

DIRECTION: THE PAPER CONTAINS MATHEMATICS EXAMINATION FOR GRADE 12 FOR NATURAL SCIENCE STREAM ONLY. THE EXAMINATION CONTAINS 65 ITEMS. THERE IS ONLY ONE BEST ANSWER FOR EACH ITEM. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND WRITE THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET.

YOU ARE ALLOWED TO WORK ON THE EXAM FOR 3 HOURS.

1. A function f is continuous on the closed interval $[2, 5]$ with $f(2)=17 \wedge f(5)=17$. Which one of the following additional conditions guarantees that there is a number C in the open interval $(2, 5)$ such that $f'(C)=0$?

- A. f has relative extreme on the open interval $(2, 5)$
- B. No additional conditions are necessary.
- C. f is differentiable on the open interval $(2, 5)$
- D. f is differentiable on the open interval $(5, 17)$

2. For the rational function $f(x) = \frac{x^3 + 2x^2 - 3x}{3x^2 + 3x - 6}$ which one of the following is NOT true?

- A. Vertical asymptote are the lines $x = -2 \wedge x = 1$
- B. Zeros of $f(x)$ are $x = -3, x = 0 \wedge x = 1$
- C. Oblique asymptote is $y = \frac{1}{3}x + \frac{1}{3}$
- D. $\lim_{x \rightarrow \infty} f(x) = \infty$

3. Two sides of a triangle have lengths of $10m \wedge 13m$. The angle between them is increasing at a rate of $\frac{\pi}{60} \frac{rad}{min}$. How fast is the length of the third side

increasing when the angle between the sides of fixed length is $\frac{\pi}{3} rad$?

- A. $\frac{13\sqrt{3}}{12\sqrt{139}} \pi \frac{m}{min}$
- B. $\frac{12\sqrt{3}}{13\sqrt{139}} \pi \frac{m}{min}$
- C. $\frac{13\sqrt{139}}{12\sqrt{13}} \pi \frac{m}{min}$
- D. $\frac{\sqrt{3}}{12\sqrt{139}} \pi \frac{m}{min}$

4. What is the equation of the tangent line to the graph of $f(x) = \ln(xe^x)$ at the point $(1, 1)$

- A. $2x - y - 1 = 0$
- B. $2x + y - 1 = 0$
- C. $2x - y + 1 = 0$
- D. $2x + y + 1 = 0$

5. For $f(x) = 2xe^{2x}$ which of the following is not true?

- A. The function $f(x)$ has local minimum at $x = \frac{-1}{2}$
- B. The function $f(x)$ is strictly increasing on $\mathbb{R}$
- C. The function $f(x)$ is strictly decreasing on $\mathbb{R}$
- D. The function $f(x)$ is concave down on $\mathbb{R}$

6. If the sequence recursively defined by $a_1 = 1, a_2 = -2$ and for $n \geq 3$.

- $a_n = a_{n-1} + (a_{n-2})^2 + n$, then the a_6 term is:
 A. 125 B. 175 C. 41 D. 21
7. What is the value of K so that $f(x) = \begin{cases} \frac{3\sin(kx)}{x}, & \text{if } x > 0 \\ 6k^2 + 5, & \text{if } x \leq 0 \end{cases}$ is continuous at $x=0$?
 A. $\frac{1}{2}$ B. 2 C. $-\frac{1}{2}$ D. -1
8. What is the value of $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$?
 A. $\sin x$ B. $\cos x$ C. $-\sin x$ D. $\cot x$
9. If $f(x) = e^{2x} \sin x$, then the second derivative $f''(x)$ is equal to ____
 A. $-\sin x e^{2x}$ C. $-e^{2x} \sin x - 2e^{2x} \cos 2x$
 B. $-4e^{2x} \sin x$ D. $3e^{2x} \sin x + 4e^{2x} \cos x$
10. Which one of the following is a divergent sequence?
 A. $\left\{1 + \frac{(-1)^7}{7^n}\right\}$ B. $\left\{\frac{2n+1}{n-1}\right\}$ C. $\left\{\frac{2n}{n-1}\right\}$ D. $\left\{\left(\frac{7}{3}\right)^n\right\}$
11. Suppose $S(t) = 25t^2 + 30t + 14$ is the position of an object at time t which is moving along a straight line. Then which of the following is TRUE about $S(t)$?
 A. The average velocity over the time interval $[a, a+h]$ is $50a + 25h + 14$.
 B. The instantaneous velocity at $t=a$ is $50a + 30$
 C. The average velocity over the time interval $[a, a+h]$ is $25a + 25h$
 D. The instantaneous velocity at $t=a$ is $50a + 4$
12. If $y = (e+1)^2$, then what is the value of $\frac{dy}{dx}$?
 A. $e+1$ B. 0 C. $2(e+1)$ D. 1
13. Which one of the following is equal to $\lim_{x \rightarrow 1} \frac{x^4 - \sqrt{x}}{\sqrt{x} - 1}$?
 A. 1 B. -2 C. 7 D. 6
14. If $f(x) = x^2 + 2x$ then $\frac{d}{dx}(f(\ln x))$ is equal to
 A. $\frac{2\ln x + 2}{x}$ B. $2x \ln x + 2$ C. $2\ln x + 2$ D. $\frac{2x+2}{x}$
15. Students created a trapezoid from the cans they had collected for the food bank. There were 10 rows in the trapezoid. The bottom row had 100 cans each consecutive row had five fewer cans than the previous row. How many cans were in the trapezoid?
 A. 995 B. 565 C. 125 D. 775
16. Which of the following relations holds for the sequence: $9, -6, 4, -\frac{8}{3}, \dots$

A. $9\left[\frac{2}{3}\right]^{n-1}$ B. $\frac{2}{3}[9]^{n-1}$ C. $9\left[\frac{-2}{3}\right]^{n-1}$ D. $\frac{-2}{3}[9]^{n-1}$

17. Which of the following arithmetic series the sum of 10 terms is 115?

A. $39+34+29+24+19+\dots$ C. $11+6+1-4-9-\dots$
B. $-11-6-1+4+9+\dots$ D. $39+31+23+15+7+\dots$

18. What is the simplified and lowest form of the following rational expression?

$$\frac{\frac{1}{x+3} - \frac{1}{x-3}}{1 + \frac{9}{x^2-9}} \text{ for } x \neq -3, 0, 3$$

A. $\frac{5}{2}$ B. $\frac{-x}{6}$ C. $\frac{x^2}{6}$ D. $\frac{-x^2}{6}$

19. If appoint p does not lie on a line l in the coordinate plane, how far is the point $p(-5, 2)$ form the line $l: 3x + 8y - 5 = 0$?

A. $\frac{5}{\sqrt{31}}$ B. $\frac{4}{\sqrt{73}}$ C. $\frac{12}{\sqrt{7}}$ D. $\frac{\sqrt{5}}{2}$

20. The equation of the line tangent to the circle with equation

$$(x-2)^2 + (y-3)^2 = 10 \text{ at appoint } p(-1, 2) \text{ is:}$$

A. $y = -3x - 1$ C. $y = \frac{-2}{3}x + \frac{5}{3}$

B. $y = 3x + \frac{5}{18}$ D. $y = -3x - 2$

21. Which one of the following is one to one corresponding function?

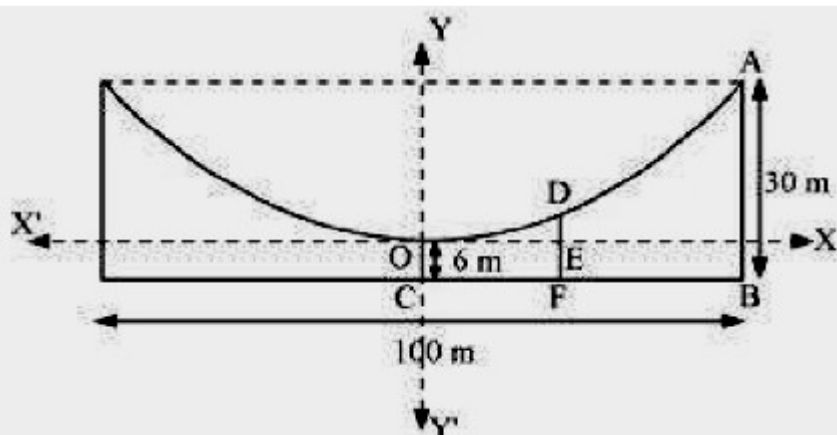
A. $f: [0, \infty) \rightarrow \mathbb{R}$ and $f(x) = |x - 5|$

B. $f: [0, \pi] \rightarrow (-1, 1) \wedge f(x) = \cos x$

C. $f: \mathbb{R} \rightarrow \mathbb{R} \wedge f(x) = \log_3 x$

D. $f: \mathbb{R} \rightarrow [0, \infty) \wedge f(x) = \left(\frac{3}{2}\right)^x$

22. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola as shown in the figure below. The road way which is horizontal and $100m$ long is supported by vertical wires attached to the cable the longest wire being $30m$ and the shortest being $6m$. What is the length of a supporting wire (that means the length DF) attached to the road way $18m$ from the middle.



- A. $\frac{5694}{625}$ B. $\frac{569}{625}$ C. $\frac{694}{5625}$ D. $\frac{625}{5694}$

23. Let p , q and r are three propositions whose truth values are $F, T \wedge F$ respectively, then which one of the following has a truth value false?
 A. $(p \wedge q) \Rightarrow r \vee q$ B. $\neg(\neg r \vee p) \Rightarrow (\neg p \Rightarrow r)$ C. $(\neg p \Rightarrow r) \vee (p \Leftrightarrow r)$ D. $(q \Leftrightarrow \neg p) \Leftrightarrow (p \wedge \neg r)$
24. Which one of the following combinations of questions matches with its truth value in the parenthesis. If $p(x, y): x = y + 1$, where the universe is the set of real number.
 A. $(\exists x)(\exists y)p(x, y)$ is true. C. $(\forall x)(\forall y)p(x, y)$ is true
 B. $(\forall x)(\exists y)p(x, y)$ is false. D. $(\exists x)(\forall y)p(x, y)$ is false
25. The following table shows the mark of 20 students. What is the modal mark?

Mark	5-9	10-14	15-19	20-24
No. of students	3	7	4	6

- A. 13.1 B. 14.8 C. 12.4 D. 11.3
26. A basket contains 6 yellow, 3 green and 5 white balls. If one ball is drawn from the basket, what is the ball is not green?
 A. $\frac{3}{14}$ B. $\frac{11}{14}$ C. $\frac{9}{14}$ D. $\frac{13}{14}$

27. In the following matrices A, B and C, if $AB=C$ then what is the value of $e \wedge f$ respectively? $A = \begin{bmatrix} -3 & 0 & 3 \\ 2 & e & -1 \\ 1 & 4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 & 6 \\ -1 & 1 & -3 \\ 0 & -3 & f \end{bmatrix}$ $C = \begin{bmatrix} -15 & -15 & -6 \\ 11 & 6 & 11 \\ 1 & 12 & -14 \end{bmatrix}$
 A. $7 \wedge -5$ B. $3 \wedge 5$ C. $-1 \wedge -8$ D. $-1 \wedge 4$
28. In how many ways can the letters of the word GOSSAMERE be arranged such that all S's $\wedge$ M's appear in the middle?
 A. $\frac{9!}{2!2!}$ B. $\frac{7!}{2!}$ C. $\frac{7!3!}{2!2!}$ D. $\frac{6!3!}{2!2!}$
29. Suppose A is 3×3 matrix and $\det(2A^{-1}) = -4 = \det(A^3(B^{-1}))^T$, then what is the value of $\det A \wedge \det B$ respectively?
 A. -2, 4 B. -2, 2 C. 2, -2 D. -4, -2
30. What is the solution set of the system $\begin{cases} 2x - 3y + z = 4 \\ x - 2y + 3z = -1 \\ 4x + y - z = 2 \end{cases}$
 A. $\left\{ \left(\frac{1}{2}, \frac{-3}{2}, \frac{-3}{2} \right) \right\}$ C. $\left\{ \left(\frac{2}{5}, \frac{-6}{5}, \frac{12}{5} \right) \right\}$
 B. $\left\{ \left(\frac{-11}{6}, \frac{4}{5}, \frac{7}{4} \right) \right\}$ D. $\left\{ \left(\frac{1}{6}, \frac{-7}{6}, \frac{5}{6} \right) \right\}$
31. In how many ways can a committee of 5 members be formed from 4 women and 6 men such that at least one woman is member of the committee?
 A. 112 B. 156 C. 208 D. 246
32. What is the exact value of the argument of w where $z = 1 + \sqrt{3}i \wedge \frac{w}{z} = 2 + 2i$?
 A. $\frac{7\pi}{12}$ B. $\frac{7\pi}{5}$ C. $\frac{6\pi}{7}$ D. $\frac{3\pi}{4}$
33. If $x^2 + 2xy = y^2$, then what is the value of $\frac{dy}{dx}$?
 A. $\frac{x+y}{y-x}$ B. $2x+2y$ C. $\frac{x+y}{y}$ D. $-x$
34. An ice cream cone has the shape of an inverted circular cone with a base of radius 2 cm and height of 4 cm. If soft serve ice cream is being pumped in to the cone at a rate of $2 \frac{\text{cm}^3}{\text{min}}$, what the rate at which the ice cream level is rising when the ice cream is 3 cm in deep?
 A. $\frac{9}{8\pi}$ B. $\frac{9\pi}{8}$ C. $\frac{8}{9\pi}$ D. $\frac{8\pi}{9}$

35. What is the value of $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4+1}}{x^2-3x+5}$?
- A. 1 B. 3 C. 9 D. ∞
36. What the area bounded by the coordinate axes the graph of $y=4e^{-2x}$ and the line $x=1$?
- A. $2(1-e^{-2})$ square unit C. $-2(1-e^{-2})$ square unit
B. $2(1+e^{-2})$ square unit D. $-2(1+e^{-2})$ square unit
37. Let y be a differentiable function with $f(-3)=12, f'(-3)=9, h(-3)=-2$ and $h'(-3)=5$. If $y=[x-f(x)]h(x)$, then what is the value of $\frac{dy}{dx}$ at $x=-3$?
- A. -2 B. 16 C. -15 D. -59
38. If i is an imaginary number what is the simplified form of $(i^{387} x i^{266}) \div (i^{69} x i^{152})$?
- A. 1 B. i C. -1 D. $-i$
39. What is the point on the curve $f(x)=3x^2-4x+5$ where the tangent line is parallel to the line $f(x)=-22x+7$.
- A. (0,5) B. (-1,12) C. (-3,44) D. (-2,25)
40. Which of the following is Not true?
- A. If $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ exist, then f is differentiable at a
B. If f is differentiable at a , then $\lim_{x \rightarrow a} f(x)=f(a)$
C. If $\lim_{x \rightarrow a} f(x)$ exists, then f is continuous at a
D. If f is differentiable at a , then f is continuous at a .
41. Which one of the following is equal to $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$?
- A. $\sqrt{2}$ B. 0 C. 2 D. $3\sqrt{2}$
42. Which of the following is the form of the partial fraction decomposition of $\frac{2x^3+2x^2+x+1}{x^2(x^2+1)}$?
- A. $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$ C. $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{x^2+1}$
B. $\frac{A}{x} + \frac{Bx}{x^2} + \frac{Cx+D}{x^2+1}$ D. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+1}$

43. Suppose $\lim_{x \rightarrow -3^-} f(x) = -1$ and $\lim_{x \rightarrow -3^+} f(x) = -1$ is not defined. Which one of the following is not true?
- A. $\lim_{x \rightarrow -3} f(x) = -1$
 B. f has discontinuity at $x = -3$
 C. If we redefine $f(-3)$ to be equal to -1 , then the new function will be continuous.
 D. f is continuous everywhere except at $x = -3$
44. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ are equal to:
- A. $(-1, 2) \wedge (-1, -6)$ C. $(-1, -2) \wedge (-2, -1)$
 B. $(-2, 1) \wedge (-2, 6)$ D. $(-1, -2) \wedge (-1, -6)$
45. What is the center of the hyperbola $89 = 16y^2 - 9x^2 + 18x + 64y$ is?
- A. $(1, -2)$ B. $(-1, -2)$ C. $(1, 2)$ D. $(-1, 2)$
46. For $f(x) = e^{4-x^2}$ which of the following is not true?
- A. The function $f(x)$ has local minimum at $x = 0$
 B. The function $f(x)$ is strictly increasing on $\mathbb{R}$
 C. The function $f(x)$ is concave down on $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
 D. The function $f(x)$ is concave up on $(-\infty, -\frac{1}{\sqrt{2}}) \wedge (\frac{1}{\sqrt{2}}, \infty)$
47. What is the value of $\int_2^9 f(x) dx$ given that $\int_2^5 f(x) dx = 3$ and $\int_5^9 f(x) dx = 8$?
- A. 8 B. -3 C. 6 D. 5
48. If $\frac{d}{dx} f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx} f(h(x))$ is equal to _____
- A. $g(\sin x)$ B. $g(x) \cos x$ C. $\sin x g(\sin x)$ D. $\cos x g(\sin x)$
49. What is the value of $\int \frac{2x}{4x-3} dx$?
- A. $\frac{x}{2} + \frac{3}{8} \ln|4x-3| + c$ C. $\frac{4x-3}{8} + \frac{8}{3} \ln|4x-3| + c$
 B. $\frac{4x-3}{8} - \frac{3}{8} \ln|4x-3| + c$ D. $\frac{4x+3}{8} - \frac{3}{8} \ln|4x-3| + c$
50. Which of the following is equal to $\int \frac{4}{49-36x^2} dx$?
- A. $\frac{1}{21} \ln \left| \frac{7-6x}{7+6x} \right| + c$ C. $\frac{1}{21} \ln \left| \frac{-7+6x}{7-6x} \right| + c$
 B. $\frac{1}{21} \ln \left| \frac{7+6x}{7-6x} \right| + c$ D. $\frac{1}{21} \ln \left| \frac{7+6x}{-7-6x} \right| + c$

51. Which one of the following is the set of critical numbers of

$$f(x) = x - 2 \ln(1 + x^2) ?$$

- A. $[2 + \sqrt{3}, 2 - \sqrt{3}]$ C. $[-2 + \sqrt{3}, -2 + \sqrt{3}]$
B. $[-2 + \sqrt{3}, 2 + \sqrt{3}]$ D. $[-2 - \sqrt{3}, 2 - \sqrt{3}]$

52. A translation takes $(x-2)^2 + (y-1)^2 = 9$ to $(x-3)^2 + (y+2)^2 = 9$ what is the image of point $p(3, 1)$ under this translation?

- A. $(3, -1)$ B. $(4, 2)$ C. $(-3, 1)$ D. $(4, -2)$

53. What is the amplitude and period, respectively, of the graph of

$$f(x) = 2 \left[\cos \frac{\pi}{3} \cos \frac{x}{3} + \sin \frac{\pi}{3} \sin \frac{x}{3} \right] ?$$

- A. $2, 6\pi$ B. $-2, \frac{\pi}{3}$ C. $2, 4\pi$ D. $2, \frac{\pi}{2}$

54. If $a - b = 2c$, $a + b = 4c$ and $c = 3i + 4j$, then what are a and b respectively?

- A. $a = 9i - 12j$ and $b = 3i - 4j$
B. $a = -9i + 12j$ and $b = 3i - 4j$
C. $a = 9i + 12j$ and $b = 3i + 4j$
D. $a = 9i - 12j$ and $b = -3i + 4j$

55. What is the equation of the sphere concentric with

$$x^2 + y^2 + z^2 + 2x + 2y + 2z - 1 = 0 \text{ and having radius as 3 units?}$$

- A. $x^2 + y^2 + z^2 + 2x + 2y + 2z + 6 = 0$
B. $x^2 + y^2 + z^2 + 2x + 2y + 2z - 6 = 0$
C. $x^2 + y^2 + z^2 - 2x + 2y + 2z - 6 = 0$
D. $x^2 + y^2 + z^2 + 2x - 2y + 2z - 6 = 0$

56. What is the image of the line given by $(x, y) = (1, 0) + t(3, 6)$ under the translation that takes $(1, 0)$ to $(0, 1)$ followed by under the rotation of 90° anticlockwise about the origin?

- A. $y = 2x - 3$ B. $y = \frac{1}{2}x + \frac{5}{2}$ C. $y = 2x + 3$ D. $y = -\frac{1}{2}x - \frac{5}{2}$

57. If $(2, 3, 4)$ is one end of the diameter of sphere $x^2 + y^2 + z^2 + 2x - 2y + 4z - 1 = 0$, then what is the other end points?

- A. $(-2, -1, -8)$ B. $(0, -1, 8)$ C. $(0, -3, -8)$ D. $(0, -1, -8)$

58. Which one of the following is valid logical argument?

- A. $p \Rightarrow q, \neg r \Rightarrow p, \neg s \Rightarrow \neg r \vdash \neg q \Rightarrow s$ C. $p \Rightarrow (q \wedge r), p \vdash \neg q$
B. $\neg p \Rightarrow \neg r, p \Rightarrow q, \neg q \vdash r$ D. $\neg p, p \vee q, r \Rightarrow q \vdash r$

59. What is the exact value of $\sin[\tan^{-1}(1) + \sin^{-1}(1)]$?

- A. $\frac{\sqrt{2}}{2}$ B. $\frac{-\sqrt{3}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\frac{-\sqrt{2}}{2}$

60. What the exact value of $(2 \cos^{-1} x + \sin^{-1} x)$ when $x = \frac{1}{5}$?
- A. 5 B. $\frac{1}{2}$ C. $\frac{1}{5}$ D. $\frac{-1}{5}$
61. Which of the following is the solution set of $1 + \cos x = \sin x$ in $[0, 2\pi]$?
- A. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \pi\right\}$ B. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ C. $\left\{\frac{\pi}{2}, \pi\right\}$ D. $[0, \pi, 2\pi]$
62. Which one of the following compound propositions is a contradiction?
- A. $(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$ C. $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$)
- B. $p \Rightarrow \neg q$ D. $(\neg p \wedge \neg q) \Rightarrow \neg(p \vee q)$
63. If $A = (2, -4)$, $B = (-6, 4) \wedge \vec{u}$ is apposition vector such that $2\vec{u} + \vec{AB} = 0$, then $\vec{u}$ is equal to ____
- A. $(2, -4)$ B. $(4, -2)$ C. $(4, -4)$ D. $(-2, 2)$
64. Which of the following is necessary true?
- A. If $|\vec{KA}| = K|\vec{A}|$, for any real number K
- B. If $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$
- C. $\vec{A} \cdot \vec{B} = 0$ if and only if $\vec{A} = 0 \vee \vec{B} = 0 \vee \vec{A}$ perpendicular $\vec{B}$
- D. If $|\vec{A}| = |\vec{B}|$, then $\vec{A} = \vec{B}$
65. Suppose $A = -2i + 2j + k$ and B is a vector in space such that $|\vec{B}| = \vec{A} \cdot \vec{B}$. If $\vec{u}$ is in the direction of $\vec{B}$, then $|\vec{A} + \vec{u}|^2$ is equal to:
- A. 12 B. 10 C. 14 D. 16

