

Chapter 6

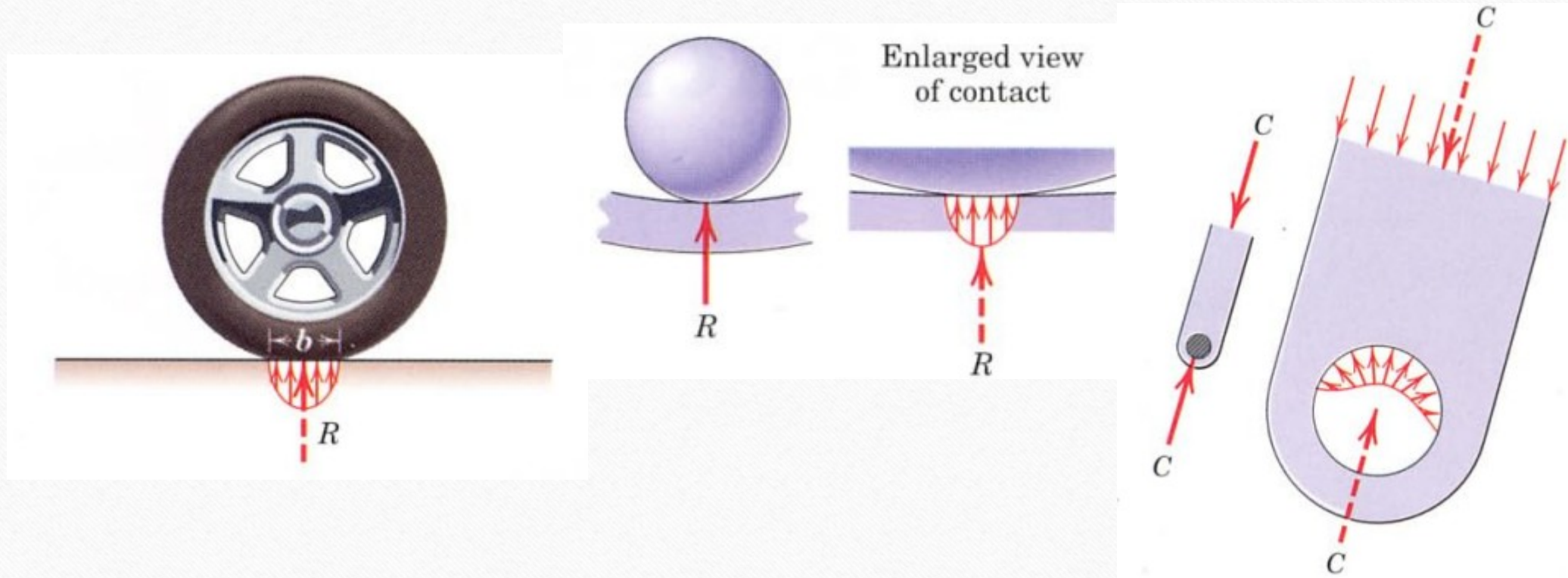
CENTROIDS

By
Bayelign H.

6.1 Introduction

- ❑ In the previous chapter we treated all as concentrated at their points of application and along their line of action. This treatment provided a reasonable model for those forces.
- ❑ Actually, concentrated force don't exist in the exact sense since every external force is applied mechanically to a body is distributed over a finite contact area ,however small.
- ❑ Example:- the force exerted by the pavement on automobile tire
 - contact force between a hardened steel ball and its race
 - the force applied to a two force member of truss.

6.1 Introduction



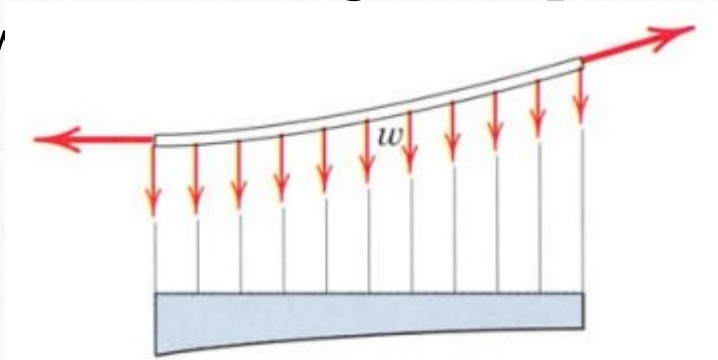
6.1 Introduction

- ❑ When forces are applied over a region whose dimensions are not negligible compared with pertinent dimensions, then we must account for the actual manner in which the force is distributed .
- ❑ We do this by summing of the effective of the distributive force over the entire region using mathematical integration.
- ❑ This requires that we know the intensity of the force at any location.
- ❑ There are three categories of such problems.

6.1 Introduction

1. Line Distribution

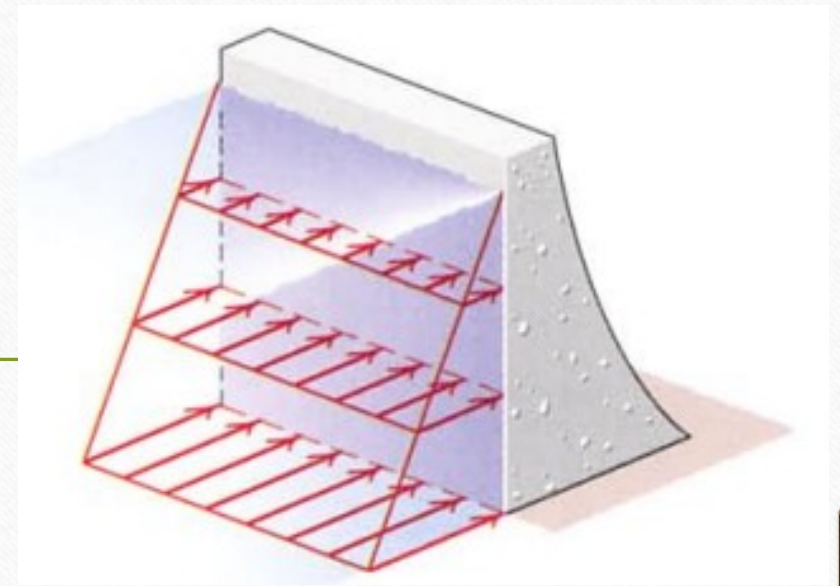
when a force is distributed along a line, as in the continuous vertical load supported by a suspended cable, the intensity w of the loading is expressed as force per unit length of line, new



6.1 Introduction

2. Area Distribution

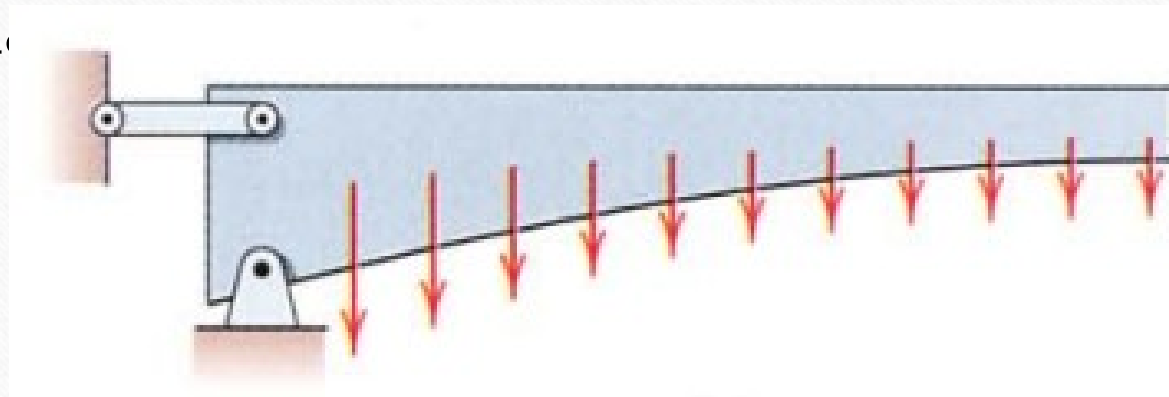
- when a force is distributed over an area, as with the hydraulic pressure of water against the inner face of a section of dam the intensity is expressed as force per unit area.
- This intensity is called pressure for the action of fluid force and stress for the internal distribution of forces in solids.



6.1 Introduction

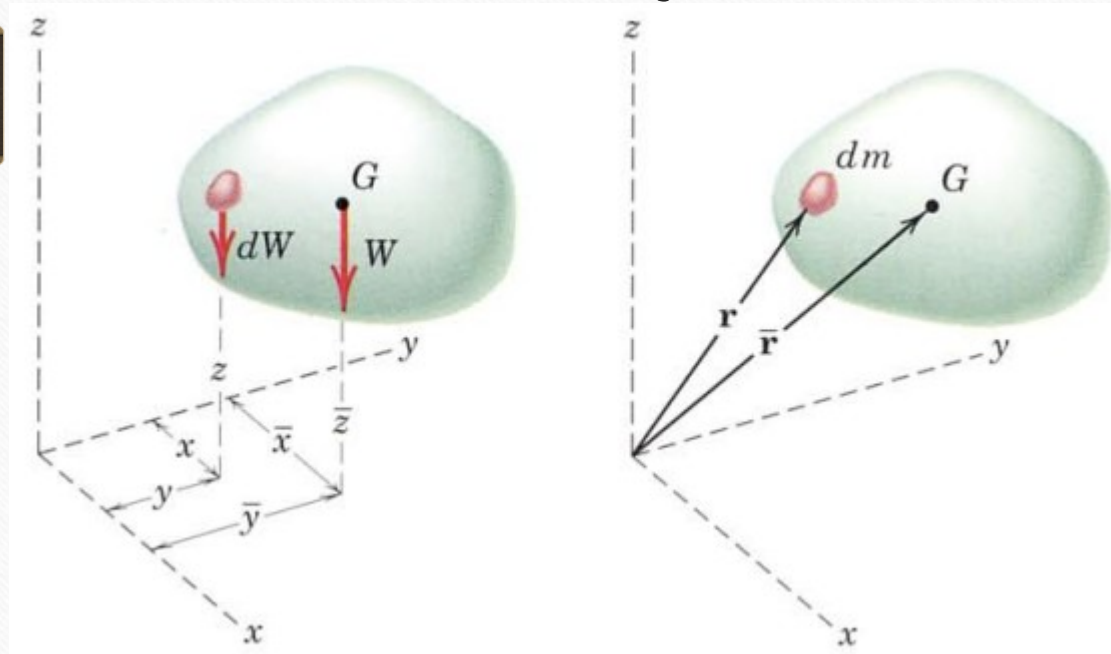
3. Volume Distribution

- A force which is distributed over the volume of a body is called a body force. The most common body force is the force of gravitational attraction, which acts on all elements of mass.



6.2 Centers of Mass, Center of Gravity and Centroid

By applying principle of moments
Center of Gravity



$$\bar{x} = \frac{\int x dW}{W} \quad \bar{y} = \frac{\int y dW}{W} \quad \bar{z} = \frac{\int z dW}{W}$$

$$\bar{x} = \frac{\int x dm}{m} \quad \bar{y} = \frac{\int y dm}{m} \quad \bar{z} = \frac{\int z dm}{m}$$

$$\bar{\mathbf{r}} = \frac{\int \mathbf{r} dm}{m}$$

6.2 Centers of Mass, Center of Gravity and Centroid

$$\begin{aligned}\bar{x} &= \frac{\int x \rho \, dV}{\int \rho \, dV} & \bar{y} &= \frac{\int y \rho \, dV}{\int \rho \, dV} & \bar{z} &= \frac{\int z \rho \, dV}{\int \rho \, dV} \\ \bar{x} &= \frac{\int x \, dV}{\int dV} & \bar{y} &= \frac{\int y \, dV}{\int dV} & \bar{z} &= \frac{\int z \, dV}{\int dV}\end{aligned}$$

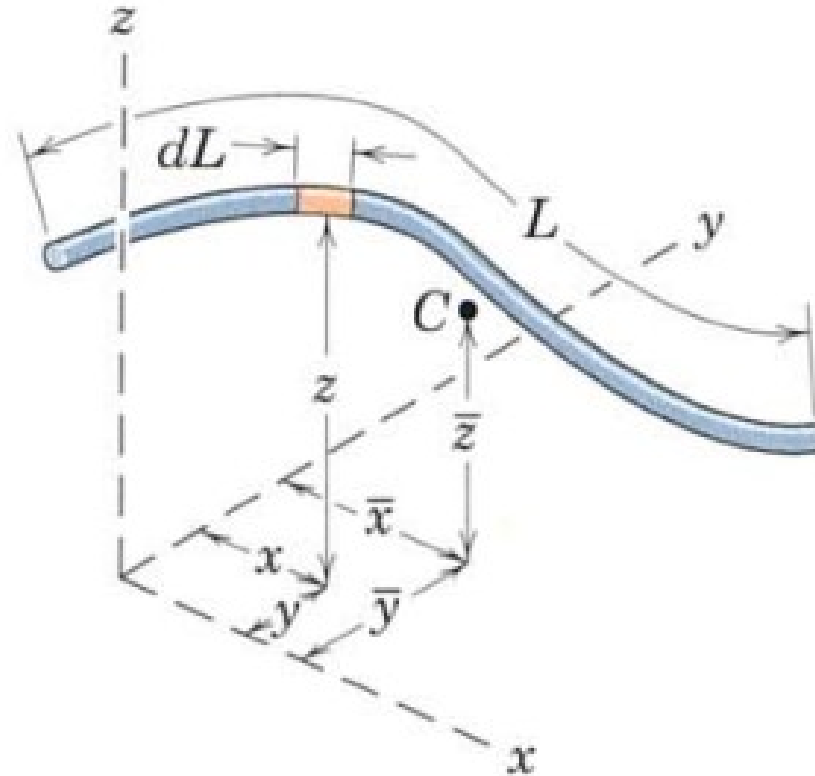
Centroids

6.2.1 Centroids of Line, Areas and Volume

Centroid of Line

$$dm = \rho A dL.$$

$$\bar{x} = \frac{\int x dL}{L} \quad \bar{y} = \frac{\int y dL}{L} \quad \bar{z} = \frac{\int z dL}{L}$$

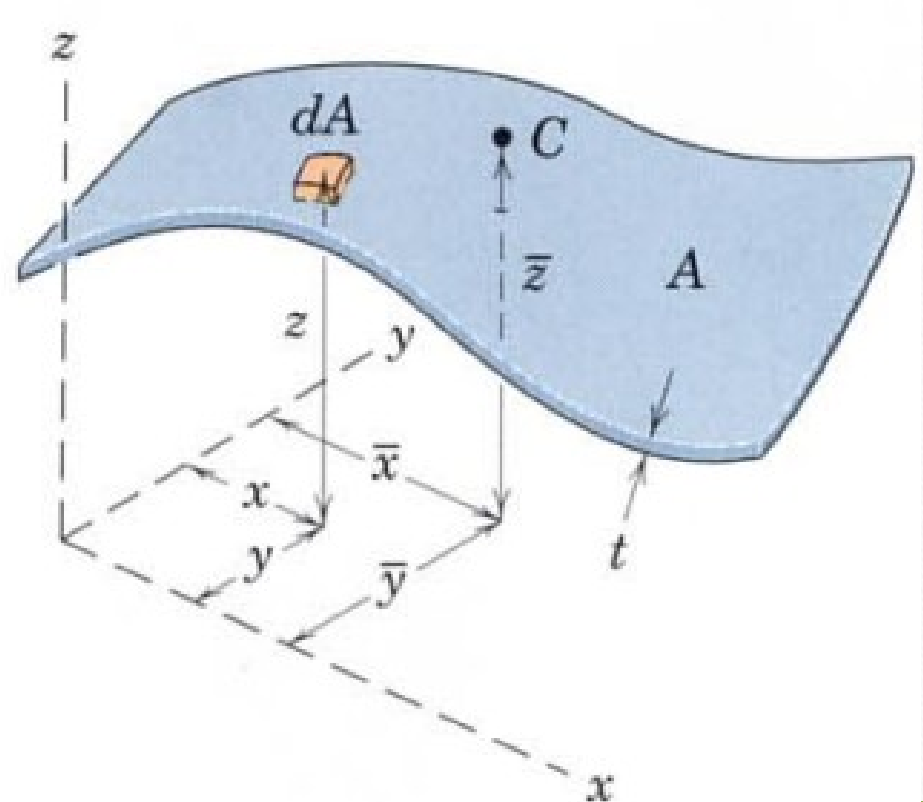


6.2.1 Centroids of Line, Areas and Volume

Centroid of Area

$$dm = \rho t dA.$$

$$\bar{x} = \frac{\int x dA}{A} \quad \bar{y} = \frac{\int y dA}{A} \quad \bar{z} = \frac{\int z dA}{A}$$

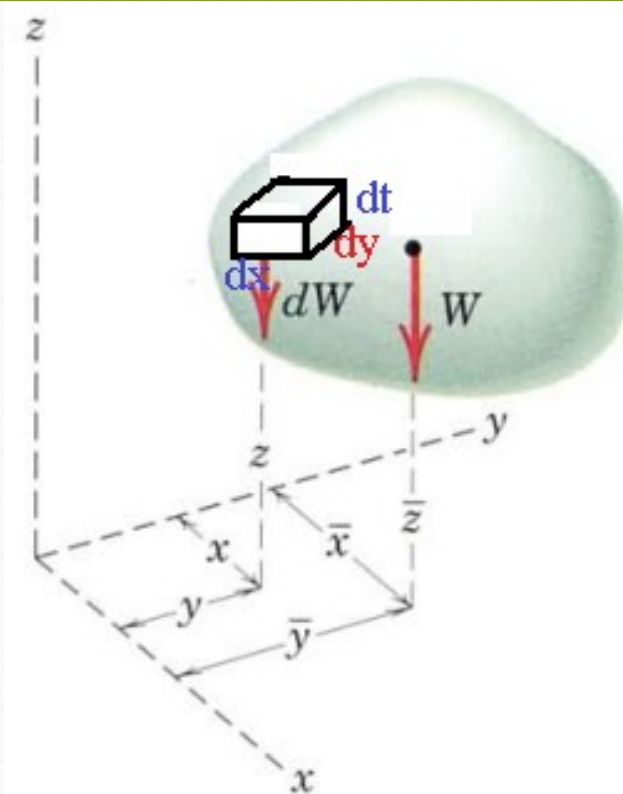


6.2.1 Centroids of Line, Areas and Volume

Centroid of Volume

$$dm = \rho dV.$$

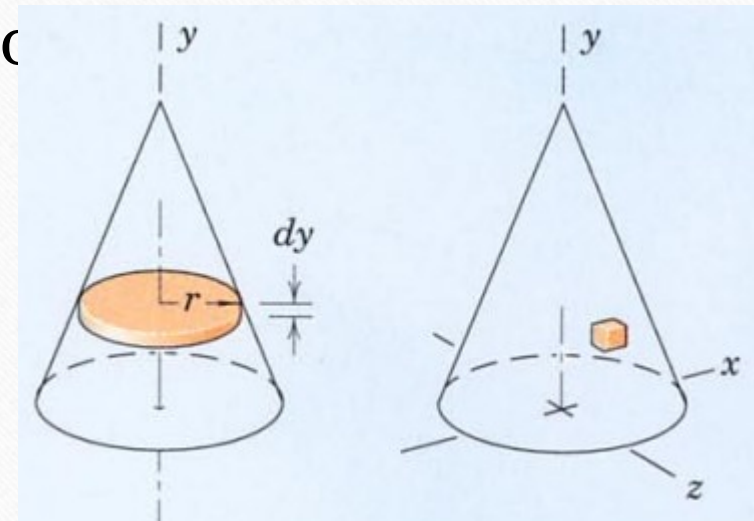
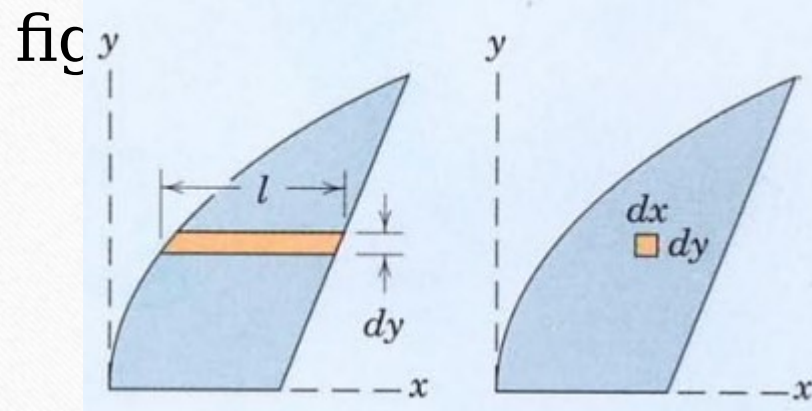
$$\bar{x} = \frac{\int x dV}{V} \quad \bar{y} = \frac{\int y dV}{V} \quad \bar{z} = \frac{\int z dV}{V}$$



6.2.2 Choice of Element for Integration

1. Order of Element

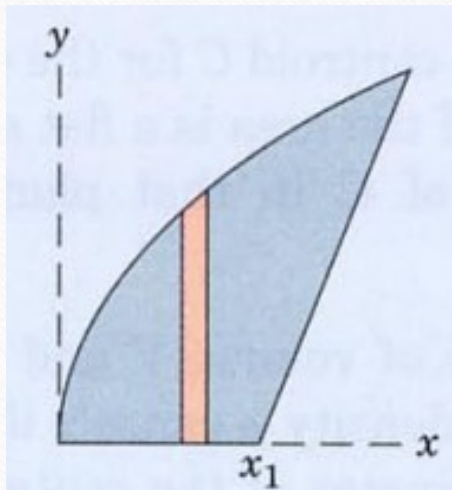
Whenever possible, a first-order differential element should be selected in preference to a higher-order element so that only one integration will be required to



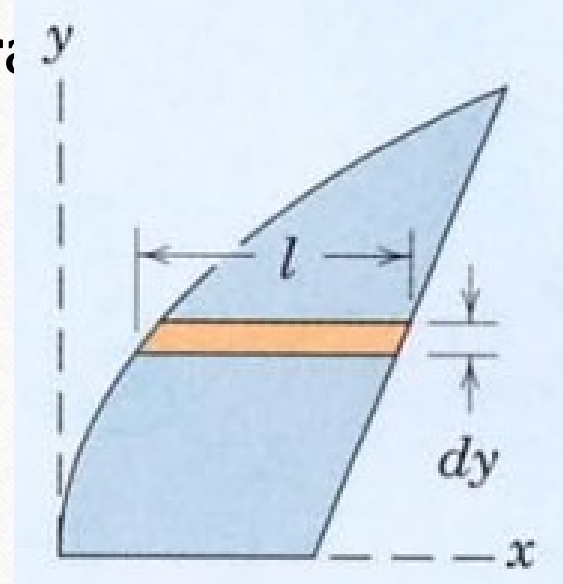
6.2.2 Choice of Element for Integration

2. Continuity

Whenever possible, we choose an element which can be integrated in one continuous operation figure.



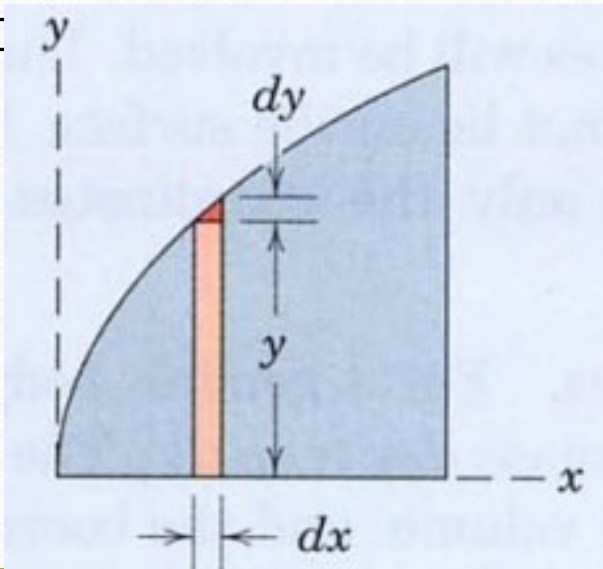
X



6.2.2 Choice of Element for Integration

3. Discarding Higher Order Term

Higher order term always be dropped compared with lower order:

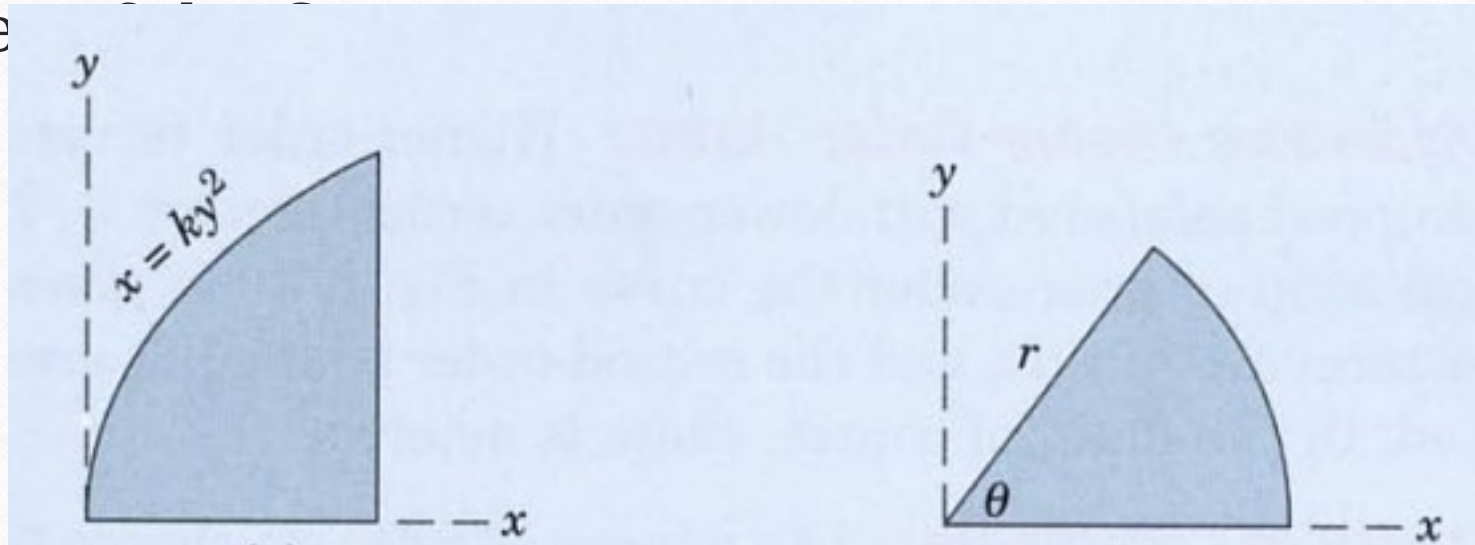


$$dA = y \, dx + \cancel{\frac{1}{2} dx \, dy} \quad \text{Dropped}$$

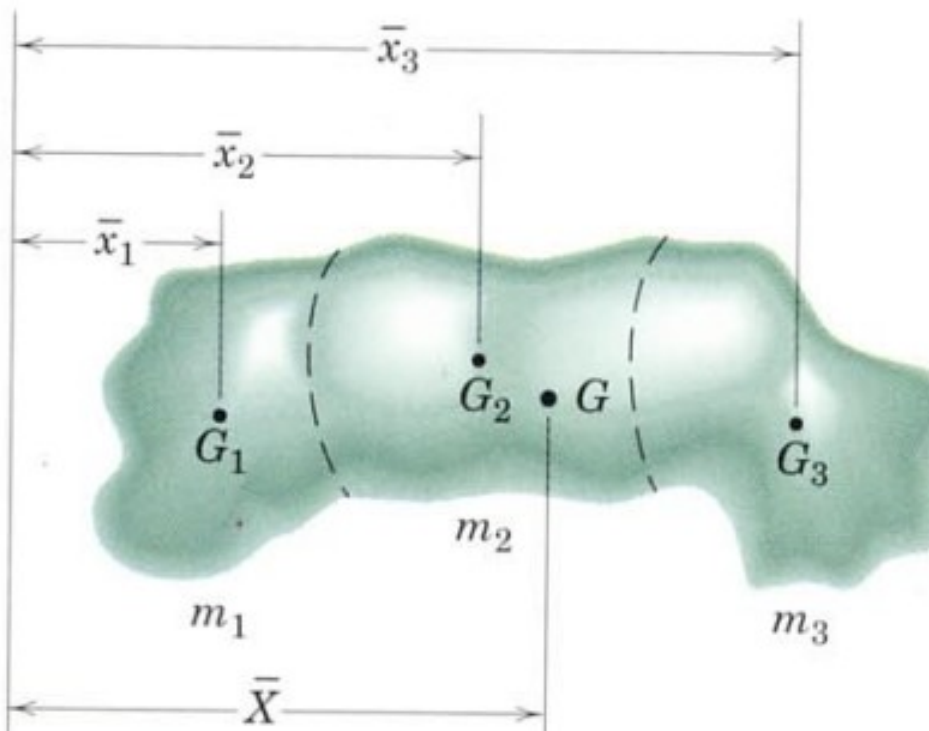
6.2.2 Choice of Element for Integration

4. Choice of Coordinates

Choose the coordinate system which best matches the boundaries



6.3 Composite Bodies and Figures, Approximation



$$(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$$

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} \quad \bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} \quad \bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m}$$

THANK YOU!!!

**QUESTION????????????????
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