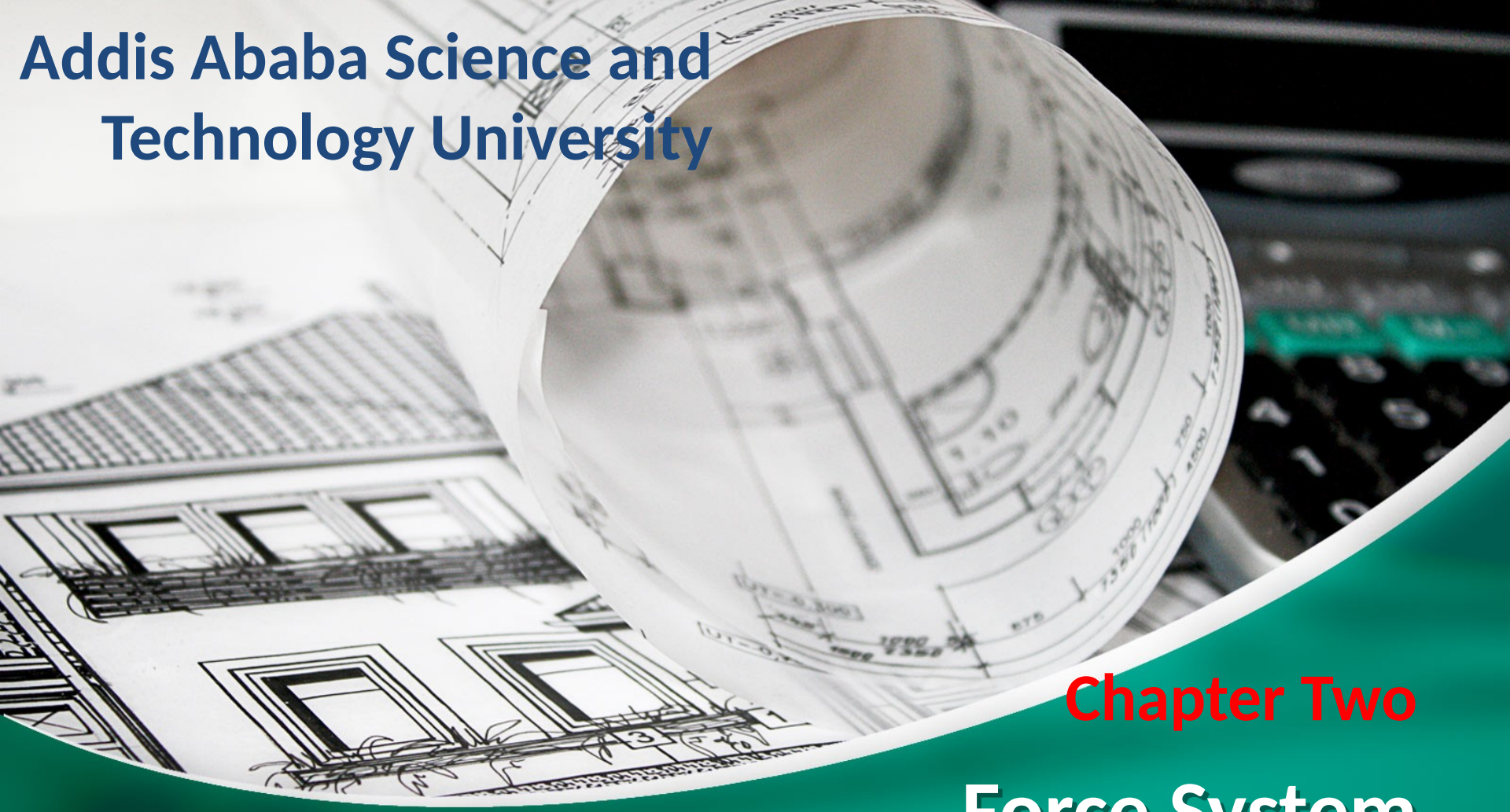
Addis Ababa Science and Technology University



Chapter Two

Force System

Engineering Mechanics

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Content

2. FORCE SYSTEMS

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2.3. Moment, Couple & Force-Couple systems

2.3.1. 2-D

2.3.2. 3-D

2.4. Resultants

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2.1 Introduction

- A force can be defined as the action of one body on another that changes/tends to changes the state of the body acted on.
- **A force can be applied on a body as;**
- **Contact force**:- Applied by direct mechanical contact of the acting body on the acted one (Created by push and pull).



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2.1 Introduction

- **Remote action (Body force):-** Applied by remote action as in gravitational, electrical, Magnetic, etc. forces.



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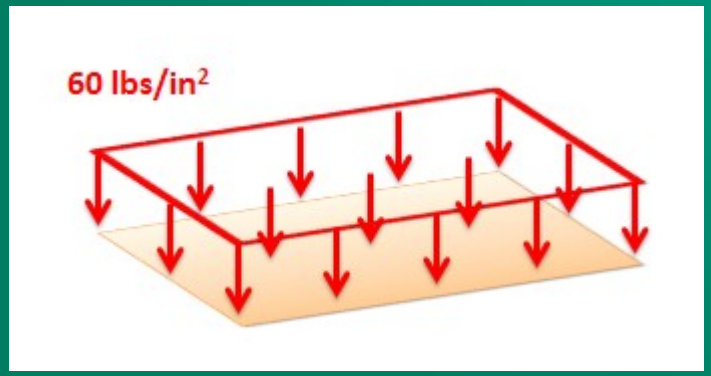
2.1 Introduction

- A force may also be classified as
  - **Concentrated and**
  - **Distributed**
- **Concentrated** when area of application is negligible
- A force can be distributed over
  - **Area**, as in the case of mechanical contact
  - **Volume**, when considering body forces such as weight
  - **A line**, in the case of the weight of a suspended cable.

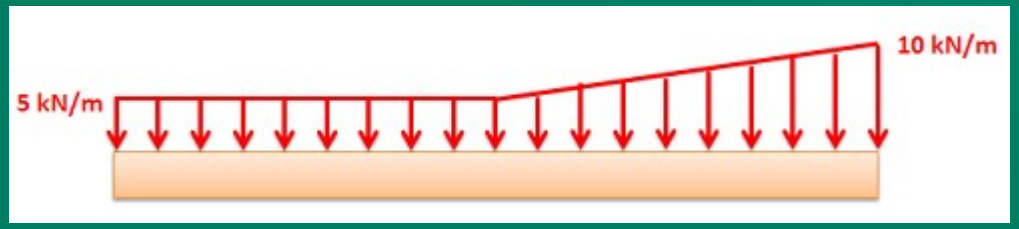
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The gravitational force this plane is distributed over the entire volume of the airplane
KN/m3



Representation of a force distributed over an area
KN/m2

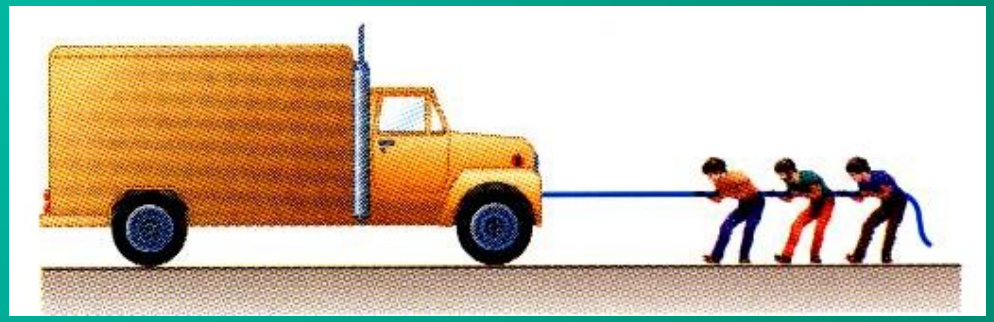


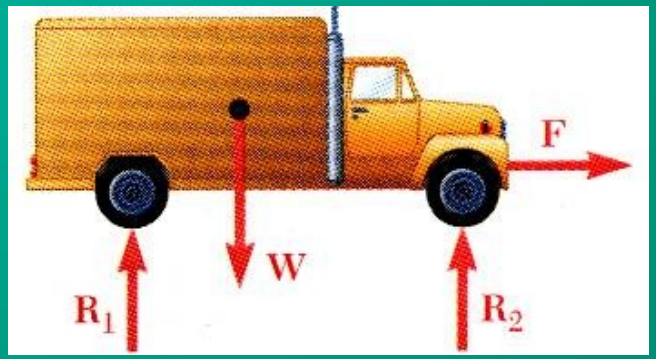
Representation of a force distributed over a line
KN/m

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2.1 Introduction

- **External and Internal Effects**
- We can separate the action of a force on a body into two effects, internal and external.

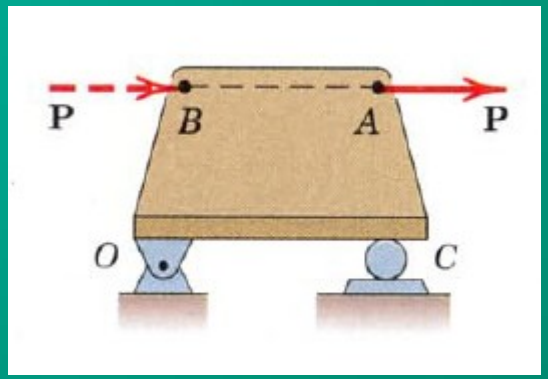




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2.1 Introduction

- **Principle of Transmissibility**
- When dealing with mechanics of rigid body, we are only concerned about the net effects of external
- In such cases it is not necessary to restrict the action of an applied force to a given point.



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2.1 Introduction

- **Principle of transmissibility** states that a force may be **applied at any point on its given line of action** without altering the resultant effect of the force external to the rigid body on which it acts.

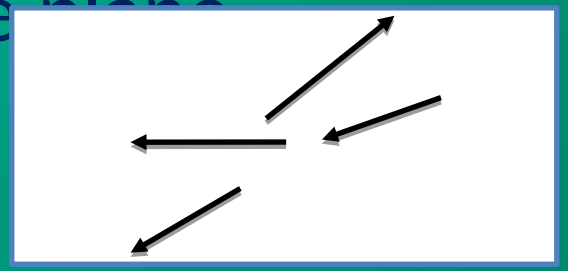
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2.1 Introduction

- **Force System**
- A system of forces can be grouped into different categories **depending on their arrangement** in space
- **Collinear forces**:- are coplanar forces acting on the same line



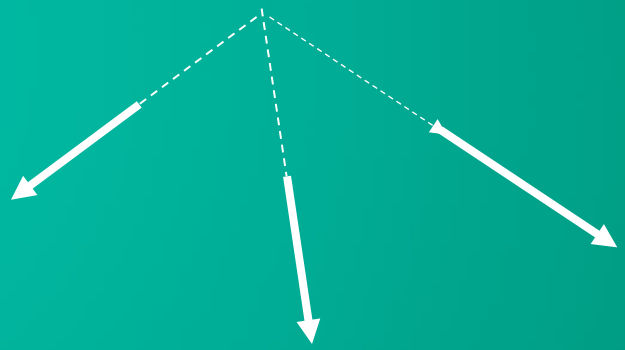
- **Coplanar Forces**:- are forces which act on the same plane



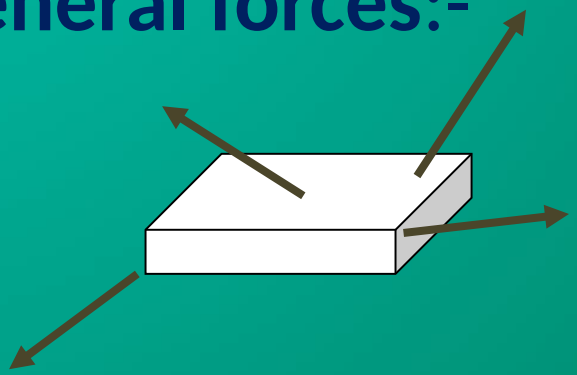
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2.1 Introduction

- **Concurrent forces**:-Are forces whose lines of action intersect at a point



- **General forces**:-

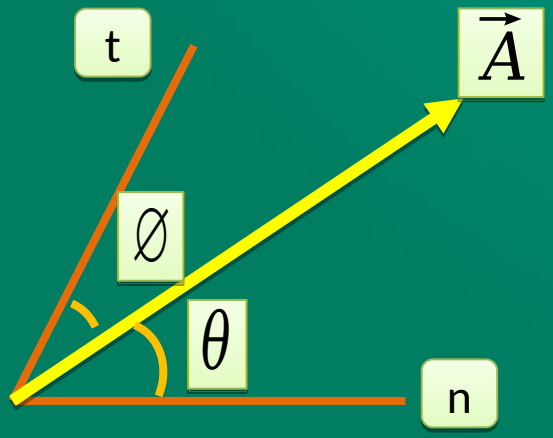


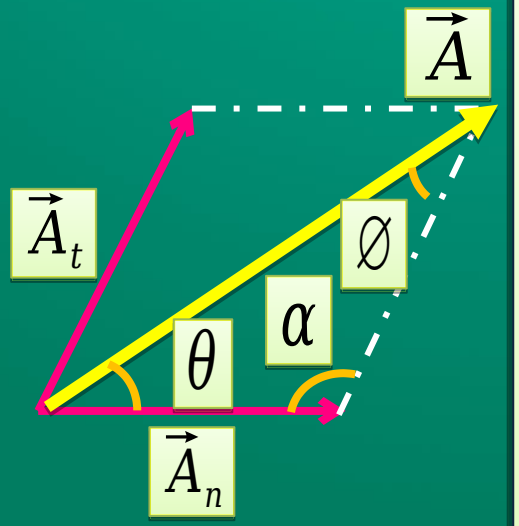
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2.2. Resolution of a Force

2.2.1. Coplanar Force Systems (2-D)

Resolution is the reverse of composition. It is the process of getting the components of a vector along different axes





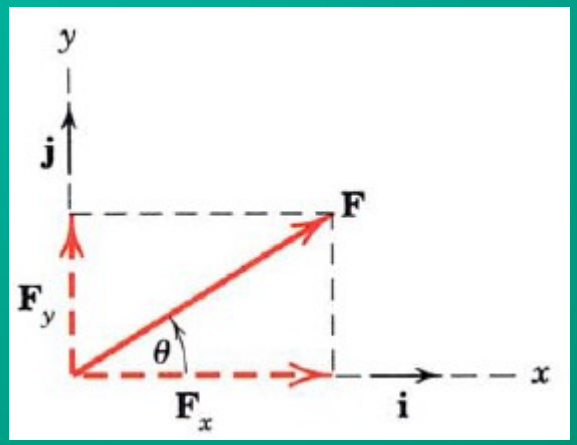
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For rectangular coordinate system

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2.2.1. Resolution of a Force

- **Rectangular Component**
- The **most common** two-dimensional resolution
- Rectangular components of a force are the components of the force along the rectangular coordinate axes.
- 

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

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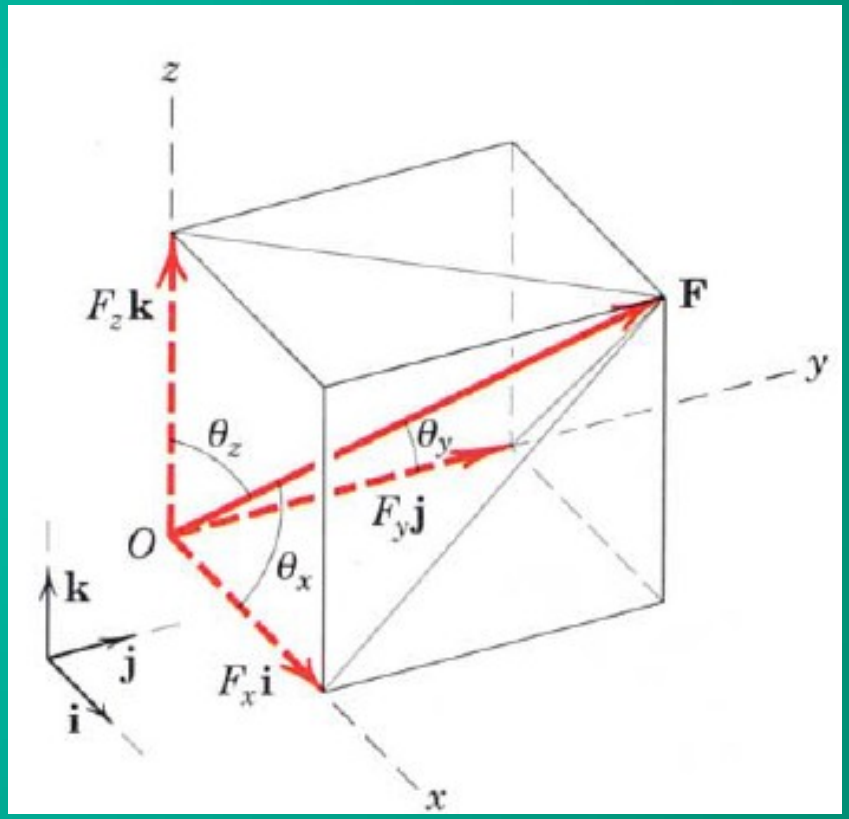


2.2.2. Non-Coplanar Force Systems (3-D)

- **Rectangular components**
- Many problems in mechanics requires analysis in 3 dimension
- To solve such problems it is necessary to resolve the force into its three mutually perpendicular components.
- A force F in 3-d system has a rectangular components Fx, Fy, Fz

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Rectangular Components



$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

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Rectangular Components

- In solving three-dimensional problems, one must usually find the x, y and z scalar components of a given or unknown force. In most cases, the direction of a force is described;

A. By two points on the line of action of the force, or

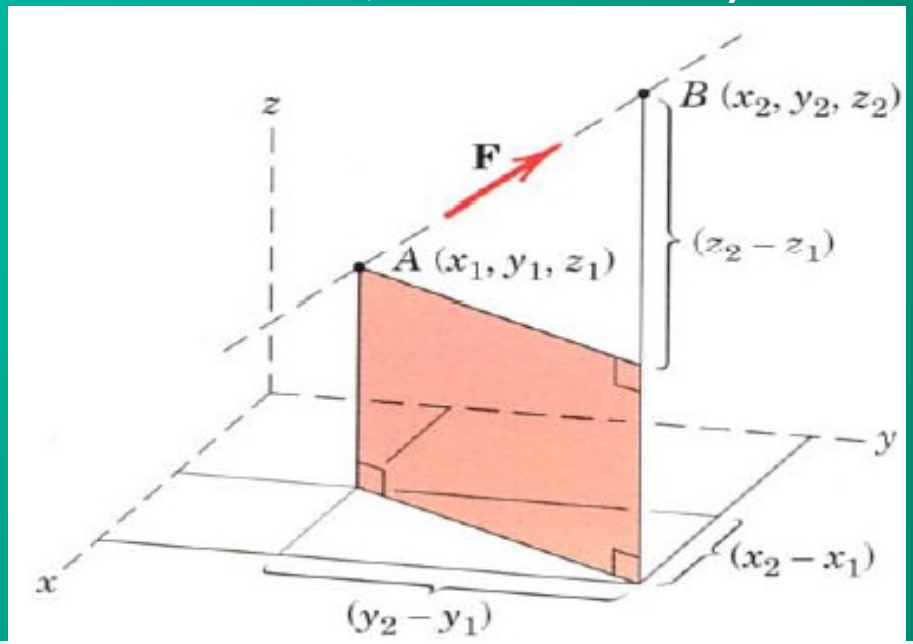
B. By two angles which orient the line of action.

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Rectangular Components

A. By two points on the line of action of the force, or

- If the coordinates of two point, say A&B, on the line of action of the force are known and the direction of the force is from A to B, the force may be written as

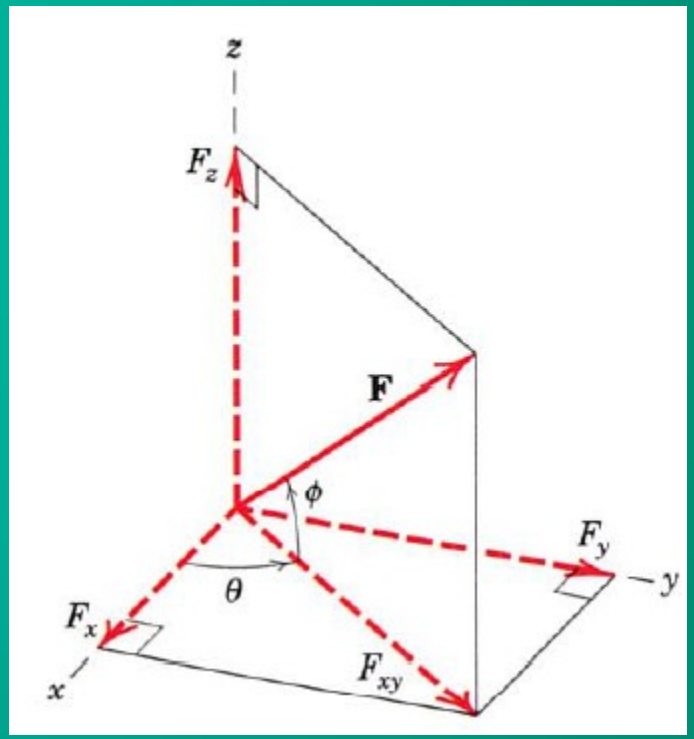


$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

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Rectangular Components

- B. By two angles which orient the line of action.



$$F_{xy} = F\cos\phi$$
$$F_z = F\sin\phi$$

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$
$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$

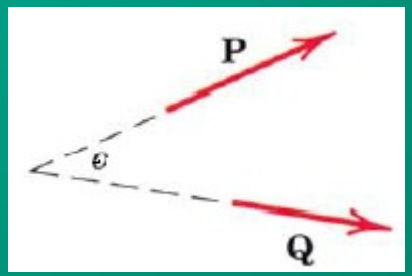
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Rectangular Components
Dot product

- It is possible to express the rectangular components of a force with the aid of the vector operation known as the dot or scalar product.

- This product can be viewed as either

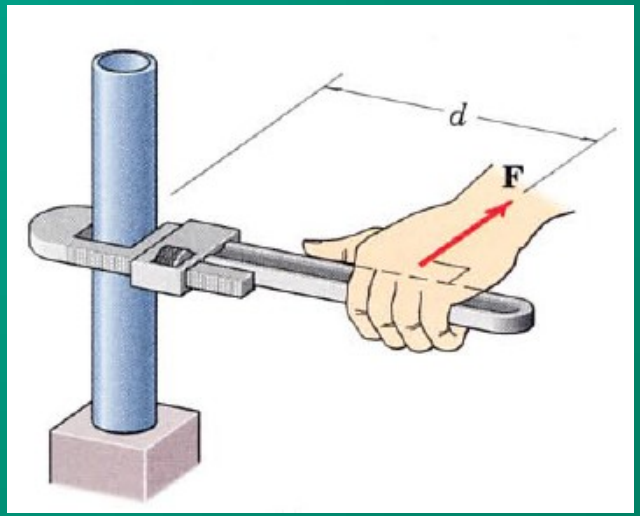
1. The orthogonal projection of of P on the direction of Q multiplied by Q, or
2. As the orthogonal projection of of Q in the direction of P multiplied by *P*



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2.3. Moment, Couple & Force-Couple systems

- **Moment**
- In addition to moving a body in the direction of application, a **force also tends to rotate**
- This **tendency of a force to rotate** a body about a given axis is known as the **moment**
- The axis of rotation maybe any line which neither intersect nor is parallel to the line of action of the force



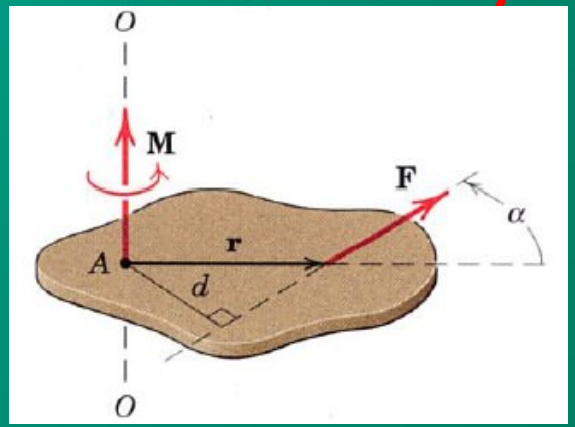
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Moment

- The moment or rotational tendency of a force is proportional to magnitude of the force and the moment arm d, which is the perpendicular distance from the axis to the line of action of the force.
- The axis of rotation, axis O-O, of the body is normal to the plane of the body formed by the force and position vector.

magnitude of moment can be obtained by:

$$M = F \times d$$



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Moment

The cross product

- Useful when dealing with complex 2-dimensional and 3-dimensional problems.
- The moment of a force F about a point maybe represented by the cross-product expression.

$$M = r \times F$$

- Where r is the position vector from point of rotation to any point on the line of application of the force
- The magnitude of this expression is

$$M = rF\sin$$

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Moment

Varignon's theorem

- One of the most important principles in mechanics
- states “the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.”

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$,

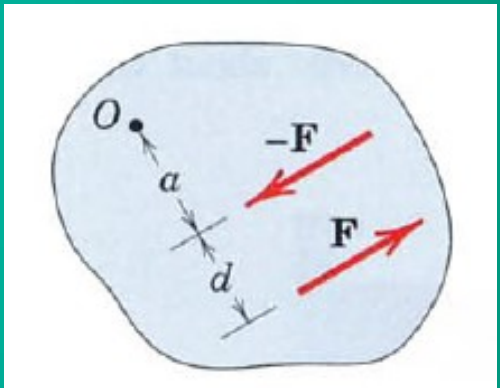
$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

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Couple

- The moment produced by two equal and opposite and non-collinear forces

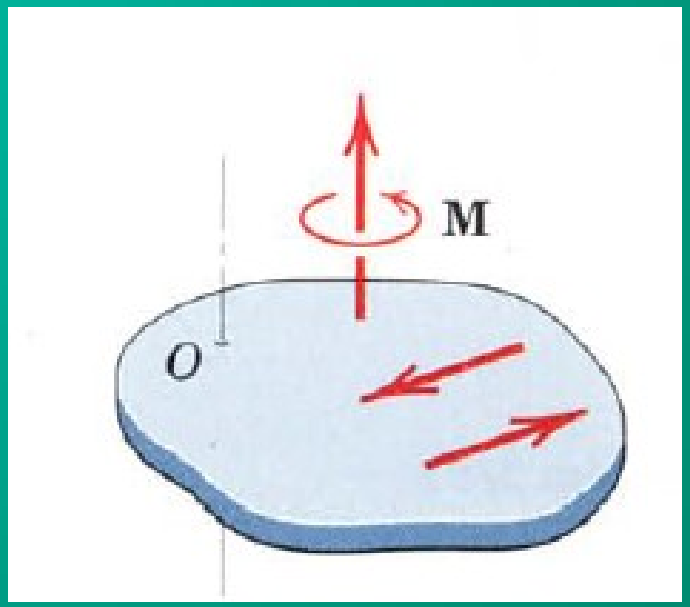


- These two forces can’t be combined in to a single force of the same effect on the body, as their sum in every direction is zero
- Their only effect is to create a tendency of rotation

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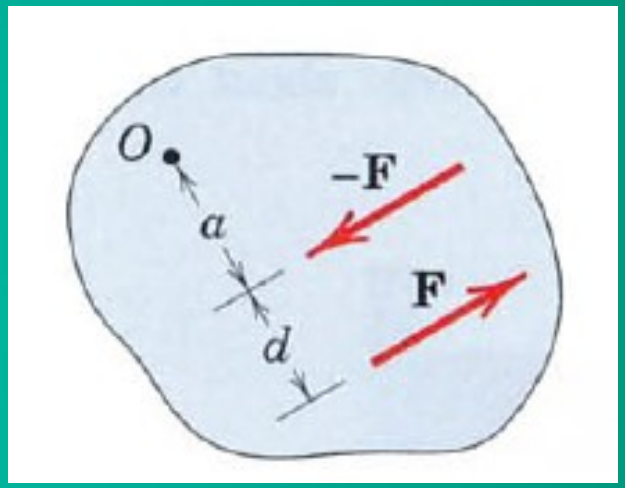
Couple

- The combined **moment** of the two forces about an axis normal to their plane and passing through any point such as o in their plane is **the couple, M.**

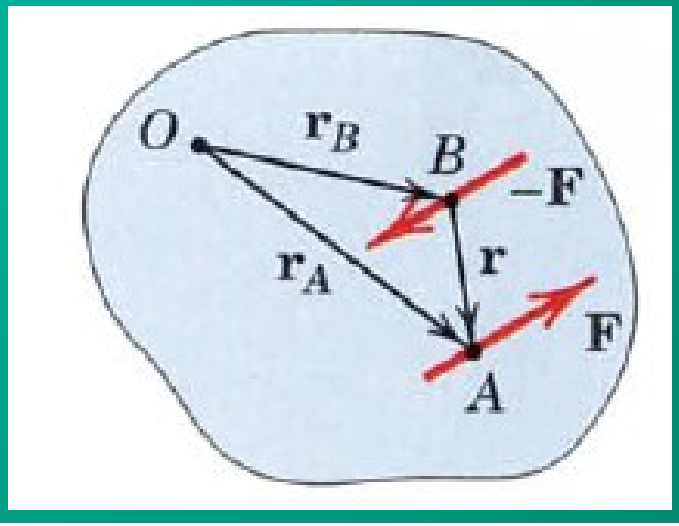


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Couple



$$\text{The magnitude of the couple } M = F(a+d) - Fa$$
$$= Fd$$

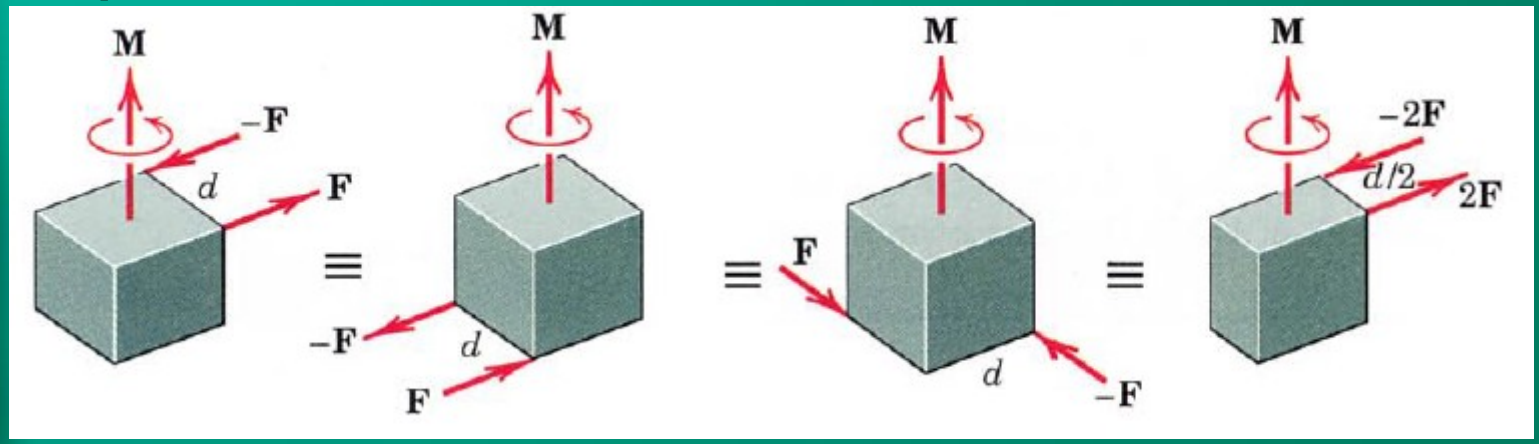


$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

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Couple

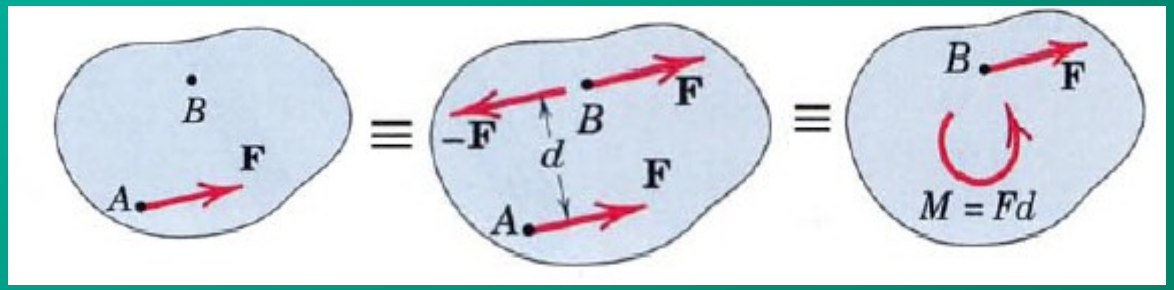
- **Equivalent Couples**
- Changing the value of *F* and *d* does not change a given couple as long as the product *Fad* remains the same.
- Likewise, a couple is not affected if the forces act in a different but parallel plane.



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Force-Couple System

- The effect of a force on a body is the tendency to push or pull and to rotate the body.
- The representation of this dual effect can be facilitated by **replacing the given force by an equal and parallel force and a couple** to compensate for the change in the moment of the forces.



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2.3.2. Moment and Couple 3-D

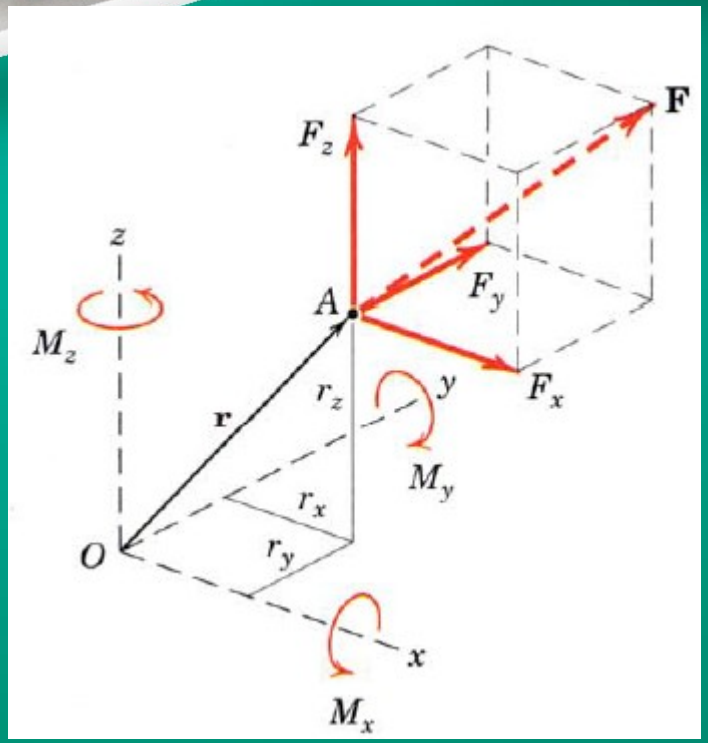
- Moments in three dimensions
- In two dimensional analysis it is convenient to use the moment arm rule (M=Fd)
- However, In three dimensions, the determination of the perpendicular distance between a point or line and the line of action of the force can be a tedious computation.
- In such cases the use of a vector approach with cross-product multiplication becomes advantageous

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Cross product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

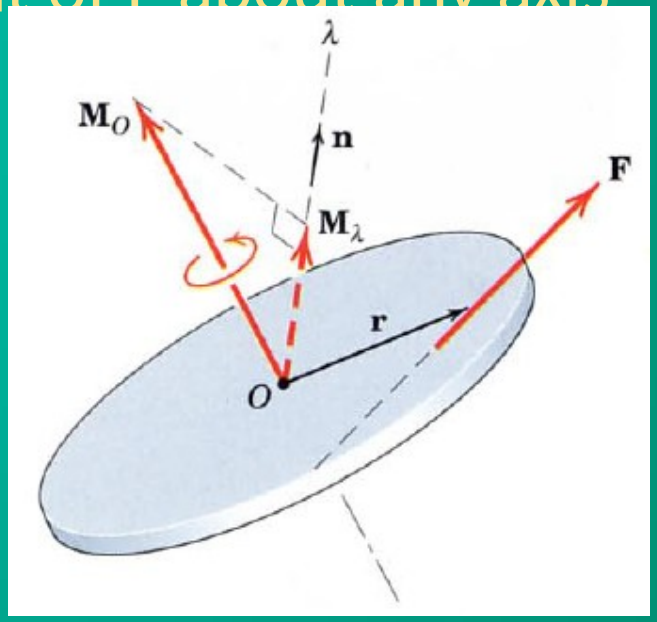
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



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2.3.2. Moment and Couple 3-D

- Moment about an Arbitrary Axis
- It is possible to obtain an expression for moment of F about any axis ¬ through O

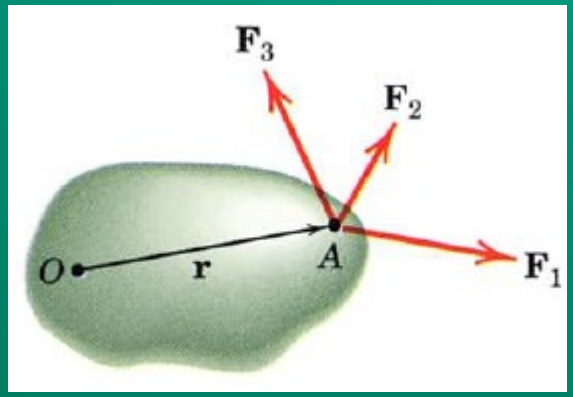


- We can use dot product expression
-
- Vector expression

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2.3.2. Moment and Couple 3-D

- **Varignon's theorem**
- Varignon's theorem introduced in two-dimension can be easily extended to three dimensions.



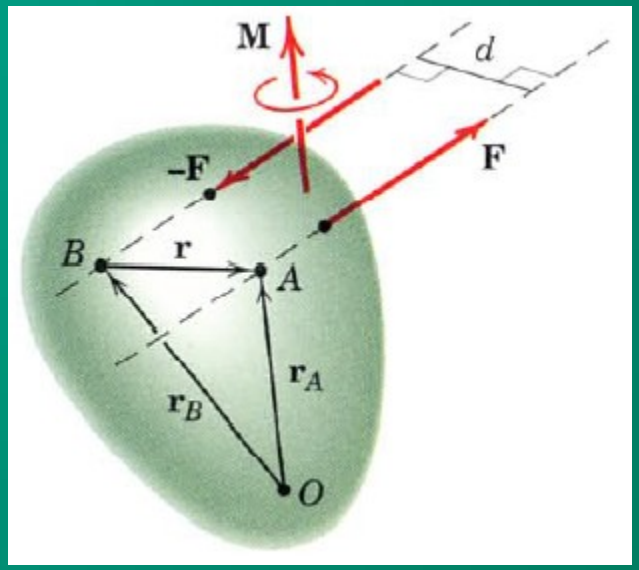
$$\mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \cdots = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots)$$
$$= \mathbf{r} \times \Sigma \mathbf{F}$$

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2.3.2. Moment and Couple 3-D

- **Couples in Three Dimensions**
- If the vector r joins any point B on the line of action of to any point A on the line of action of . The combined moment (couple) of the two forces about O is;

- **Magnitude of M**



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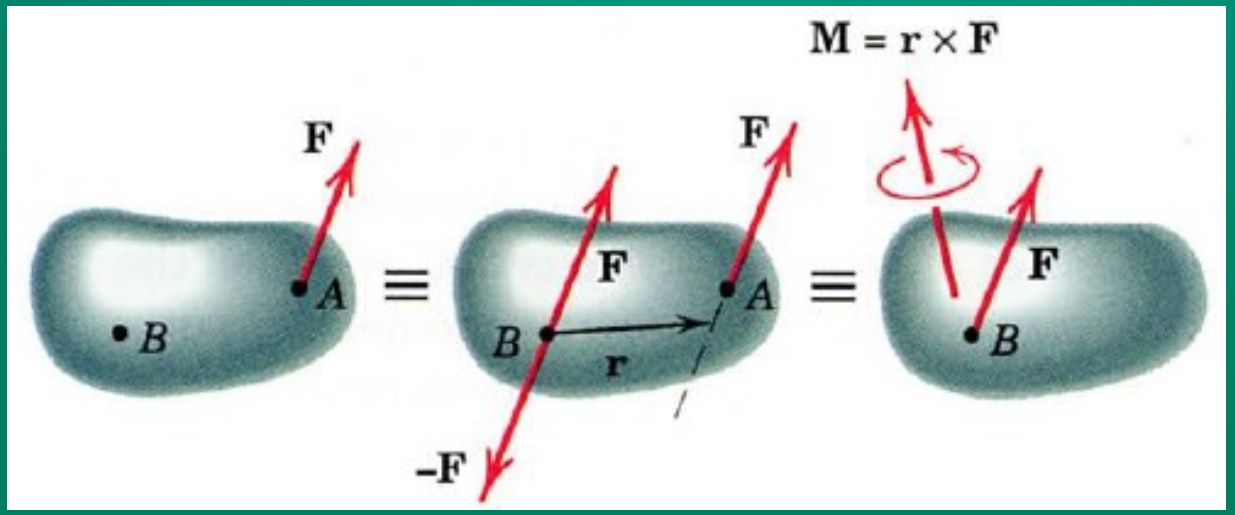
2.3.2. Moment and Couple 3-D

- The moment of the couple, , is the same about all points. Thus,
  - ✓ The moment of a couple is a free vector,
  - ✓ whereas the moment of a force about a point (which is also the moment about a defined axis through the point) is a sliding vector whose direction is along the axis through the point.
  - ✓ A couple tends to produce a pure rotation of the body about an axis normal to the plane of the forces which constitute the couple.

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2.3.2. Moment and Couple 3-D

- Force-Couple System
- It is possible to carry out replacement of a force by its equivalent force-couple system in three dimensions.



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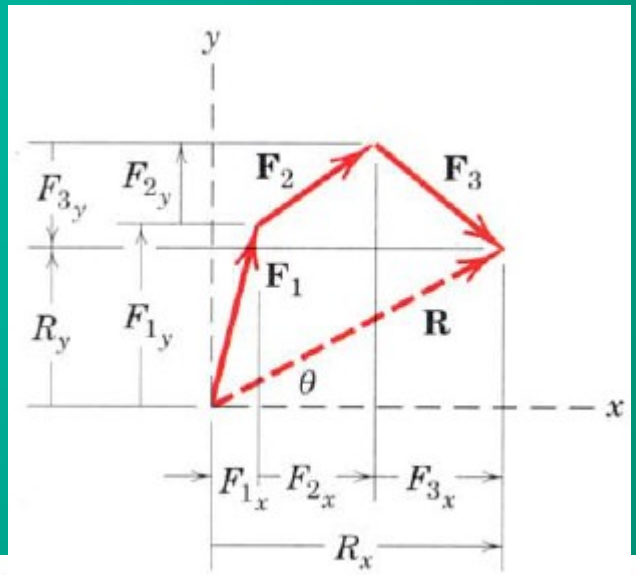
2.4. Resultants

- The resultant of a force system is the simplest force combination that can replace the original forces without altering the external effect of the system on the rigid body to which the forces are applied.
- The equilibrium of a body is the condition where the resultant if all forces that act on it is zero.

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2.4. Resultants

- The resultant can be computed by using the parallelogram rule or using analytical methods.



$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

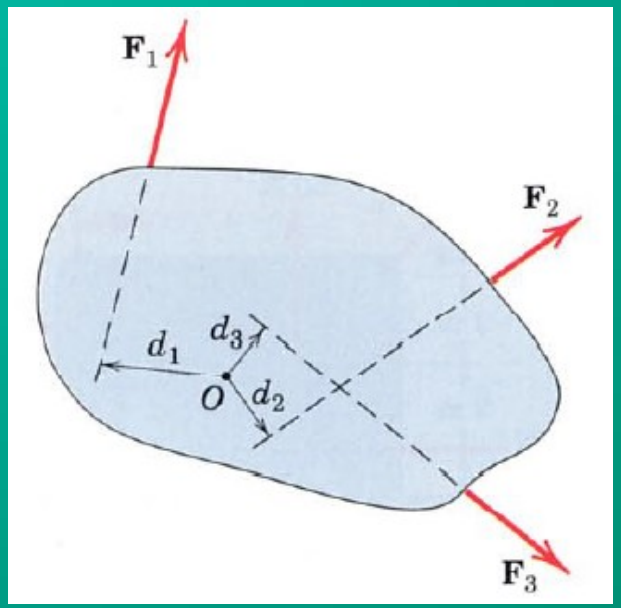
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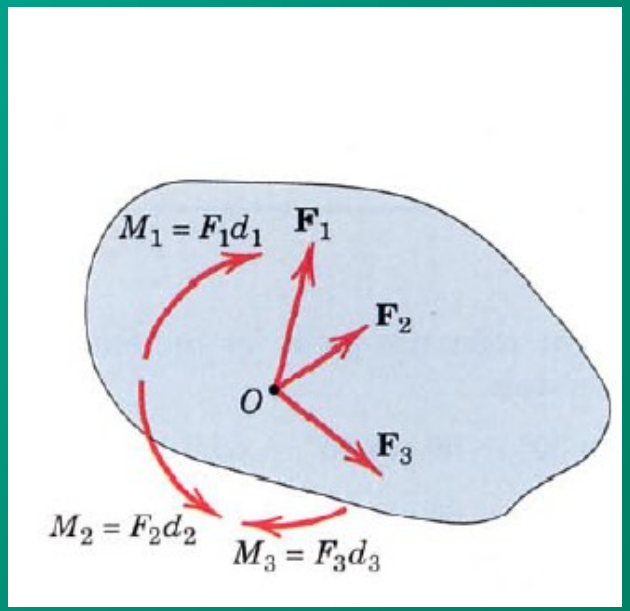
2.4. Resultants

- The algebraic method can be used to obtain the resultant force and its line of action.

Steps

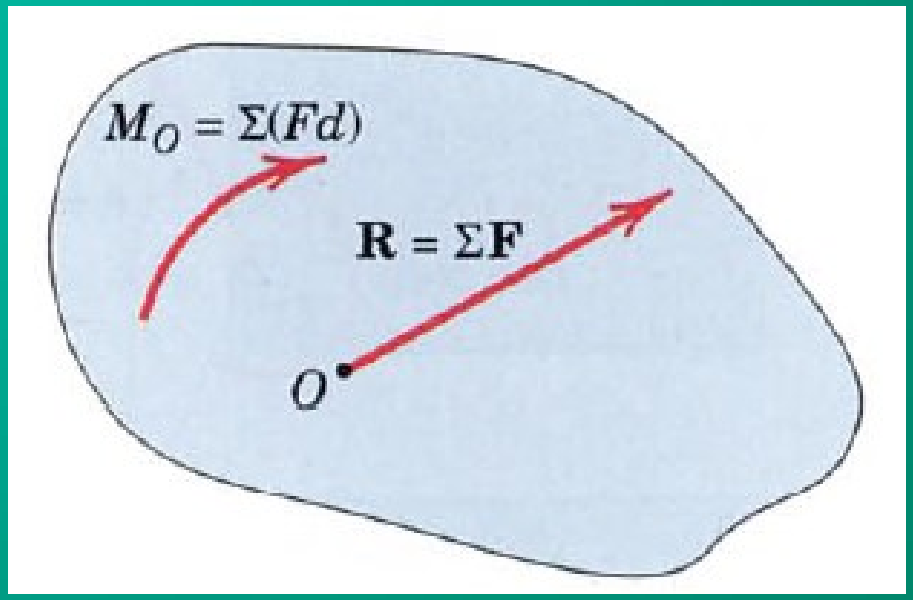
1. Choose a convenient reference point and move all forces to that point. Also determine the couple resulting from the transfer of forces.





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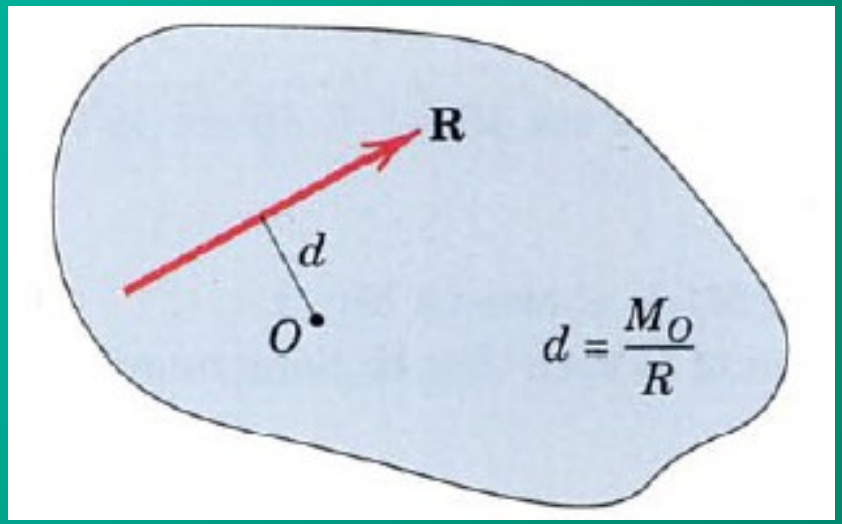
2. Add all forces at the point chosen to form the resultant force R and add all the couples to form the resultant couple M. Then the single force-couple system can be obtained.



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2.4. Resultants

3. The line of action of the resultant force R can be obtained by determining the arm distance d required to create the moment M



$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma (Fd)$$

$$Rd = M_O$$

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2.4. Resultants

- The resultant of a force system is the simplest force combination
- Any system of forces may be replaced by its resultant force R and the resultant couple M

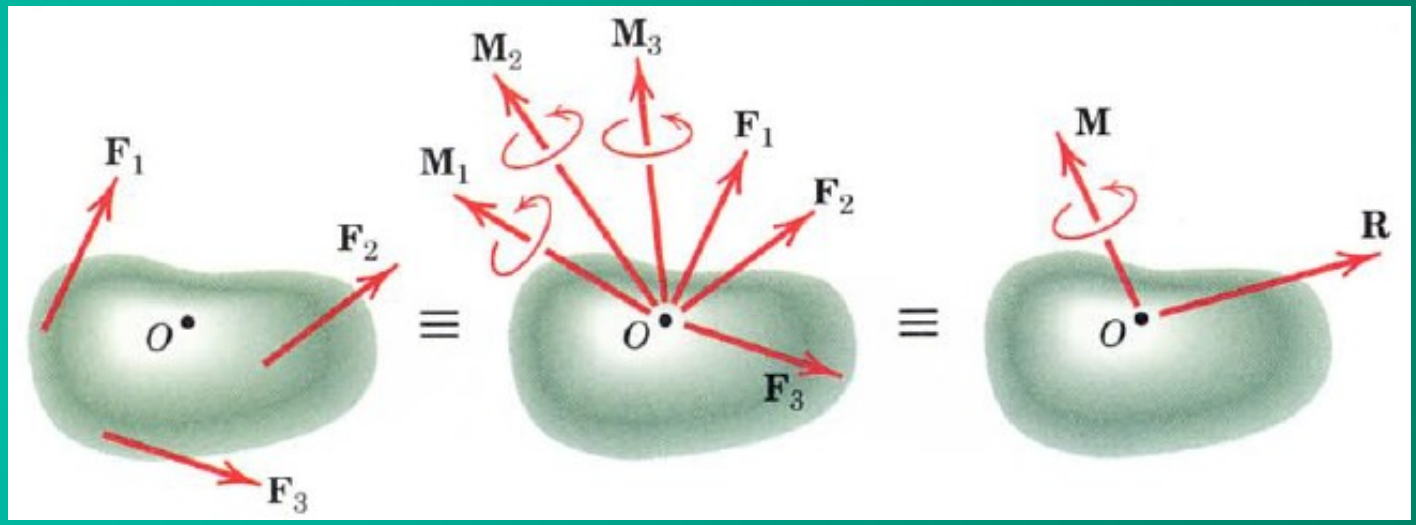
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

- The magnitude of the resultants and their components are

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$
$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$
$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

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2.4. Resultants



- The point O selected as the point of concurrency for the force is arbitrary
- The magnitude and direction of M depend on the particular point O selected
- However the magnitude and direction of R are the same no matter which point is selected.

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2.4. Resultants

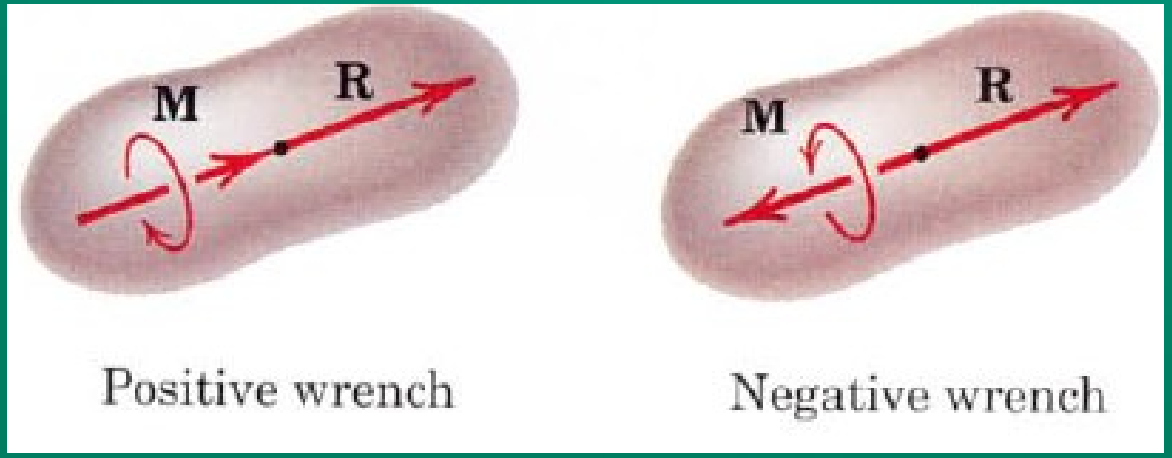
- **Resultant of special force system**
- **Concurrent Forces.** When forces are concurrent at a point only is used because there is no moment about the point of concurrency.
- **Parallel Forces.** For a of parallel forces not all in the same plane, the magnitude of the resultant force *R* is simply the algebraic sum of the given forces.

  The position of its line of action is obtained from the principle of moments by requiring that
- **Coplanar Forces.**

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2.4. Resultants

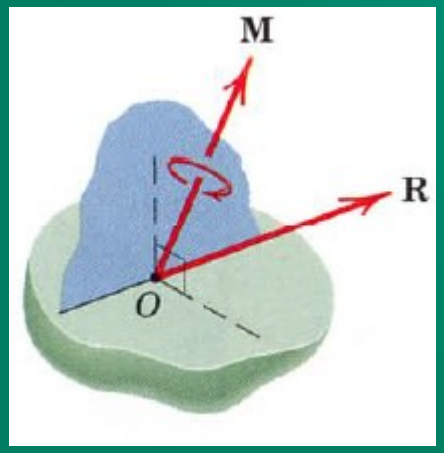
- **Wrench Resultant. When the resultant couple vector is parallel to the resultant force R the resultant is called a wrench.**

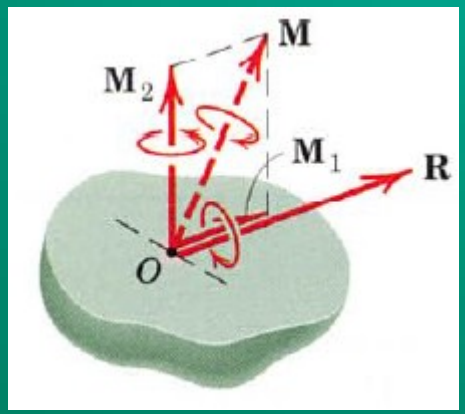


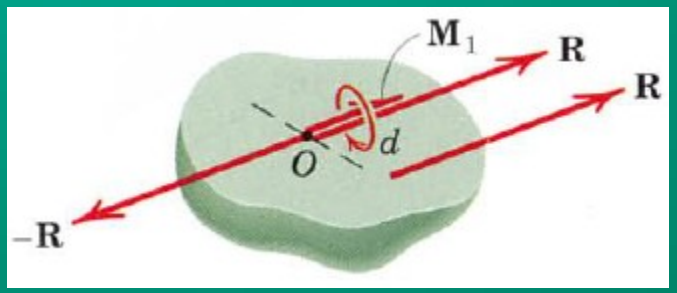
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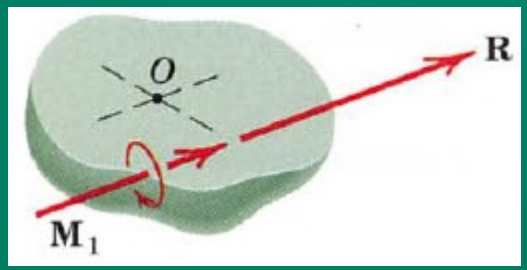
2.4. Resultants

- **Any general force system may be represented by a wrench applied along a unique line of**









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THANK YOU

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