NUMBER OF ITEMS: 65 **SUBJECT: Maths (N.S)**

First self assesment MODEL EXAM 2012E.C **GRADE : 12**

TIME ALLOWED: 3:00 HOURS



INSTRUCTION:

This Maths exam contains 65 question items with four alternative answers each and represented by the capital letters A, B, C and D. Choose the best answer to each question item and blacken zero under the letter that represents your chosen answer.

1. What is the range of the inverse of $R=\{(x,y): y=\sqrt{1-|x+2|}\}$?

   $A.\{y: y\geq -2\}$ B. $\{y: -3\leq y\leq -1\}$ C. $\{y: y\leq -2\}$ D. $\{y: 1\leq y\leq 3\}$

2. Which one of the following is true about Signum, absolute value and greatest integer functions?

   A. $\text{Sgn}(x)=\pm|x|$, for all $x\in R$.
   C. $\text{Sgn}(x)\leq \lfloor x \rfloor$, for all $x\leq 0$.
   B. $\text{sgn}(x)\leq \lfloor x \rfloor$, for all $x\geq 0$.
   D. $|x|=x\text{Sgn}(x)$, for all $x\in R$.

3. Let $f(x)=3x-4$ and $g(x)=mx+b$. Then for what values of m and b is $(fog)_{(x)}=x=(gof)_{(x)}$?

   A. $m=\frac{1}{3}$ and $b=\frac{4}{3}$
   C. $m=\frac{1}{3}$ and $b=4$
   B. $m=-3$ and $b=4$
   D. $m=\frac{1}{3}$ and $b=\frac{3}{4}$

4. What is the partial fraction form of $\frac{3x^2-2x-4}{x^3+2x}$?

   A. $\frac{2}{x}+\frac{5x+2}{x^2+2}$ B. $\frac{-2}{x}+\frac{5x+3}{x^2+2}$ C. $\frac{-2}{x}+\frac{5x-2}{x^2+2}$ D. $\frac{2}{x}+\frac{5x-2}{x^2+2}$

5. Which one of the following is not true about the graph of $f(x)=\frac{x-x^3}{x^3(x-1)}$?

   A. The vertical asymptotes of the graph is $x=0$.
   B. The horizontal asymptotes of the graph is $y=0$.
   C. The graph intersects its horizontal asymptotes at the point (-1,0).
   D. The graph intersects the vertical line $x=0$.

6. Which one of the following equation of a line that passes through the intersection of $x-2y+1=0$ and $2x+y-3=0$ and parallel to $3x-4y+2=0$.

   A. $4x-3y-1=0$
   C. $3x+4y+1=0$
   B. $3x-4y+1=0$
   D. $3x-4y-1=0$

7. A circle of radius 1unit is internally tangent to the circle $x^2+y^2-2y=8$ at (3,1).Then, the equation of the smaller circle is

   A. $x^2+(y-1)^2=1$
   C. $(x-2)^2+(y-1)^2=1$
   B. $(x-1)^2+(y-2)^2=1$
   D. $(x-4)^2+(y-1)^2=1$

8. Which of the following is not true about the hyperbola whose equation is given by $4x^2-y^2+24x+2y+19=0$?

   A. Its vertices are (-5,1) and (-1,1)
   B. The length of the transverse axis is 8
   C. Its eccentricity is $\sqrt{5}$
   D. The asymptotes are $y=2x+7$and $y=-2x-5$

9. The length of the minor axis of the ellipse $4x^2+y^2-6y+5=0$ is

A. 2 B. 1 C. 4 D. $\frac{1}{2}$

10. Let U=R (the set of real numbers) be the universe. Which one of the following proposition is false?
    A. $(\exists_x)(\forall_y)(x<y^2-1)$ C. $(\forall_x)(\forall_y)(x^2+y^2\geq 2xy)$
    B. $(\forall_x)(\exists_y)(x^2-y^2=x+y)$ D. $(\exists_x)(\forall_y)(x>0\Rightarrow x^2+y^2\geq 2)$
11. The valid conclusion from the premises: $P\vee Q, P\Rightarrow R, \neg R$ is :
    A. P B. $Q\Rightarrow P$ C. $P\wedge\neg R$ D. Q
12. Which one of the following is neither tautology nor contradiction?
    A. $[(p\wedge q)\wedge\neg p]\Rightarrow p$ C. $p\vee(q\Rightarrow\neg p)$
    B. $(\neg p\Rightarrow\neg q)\wedge(p\Rightarrow q)$ D. $(q\wedge\neg p)\Leftrightarrow(p\vee\neg q)$
13. In a class three groups A, B and C are formed. Group A with 15 students scored average of 16 points; Group B with 14 students scored average of 17 points; Group C with 11 students scored average of 14 points on group class quiz out of 20 points. What is the average of the whole students in the class?
    A. 15.3 B. 16.2 C. 15.8 D. 15.4
14. The following table gives the height distribution of students in a certain class.

Height	140-145	146-151	152-157	158-163	164-169
No. of students (f)	2	5	9	8	6

Which one of the following is not true about the distribution?
    A. The modal value is 156.3 C. The 5^{th} decile(D_5) is 156.83
    B. The first quartile (Q_1) is 154.83 D. The 60^{th} percentile (P_{60}) is 159
15. A box contains 4 red, 3 white and 5 blue balls. Three balls are drawn one after the other without replacement. What is the probability of getting all three balls are blue?

    A. $\frac{1}{22}$ B. $\frac{1}{55}$ C. $\frac{1}{27}$ D. $\frac{5}{12}$

16. The variance of 20 observations is 16. If each observation is multiplied by $\frac{3}{2}$, then what is the standard deviation of the resulting observations?
    A. 36 B. 24 C. 9 D. 6
17. In a group of 45 students, 25 play basket ball, 30 play football and 19 play both basketball and football. What is the probability of the students that play neither?
    A. $\frac{4}{5}$ B. $\frac{2}{5}$ C. $\frac{1}{5}$ D. $\frac{2}{3}$

18. If $A=\begin{pmatrix}0&4&1\\2&1&5\\0&3&2\end{pmatrix}$ and B is a 3×3 matrix such that $\det(2B)=40$, then $\det(AB)$ is equal to:

A. 200 B. -200 C. 50 D. -50

19. If the inverse of $A=\begin{pmatrix}-1&0&1\\0&y&0\\-1&x&\frac{1}{2}\end{pmatrix}$ is $B=\begin{pmatrix}1&0&y\\0&\frac{-1}{2}&x\\2&0&y\end{pmatrix}$, then the value of x and y respectively are:

A. 1 and -2 B. 0 and -2 C. 1 and 2 D. -2 and 1

20. Consider the following system of equations: $\begin{cases}\alpha y+\beta z=6\\2x+2y+z=0\\3x+y+2z=0\end{cases}$. If the determinant of the coefficient matrix is 3, then what is the solution set of the system?

A. $\{(6,-2,-8)\}$ B. $\{(-2,6,10)\}$ C. $\{(1,-2,-5)\}$ D. $\{(-6,2,-8)\}$

21. Let $A=\begin{pmatrix}1&2&1\\1&-1&3\\-2&2&0\end{pmatrix}$ and $B=\begin{pmatrix}2&-1&0\\-1&2&1\\1&2&3\end{pmatrix}$. If $C=3BA-2A^T$, then the $(2,3)$ entry of C is:

A. 20 B. 18 C. 11 D. 14

22. If $A=\begin{pmatrix}1&2\\2&3\end{pmatrix}$ and $B=\begin{pmatrix}-3&2\\2&-1\end{pmatrix}$, then which one of the following is not true?

A. $AB=BA$ C. $A+B$ is a symmetric matrix.

B. $B-A$ is a scalar matrix. D. $A+B=A-B$

23. Which one of the following is not true about the complex number $Z=\frac{\sqrt{3}+i}{\sqrt{3}-i}$?

A. Its conjugate is $\frac{1}{2}-\frac{\sqrt{3}i}{2}$. C. Its polar form is $\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)$.

B. Its modulus is 1. D. The value of Z^{60} is -1.

24. In the set of complex numbers, the solution set of $Z^3+8=0$ is

A. $\{-2\}$ C. $\{2,-1+i\sqrt{3},1-i\sqrt{3}\}$

B. $\{-2,1+i\sqrt{3},1-i\sqrt{3}\}$ D. $\{-2,1+i\sqrt{2},1-i\sqrt{2}\}$

25. What is the principal argument of the complex number $(1+i\sqrt{3})^{16}$?

A. $\frac{-2\pi}{3}$ B. $\frac{\pi}{3}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{6}$

26. Which one of the following is false about the sequence $\left\{1,-\frac{1}{3},-\frac{5}{3},-3,-\frac{13}{3},-\frac{17}{3},-7,\ldots.\right\}$?

$d=-\frac{4}{3}$

A. It is an A.P with common difference .

B. The n^{th} tem is $a_n=\frac{1}{3}(7-4n),\ for\ n\geq 1$.

C. The sum of the first 10 terms (S_{10}) is -25.

D. The largest term of the sequence less than -190 is -189.

27. Which one of the following is the n^{th} term of the sequence $\{0.5, 0.55, 0.555, ...\}$?

A. $5\left(1-\frac{1}{10^n}\right)$ B. $\frac{1}{9}\left(1-\frac{1}{10^n}\right)$ C. $\frac{5}{9}\left(1-\frac{1}{10^n}\right)$ D. $\frac{5}{9}\left(5-\frac{1}{10^n}\right)$

28. The series $1-\frac{3}{5}+\frac{9}{25}-\frac{27}{125}+.....$ is equal to:

A. 1 B. $\frac{4}{25}$ C. $\frac{5}{8}$ D. $\frac{5}{2}$

29. A boat is now worth Birr 125,000 and losses 15% of its value each year. What will it be worth after 5 years?

$\{(0.85)^5=0.4437,\ (1.15)^5=2.011357,\ (0.85)^4=0.5220,\ (1.15)^4=1.7490\}$

A. Birr 65,250.78
B. Birr 55,463.164
C. Birr 218,625.78
D. Birr 251,419.648

30. Let $\{a_n\}$ be a sequence with $a_1=-2\ and\ a_n=\frac{1}{a_{n-1}},\ for\ n\geq 2$, then what is the value of a_{98}?

A. -2 B. $-\frac{1}{2}$ C. $\frac{1}{97}$ D. $-\frac{1}{4}$

31. Which one of the following is not true about the sequence $a_n=\left\{\frac{1+3+5+\ldots(2n-1)}{3n^2-1}\right\}$?

A. $\frac{1}{2}<a_n<\frac{1}{3}$ B. $\text{lub}=\frac{1}{2}$ C. $\text{glb}=\frac{1}{3}$ D. $\lim_{n\to\infty} a_n=\frac{1}{3}$

32. The $\lim_{x\to 3}\left(\frac{\sqrt{x}-\sqrt{3}}{x-3}\right)$ is equal to:

A. $\frac{\sqrt{3}}{6}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{2\sqrt{3}}{3}$ D. $\frac{\sqrt{3}}{3}$

33. The $\lim_{x\to 1}\left(\frac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}\right)$ is equal to:

A. $\frac{\sqrt{3}}{2}$ B. $\frac{4}{3}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$

34. The $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{-\frac{x}{2}}$ is equal to:

A. e^2 B. $\sqrt{e}$ C. e^{-1} D. ∞

35. Which one of the following sequence is divergent sequence?

A. $a_n=\left\{\frac{1}{\sqrt[n]{5}}\right\}$ B. $a_n=\left\{5^{\left(\frac{n+1}{n-1}\right)}\right\}$ C. $a_n=\left\{\ln\left(\frac{1}{e^n}\right)\right\}$ D. $a_n=\{0.1, 0.11, 0.111, \ldots\}$

36. If $f(x)=\begin{cases}3^x+k, & \text{if } x\le 0\\ \end{cases}$ is continuous at $x=0$, then what is the value of K?

A. 5 B. $\frac{11}{5}$ C. $\frac{1}{5}$ D. $-\frac{9}{5}$

37. Which one of the following is true about the function $f(x)=\begin{cases}x^2, & \text{if } |x|\le 1\\ \end{cases}$?

A. It is differentiable everywhere.

B. It is continuous everywhere.

C. $\lim_{x\to 0} f(x)=1$

D. $f'(-1)=1$

38. Let $f(x)=\frac{kx+3}{2x-k}$. If $f'(1)=-1$, then value of k?

A. 2 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. -5

39. Let $f(x)=\ln(x\sec x)$. What is the derivative of $f(x)$?

A. $f'(x)=x-\tan x$ C. $f'(x)=\frac{1}{x}+\tan x$

B. $f'(x)=x+\tan x$ D. $f'(x)=\frac{1}{x}-\tan x$

40. What is the value of k so that the line $y=-3x+k$ is tangent to the curve $y=2x^2-3x+1$?

A. -11 B. 16 C. -16 D. 5

$f(x)=\frac{1}{3}x-|x^2-4|$

41. Let . What are the set of all the critical numbers of the function $f(x)$?

A. $\{-2,\frac{1}{6},2\}$ B. $\{-2,-\frac{1}{6},0,\frac{1}{6},2\}$ C. $\{-2,-\frac{1}{6},\frac{1}{6},2\}$ D. $\left\{-2,-\frac{1}{6},2\right\}$

42. Which one of the following is false about the function $f(x)=x^3-3x^2+3x+1$?

A. $f(x)$ is concave upward on $[1,\infty]$.

B. $f(x)$ is strictly increasing on $[-\infty,\infty]$.

C. $f(x)$ has local extreme value at $x=1$.

D. $(1,2)$ is the inflection point of $f(x)$.

43. What is the maximum possible area of an isosceles triangle whose perimeter is 6m?

A. $2\ m^2$ B. $\sqrt{3}\ m^2$ C. $2\sqrt{3}\ m^2$ D. $3\ m^2$

44. A ladder 6m long rests against a vertical wall and the bottom of the ladder slides away from the wall at the rate of $\frac{3}{4}m/\sec$. What is the angle between the top of the ladder and the wall, when the angle is changing at the rate of $\frac{\sqrt{2}}{8}rad/\sec$?

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{4}$

45. Which one of the following point on the graph of the function $f(x)=2x-1$ is closest to the point (1,2)?

A. $\left(\frac{8}{5},-\frac{4}{5}\right)$ B. $\left(\frac{7}{5},\frac{9}{5}\right)$ C. $\left(\frac{4}{5},\frac{3}{5}\right)$ D. $\left(\frac{8}{5},\frac{4}{5}\right)$

46. The $\int_0^{x^2}\sin t\,dt$, is equal to:

A. $\cos^2 x-1$ B. $\sin^2 x$ C. $-\cos^2 x$ D. $1-\sin^2 x$

47. $\int \frac{x^2-1}{x^3+x}\,dx$ is equal to:

A. $\ln(x^2+1)-\ln|x|+c$ C. $\frac{1}{2}\ln(x^2+1)-\ln|x|+c$

B. $\ln(x^2+1)+\ln|x|+c$ D. $2\ln(x^2+1)-\ln|x|+c$

48. What is an anti-derivative of $f(x)=\frac{e^{\tan x}}{\cos^2 x}$?

A. $e^{\cos x}+c$ B. $e^{\cot x}+c$ C. $e^{\tan x}+c$ D. $e^{\tan^2 x}+c$

49. Which one of the following integral value is false?

A. $\int \sin^2 x\,dx=\frac{1}{2}x-\frac{1}{4}\sin(2x)+c$ C. $\int \tan^2 x\,dx=\tan x-x+c$

B. $\int \cos^2 x\,dx=\frac{1}{2}x+\frac{1}{4}\sin(2x)+c$ D. $\int \ln x\,dx=\frac{1}{x}+c$

50. What is the area of the region between the graphs of the functions $f(x)=|x|$ and $g(x)=-x^2+3$?

A. $\frac{3}{2}$ sq.unit B. $\frac{25}{6}$ sq.unit C. $\frac{9}{2}$ sq.unit D. $\frac{13}{6}$ sq.unit

51. The volume of the solid generated, when the region enclosed by the graphs of the functions $f(x)=x^2$ and $g(x)=4x-x^2$ revolving about the $x-$ axis is:

A. $\frac{224}{3}\pi$ c.unit B. $\frac{22}{3}\pi$ c.unit C. $\frac{32}{3}\pi$ c.unit D. $\frac{24}{3}\pi$ c.unit

52. What is the angle between vectors $\vec{U}=mi+m\sqrt{3}\,j$ and $\vec{V}=m\sqrt{3}i+mj$?

A. 90^0 B. 60^0 C. 30^0 D. 53^0

53. Suppose that the standard form of equation of a line is given by $l: 3x-5y=6$.

What is the vector form of the equation of the line l?

A. $(x,y)=(2,0)+\lambda(5,3)$ C. $(x,y)=(2,0)+\lambda(3,5)$

B. $(x,y)=(-3,2)+\lambda(5,-2)$ D. $(x,y)=(0,2)+\lambda(3,2)$

54. A translation T takes the circle $x^2+y^2+2x+4y=0$ in to the circle $x^2+y^2-4x+2y=0$. Then what is the image of the ellipse $2(x+3)^2-3(y-2)^2=12$ under the translation T?

A. $2(x-3)^2-3(y+2)^2=12$ C. $2x^2-3(y-3)^2=12$

B. $2(y-3)^2-3x^2=12$ D. $2(y+3)^2-3(x-3)^2=12$

55. Which one of the following is the image of the line $l:3x+2y=6$ after a rotation of 90^0about the point $p(2,3)$?

A. $l^{'}: 2x-3y=6$ C. $l^{'}:3x-2y=-6$

B. $l^{'}:3x-2y=1$ D. $l^{'}: 2x-3y=1$

56. If the image of the point $P(-2,3)$is $P^{'}(6,7)$ under a line of reflection, then what is the line of reflection?

A. $2y=x+9$ B. $y=-2x+9$ C. $y=2x-9$ D. $y=-x+9$

57. Given a certain sinusoidal wave equation as $f(x)=6+\frac{3}{5}\sin\left(\frac{\pi x}{20}\right)\cos\left(\frac{\pi x}{20}\right)$, what is the amplitude, period and the range of $f(x)$ respectively?

A. $\frac{3}{5},\frac{\pi}{20}and[-6,6]$ C. $\frac{3}{10},20$ and $\left[\frac{57}{10},\frac{63}{10}\right]$

B. $\frac{3}{10},20\pi$and$\left[\frac{27}{5},\frac{33}{5}\right]$ D. $\frac{3}{5},\frac{1}{20}$ and $\left[\frac{-3}{5},\frac{3}{5}\right]$

58. Which one of the following is equal to $\tan\left[\cos^{-1}\left(\frac{1}{5\sqrt{2}}\right)-\sin^{-1}\left(\frac{4}{\sqrt{17}}\right)\right]$?

A. $\frac{3}{29}$ B. $\frac{13}{29}$ C. $\frac{-15}{29}$ D. $\frac{-11}{27}$

59. What is the solution set of the equation $4\sin^3x-\cos(2x)+1=0$ on the interval $[0,2\pi]$?

A. $\left\{0,\frac{7\pi}{6},\frac{11\pi}{6}\right\}$ C. $\left\{0,\frac{5\pi}{6},\frac{5\pi}{6},2\pi\right\}$

B. $\left\{0,\pi,\frac{7\pi}{6},\frac{11\pi}{6},2\pi\right\}$ D. $\left\{0,\pi,\frac{11\pi}{6},2\pi\right\}$

60. If in ΔABC, $AB=7$, $BC=8$and $m(\angle B)=120^0$, then what are the length of AC and the sine of $\angle A$,respectively?

A. $\sqrt{57}$and$\frac{4\sqrt{3}}{13}$ C. $13$and $\frac{23}{26}$

B. $\sqrt{57}$and$\frac{-4\sqrt{3}}{13}$ D. 13 and $\frac{4\sqrt{3}}{13}$

61. Let $\vec{a}=3i+(x-2)j+4k$ and $\vec{b}=2i-2j+yk$ be vectors. If $\vec{a}\cdot\vec{b}=0$ and $|\vec{a}|=5$, which one of the following is a possible value of y?

A. 2 B. $-\frac{3}{2}$ C. -2 D. $\frac{3}{2}$

62. If P(3,3,2) and Q(2,0,4) are points on a sphere whose center is on z- axis, then which one of the following points is outside of the sphere?

A. (-4,3,0) B. (2,-2,1) C. (3,1,1) D. (0,0,0)

63. Suppose $A=2j-k$ and $B=5i+15k$, where i, j and k are the standard unit vectors in the directions of positive x, y and z – axis respectively. Which one of the following is the unit vector in the direction of A+15B?

A. $35i+45k$ B. $13i+23j+23k$ C. $45j-35k$ D. $23i-13j+23k$

64. Consider the assertion "the sum of positive irrational numbers is positive irrational number." Which one of the following is correct about the assertion?

A. Taking irrational numbers $\sqrt{2},\sqrt{3},\sqrt{5},\sqrt{6}$ and so on. If we add any two of them, the sum is irrational. There fore the assertion is true.

B. The sum of $1+\sqrt{2}$ and $1-\sqrt{2}$ is 2, which is rational. This is a counter example that disproves the assertion.

C. The sum of $\sqrt{7}$ and $\sqrt{2}$ is a counter example that shows the assertion is false

D. The assertion can be disproved by taking the sum of $1+\sqrt{2}$ and $2-\sqrt{2}$ as counter example

65. Consider the following argument

"if he doesn't love her, she will not marry him. He loves her. Therefore, she will marry

him." If p ≡he loves her and q ≡ she will marry him, which one of the following is the

Correct representation of the argument and its validity?

A. $\neg p\Rightarrow\neg q, p|-q$; valid argument

B. $\neg p\Rightarrow\neg q, p|-q$; invalid argument

C. $p\Rightarrow q, p|-q$; valid argument

D. $p\Rightarrow q, p|-q$; invalid argument