible state of nature. In this situation, there are more than one state of nature. This kind of decision environment is the most common in the business world. The information that the manager have can be past records or subjective judgments. There are three criteria for decision making under condition of risk: Expand value criterion, criterion of rationality and criterion of maximum likelihood.

Expected value criterion: A criterion which requires the decision maker to calculate the expected value for each decision alternative and choose that decision which has the highest (lowest) expected value among all of the available decision alternative.

Criterion of Rationality: In decision-making situations where we have little or no data on past demand, we can apply the criterion of rationality (also

called the principle of insufficient reason). This criterion says that in the absence of any strong information to the contrary, you might as well assume that all states of nature equally likely.

Maximum likelihood criterion: A criterion which selects that the state of nature which has the highest probability of occurring and picks the decision alternative which yields the highest payoff for that state. This decision criteria is rather widely used and will produce valid results when are state of nature is much more probable than any other; however, it is possible to make some serious errors if we use this criterion in a situation where a large number of states of nature exist and each of them has a small, nearly equal probability of occurrence.

8.1.3 Decision criterion under uncertainty

This is the decision environment in which more than one state of nature exists but in which the decision maker cannot assign probabilities to the various states i.e. the decision maker has no knowledge about the various states, not even sufficient knowledge to permit the assignment of probabilities to the states of nature. The criteria used for making decision under the condition of uncertainty includes the maximax criteria, maxmini criteria and minimax regret criteria.

Maximax criterion: An optimistic criterion that maximizes the maximum payoffs. In case of cost, minimize the minimum payoffs.

Minimax regret criterion: A criterion that selects that decision alternative which minimizes the maximum regret. Regret or opportunity loss is the difference between the optimal payoff and the payoff experienced. This criterion is neither purely optimistic nor purely pessimistic in its approach.

Maxmini criterion: A pessimistic criterion that maximizes the minimum payoffs. It is different for cost cases.

8.2 Tools used to aid Decision Analysis

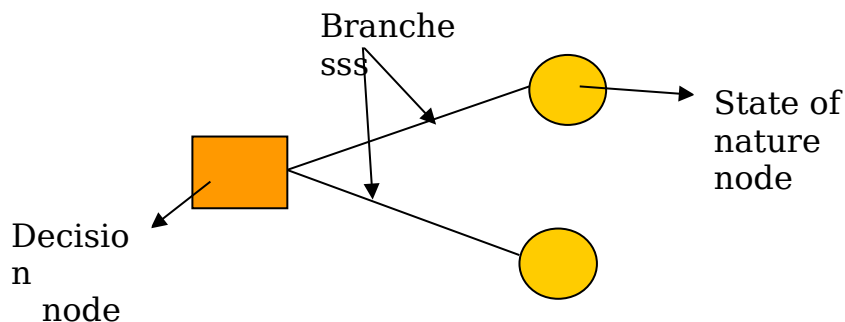
For the purpose of decision analysis, a number of tools are used by decision makers. Let's look at some of the most common tools that help the decision analysis process to be easier and understood.

8.2.1 Payoff Table

It is a table that illustrates the payoffs (outcomes) which would result from each possible combination of decision alternatives and state of nature in decision analysis terminology, the outcomes resulting from making a certain decision and the occurrence of a particular state of nature as the payoff. In general, entries in a payoff table can be stated in terms of profit, cost, or any other measure of output that may be appropriate for the particular situation being analyzed. The notation we will use for the entries in the payoff table is $v(d_i, s_j)$ which denotes the payoffs associated with decision alternative d_i and state of nature s_j .

8.2.2 Decision Tree

This is a graphical representation of the decision making process from decision to state of nature to payoffs. Decision tree indicates decision alternatives, states of nature, probabilities attached to the states of nature, and conditional benefits and losses. In decision analysis terminology, a node is a junction point at which branching takes place in a decision tree. The node can be decision node which is denoted by square or state of nature node which is denoted by a circle. Note that, the tree shows the natural or logical progression that will occur .i.e. first a form should make a decision; and then, once the decision is implemented, the state of nature will occur. The number of each end point of the decision tree represents the payoffs associated with a particular chain of events.

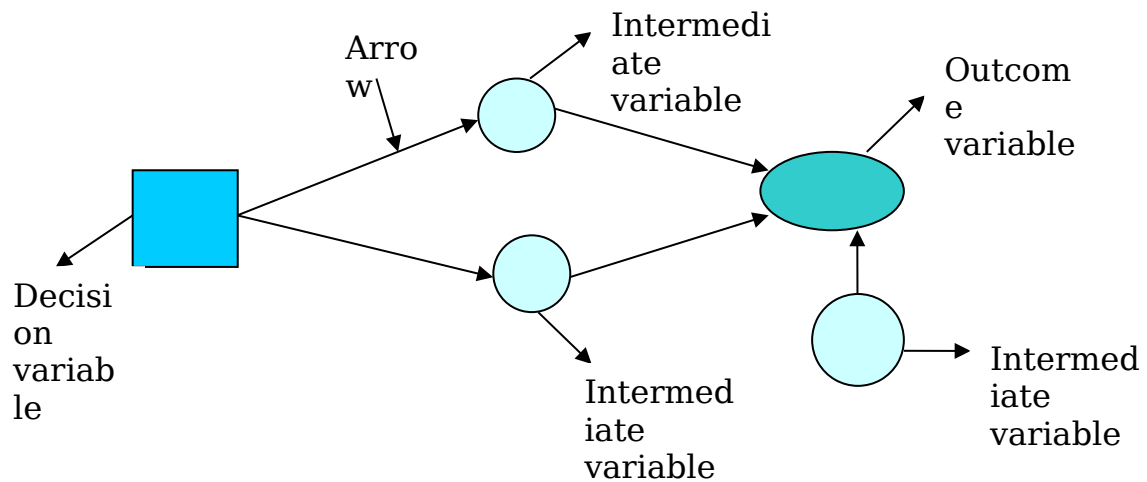


The merit of using decision analysis is as a tool in aiding a decision analysis process includes:

- ◆ Decision tree structures the decision process in an orderly, sequential fashion.
- ◆ Decision tree requires the decision maker to examine all possible outcomes.
- ◆ Decision tree communicates the decision making process to others.
- ◆ Decision tree allows a group to discuss alternatives by focusing on each individual financial figure, probability, and underlying assumptions—one at a time.
- ◆ Decision tree can be used with a computer.

8.2.3 Influence Diagram

Influence diagram is a graphical device which fills the gap between the problem and the mathematical model. In the decision analysis terminology, the word influence means dependency of a variable on the level of another variable. In the case of influence diagram, there are three variables: Decision variable (under the control of the decision maker) which is denoted by square; Intermediate variable (any variable that necessary to link the decision variable to the outcome) which is denoted by circle and Outcome variables (attributes used by the manager to measure the performance) which are denoted by oval shape.



- ❖ Influence diagram can be developed from the decision tree. Unlike, the decision tree, influence diagram is flexible, expandable and it can indicate a wider range of complex relationships among different variables.

8.3 DECISION MAKING ENVIRONMENT

8.3.1 Decision Making Under Certainty

The decision environment, in which any one state of nature exists under conditions of complete certainty, it is easy to analyze the situation and make good decision. For example, if the manager knows for sure that the demand of a product is moderate, he/she will choose the decision alternative that maximizes the payoff under the moderate demand state of nature.

8.3.2. Decision Making Under an Uncertain Environment

As we have already seen, decision-making environment is categorized in to two: decision-making under certain environment, and decision-making under uncertainty or risk-filled environment. This section explains the second environment in which decision is made under uncertain situations, which is also the main essence of the **decision analysis model** of management science.

There are two ways of approaching the decision-making process under uncertain environment. The first one is decision-making without probabilities, and the other is decision-making using probability. We shall see each of them in detail as below.

8.3.2.1 Decision Making Without Probabilities

In situations where the decision-maker has little confidence in his or her ability to assess the probabilities, or in which a simple best-case or worst-case analysis is required, there are decision-making approaches that do not require application of probabilities. These approaches do not require

knowledge of probabilities. The following three non-probability approaches are presented below and we shall see them in detail.

Optimistic /Maximax/ Approach

The optimistic approach evaluates each decision alternative in terms of the **best payoff** that can be obtained. The decision alternative that is recommended is the one that provides the best possible payoff. For a problem in which maximum profit is desired, the optimistic approach would lead the decision maker to choose the alternative corresponding to the largest profit. For problems involving minimization, this approach leads to choosing the alternative with the smallest payoff. Let us see the application of this approach by taking the following example.

Suppose XYZ Company, a real estate developer, is encountered with three decision alternatives to construct different-sized condominium complex. The decision alternatives are to construct buildings with 30 condominiums, 60 condominiums or 90 condominiums. The objective of XYZ Company is to choose the size of the complex that provides the maximum profit given the uncertainty concerning the demand for the condominiums.

Given the statement of the problem, the decision is to select the best size for the condominium complex that will give the maximum profit to XYZ Company. The decision alternatives are presented below:

- d₁ a small complex with 30 condominiums
- d₂ a medium complex with 60 condominiums
- d₃ a large complex with 90 condominiums

A factor in selecting the best decision alternative is the uncertainty associated with the chance event concerning the demand for the condominiums. XYZ Company identified two possible chance-event outcomes: a strong demand and a weak demand. In a decision analysis, possible outcomes for chance events are referred to as the **states of nature**. The states of nature are defined so that one and only one of the possible states of nature will occur. In our example, the chance event concerning the demand for the condominiums has two states of nature:

- s₁ strong demand for the condominiums
- s₂ weak demand for the condominiums

Therefore, the summary of the problem is to select a decision alternative (size of the complex). Once the size of the complex is chosen, then XYZ's profit will depend on the outcome of the states of nature (strong demand or weak demand).

Continuing with the example, suppose that the management of XYZ Company has a preliminary information on the possible profit levels corresponding to the three decision alternatives in each of the states of nature. The data is presented in the payoff (Table A) below:

Table A Payoff Table For Xyz Condominium Project
(Profits In \$ Million)

Decision Alternative	State of Nature	
	Strong Demand (s_1)	Weak Demand (s_2)
Small complex (d_1)	8	7
Medium complex (d_2)	14	5
Large complex (d_3)	20	-9

To illustrate the optimistic approach, we first determine the maximum payoff for each decision alternative; then we select the decision alternative that provides the overall maximum payoff. These steps systematically identify the decision alternative that provides the largest possible profit. Table A.1 below shows these steps.

TABLE A.1 MAXIMUM PAYOFF FOR EACH DECISION ALTERNATIVE

Decision Alternative	Maximum Payoff	
Small complex (d_1)	8	
Medium complex (d_2)	14	
Large complex (d_3)	20	← Maximum of the maximum payoff values

Because 20, corresponding to d_3 , is the largest payoff, the decision to construct the large condominium complex is the recommended decision alternative using the optimistic approach.

Conservative/Maximin/ Approach

The conservative approach evaluates each decision alternative in terms of the **worst payoff** that can occur. The decision alternative recommended is the one that provides the **best of the worst** possible payoff. For a problem in which the output measure is profit, as in our example, the conservative approach would lead the decision maker to choose the alternative that maximizes the minimum possible profit that could be obtained. For problems involving minimization, this approach identifies the alternative that will minimize the maximum payoff.

To illustrate the conservative approach, we first identify the minimum payoff for each decision alternative; then we select the decision alternative that maximizes the minimum payoff. Table A.2 below shows these steps for XYZ Company.

Table A.2 Minimum Payoff For Each Decision Alternative

Decision Alternative	Minimum Payoff	
Small complex (d_1)	7	← Maximum of the minimum

		payoff values
Medium complex (d_2)	5	
Large complex (d_3)	-9	

Because 7, corresponding to d_1 , yields the maximum of the minimum payoffs, the decision alternatives of a small condominium complex is recommended. This decision approach is considered conservative because it identifies the worst possible payoffs and then recommends the decision alternative that avoids the possibility of extremely 'bad' payoffs.

Minimax Regret Approach

Minimax regret is an approach to decision making that is neither purely optimistic nor purely conservative. To illustrate this approach, we continue to use the example of XYZ Company's decision alternatives.

Suppose XYZ constructs a small condominium complex (d_1) and demand turns out to be strong (s_1). From our original Table A, it is shown that the resulting profit would be \$8 million. However, given that the strong demand state of nature (s_1) has occurred, we realize that the decision to construct a large condominium complex (d_3), yielding a profit of \$20 million, would have been the best decision. The difference between the payoff for the best decision alternative (\$20 million) and the payoff for the decision to construct a small condominium complex (\$8 million) is the **opportunity loss**, or **regret**, associated with decision alternative d_1 when state of nature s_1 occurs. For this case, the opportunity loss or regret is \$12 million (\$20 million - \$8 million).

The following expression represents the opportunity loss or regret.

$$R_{ij} = V_j - V_{ij}$$

where

R_{ij} = the regret associated with decision alternative d_i and state of nature s_j

V_j = the payoff value corresponding to the best decision for the state of nature s_j

V_{ij} = the payoff corresponding to decision alternative d_i and state of nature s_j

Applying the above equation on our example in Table A, we can compute the regret associated with each combination of decision alternative d_i and state of nature s_j . Table A.3 below shows the opportunity loss, or regret, table for XYZ Company.

Table A.3 Opportunity Loss, Or Regret, Table For Xyz Condominium Project (\$ Million)

Decision Alternative	State of Nature	
	Strong Demand (s_1)	Weak Demand (s_2)
Small complex (d_1)	12	0

Medium complex (d_2)	6	2
Large complex (d_3)	0	16

The next step in applying the minimax regret approach is to list the maximum regret for each decision alternative, as shown in Table A.4 below. Selecting the decision alternative with *minimum* of the *maximum* regret values yields the minimax regret decision.

Table A.4 Maximum Regret For Each Decision Alternative

Decision Alternative	Maximum Regret	
Small complex (d_1)	12	
Medium complex (d_2)	6	← Minimum of the maximum regret
Large complex (d_3)	16	

The three approaches discussed so far provide different recommendations. They simply show the difference in decision-making philosophies. The main drawback of these approaches is that they do not consider any information about the probabilities of the various states of nature.

8.3.2.2 Decision Making With Probabilities

In many decision-making situations, we can obtain probability assessments for the states of nature. When such probabilities are available, we can use the **expected value approach** to identify the best decision alternative.

The **expected value (EV)** of decision alternative d_i is defined as:

$$EV(d_i) = \sum_{j=1}^N P(s_j) V_{ij}$$

where

N = the number of states of nature

$P(s_j)$ = the probability of state of nature s_j

The probabilities must satisfy two conditions:

$$P(s_j) \geq 0 \quad \text{for all states of nature}$$

$$P(s_1) + P(s_2) + \dots + P(s_N) = 1$$

In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative. The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur.

To illustrate the application of the expected value approach, we assume an initial subjective probability assessment of 0.8 that demand will be strong (s_1) and a corresponding probability of 0.2 that demand will be weak (s_2). Using the EV formula, the expected value of each decision alternative will be as follows:

$$EV(d_1) = 0.8(8) + 0.2(7) = 7.8$$

$$EV(d_2) = 0.8(14) + 0.2(5) = 12.2$$

$$EV(d_3) = 0.8(20) + 0.2(-9) = 14.2$$

Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

Expected Value of Perfect Information

Suppose that XYZ Company has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature. To determine the potential value of this information, suppose that the study could provide **perfect information** regarding the states of nature; that is, we assume for the time being that XYZ could determine with certainty, prior to making a decision, which state of nature is going to occur. To make use of this perfect information, we will develop a decision strategy that XYZ should follow once it knows which state of nature will occur. A **decision strategy** is a decision rule that specifies the decision alternative to be selected after new information becomes available.

To help determine the decision strategy for XYZ, we again use Table A. If XYZ knew for sure that state of nature s_1 would occur, the best decision alternative would be d_3 , with a payoff of \$20 million. Similarly, if XYZ knew for sure that state of nature s_2 would occur, the best decision alternative would be d_1 , with a payoff of \$7 million. Thus we can state XYZ's optimal decision strategy when the perfect information becomes available as follows:

If s_1 , select d_3 and receive a payoff of \$20 million.

If s_2 , select d_1 and receive a payoff of \$7 million.

Then the question follows: What is the expected value for this strategy? To compute the expected value with perfect information, we will take the previous probabilities for the states of nature: $P(s_1) = 0.8$, and $P(s_2) = 0.2$. Thus, there is a 0.8 probability that the perfect information will indicate state of nature s_1 and the resulting decision alternative d_3 will provide a \$20 million profit. Similarly, with a 0.2 probability for state of nature s_2 , the optimal decision alternative d_1 will provide a \$7 million profit. Thus, the expected value of the decision strategy that uses perfect information is:

$$0.8(20) + 0.2(7) = 17.4$$

We refer to the expected value of \$17.4 million as **the expected value with perfect information** (EVwPI).

Previously, we showed that the recommended decision using the expected value approach is decision alternative d_3 , with a expected value of \$14.2 million. Because this decision recommendation and the expected value computation were made without the benefit of perfect information, \$14.2 million is referred to as the **expected value without perfect information** (EVwoPI).

We have now obtained the expected value with perfect information as \$17.4 million, and the expected value without perfect information as \$14.2 million. Therefore, the expected value of the perfect information (EVPI) is \$17.4 - \$14.2 = \$3.2 million. In other words, \$3.2 million represents the additional expected value that can be obtained if perfect information were available about the states of nature.

Generally speaking, a market research study will not provide “perfect” information. However, if the market research study is a good one, the information gathered might be worth a sizable portion of the \$3.2 million. Given the EVPI of \$3.2 million, XYZ Company should seriously consider the market survey as a way to obtain more information about the states of nature.

In general, the **expected value of perfect information** is computed as follows:

$$EVPI = EVwPI - EVwoPI$$

where

EVPI = expected value of perfect information

EVwPI = expected value with perfect information about the states of nature

EVwoPI = expected value without perfect information about the states of nature

8.4 DECISION ANALYSIS WITH SAMPLE INFORMATION

In applying the expected value approach, we have seen how probability information about the states of nature affects the expected value calculations and thus the decision recommendation. Most of the time, decision-makers have preliminary or prior probability estimates for states of nature that are the best probability values available. However, to make the best possible decision, the decision-maker may want to seek additional information about the states of nature.

This new information can be used to revise or update the prior probabilities so that the final decision is based on better probability estimates for the states of nature. The seeking of additional information is most often accomplished through experiments designed to provide sample information about the states of nature. Raw material sampling, product testing, and market research studies are examples of experiments that may enable a revision or updating of the states of nature probabilities.

To illustrate the decision analysis approach, let us consider the case of a newly formed computer service which has the problem of selecting one of the three computer systems; large, medium, or small. They differ in size and capacity. The three decision alternatives are denoted by d1, d2 and d3, as shown in the payoff table below:

Decision Alternative	State of Nature	
	High Acceptance (s ₁)	Low Acceptance (s ₂)

Large system (d_1)	200,000	-20,000
Medium system (d_2)	150,000	20,000
Small system (d_3)	100,000	60,000

Suppose that management believes that s_1 has a 0.3 probability of occurrence and that s_2 has a 0.7 probability of occurrence. Thus $P(s_1) = 0.3$ and $P(s_2) = 0.7$. At this point, we will refer to these initial probability estimates as the prior probabilities for the states of nature. Using these probabilities, we find that d_3 , the decision to select the small system is optimal, yielding an expected value of \$72,000. From this figure, we can also expect that the value of the perfect information about the states of nature could potentially worth as much as $EVPI = \$30,000$.

Suppose that management of the newly formed computer service decides to consider hiring a market research firm to study the acceptance of that service. The market research study will provide new information that can be combined with the prior probabilities through the Bayesian procedure to obtain updated or revised probability estimates for the states of nature. These revised probabilities are called posterior probabilities. We refer to the new information obtained through research or experiment as an **indicator** and is often referred to as sample information.

Using the indicator terminology, we can denote the outcomes of the marketing research study as follows:

- I_1 = favorable market research report
- I_2 = unfavorable market research report

To make effective use of this indicator information, we must know something about the probability relationships between the indicators and the states of nature. For example, in our case problem, given that the state of nature ultimately turns out to be high customer acceptance, what is the probability that the market research study will result in a favorable report? In this case, we are asking about the conditional probability indicator I_1 , given state of nature s_1 , written as $P(I_1|s_1)$. To carry out the analysis, we will need conditional probabilities for all indicators given all states of nature,

i.e., $P(I_1|s_1)$, $P(I_1|s_2)$, $P(I_2|s_1)$, and $P(I_2|s_2)$.

In our problem, assume that the following estimates are available for the conditional probabilities:

State of Nature	Market Research Report	
	Favorable (I_1)	Unfavorable (I_2)
High acceptance (s_1)	0.8	0.2
Low acceptance (s_2)	0.1	0.9

Note that these probability estimates indicate that a good degree of confidence can be placed in the market research report.

8.4.1. Developing a Decision Strategy

a decision strategy is a rule that is to be followed by the decision maker. In our case problem, it is a rule that recommends a particular decision based on whether the market research report is favorable or unfavorable.

Using the decision tree terminology, we have now introduced an indicator node, node 1, and indicator branches, I1 and I2. Selecting the best decision branch is equivalent to making the best decision. All states of nature probabilities will be expressed in a $P(s_j|I_k)$ form.

An Optimal Decision Strategy

An optimal decision strategy is the use of the expected value (EV) approach to determine the decision strategy. Working backward through the decision tree, we first compute the EV of each state of nature node. Since the decision maker controls the branch leaving a decision node and since we are trying to maximize expected profits, the optimal decision at node 2 is d1, since d1 leads to an EV of \$150,324. A similar analysis of decision node 3 shows that the optimal decision branch at this node d3, with EV of \$63,480.

As a final step, we continue working backward to the indicator node and establish its EV. We see that since node 1 has probability branches, we can not select the best branch. Rather, we must compute the EV. Thus, we have,

$$\begin{aligned} \text{EV}(\text{node 1}) &= (0.31) \text{EV}(\text{node 2}) + (0.69) \text{EV}(\text{node 3}) \\ &= (0.31) (150,324) + (0.69) (63,480) \\ &= \$90,402. \end{aligned}$$

The value \$90,402 is viewed as the EV of the optimal decision strategy when the market research study is used.

The results of the decision analysis at this point have provided the following optimal decision strategy:

If report favorable (I1), then select large system (d1)

If report unfavorable (I2), then select small system (d3)

Since the additional information provided by the market research firm will result in added cost in terms of the fee paid to the research firm, the management of the newly formed computer service may question the value of this market research information.

The value of sample information is often measured by calculating what is referred to as the EV of sample information (EVSI).

$$\text{EVSI} = \begin{array}{cc} \text{EV of the optimal decision} & - \text{EV of the optimal decision} \\ \text{with sample information} & \text{without sample information} \end{array}$$

The decision tree calculations indicated that the EV of the optimal decision with the market research information was \$90,402, thus, this value is the EV with sample information; i.e., $\text{EV}_{\text{wSI}} = \$90,402$. When sample information is not available, the EV approach resulted in recommending decision alternative d3 with an EV of \$72,000; thus, this value is the EV without sample information; i.e., $\text{EV}_{\text{woSI}} = \$72,000$. Therefore,

$$\text{EVSI} = \$90,402 - \$72,000 = \$18,402.$$

Thus, the newly formed computer service should be willing to pay up to \$18,402 for the market research information.

Efficiency of Sample Information

From our previous analysis of EV of perfect information, we know that EVPI was \$30,000. While we never expected the market research report to obtain perfect information, we can use an efficiency measure to express the value of the market research information. With perfect information having an efficiency rating of 100%, the efficiency rating E for sample information is computed as follows:

$$\begin{aligned}\text{Efficiency of sample information (E)} &= (\text{EVSI}/\text{EVPI}) \times 100 \\ &= (\$18,402/\$30,000) \times 100 \\ &= 61.3\%\end{aligned}$$

This means that, the information from the market research firm is 61.3% as efficient as perfect information.

Low efficiency ratings for sample information (SI) might lead the decision maker to look for other types of information. On the other hand, high efficiency ratings indicate that the SI is almost as good as perfect information (PI), and additional sources of information should not be worthwhile.

8.5. SENSITIVITY ANALYSIS

Analyzing decisions under risk requires working with estimated values: Both the payoff and the probabilities for the states of nature are typically estimated values. Inaccuracies in the estimates can have an impact on the choice of an alternative, and ultimately, on the outcome of the decision. Sensitivity analysis can be used to determine how changes in the probabilities for the states of nature and/or changes in the payoffs affect the recommended decision alternative. In many cases, the probabilities for the states of nature and the payoffs are based on subjective assessments. Sensitivity analysis helps the decision maker understand which of these inputs are critical to the choice of the best decision alternative. If a small change in the value of one of the decision analysis problem is sensitive to that particular input. Extra effort and care should be taken to make sure the input value is accurate as possible. On the other hand, if a modest to large change in the value of one of the inputs does not cause a change in the recommended decision alternative, the solution to the decision analysis problem is not sensitive to that particular input. No extra time or effort would be needed to refine the estimated input value. Sensitivity analysis helps you to identify the most important parameters (the probabilities and outcomes) in the model.

8.5.1. Algebraic Presentation

Sensitivity to changes in the probability of the states of nature. One approach to sensitivity analysis is to select different values for the probabilities of the states of nature and/or the payoffs and then resolve the decision analysis problem. If the recommended decision alternative changes, we know that the solution is sensitive to the changes made. The drawback to

this approach is the numerous calculations required to evaluate the effects of several possible changes in the state of nature probabilities.

Consider an example of a condominium project shown in the table below.

Table 1. Payoff (in \$ million) using expected value approach for a condominium project.

Alternative	State of Nature (Demand)		Expected Value
	S_1 (strong)	S_2 (Weak)	EV
small complex (a ₁)	8	7	7.8
Medium complex (a ₂)	14	5	12.2
Large complex (a ₃)	20	-9	14.2
Prior probability	0.8	0.2	

The revised EV values for the three decision alternatives are

$$EV(a_1) = 0.2(8) + 0.8(7) = 7.2$$

$$EV(a_2) = 0.2(14) + 0.8(5) = 6.8$$

$$EV(a_3) = 0.2(20) + 0.8(-9) = -3.2$$

From the results, it can be shown that the recommended alternative for the project would be to construct small condominium (a₁). The change in the probabilities of the states of nature has affected the previous assessment; i.e., alternative (a₃) is highly sensitive to the change among the other alternatives.

8.5.2. Graphical Presentation

For the special case of two states of nature, a graphical procedure can be used to determine how changes for probabilities affect the recommended decision alternative. As illustrated in the example below

The two states of nature in table1 s_1 and s_2 have probabilities of occurrence $p(s_1)$ and $p(s_2)$ respectively.

Let $p(s_1) = p$, then $p(s_2) = 1 - p$. The expected value of three alternatives

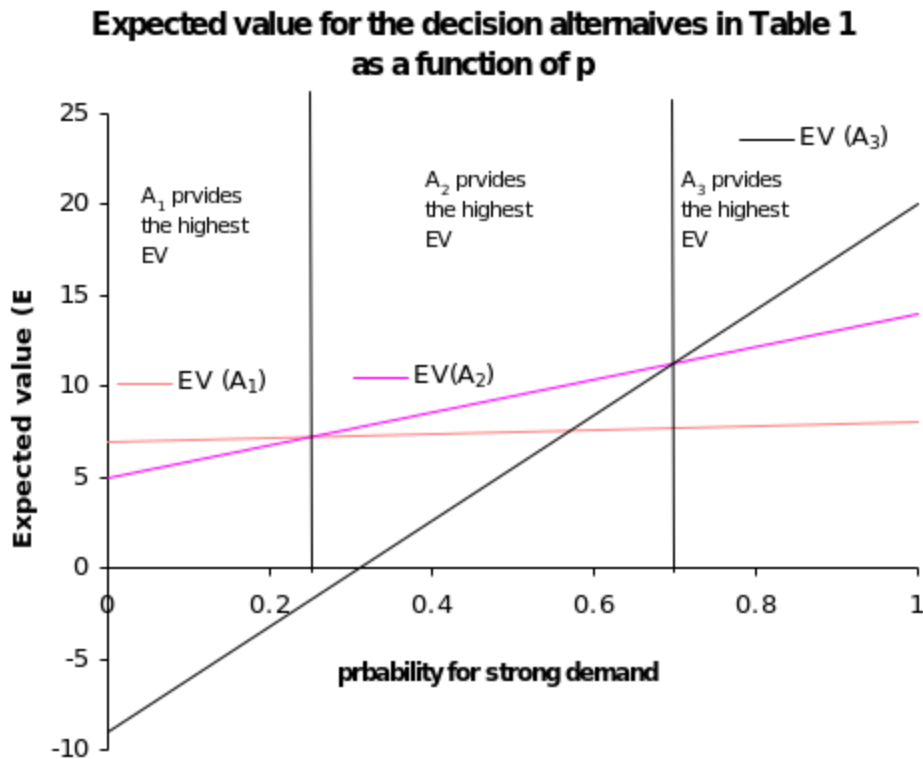
$$EV(a_1) = p + 7$$

$$EV(a_2) = 9p + 5$$

$$EV(a_3) = 29p - 9$$

Plotting the graph for this values, we can conclude that the decision alternative A1 provides the largest value for $p \leq 0.25$, decision alternative A2

provides the largest expected value for $0.25 \leq p \leq 0.70$, and decision alternative A3 provides the largest expected value for $p \geq 0.7$. Because p is the probability of the state of nature s_1 and $(1-p)$ is the probability of the state of nature s_2 , we now have the sensitivity analysis information that tell us how changes in the state of nature probabilities affect the recommended decision alternative.



Graphical sensitivity analysis shows how changes in the probabilities for the states of nature affect the recommended decision alternative. Sensitivity analysis can assist management in deciding whether more time and effort should be spent obtaining better estimates of payoffs and probabilities.

8.6. UTILITY AND GAME THEORY

8.6.1 Utility

Although monetary value is a common basis for decision making, it is not the only basis, even for business decisions. In certain instances, decision makers use multiple criteria, one of which is the potential satisfaction or dissatisfaction associated with possible payoffs.

For example, a great many people participate in state lottery. However, from a strict standpoint, lotteries have a negative expected value; the expected return is less than the cost of buying the lottery. If it were not, the states would lose money by running lotteries. Why do people play lotteries then? The answer is that they are hoping to win a large amount of money, and they

are willing to sacrifice a relatively small amount of money to have that chance. In other words, even though their chances of winning are close to zero, they have a great utility for the potential winnings despite a negative expected value than for the amount of money they have to give up (pay) to participate in the lottery. Similar arguments can be made for other forms of wagering. People who behave in this fashion, whether for purposes of wagering or in other forms of decision making, are sometimes referred to as **risk takers**.

Just the opposite happens when a person buying insurance giving up a fixed dollar amount to insure against an event (eg., a fire accident) that has very little chance of occurring. Even so, if a fire or other insured event did occur, the consequences would be so catastrophic that an individual would not want to be exposed to that degree of risk. Thus, although buying insurance carries a negative monetary value, most individuals recognize the merit of doing so. We refer to such individuals as risk averters. Of course, some individuals exhibit both forms of behavior in their decision making; they are risk takers for certain kinds of decisions, but risk averters for others. A lottery player who owns a life insurance policy would be an example of this.

Thus, utility is a measure of the potential satisfaction derived from money. Although utility can be an important factor in certain kinds of decision making, assessing and using utility values can be rather complex. Not only does utility varies with an individual for different types of situations, but it also seems to vary among individuals for the same situation. That is, different people might choose different alternatives in a given instance because of utility considerations.

8.6.2 Game Theory

Game theory deals with decision situations in which two intelligent opponents have conflicting objectives. Typical examples include launching advertisement campaigns for competing products, and planning war strategies for opposing armies. These situations are in contrast with the ones we have studied so far, where nature is viewed as a non-malevolent opponent.

In a game conflict, two opponents, known as **players**, will each have a finite or infinite number of alternatives or strategies. Associated with each pair of strategies is a payoff that one player pays to the other.

In games, the success of one player is at the expense of the other. Each participant selects and executes those strategies, which he believes will result in his "winning the game". In games, the participants make use of deductive and inductive logic in determining a strategy for winning. The theory of games is of interest to us for this reason.

8.6.2.1. Two-Person Zero-Sum Games

Our treatment of the theory of games will be limited to two-person games, that is, situations of conflict where there are only two participants. Many management situations obviously involve the participation of many persons and thus are not examples of two-person games. The mathematics for three-person and larger games, however, is too complex to include in this paper.

We should remember that the basis underlying the generation of optimal or winning strategies in conflict situations must respect the same principle of inductive and deductive logic, regardless of the number of participants. A two-person game is illustrated as follows:

If a player A has m strategies represented by the subscripted letter $A_1, A_2, A_3, \dots, A_m$ and player B has n strategies represented by the subscripted letter $B_1, B_2, B_3, \dots, B_n$. The number m and n need to be equal. The total number of possible outcomes is therefore, $m \times n$. Here it is assumed that each player knows not only his own list of possible courses of action but also, of his opponent. For convenience, it is assumed that player A is always a gainer whereas player B is a loser. Let a_{ij} be the pay off which player A gains from player B if player A chooses strategy i and player B chooses strategy j . Then the payoff matrix is shown below:

Player A's strategy	Player B's Strategy		
	B_1	$B_2 \dots$	B_n
A_1	a_{11}	$a_{12} \dots$	a_{1n}
A_2	a_{21}	$a_{22} \dots$	a_{2n}
.	.	.	.
.	.	.	.
.	.	.	.
A_m	a_{m1}	a_{m2}	a_{mn}

By convention, the rows of the payoff matrix denote player A's strategies and the column denote player B's strategies. Since player A is assumed to be the gainer always, so he wishes to gain as large a payoff a_{ij} as possible while player B will do his best to reach a small a value of a_{ij} as possible. Of course, the gain to player B and loss to A must be $-a_{ij}$.

Example: For the game with payoff matrix

Player A's	Player B's		
	B_1	B_2	B_3
A_1	-1	2	-2
A_2	6	4	-6

- Determine the best strategies for player A and B.
- Determine the value of game
- Is the game fair? Strictly determinable?

Solution:

In this example, gains to player A or losses to player B are represented by the positive quantities. Whereas, losses to A and gains to B are represented by negative quantities. It is assumed that A wants to maximize his minimum gains from B. Since the payoffs given in the matrix are what A receives, therefore, he is concerned with the quantities which represent the row minimums. Now A can do now worse than receive one of these values, and best of them occurs when he chooses strategy A_1 . This choice provides a payoff of -2 to A when B chooses strategy B_3 . This refers to A's choice of A_1

as his maximum payoff strategy because this row contains the maximum of A's minimum possible payoff from his competitor B.

Player A's	Player B's			Row minimum	
	B ₁	B ₂	B ₃		
A ₁	-1	2	-2 min	-2	maxi
A ₂	6	4	-6	-6	
A ₃					
Column maximum	6	4	-2 ← Mini max		

Similarly, it is assumed that B wants to minimize his losses and wishes his losses to A be as small as possible. There are column maximums that represent the greatest payments, B might have to make to A. The smallest of these losses is -2, which occurs when A chooses his course of action, A₁ and B chooses his course of action B₃. This choice of B₃ by B is his mini max loss strategy because; this column amount is the minimum of the maximum possible losses. In the immediately preceding table, the quantity -2 in the A₁ row and B₃ column is enclosed both in a box and a circle. That is, it is both the minimum of the column maxima and the maximum of the row minima. This value is referred to as the saddle point.

The payoff amount in the saddle point position is also called value of the game. For this game value of the game $V = -2$, for player A. The value of the game is always expressed from the point of view of the player whose strategies are listed in the rows. The game is strictly determinable. Also since the value of the game is not zero, the game is not fair.

Exercise:

Assume that two firms are competing for market Share for a particular product. Each firm is considering what promotional strategy to employ for the coming period. Assume that the following payoff matrix describes the increase in market share of firm A and the decrease in the market share of firm B. Determine the optimal strategies for each firm.

Firm A	Firm B		
	No promotion	Moderate promotion	Much promotion
No promotion	5	0	-10
Moderate Promotion	10	6	2
Much promotion	20	15	10

- i. Which firm would be the winner in terms of market share?

- ii. Would the solution strategies necessarily maximize profits for either of the firm?
- iii. What might the two firms do to maximize the profit?

CHAPTER -9

NETWORK TECHNIQUE

A project consists of number of activities involving the consumption of labour, time, material and money etc. Many of these activities will be interdependent and the rest can be carried out independently in order to achieve the desired objectives. The project management should not only plan, schedule, co-ordinate and optimize the activities of the various participants in the project effectively but should also identify and watch closely those critical activities which must be completed within their allotted duration, otherwise, the total project completion time will require extension. On the there hand, management should take advantage of those activities which have some spare time and allow flexibility of planning and economic utilization of the resources.

Unless the project happens to be a very small one, the number of activities involved will be so large that it will be very difficult for the management to keep effective control. Therefore, to deal with large and complex projects or works, a number of management techniques called as 'Net-Work Techniques' have been developed.

A few of the more popular techniques are:

- a) Critical Path Method (C.P.M.)
- b) Program Evaluation and Review Technique. (P.E.R.T.)
- c) Resource allocation and Multi-Project Scheduling (R.A.M.P.S.)

Among these, C.P.M. and P.E.R.T. are considered to be most convenient for application to various engineering projects.

The use of this management technique of modern network system has become very essential in the developing countries. This was developed in 1957 and is suitable for the construction of engineering projects and for scheduling plant maintenance etc. C.P.M. was first used by the research team lead by Morgan R. Walker to determine how best to reduce the time required to perform routine plant overhauling, maintenance and construction work.

With the help of C.P.M., a manager can know that which operation should be started after completing a particular operation and what is the status of the work as related to the scheduled timings. Although, an experienced person may know it but this information, available from the network, helps all concerned. It also tells that at what moment and in which operations, time

schedules are adhered strictly so that future programme is not affected and where it can be delayed and to what extent, without affecting the last date of completion of project. The network can be prepared in the following phases:

- Preparation of network.
- Estimation of expected time to perform each activity.
- To compute the critical path schedule.
- Interpretation of results.

OBJECTIVES OF NETWORK ANALYSIS

Network analysis entails a group of techniques for presenting information relation to time and resources so as to assist in the planning, scheduling, and controlling of projects. The information usually represented by a network includes the sequences, interdependencies, interrelationship, and criticality of various activities of the project. A network analysis has following objectives:

1. Powerful tool of planning, scheduling and control.
2. Shows the inter-relationships of the activities of a project or a programme.
3. Minimize total cost where the cost of delays and cost of resources required to carry out the tasks can be measured.
4. Minimize total time where required e.g. in maintenance of production line machinery in a factory.
5. Minimization of idle resources.
6. Minimize production delays
7. To provide systematic approach in planning and scheduling.
8. Follow as integrated approach and bring about better coordination between the departments.
9. Focuses attention on critical activities of the project.
10. Provides up-to-date status information.
11. Suggests areas for increasing efficiency, reduction of cost.

Applications Of Network Techniques

As explained earlier, network analysis is the systematic application for planning, scheduling, monitoring and controlling projects or programmes.

Network techniques can be applied to chain of events involving the use of time, labour and physical resources.

Some of other important applications are:

- a) Construction projects

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- b) planning
- c) Assembly line scheduling
- d) Budget and auditing
- e) Programmers involving several activities
- f) Installation of a complex new plant/ equipment
- g) Research and development
- h) Market penetration programmes
- i) Strategic and tactical planning

- j) Maintenance and overhauling complicated equipment in the plants/factories.
- k) Organizing of large functions etc.

BASIC NETWORK CONSTRUCTION

Activity and Events

Activity. A project consists of several activities or operations, and the project is said to be completed when all such activities are completed. Activity is the actual performance of task. This consumes resources, i.e. time, manpower, space, material, money etc.

Event. It is a start or completion of a task. In other words, an event is an instantaneous point of time at which activity or activities start or finish. This do not consume time or resources.

Following example will clarify the differences between an event and activity:

Start Machine Installation: An activity

Completion of Machine installation: An event

CRITICAL PATH METHOD (C.P.M.)

Mainly there are two systems of network construction:

1. AOA system, i.e., 'Activity on the Arrow system'.
2. AON system, i.e., 'Activity on the Node system'.

To avoid confusion, the discussion here will focus primarily on the activity-on-arrow convention.

A.O.A. SYSTEM

In the 'Activity on the Arrow system' (Arrow Diagram Method) an activity is graphically represented by an 'arrow' as shown on the left -side of Fig. Fig 9.1

The tail end of the arrow represents the start and the head of the arrow represents the finish or end of the activity. The description of the activity is written above the arrow. Events are represented by circles or nodes at the beginning and the end of activity arrow/ arrows as shown on the right hand side of Fig 9.1 . An event is a junction of two or more activities.

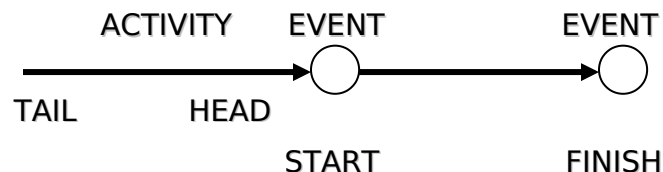


Fig.9.1. Representation of activity and event in AOA system.

The simple network shown in Fig. 9.2 clarifies the representation of activities and events. In any project, certain activities have to be completed before the commencement of certain other activities. Some of the activities can be started concurrently. If all the activities and events in the project are graphically represented showing their interdependencies, then a Network is obtained. Such a network represents the plan for the entire project.

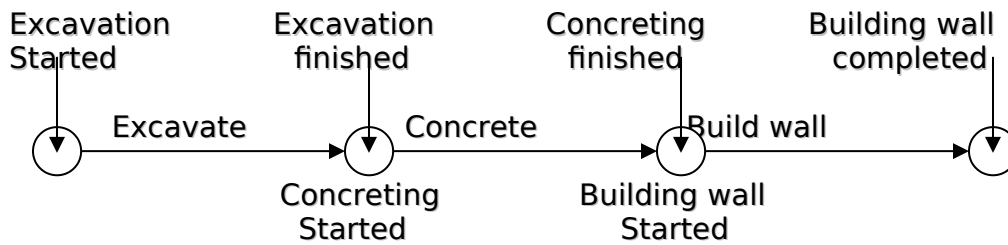


Fig. 9.2 A simple Network.

RULES FOR NETWORK CONSTRUCTION

- The length of the arrow bears no relationship to the time which the activity takes or the resources which the activity consumes.
- The arrow in a net work identifies the logical conditions of dependency. The geometry of arrow diagram is of no significance except that it should be aesthetically pleasing.
- The direction of the arrow indicates the direction of work flow. The normal convention is to go from left to right.
- All networks are constructed logically on the principle of dependency.
- No event can be reached in a project before the activity which immediately precedes is completed. Similarly no activity can be started until the event which immediately precedes it, has been reached.
- Every activity in the net work should be completed to reach the objective or the end event.
- No set of activities can form a circular loop.

Numbering the events and activity identification

Rules for numbering events and identification of activities are as follows:

- All events must be numbered.
- The same number cannot be used for more than one event. This is necessary in order to identify each activity.
- Events can be numbered in any order but usually forward numbering system is more familiar. In this method, first event in the network is numbered 1 (or zero) and the second 2 and so on. In such a way the event number at the tail of each activity arrow is always smaller than the event number at the head of that activity arrow. This implies that the final event of the network will have the largest number and will be equal to the number of the nodes in the network.

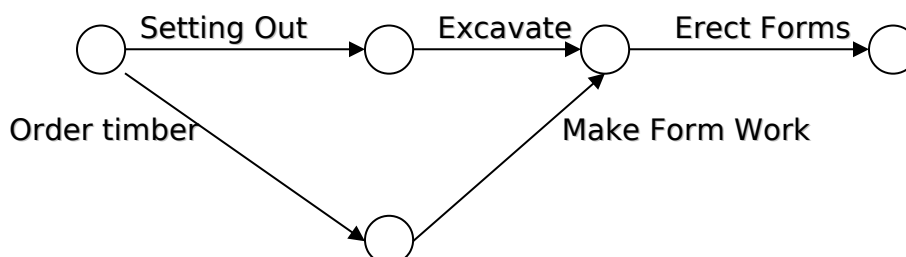


Fig.9.3. Numbering of events and activity identification.

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Fig. 9.3. illustrates the forward numbering of events and activities identification in a network.

Activity Description	Activity Identification
Setting out	1-2
Order timber	1-3
Excavate	2-4
Make Form Work	3-4
Erect Forms	4-5

Use of Dummy Activities

A dummy is an artificial activity introduced in a net work to:

- Maintain a unique numbering system for the different activities.
- Keep the logical sequence of activities and their interrelationships correct.

A dummy is shown by a dotted arrow as indicated in Fig 9.4 .



Fig 9.4 .

Dummy does not consume Time or Resources.

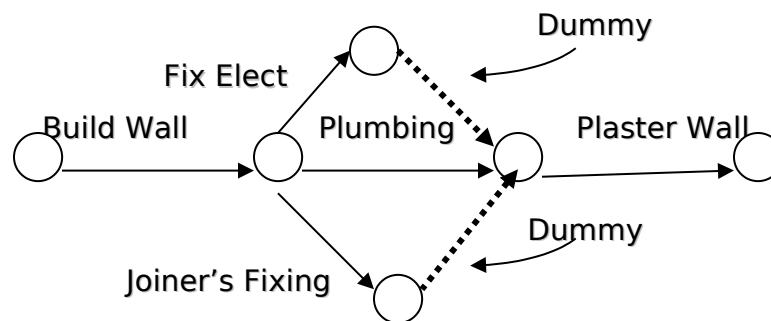


Fig. 9.5

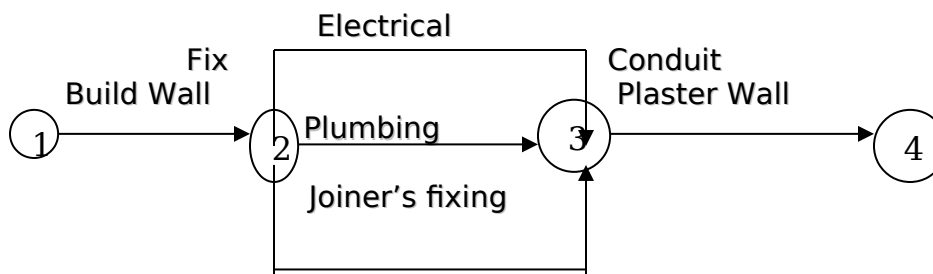
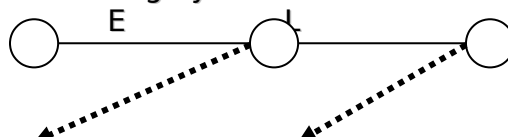


Fig 9.6 . Use of Dummies to identify activities.

Consider the network shown in Fig.9.5. Three of the activities in the network of electrical conduits, plumbing and joiner's fixing bear the same number 2 identification of activities is not unique. This situation can be corrected by using as shown in Fig. 9.6 . Now fixing electrical conduits is identified by 2-3, plumbing joiner's fixing by 2-4. Activities 3-5 and 4-5 are dummies.



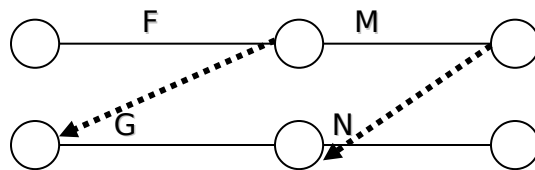


Fig.9.7 . Use of Dummies to show dependences.

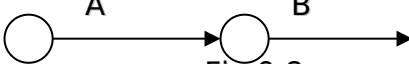
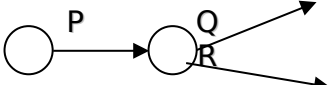
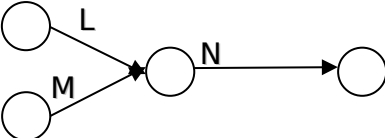
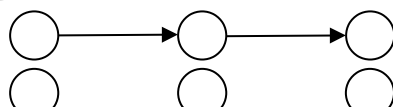
Consider the network shown in Fig.9.7 . Let EFG represent the shuttering on of a retaining wall and L,M and N, the corresponding concreting operations. In nature of the activities, it is reasonable to presuppose that in each case, the shutter precede concreting. As regards the labour, only one forward crew and one concrete available. Dummies are used here to represent the restraint and consequent interdependence.

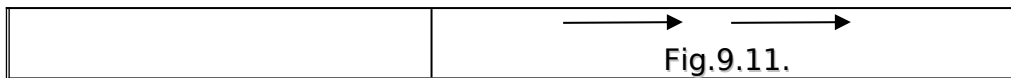
NET WORK DEVELOPMENT

In developing a network diagram, the following questions must be asked for each

- What activity must immediately precede this operation?
- What activity should immediately follow this operation?
- What activities can take place concurrently with this operation?

Based on these questions, each activity is connected with one or several of the way in Figs. 9.8. to 9.11.

Network Representation	Interpretation
a) Sequential	Activity B cannot start before activity A is completed or activity B follows activity A precedes B.  Fig 9.8
b) Diverging	Activity Q and R cannot start before the completion of activity P. Activity Q and R start at the same time.  Fig. 9.9
c) Converging	Activity N cannot start before both the activities L and M are completed.  Fig. 9.10
d) Parallel	In these activity chains, B follows A and F follows E but the activity chains are completely independent of each other. 



In the first instance, it is important to prepare a network that represents the time logical dependency of activities on one another, assuming availability of unlimited resources.

It is useful to show each arrow with a horizontal portion being used for activity description that should always be included in every network. This layout with horizontal zones representing either separate sections of the project or perhaps different types of work such as design, contracting, civil works, etc.

With a more deterministic system as compared to that in P.E.R.T. This point has been explained in the figure. The range and hence the uncertainty is more and when the range is small, and the system tends to be a more deterministic model.

How to Draw the Network Diagram

Before starting to draw the network, following measures should be taken serially:

1. Prepare a list of the activities included in the project and their dependencies on other activities.
2. Draft the network roughly, usually by soft pencil and an eraser on a large piece of paper.
3. The estimate of time required for each activity is made considering manpower and equipments available and in certain cases assumptions are based on statistical approach and experience.
4. These time estimates are then written on each activity.
5. Then scheduling computations are done to get earliest and latest allowable start and finish times for each activity, to identify critical path and to indicate the amount of slack on non-critical paths.
6. Now this is prepared in final form for use in the field. Project is controlled by checking the progress against the schedule.

ADVANTAGES OF CRITICAL PATH METHODS

Some of the important advantages are summarized here:

- It helps in ascertaining time schedules.
- With its help, control by the management becomes easy.
- It makes better and detailed planning possible.
- It encourages discipline.
- It provides a standard method for communicating project plans, schedules and cost performance.
- It identifies most critical activities and thus more attention can be paid on these activities. The CPM technique requires greater planning than required otherwise. Thus, this method increases the planning cost but this cost can easily be justified by concentrating attention on critical paths only and avoiding unnecessary expenses on the strict supervision over the whole programme.

PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

P.E.R.T. (Programme Evaluation and Review Technique) is a technique used for scheduling and controlling the projects. It is helpful in the programmes, where activities are subject to considerable degree of uncertainty in performance time. It is very essential for timely execution of the projects. Originally, it was used in designing and producing major Weapon Systems for the U.S. Military Establishments. Then, it was used in large concerns, which due to their size were using computers. After its becoming public property in 1959, producers also thought it to use due to the advantage that it has disciplined approach to planning, scheduling, controlling and monitoring the progress of variety of projects.

C.P.M uses only one-time estimate for each activity while P.E.R.T., uses three times estimates. But one time estimate is sufficient for small projects.

Difference between C.P.M. and P.E.R.T.

P.E.R.T. is a technique used for scheduling and controlling the projects, where activities are subject to considerable degree of uncertainty in the performance time. The methods of calculating time of start, finish, critical path and project time of P.E.R.T. and C.P.M. are similar, still there are some differences between the two. These differences are:

- (i) C.P.M. is the activity oriented technique while P.E.R.T. is the event oriented technique. Event means the beginning or ending of one or more activities. P.E.R.T. network diagram uses the line and arrow to connect the events (shown by circles) which are related to each other.

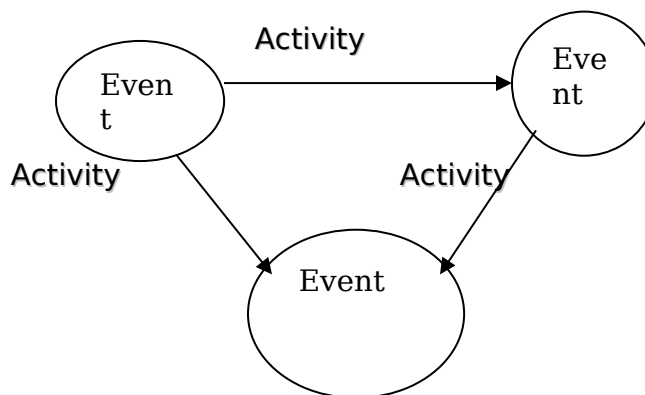


Fig.9.12 . Relation between event and activity.

- (ii). C.P.M. has one time estimate while P.E.R.T. has three time estimates.

Three-time Estimates for P.E.R.T.

P.E.R.T. is based on the assumption that operation time cannot be estimated accurately, because of the various uncertainties involved in the working. To take these uncertainties into account, the P.E.R.T. uses the following three time estimates:

NETWORK TECHNIQUES

- Optimistic time (a)
- Most Likely time (m)

- Pessimistic time (b)

Optimistic time. In this, it is assumed that every thing will go well, therefore it is the estimate of the minimum possible time, which an activity takes for completion under ideal conditions. In this, no provision is made for delays or breakdowns etc. Always conditions better than normal are assumed. This estimate of time is denoted by “a”.

Most likely time. It is the value, which will probably be the actual time. In this, it is assumed that things go in the normal way, with a few delays or set backs etc. This estimate of time, lies between the optimistic and pessimistic time. It is denoted by “m”

Pessimistic time. This time is based on the assumption that every thing will go badly. Thus, it is the maximum possible time taken for the completion of an activity. This does not include major catastrophes like labour strikes, acts of God and unrest etc. This is denoted by “b”

Expected time or Average time

According to B distribution, we can get the expected time (t_e) by adding together $\frac{1}{6}$ th of the optimistic, $\frac{3}{3}$ re of the most likely and $\frac{1}{6}$ th of the pessimistic time estimates. Thus:

$$t_e = \frac{1}{6}a + \frac{2}{3}m + \frac{1}{6}b$$

Or

$$t_e = \frac{a + 4m + b}{6}$$

Performance of these activities depends upon the several factors like weather, equipment and other personnel problems. To stick to schedule, if difficulties are experienced in any particular activity, then additional resources must be applied or specifications can also be related.

Equation (1) is important for P.E.R.T. analysis. It indicates as how to calculate the expected time from the three- time estimates. The expected time indicates that there is a fifty- fifty chance of getting the job done within that time.

Some Definitions used in P.E.R.T.

- 1) **Event.** It is the start or completion of a task. It is represented by a circle and do not consume time or resources.
- 2) **Activity.** It is the actual performance of a task. It consumes time and resources, such as manpower, space, material etc. It is represented by the line and arrow.
- 3) **Successor Event.** It is the event or events that immediately follow another event.
- 4) **Predecessor Event.** It is an event which comes immediately before another event.
- 5) **Earliest Expected Time (T_E).** It is the earliest expected completion time of the project and is equal to the sum of expected time (t_e) of all the activities along the longest path of the project.
- 6) **Latest Allowable Time (T_E).** It is the largest possible time an event can take with out delaying the final completion date.

T_L for an event = Total expected time for the end event of the project from longest path- total expected time for the event under consideration from longest path.

- 7) **Slack.** It is the difference of Latest Allowable Time (T_L) and Earliest Expected Time (T_E) i.e., $\text{Slack} = T_L - T_E$. It is the range of time available for the performance of an activity without delaying the completion date of the project. This value of slack indicates that how much critical that event is and how much stress has to be given on that activity.
- 8) **Critical Path.** This is a path on the network along which no slippage is allowed. In this path, slack is either negative or zero.
- 9) **Variance.** $\text{Variance} = \sigma^2 = \left\{ \frac{b-a}{6} \right\}^2$ or $\sigma^2 = \left\{ \frac{t_p - t_o}{6} \right\}^2$

Where, $a = t_o$ = Optimistic time; $b = t_p$ = Pessimistic time; σ = Standard deviation.

Example -1 Activities

PERT NETWORK Preceding Activity

Time taken

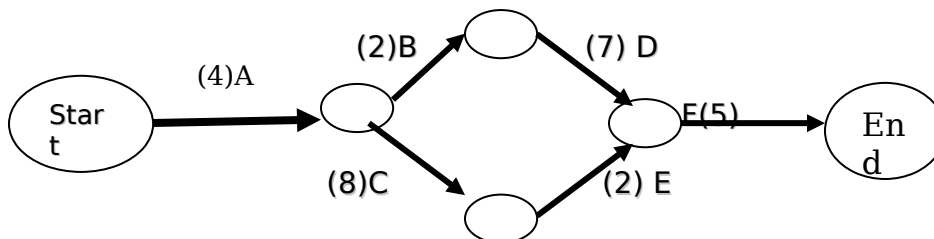
A	-	4 weeks
B	A	2 "
C	A	8 "
D	B	7 "
E	C	2 "
F	D,E	5 "

Required: Draw PERT network and determine critical activities

O nodes (starting and ending Pts)

→

Movement of activity



Possible paths:

$$A \rightarrow B \rightarrow D \rightarrow F = 4 + 2 + 7 + 5 = 18 \text{ wks}$$

Critical path $A \rightarrow C \rightarrow E \rightarrow F = 4 + 8 + 2 + 5 = 19 \text{ wks}$

Example -2

Activities

I

Preceding activity

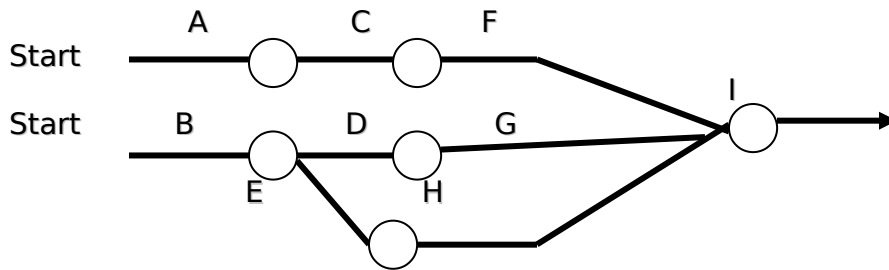
A	-
B	-
C	A
D	B
E	B
F	C
G	D
H	E

F,G,H

Time	
10	weeks
5	"
12	"
15	"
14	"
2	"
5	"
7	"
12	"

Required: Draw PERT network and determine critical activities

Solution



$$A \rightarrow C \rightarrow F \rightarrow I = 10 + 12 + 2 + 12 = 36 \text{ weeks}$$

$$B \rightarrow D \rightarrow G \rightarrow I = 5 + 15 + 5 + 12 = 37 \text{ weeks}$$

$$B \rightarrow E \rightarrow H \rightarrow I = 5 + 14 + 7 + 12 = 38 \text{ weeks}$$

∴ path BEH is the critical path

Beta Distribution of a project

Beta distribution is used when the exact time taken to complete an activity is not exactly known. The calculation of beta distribution is based on optimistic time, most likely time and pessimistic time

$$\text{Mean expected time (tej)} = \frac{a + 4m + b}{6}$$

$$\text{Standard deviation } (\delta) = \frac{b-a}{6}, \text{ where}$$

a = optimistic time
m = most likely time
b = pessimistic time

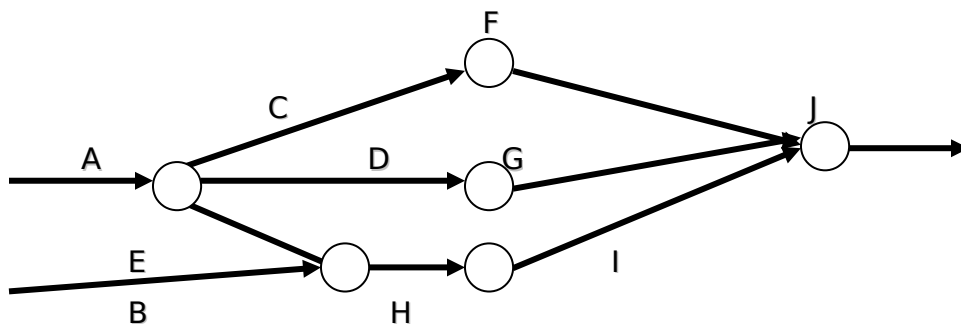
Example -3

Act	Preceding activity	a	m	b	tej	σ
A	-	4 weeks	5 weeks	12 weeks	6 weeks	$\frac{4}{3}$ weeks
B	-	1 »	1.5 »	5 »	2 »	$\frac{2}{3}$ »
C	A	2 »	3 »	4 »	3 »	$\frac{1}{3}$ »
D	A	3 »	4 »	11 »	5 »	$\frac{4}{3}$ »
E	A	2 »	3 »	4 »	3 »	$\frac{1}{3}$ »
F	C	1.5 »	2 »	2.5 »	2 »	»
G	D	1.5 »	3 »	4.5 »	3 »	$\frac{1}{2}$ »
H	B,E	2.5 »	3.5 »	7.5 »	4 »	$\frac{5}{6}$ »
I	H	1.5 »	2 »	2.5 »	2 »	$\frac{1}{6}$ »
J	F,G,I	1 »	2 »	3 »	2 »	$\frac{1}{3}$ »

Required: = 1. Draw PERT network

2. Calculate expected project completion time

3. Draw beta distribution of the project and chances that project can be completed with in 20 weeks.



A → C → F → J = 6+3+2+2 = 13 weeks

A → D → G → J = 6+5+3+2 = 16 weeks

A → E → H → I → J = 6+3+4+2+2 = 17 weeks

B → H → I → J = 2+4+2+2 = 10 weeks

$$Z = \frac{x - \bar{x}}{\sigma_{CP}}$$

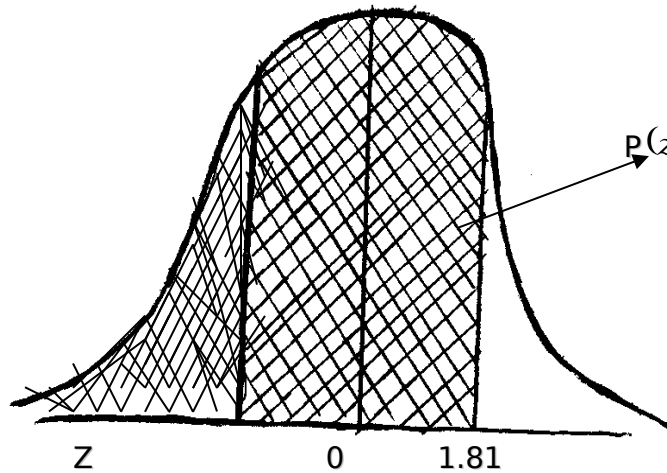
$$\sigma^2 = \sum \left(\frac{x_i - \bar{x}}{n-1} \right)$$

$$\sigma = \sqrt{\sigma^2}$$

$\sigma_{CP} = \sqrt{\text{sum total of square of s.d of all critical activities}}$

$$\sigma_{CP} = \sqrt{\sigma_A^2 + \sigma_E^2 + \sigma_H^2 + \sigma_I^2 + \sigma_J^2}$$

$$= \sqrt{\left(\frac{3}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2} = 1.65$$



$$z = \frac{20 - 17}{\sqrt{2.75}} = \frac{3}{1.65}$$

$$= 1.818$$

$$\begin{aligned} P(z \leq 1.81) &= P(z \leq 0) + P[0 \leq z \leq 1.81] \\ &= 0.5 + 0.4652 \\ &= 96.52\% \end{aligned}$$

$$1817 = 1.82$$

1). Question:

What is the probability that the project will be completed is 15 Wks? Exactly 17 wks?

2). In order to be 96.4 % sure, what should be the length of project completion time?

Example -4

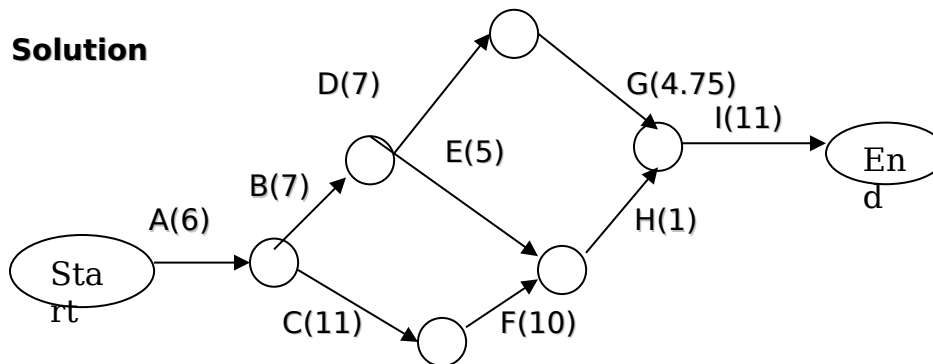
Act Preceding a m b tei 6

activity						
A	-	4	5	12	6	$\frac{8}{6}$
B	A	3	7	11	7	$\frac{8}{6}$
C	A	8	10	18	11	$\frac{10}{6}$
D	B	5	7	9	7	$\frac{4}{6}$
E	B	3	4	11	5	$\frac{8}{6}$
F	C	7	9	17	10	$\frac{10}{6}$
G	D	5	3.5	9.5	4.75	$\frac{6}{4.5}$
H	E,F	1	1	1	1	0
I	G,H	5	11	17	11	2

Required 1). Draw PERT network, identify critical path and project completion time, and the critical expected project complete time and P of probability that the project will be completed between 35 to 45 weeks?

2). If one wants to be 95.05% sure how many weeks before the project should be started?

1). **Solution**



Alternative paths

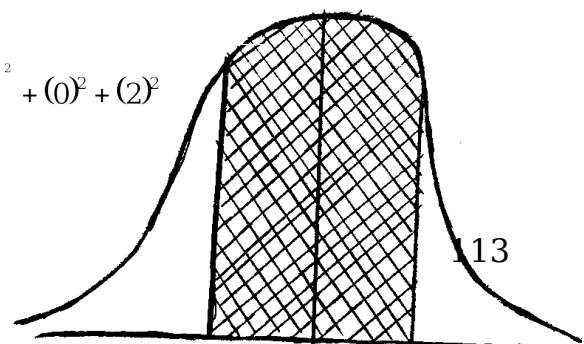
$$ABDGI = 6 + 7 + 7 + 4.75 + 11 = 35.75$$

$$ABEHI = 6 + 7 + 5 + 1 + 11 = 30$$

$$ACFHI = 6 + 11 + 10 + 1 + 11 = 39$$

$$\sigma_{cp} = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_D^2 + \sigma_G^2 + \sigma_I^2}$$

$$= \sqrt{\left(\frac{8}{6}\right)^2 + \left(\frac{10}{6}\right)^2 + \left(\frac{10}{6}\right)^2 + (0)^2 + (2)^2}$$



$$\begin{aligned}
 &= \sqrt{\frac{64}{36} + \frac{100}{36} + \frac{100}{36} + \frac{0}{36} + \frac{144}{36}} \\
 &= \underline{\underline{3.367}} \\
 Z_1 &= \frac{x_1 - \bar{x}}{\sigma_p} , \quad Z_2 = \frac{x_2 - \bar{x}}{\sigma_p} \\
 &= \frac{35 - 39}{3.367} = \frac{54 - 39}{3.367} \\
 &= \frac{-4}{3.367} = \frac{6}{3.367} \\
 &= -1.19 \quad = 1.78
 \end{aligned}$$

Required

$$\begin{aligned}
 P(z_1 \leq z \leq z_2) &= P(z_1) + P(z_2) \\
 &= p(z=-1.19) + p(z=1.78) \\
 &= .3830 + .4625 \\
 &= .8455 \\
 &= \underline{\underline{84.55\%}}
 \end{aligned}$$

∴ Probability of completing the project with in 35-45 wks is 84.55%

2). Solution

Given: P = .9505 = 95.05%

Required = How many weeks before the project should start to be 95.05% confident?

Solⁿ

$$P(z) = .4505$$

$$\Rightarrow z = 1.65$$

And

$$Z = \frac{x - \bar{x}}{\sigma_p}$$

$$\Rightarrow 1.65 = \frac{x - 39}{3.367}$$

$$1.65(3.367) = x - 39$$

$$x = 44.55 \text{ weeks}$$

∴ To be 95-05% sure the project should be started before 5.55 (44.55-39) wks of the average completion time i.e. 39 wks.

Exercise on Floating and Crashing

Float Time

Definition: The time by which an activity can be delayed without affecting total project completion time.

Example: 1

Activity	Preceding Activity	Duration	Earliest Start	Latest Start	Float
A	-	5	0	0	0
B	A	2	5	2	2
C	A	6	5	5	0
D	B	12	7	7	0
E	D	10	19	19	0
F	D	9	19	21	2
G	D	5	9	22	11
H	B	9	7	18	11
I	C,E	1	29	29	0
J	G	2	24	28	4
K	F,I,J	3	30	30	0
L	K	9	33	33	0
M	H,G	7	24	27	3
N	M	8	31	34	3

Required: Find Float

Solution

Float = Latest start - Earliest Start or
= Latest Finish - Earliest Finish

Crashing

Definition: Process of completing an activity before normal time

Example - 2

		Normal Time (NT)	Crashin g Time (CT)	Normal Cost (NC)	Crashin g Cost (CC)	Average Crashing cost (ACC)
A	-	10	8	8,000	12,000	2,000
B	-	7	6	20,000	25,000	5,000
C	A	12	7	20,000	25,000	1,000
D	B	2	2	11,000	11,000	-
E	A	10	6	15,000	23,000	2,000
F	D	20	16	10,000	12,000	500
G	E	5	4	6,000	9,000	3,000

Required: a) Draw PERT network, b) identify critical path, c) Project completion time and minimum project completion time after crashing.

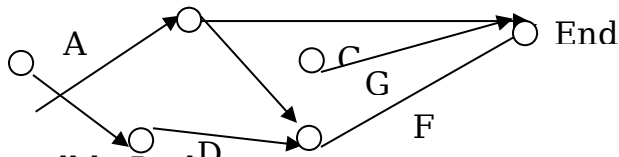
Points to be Noted

1. We always crash only critical activities which helps to reduce the total project completion time.
2. First we crash those critical activities which have minimum average crashing cost
3. Should try to crash activities which are most common (if we have two critical paths).

Given: Normal total cost of project is given as Birr 90,000

Solution

a) PERT Diagram



b) Possible Paths

$$A \rightarrow C \ 10+12 = 22$$

$$A \rightarrow E \rightarrow G \ 10+10+5 = 25$$

$$B \rightarrow D \rightarrow F \ 7+2+20 = 29$$

$$c) \quad \text{Av. Crashing Cost} = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}$$

Crashing of the project

Stage of Crashing	Alternative Paths	Project Completion time	Normal Cost	Additional Crashing Cost	Total Project Cost
1. No crashing	A→C=22 wks A→E→G=25wks B→D→F=29wks	29wks 29wks 29wks	Br.90,000	-	Br.90,000
2. Crash activity F by 4 weeks	A→C = 22 wks A→E→G = 25wks B→D→F = 25 wks	25wks 25wks 25wks	Br.90,000	500x4= 2000	Br.92,000
3. Crash A by 1 week	A→C= 21wks A→E→G = 24wks B→D→F = 25wks	25wks 25wks 25wks	Br.90,000	2000x1=2000	Br.94,000
4. Crash B by 1 week	A→C= 21wks A→E→G = 24wks B→D→F = 24 wks	24wks 24wks 24wks	Br.90,000	5000x1=5000	Br.99,000

Example 3

		NT	CT	NC	CC	ACC
A	-	8	6	8,000	10,000	1,000
B	-	7	5	6,000	8,400	1,200
C	A	5	4	7,000	8,500	1,500
D	B	4	3	3,000	3,800	800
E	A	3	2	2,000	2,600	600
F	D,E	5	3	5,000	6,600	800

G C 4 3 6,000 7,000 1,000

Required

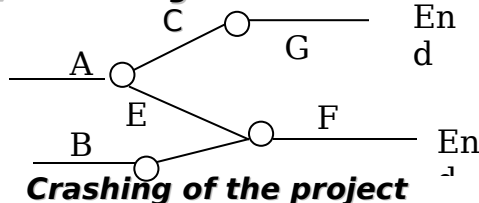
- Draw PERT Network
- Find total min. Cost

Additional Information

The manager wants to complete the project with minimum time but not authorized to use crashing more than Birr 5,000.

Solution

i) PERT Diagram



$$A \ C \ G = 8 + 5 + 4 = 17$$

$$A \ E \ F = 8 + 3 + 5 =$$

Stage of Crashing	Alternative Paths	Project Completion time	Normal Cost	Additional Crashing Cost	Total Project Cost
1.No crashing	ACG = 17 wks AEF = 16 wks BDF = 16 wks	17 wks	Br. 37000	-	Br. 37000
2.Crash activity A by 1 wk	ACG = 16 wks AEF = 15 wks BDF = 16 wks	16 wks	Br. 37000	1000x1=1000	Br. 38000
3. Crash F by 1 wk	ACG = 16wks AEF = 14 wks BDF = 15 wks	16 wks	Br. 37000	800x1=800	Br. 38800
4. Crash 'A' by 1 wk	ACG = 15 wks AEF = 13 wks BDF = 15 wks	15 wks	Br. 37000	1000x1=1000	Br. 39800
5. Crash 'F' by 1 wk	ACG = 15wks AEF = 12 wks BDF = 14 wks	15 wks	Br. 37000	8000x1=8000	Br. 40600
6.Crash G by 1 wk	ACG = 14 wks AEF = 12 wks BDF = 14 wks	14 wks	Br. 37000	1000x1=1000	Br 41600

∴ The minimum project completion time given the constraint will be 14 weeks.