

MATHEMATICS MODEL EXAM FOR GRADE 12

DIRECTION: THE PAPER CONTAINS MATHEMATICS EXAMINATION FOR GRADE **12** FOR **NATURAL SCIENCE STREAM ONLY**. THE EXAMINATION CONTAINS **65** ITEMS. THERE IS ONLY ONE BEST ANSWER FOR EACH ITEM. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND WRITE THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET.

YOU ARE ALLOWED TO WORK ON THE EXAM FOR **3 HOURS**.

1. Which one of the following is geometric sequence?

- A. 1,3,5,7,... B. 1,4,9,16,... C. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \dots$ D. 5, 9, 13, 17, ...

2. Which of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right)^{2x+1}$?

- A. e B. e^{-2} C. e^{-1} D. 1

3. If $\log_{\sqrt{5}} x + \log_{\sqrt[3]{5}} x + \log_{\sqrt[4]{5}} x + \dots$ up to 7 terms is equal to 35, then x is equal to:

- A. 5 B. 25 C. 125 D. 175

4. Let $f(x) = \frac{X}{X+1}$ and $g(x) = \sqrt{x-1}$, then what is the value of $f(a-1) + g(a+1)$?

- A. $\frac{a+a\sqrt{a-1}}{a}$ B. $\frac{a-a\sqrt{a-1}}{a}$ C. $\frac{a+a\sqrt{a+1}}{a}$ D. $\frac{a+a\sqrt{a-1}}{a^2}$

5. Let $f(x) = \begin{cases} x+a\sqrt{2}\sin x & \text{if } 0 \leq x < \frac{\pi}{4} \\ 2x\cot x + b & \text{if } \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \\ a\cos 2x - b\sin x & \text{if } \frac{\pi}{2} < x \leq \pi \end{cases}$ If f is continuous at

$0 \leq x \leq \pi$, then what are the values of **a** and **b** respectively?

- A. $\frac{\frac{\pi}{6} \wedge -\pi}{12}$ B. $\frac{\frac{\pi}{3} \wedge \pi}{12}$ C. $\frac{\frac{\pi}{6} \wedge \pi}{12}$ D. $\frac{-\frac{\pi}{3} \wedge -\pi}{2}$

6. Let $\lim_{x \rightarrow b} f(x) = 4$ and $\lim_{x \rightarrow b} g(x) = 2$, then $\lim_{x \rightarrow b} \frac{[f(x)]^2 - 4[g(x)]^2}{f(x) - 2g(x)}$ is equal to:

- A. 1 B. 8 C. 3 D. -8

7. Which one of the following is equal to $\lim_{x \rightarrow \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$?

- A. 0 B. ∞ C. 1 D. -1
8. Which one the following is equal to $\lim_{x \rightarrow \infty} 3x \ln\left(1 + \frac{1}{x}\right)$?
- A. $\ln e$ B. 3 C. e^3 D. e
9. If $f(x) = \log_3 81$ then $f'(1)$ is equal to:
- A. $\frac{1}{81 \ln 3}$ B. 1 C. 0 D. e^4
10. If $f(x) = [\ln(5x^2 + 9)]^3$, then which one of the following is equal to $f'(x)$?
- A. $\frac{30x[\ln(5x^2 + 9)]^2}{5x^2 + 9}$ C. $60x \ln(5x^2 + 9)$
- B. $\frac{60x \ln(5x^2 + 9)}{5x^2 + 9}$ D. $30x \ln(5x^2 + 9)$
11. The inverse of the function defined by $f(x) = \frac{2x+1}{1-3x}$ is
- A. $f^{-1}(X) = \frac{x-1}{3x+2}$ C. $f^{-1}(X) = \frac{x+1}{3x-2}$
- B. $f^{-1}(X) = \frac{3x+2}{X-1}$ D. $f^{-1}(X) = \frac{2x+1}{1-3x}$
12. If $f(x) = x^2 + x - 2$ and $\frac{1}{2}(g \circ f)(x) = 2x^2 - 5x + 2$ then $g(x)$ is
- A. $2x - 3$ B. $2x + 3$ C. $x^3 + 3x + 1$ D. $2x^2 - 3x + 1$
13. The first, second and middle terms of an arithmetic sequence are **a** , **b** and **c** respectively. Their sum is?
- A. $\frac{2(c-a)}{b-a}$ B. $\frac{2c(c-a)}{b-a} + c$ C. $\frac{2c(b-a)}{c-a}$ D. $\frac{2b(c-a)}{b-a}$
14. Which one of the following functions is one to one correspondence?
- A. $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g(x) = \sin x$
- B. $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and $f(x) = 2\sqrt{x} + 5$
- C. $f[0, \pi] \rightarrow [-1, 1]$ and $f(x) = \cos x$
- D. $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g(x) = \log x$

15. If the function $f: \mathbb{R} \rightarrow A$ given by $f(x) = \frac{x^2}{x^2+1}$ is surjective then **A** is :

- A. $\mathbb{R}$ B. $[-1, 1]$ C. $[-1, 1)$ D. $(-1, 1]$

16. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangent to the same circle. The radius of the circle is.

- A. $\frac{3}{4}$ B. $\frac{3}{2}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

17. If **f** and **g** are twice differentiable functions such that

$g(x) = e^{f(x)} \wedge g''(x) = h(x)e^{f(x)}$, then $h(x)$ is equal to:

- A. $f'(x) + f''(x)$ C. $[f'(x) + f''(x)]^2$
 B. $f'(x) + [f''(x)]^2$ D. $[f'(x)]^2 + f''(x)$

18. The equation of the hyperbola with foci at $(3\sqrt{5}, 0)$, $(-3\sqrt{5}, 0)$ and the latus rectum is of length **8** is ?

- A. $4x^2 - 5y^2 = 100$ C. $4x^2 - 5y^2 = 1$
 B. $5x^2 - 4y^2 = 100$ D. $4x^2 + 5y^2 = 100$

19. An arch is the form a parabola with its axis vertical. The arch is 10m high and 5m wide at the base. How wide is it 2m from the vertex of the parabola?

- A. $\frac{\sqrt{5}}{2}$ B. $\sqrt{5}$ C. $\frac{5}{4}$ D. $\frac{\sqrt{5}}{5}$

20. The probability that a company executive will travel by train is $\frac{2}{3}$ and that he will travel by plane is $\frac{1}{5}$. The probability of his travelling by train or plane is

- A. $\frac{2}{15}$ B. $\frac{13}{15}$ C. $\frac{15}{13}$ D. $\frac{15}{2}$

21. If $y = \ln\left(\frac{e^x}{e^x - 10}\right)$, then $\frac{dy}{dx}$ is

A. $\frac{x-e^x}{e^x-10}$ B. $\frac{-1}{e^x}$ C. $\frac{e^x}{e^x-10}$ D. $\frac{10}{10-e^x}$

22. If $h(x) = (f(x))^2 - (g(x))^2$, $f'(x) = -g(x)$ and $g'(x) = f(x)$, then $h'(x)$ is equal to:

A. $-2[g(x)+f(x)]$ C. $(-g(x))^2 - (f(x))^2$
 B. $-4f(x)g(x)$ D. $-4f(x)+g(x)$

23. Suppose $f(x)$ twice differentiable function satisfying

$f(x^2) = f(x) + x^2$, then what are the values of $f'(1)$ and $f''(1)$ respectively?

A. $\frac{2 \wedge -3}{2}$ B. $\frac{-2 \wedge 3}{2}$ C. $\frac{2 \wedge -2}{3}$ D. $\frac{-2 \wedge -2}{3}$

24. Which of the following is not true about $f(x) = \frac{x^2+x-2}{x^2-x-6}$?

- A. The graph of f has no vertical asymptote
 B. $y=1$ is a horizontal asymptote to the graph of f
 C. If $x \rightarrow 3^-$, then $y \rightarrow \infty$
 D. The graph of f is discontinuous at $\left(-2, \frac{3}{5}\right)$

25. If $z = \frac{2-i}{(1-2i)^2}$ is a given complex number, then what is the conjugate $\bar{z}$ of z ?

A. $\bar{z} = \frac{2}{25} + \frac{11}{25}i$ C. $\bar{z} = \frac{-2}{25} + \frac{11}{25}i$
 B. $\bar{z} = \frac{2}{25} - \frac{11}{25}i$ D. $\bar{z} = \frac{-2}{25} - \frac{11}{25}i$

26. If $f(x) = \tan x \sin x \cos x$, then $f'(x)$ is equal to:

A. $\sin x \cos x$ B. $-\sin x \cos x$ C. $\sin 2x$ D. $\frac{\sin x \cos x}{2}$

27. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at x equal to:

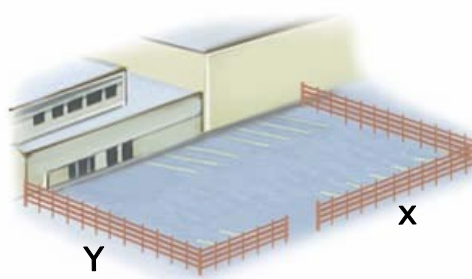
A. 4

B. 2

C. 1

D. 0

28. The manager of a department store wants to build a 600 square meter rectangular enclosure on the store's parking lot to display some equipment. Three sides of the enclosed will be built of wood fencing at a cost of birr 14 per running foot. The fourth side will be built of cement blocks, at a cost of birr 28 per running foot as shown in the figure below. What are the dimensions of enclosure that will minimize the total cost of the building materials?



A. $x=20\text{ meter} \wedge y=30\text{ meter}$

B. $x=50\text{ meter} \wedge y=12\text{ meter}$

C. $x=25\text{ meter} \wedge y=24\text{ meter}$

D. $x=15\text{ meter} \wedge y=40\text{ meter}$

29. If the function $f(x)=x^3-6x^2+ax+b$ defined on $[1,3]$ satisfies Rolle's

Theorem for $C=\left(2+\frac{1}{\sqrt{3}}\right)$, then values of **a** and **b** respectively.

A. 1 & 6

B. 11 & 6

C. 11 & any real number

D. -11 & $\frac{1}{2}$

30. If $f(x)=(x+2)e^{-x}$, then which of the following is **true** about f ?

A. Decreasing for all x

B. Decreasing in $(-\infty, -1) \wedge$ increasing in $(-1, \infty)$

C. increasing for all x

D. Decreasing in $(-1, \infty) \wedge$ increasing in $(-\infty, -1)$

31. An up turned right circular cone with semi vertical angle 45° is being filled with water at a constant rate of 30 cm^3 per second. When the depth of the water is 60 cm , what is the rate at which the area of the water surface is increasing?

- A. $1\frac{\text{cm}^2}{\text{sec}}$ B. $60\frac{\text{cm}^2}{\text{sec}}$ C. $\frac{1}{120}\frac{\text{cm}^2}{\text{sec}}$ D. $30\frac{\text{cm}^2}{\text{sec}}$

32. The modulus of the complex number $\frac{(1-i\sqrt{3})(\cos\theta+i\sin\theta)}{2(1-i)(\cos\theta-i\sin\theta)}$ is

- A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{2\sqrt{2}}$ C. $\frac{1}{\sqrt{3}}$ D. $\sqrt{3}$

33. If $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{3x^2} = \frac{1}{a}$ then what is the value of a ?

- A. 1 B. -8 C. 3 D. 8

34. If $\int f(x) \sin x \, dx = -f(x) \cos x + \int 3x^2 \cos x \, dx$, then what is the value of $f(x)$?

- A. $3x^2$ B. x^3 C. $-x^3$ D. $-3x^2$

35. The equation $|Z-1|^2 + |Z+1|^2 = 2$ represents

- A. A circle of radius 1 C. The ordered pair (0, 0)
B. A straight line D. A circle of radius 4

36. What is the area of the region between the graph of

$y = (x+1)^2 \wedge y = (x-1)^2$ and the line $y = \frac{1}{4}$ is:

- A. 4 square unit C. $\frac{4}{3}$ square unit
B. $\frac{1}{6}$ square unit D. $\frac{1}{3}$ square unit

37. Which one of the following is equal to $\int_1^e \left(\frac{x^2-1}{x} \right) dx$?

- A. $e - \frac{1}{e}$ B. $e^2 - e$ C. $\frac{e^2}{2} - e + \frac{1}{2}$ D. $\frac{1}{2}e^2 - \frac{3}{2}$

38. Which one of the following is equal to $\int \frac{dx}{(x-1)(x+3)}$?

A. $\frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + c$

C. $\frac{1}{4} \ln |(x-1)(x+3)| + c$

B. $\frac{1}{2} \ln \left| \frac{x+3}{x-1} \right| + c$

D. $\frac{-1}{2} \ln \left| \frac{x+3}{x-1} \right| + c$

39. The value of β for which the system of equations

$$2x - y - z = 12$$

$$x - 2y + z = -4 \quad \text{has no solution is:}$$

$$x + y + \beta z = 4$$

A. -3

B. 2

C. -2

D. 3

40. Which one of the following is valid logical argument?

A. $p \Rightarrow q, \neg q \Rightarrow r \vdash p$

C. $\neg q \Rightarrow \neg p, r \Rightarrow p, \neg q \vdash r$

B. $\neg q \Rightarrow \neg p, \neg r \Rightarrow \neg q \vdash \neg r \Rightarrow \neg p$

D. $p \Leftrightarrow q, p \vdash \neg q$

41. What is the sum of the series $\sum_{n=6}^{\infty} \frac{(-1)^n 2^{n-3}}{3^{n-2}}$?

A. $\frac{8}{135}$

B. $\frac{-8}{135}$

C. 135

D. 127

42. The solution set of $\frac{2}{3x+1} - \frac{1}{x} = \frac{-6x}{3x+1}$ is

A. $\left\{ \frac{-1}{3}, 2 \right\}$

B. $\left\{ \frac{-1}{3}, \frac{1}{2} \right\}$

C. $\left\{ \frac{1}{2} \right\}$

D. $\left\{ \frac{1}{2}, -1 \right\}$

43. In how many ways 7 men and 7 women can be seated around table such that no two women can sit together?

A. $7!$

B. $7! \times 6!$

C. $(7!)^2$

D. $(6!)^2$

44. The foci of the ellipse $25(x+1)^2 + 9(y+2)^2 = 225$ are equal to:

A. $(-1, -2) \wedge (-1, -6)$

C. $(-1, -2) \wedge (-2, -1)$

B. $(-2, 1) \wedge (-2, 6)$

D. $(-1, 2) \wedge (-1, -6)$

45. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then $5A - 3B + 2C$ is equal to :

- A. $\begin{bmatrix} 8 & -20 \\ 7 & -9 \end{bmatrix}$ B. $\begin{bmatrix} 8 & -10 \\ 7 & -9 \end{bmatrix}$ C. $\begin{bmatrix} 8 & 7 \\ -20 & -9 \end{bmatrix}$ D. $\begin{bmatrix} -8 & 20 \\ -7 & 9 \end{bmatrix}$

46. The sum of the first three terms of geometric sequence is 7 and sum of their squares are 21 then what are the ratios of geometric sequence?

- A. $\frac{2 \pm 3}{7}$ B. $\frac{-2 \pm -1}{2}$ C. $\frac{2 \pm 1}{2}$ D. $\frac{-2 \pm 1}{2}$

47. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added then all the balls can be arranged in the shape of a square and each of the sides then contains 8 balls less than each side of the triangle did. The initial number of the ball is?

- A. 1540 B. 55 C. 1550 D. 6650

48. For a square matrix A and a non-singular matrix B of the same order. Value of determinant of $B^{-1}AB$ is equal to:

- A. $|A^{-1}|$ B. $|B|$ C. $|B^{-1}|$ D. $|A|$

49. The following system of equations $\begin{cases} \alpha x + y + z = \alpha - 1 \\ x + \alpha y + z = \alpha - 1 \\ x + y + \alpha z = \alpha - 1 \end{cases}$ has infinitely many solution if α is?

- A. 1 B. 2 C. -1 D. -3

50. For the following frequency distribution table the value of the standard deviation is equal to :

- A. 1.6 C. 2
B. $\sqrt{1.6}$ D. 5

V	3	4	5	6	7
f	1	1	3	3	1

51. What is the image of the circle $(x-2)^2 + (y+3)^2 = 1$ under the translation that takes $(2,3)$ to $(1,4)$ followed by the reflection through the y-axis?

- A. $(x+2)^2 + (y+3)^2 = 1$ C. $(x+2)^2 + (y-3)^2 = 1$
B. $(x-1)^2 + (y-3)^2 = 1$ D. $(x+1)^2 + (y+2)^2 = 1$

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52. The point on which the line L passing through $(2, 4, -1)$ and $(5, 0, 7)$ intersect the xy -plane is equal to:
- A. $\left(\frac{7}{2}, \frac{19}{8}, 0\right)$ C. $\left(\frac{1}{8}, \frac{7}{2}, \frac{19}{8}\right)$
B. $\left(\frac{19}{8}, \frac{7}{2}, 0\right)$ D. $\left(\frac{7}{2}, \frac{19}{8}, \frac{1}{8}\right)$
53. Let two lines L_1 and L_2 are intersecting at $(0, 0)$ in a plane and passes through $(1, 1)$ and $(0, 1)$ respectively. Then the angle between them is equal to:
- A. 30° B. 45° C. 60° D. 90°
54. Consider the argument "Production is high if rain continues. Rain doesn't continue. Therefore, either production is low or rain continues." Then which of the following is true?
- A. The argument form is $q \Rightarrow p, q \vdash p \vee q$
B. The argument is valid
C. The argument form is $q \Rightarrow p, q \vdash p \vee q$
D. The argument is invalid
55. The unit vector in the opposite direction of $V = (2, 4)$ is
- A. $\left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$ C. $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$
B. $\left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)$ D. $\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$
56. Find k , so that the vector from a point $A(1, -1, 3)$ to a point $B(3, 0, 5)$ is perpendicular to the vector from point A to point (k, k, k) .
- A. $\frac{2}{5}$ B. $\frac{-8}{5}$ C. $\frac{12}{5}$ D. $\frac{7}{5}$
57. Equation of the sphere Concentric with the sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z + 10 = 0$ and which passes through $(0, 1, 0)$ is:
- A. $x^2 + y^2 + z^2 - 4x - 6y - 8z + 1 = 0$
B. $x^2 + y^2 + z^2 - 4x - 6y - 8z + 5 = 0$
C. $x^2 + y^2 + z^2 - 4x - 6y - 8z + 2 = 0$
D. $x^2 + y^2 + z^2 - 4x - 6y - 8z + 3 = 0$

58. If a bug walks on a sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 3 = 0$. Then how close and how far can it get from the origin?
- A. The largest is 3 & the smallest is $\sqrt{6}$
 B. The largest is 3 & the smallest is $3 - \sqrt{6}$
 C. The largest is $3 + \sqrt{6}$ & the smallest is $3 - \sqrt{6}$
 D. The largest is 6 & the smallest is 3
59. The image of a line given by $(x, y) : (1, 0) + t(1, 2)$ under a translation T that takes $(1, 0)$ to $(0, 1)$.
- A. $y = 2x - 2$ B. $y = 2x$ C. $y = 2x - 1$ D. $y = 2x + 1$
60. The image of a point $(1, 0)$ rotated by 60° about a point $(3, 2)$
- A. $(2 + \sqrt{3}, 1 - \sqrt{3})$ B. $(2 - \sqrt{3}, 1 - \sqrt{3})$
 C. $(2 + \sqrt{3}, 1 + \sqrt{3})$ D. $(2 - \sqrt{3}, 1 + \sqrt{3})$
61. The value of $\tan(2\sin^{-1}x)$ in terms of x is
- A. $\frac{2\sqrt{1-x^2}}{1-x^2}$ B. $\frac{2\sqrt{1-x^2}}{1+2x^2}$
 C. $\frac{2x\sqrt{1-x^2}}{1-2x^2}$ D. $\frac{2x\sqrt{1-x^2}}{1+x^2}$
62. The expression $\frac{\cos A}{1 + \sin A} + \frac{\cos A}{1 - \sin A}$ is equal to
- A. $\frac{2}{1 - \sin A}$ B. $\frac{2}{\cos A}$ C. $\frac{2}{\sin^2 A}$ D. $\frac{2}{\cos^2 A}$
63. What is the amplitude and period, respectively of the graph of $f(x) = \sin^4 x + \cos^4 x$?
- A. $\frac{3}{4}, 2\pi$ B. $\frac{1}{4}, \pi$ C. $\frac{1}{4}, \frac{\pi}{2}$ D. $\frac{1}{2}, \frac{\pi}{3}$
64. The solution set of the equation $\tan^2 \theta + \cot^2 \theta = 2 \in [0, 2\pi]$
- A. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ C. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

B. $\frac{\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{-7\pi}{4}$

D. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{-7\pi}{4}$

65. What is the possible value of X that solves the equation

$$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6} ?$$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{3}{\sqrt{2}}$

D. $\frac{\sqrt{2}}{3}$