**DIRECTION:** THE PAPER CONTAINS MATHEMATICS EXAMINATION FOR GRADE **12** FOR **NATURAL SCIENCE STREAM ONLY**. THE EXAMINATION CONTAINS **65** ITEMS. THERE IS ONLY ONE BEST ANSWER FOR EACH ITEM. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND WRITE THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET.

YOU ARE ALLOWED TO WORK ON THE EXAM FOR **3 HOURS**.

1. If $a_1, a_2, a_3 \ldots$ in arithmetic sequence such that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225, then\, a_1+a_2+a_3+a_4+\ldots+a_{23}+a_{24} is$ equal to

   A.909 B. 75 C. 750 D. 900

2. The sum of integers from 1 to 100 which are not divisible by 3 or 5 is

   A. 2489 B. 4735 C. 2317 D. 2632

3. What is the sum of the series $\sum_{n=1}^{\infty}\left(\frac{4}{2^n}-\frac{2}{n(n+1)}\right)$?

   A. 2 B. 4 C. 0 D. -2

4. If $f(x)=x^2-x^{-2}$, $x \in$-$\{0\}$, then $f\left(\frac{1}{x}\right)$ is equal to

   A. $f(x)$ B.$-f(x)$ C.$f\left(\frac{1}{x}\right)$ D. $(f(x))^2$

5. If $f(x)=x^3+3x^2+4x+5$ and $g(x)=5, then\, g¿$ is equal to

   A. $5x^2+15x+25$ C.1125

   B. $5x^3+15x^2+20x+25$ D.5

6. If $a, b, c, d, e \wedge f$ are in arithmetic sequence, then $e-c$ is equal to:

   A. $2(c-a)$ B. $2(d-c)$ C. $2(f-a)$ D. $(d-c)$

7. Let $(x)=\begin{cases} x^2-a^2x, if\, x<2 \\ 4-2x^2, if\, x\geq 2 \end{cases}$. What is the value of **a** if *f* is continuous at $x=2?$

   A.-2 B. 2 C. 2 & -2 D. 4

8. If $f(x)=\lfloor x \rfloor, x \in IR$, which of the following is **true** about *f*?

   A.$f(x)=2f(x)$ C.$f(x)=(fof)(x)$

B. $f(x)=(fxf)(x)$ D. $f(x)=x$

9. Let $\{a_n\}_{n=1}^{\infty}$ be defied recursively by $a_1=1 \wedge a_{n+1}=\left(\frac{n+2}{n}\right)a_n \text{ for } n\geq 1.$

Then $\sum_{n=1}^{5} a_i$ is equal to

A. 55 B. 45 C. 35 D. 20

10. If $f(x)=-\sqrt{25-x^2}$ , then $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$ is

A. $\frac{1}{24}$ B. $\frac{1}{5}$ C. $-\sqrt{24}$ D. $\frac{1}{\sqrt{24}}$

11. For the equation $4x^2+7y^2+32x-56y+148=0$. Which one of the following is **true**?

A. ellipse with center $(4,-4)$ and foci at $(4\pm\sqrt{3},-4)$

B. hyperbola with center $(-4,4)$ and foci at $(4,-4\pm\sqrt{3})$

C. ellipse with center $(-4,4)$ and foci at $(-4\pm\sqrt{3},4)$

D. hyperbola with center $(4,-4)$ and foci at $(-4,4\pm\sqrt{3})$

12. Which of the following is equal to $\sum_{n=1}^{\infty}\frac{(2)^{n-1}+3}{3^{n+2}}$

A. $\frac{1}{3}$ B. $\frac{1}{9}$ C. $\frac{1}{4}$ D. $\frac{1}{5}$

13. If $a,b,c,d$ are positive integers, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$ is equal to

A. $e^{\frac{d}{b}}$ B. $e^{\frac{c+d}{a+b}}$ C. $e$ D. $e^{\frac{c}{a}}$

14. A fair die is tossed once. The probability that either an even number or 3 will be appear is

A. $\frac{3}{4}$ B. $\frac{4}{5}$ C. $\frac{2}{3}$ D. $\frac{1}{6}$

15. The following is the frequency of a grouped data. The mean of the following frequency table is 50. Then what is the value of $f_1 \wedge f_2$ respectively.

| Class interval | Frequency |
|---|---|
| 0-20 | 17 |
| 20-40 | $f_1$ |
| 40-60 | 32 |
| 60-80 | $f_2$ |
| 80-100 | 19 |
| Total | 120 |

A. 27 & 25 B. 22 & 30 C. 28 & 24 D. 36 & 24

16. If $f(2)=4$ and $f'(2)=5$, then $\lim_{x\to 2} \frac{xf(x)-2f(x)}{x-2}$ is

A. – 6 B. – 4 C. 2 D. 3

17. $\int \frac{x-1}{(x-3)(x-2)}dx$ is equal to

A. $2\ln|x-3|-\ln|x-2|+c$ C. $\ln|x-3|-2\ln|x-2|+c$

B. $\ln\left|\frac{x-3}{x-2}\right|+c$ D. $2\ln|x+3|-\ln|x-2|+c$

18. If $f$ is a continuous function such that $f(0)=f(1)=0, f'(1)=2$and $g(x)=f(e^x)e^{f(x)}$ , then $g'(0)$is :

A. 0 B. 1 C. 2 D. 3

19. The value of the derivative of $|x-2|+|x-5|$ at $x=4$ is

A. 0 B. -1 C. $2x-3$ D. 2

20. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 12 & sinx & cosx \\ 13 & cosx & sinx \end{bmatrix}$, then $|A|$ is

A. $\cos 2x$ B. $-\cos 2x$ C. $\sin 2x$ D. $cosx$

21. If A and B are square matrices of the same order and A is non- singular, then for a positive integer **n**, $(A^{-1}BA)^n$ is equal to

A. $(A^{-1})^n B^n A^n$ B. $A^n B^n$¿ C. $A^{-1}B^n A$ D. $n(A^{-1}BA)$

22. Which of the following is **Not** rational expression?

A. $\frac{\log_2 16+3x^2}{2^3}$ C. $3x^{-2}+15x^{-1}+8$

B. $\frac{3^5+3x^2}{9x}$ D. $\frac{\sqrt{x^2}}{x+1}$

23. If $f(x)=\begin{cases} ax^2-b, if\ |x|<1 \\ \frac{1}{|x|}, if\ |x|\geq 1 \end{cases}$ is differentiable at $x=1$, then the value of $a$ and $b$ is :

A. $a=\frac{1}{2}, b=\frac{-1}{2}$ B. $a=\frac{-1}{2}, b=\frac{-3}{2}$ C. $a=b=\frac{1}{2}$ D. $a=b=\frac{-1}{2}$

24. A person walks at the rate of $6\frac{m}{s}$ towards a street light pole whose lamp is 5m above the base of the light pole. If the person is 1.5m tall, determine the rate of change of the length of his shadow at the rate of change of the length of his shadow at the moment he is 10m from the base of light pole.

A. $\frac{18}{7}\frac{m}{s}$ B. $\frac{30}{7}\frac{m}{s}$ C. $\frac{7}{30}\frac{m}{s}$ D. $\frac{7}{18}\frac{m}{s}$

25. The latus rectum of the ellipse $5x^2+9y^2=45$ is

A. $\frac{6}{5}$ B. $\frac{3}{5}$ C. $\frac{5}{3}$ D. $\frac{10}{3}$

26. $\int_0^1 (x+1)e^{x^2+2x}dx$ equal to :

A. $\frac{e^3}{2}$ B. $\frac{e^3-1}{2}$ C. $\frac{e^3}{2}+\frac{1}{2}$ D. $\frac{-1}{2}-\frac{e^3}{2}$

27. $\int x\sqrt{x-2}dx$ is equal to :

A. $\frac{2}{5}(x-2)^{\frac{5}{2}}+\frac{4}{3}(x-2)^{\frac{3}{2}}$ + c

B. $\frac{2}{5}(x-2)^{\frac{5}{2}}+\frac{2}{3}(x-2)^{\frac{3}{2}}$ + c

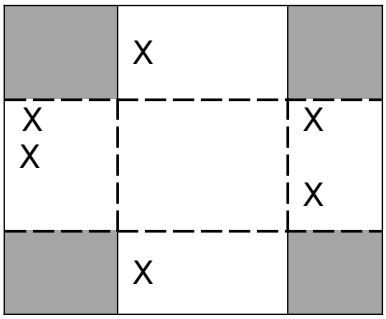
C. $\frac{5}{2}(x-2)^{\frac{5}{2}}+\frac{4}{3}(x-2)^{\frac{3}{2}}$ + c

D. $\frac{2}{5}(x-2)^{\frac{5}{2}}+\frac{4}{3}(x+2)^{\frac{3}{2}}$ + c

28. If the function $f(x)=ax^3+bx^2+11x-6$ satisfies conditions of Rolle's Theorem in $[1,3]$ and $f'\left(2+\frac{1}{\sqrt{3}}\right)=0$, then values of **a** and **b** respectively.

A.1 & -6 B. -2 & 1 C. -1 & 6 D. -1 & $\frac{1}{2}$

29. A metal box (without top) is to be constructed from a square sheet of metal that is 10cm on the side by first cutting the square pieces of the same size from the corners of the sheet and folding up sides a shown in the figure below. What size squares should be cut in order to maximize he volume of the box.




A.$\frac{5}{3}cm$ B. 5cm C. $\frac{3}{5}cm$ D. $\frac{1}{5}cm$

30. Which one of the following is a valid logical argument?

A. $p \Rightarrow q, \neg q \vdash p$

B. $p \wedge q$, p$\vdash \neg$ q

C. p$\Rightarrow$ q, $\neg$ r$\Rightarrow \neg q \vdash \neg$r$\Rightarrow \neg$p

D.$p \Leftrightarrow q$, p $\vdash \neg$q

31. Let z be complex number. Then the solution set of$z^2-z+1=0$.

A.$\{\frac{-1\pm i\sqrt{3}}{2}\}$ B.$\{\frac{1\pm i\sqrt{3}}{2}\}$ C.$\{\frac{1\pm i\sqrt{3}}{3}\}$ D.$\{1\pm\sqrt{3}\}$

32. On which of the following intervals is the graph of $f(x)=3x^4-4x^3$ concave down ward?

A. (0, $\frac{2}{3}$¿ B. $\left(\frac{2}{3},\infty\right)\wedge(-\infty,0)$ C. $[0,2]$ D. $[0,4]$

33. On which of the following intervals is the graph of $f(x)=x^3-3x^2-24x+1$ decreasing?

A. $(-\infty,-2]$ B.$[-2,4]$ C. $(4,\infty)\wedge(-\infty,-2]$ D. $(-2,3)$

34. What is an anti derivative of $f(x)=\frac{\cos(2x)}{(sinx+cosx)^2}$

A.$\ln(sinx-cosx)$ C. $\ln(sinx-cosx)^2$

B. $\ln(sinx+cosx)^2$ D. $\ln|sinx+cosx|$

35. The area of the region bounded between the graph of $y=|x-1|$ and $y=3-|x|$ is

A.2squ. Unit B. 3squ.unit C. 4squ. Unit D. 6squ. Unit

36. If A and B are two matrices such that $AB=B$ and $BA=A$ , then $A^2+B^2$ is equal to

A. $2AB$ B. $2BA$ C. $A+B$ D. $AB$

37. Let $f$ be twice differentiable function such that $f''(x)=-f(x)$ and $g(x)=f'(x), h(x)=[f(x)]^2+[g(x)]^2$. If $h(7)=12$ then $h(5)$ is equal to

A.12 B. 2x + 4 C. 12x + 1 D. 5

38. Let $f(x)=\ln(x^2)$.What is the equation of the line tangent to the graph of $f$ at$x=e^2$.

A.$y=\frac{2}{e^2}(x-2)$ B.$y+2=\frac{2}{e^2}x+2$ C. $y=\frac{-2}{e^2}(x+2)$ D. $y=e^2$

39. Equation of a circle through $(-1,-2)$ and concentric with the circle $x^2+y^2-3x+4y-c=0$ is

A. $x^2+y^2-3x+4y-1=0$ C. $x^2+y^2-3x+4y+2=0$

B. $x^2+y^2-3x+4y+1=0$ D. $x^2+y^2-3x+4y=0$

40. If the diameter of a parabolic mirror is 10m and if the mirror is 5m deep at the center, how far is the focus from the center of the mirror?

A. $\frac{5}{4}$ B. $\frac{5}{2}$ C. $\frac{4}{5}$ D. $\frac{2}{5}$

**41.** The radius of a circle with center $(7,12)$ and tangent to the line $2x-y+3=0$ is

A. 5 B. 25 C. $\sqrt{5}$ D. $\frac{\sqrt{5}}{5}$

42. $\lim_{x\to\frac{\pi}{2}} \frac{cosxsin(cosx)}{sinx-cscx}$ is equal to :

A. -1 B. 1 C. ∞ D. 4

43. Let $X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}, A=\begin{bmatrix}1&-1&2\\2&0&1\\3&2&1\end{bmatrix}$ and $B=\begin{bmatrix}3\\1\\4\end{bmatrix}$ and if $Ax=B$, then X equal to:

A. $\begin{bmatrix}1\\2\\3\end{bmatrix}$ B. $\begin{bmatrix}-1\\-2\\3\end{bmatrix}$ C. $\begin{bmatrix}-1\\-2\\-3\end{bmatrix}$ D. $\begin{bmatrix}-1\\2\\3\end{bmatrix}$

44. If all the letters of the word **MISSISSIPPI**, are written down at random, the probability that all the four **S'**s appear consecutively is

A. $\frac{3}{165}$ B. $\frac{7}{165}$ C. $\frac{4}{165}$ D. $\frac{1}{6}$

45. What is the volume of the solid generated if the region bounded by $f(x)=3-x^2$ and $g(x)=3x-1$ on $[0,1]$ by revolving about the $y-axis$

A. $\frac{\pi}{2}$ B. $\frac{3\pi}{2}$ C. $\frac{\pi}{4}$ D. $\frac{5\pi}{4}$

46. The solution set of $\frac{3}{x+1¿}=\frac{-3x}{x+1}+\frac{6}{x}$ is

A. $\{-1,2\}$ B. $\{-1\}$ C. $\{2\}$ D. $\left\{\frac{1}{2},-1\right\}$

47. $p(x,y):x+y=7$ Where $x$ and $y$ are natural numbers, then which one of the following is true?

A. $(\exists x)(\exists y)P(X,Y)$ C.$(\forall x)(\forall y)P¿$**)**

B.$(\exists x)(\forall y)P(X,Y)$ D.$(\forall x)(\exists y)P(X,Y)$

48. Polar form of $z=1-\sqrt{3}i$ is

A. $2\left[\cos\left(\frac{\pi}{3}\right)+iSin\left(\frac{\pi}{3}\right)\right]$ C. $\sqrt{2}\left[\cos(300^{\circ})+iSin(300^{\circ})\right]$

B. $2\left[\cos\left(\frac{-\pi}{3}\right)+iSin\left(\frac{-\pi}{3}\right)\right]$ D.$2\left[\cos(120^{\circ})+iSin(120^{\circ})\right]$

49. Three letters are written to different persons and corresponding addresses are written on three envelopes. However; letter are placed in envelopes without looking at the addresses. The probability that the letters go in to right envelopes is

A. $\frac{1}{6}$ B. $\frac{1}{27}$ C. $\frac{1}{9}$ D. $\frac{1}{3}$

50. $f(x)=\frac{x^2-3x}{x^2-2x-8}$ , Which of the following is **true** about $f$*?*

A. The line $y=3$ is a Horizontal asymptote.

B. The line $x=4$ and $x=-2$ are a vertical Asymptote.

C. If $x\to\infty$ , then $f(x)\to 4$

D. If $\to 4^{+¿¿}$ , then $f(x)\to-\infty$

51. What is the amplitude and period, respectively of the graph $f(x)=2\cos\frac{1}{3}(x-\pi)$

A. $2\wedge 6\pi$ B. $2\wedge 2\pi$ C. $2\wedge 2\pi$ D.$-2\wedge 6\pi$

52. The image of the line $2y+3x=1$after it has reflected by the line $y=-3x+2$ is

A. $y=-18x+17$ C. $y-9x=-10$

B. $y=x-3$ D. $2y+11x-9=0$

53. The angle of elevation of the top of tower standing on a horizontal plane from a point A is $\propto$ after walking a distance towards the foot of the tower the angle of elevation is found to be$\beta$. The height of the tower is

A. $\frac{asin\propto sin\beta}{\sin(\beta+\alpha)}$ B. $\frac{asin\propto sin\beta}{\sin(\propto-\beta)}$ C. $\frac{asin\propto sin\beta}{\sin(\beta-\alpha)}$ D. $\frac{asin\propto sin\beta}{\cos(\beta+\alpha)}$

54. Which one of the following is the image of the circle $x^2+(y-2)^2=1$ under rotation by $90^\circ$ in anticlockwise direction about the origin?

A. $(x+2)^2+y^2=1$ C. $x^2+(y+2)^2=1$

B. $(x-2)^2+y^2=1$ D. $(x+2)^2+(y-2)^2=1$

55. What is the image of the line given by $(x,y)=(1,3)+t(1,2)t\in R,$ under the translation that takes $(2,0)¿(0,2)$ followed by the reflection about the line $y=2x+1?$

A. $y=2x-4$ C. $y=2x+1$

B. $y=2x+4$ D. $y=2x+7$

56. The angle between the vector $\vec{a}=\vec{i}-\vec{j}$ and $\vec{b}=2\vec{j}$ is

A. $90^\circ$ B. $45^\circ$ C. $135^\circ$ D. $120^\circ$

57. Which one of the following is not correct identity

A. $\frac{\cos x}{1+sinx}+\frac{cosx}{1-\sin x}=2secx$ C. $\sin x\cos^2 x-\sin x=-\sin^3 x$

B. $\sin 2x(1+\tan^2 x)=2tanx$ D. $\cos^4 x-\sin^4 x=\sin 2x$

58. What is the value of $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)$

A. 1 B. -1 C. $\frac{\pi}{2}$ D. $\frac{\pi}{6}$

59. If ¿ is false, which of the following is TRUE?

A. $s\Longrightarrow r$ B. $\neg ¿$ C. $(p\wedge\neg q¿\Longleftrightarrow r$ D. $(p\wedge r)$

**60.** If $\vec{a}=3\vec{i}+2\vec{j}+\vec{k}$ and $\vec{b}=2\vec{i}-4\vec{j}-3\vec{k}$ , what is the value of $|\vec{a}-2\vec{b}|$

A. $\sqrt{150}$ B. $\sqrt{160}$ C. $10\sqrt{150}$ D. $5\sqrt{150}$

61. Let $\vec{a}=2\vec{i}-\vec{j}+\vec{k}$ and $\vec{b}=2\vec{i}-4\vec{j}+t\vec{k}$ are perpendicular. Then what is the value of $t$?

A. 16 B. 4 C. -8 D. 8

62. The distance of the point $p(4,-3,5)$ from $xz-plane$ is

A. $5\sqrt{2}\,unit$ B. 5 unit C. 3unit D. 4 unit

63. What is the image of the ellipse whose equation is $25(x+1)^2+(y-3)^2=25$ under a translation that takes that $(2,-1)$ to $(3,-4)$ followed by rotation $90^\circ$.

A. $x^2+25y^2=25$ C. $25x^2+y^2=25$

B. $x^2+\frac{y}{25}^2=1$ D. $25x^2+y^2=1$

64. The angle between the vectors $\vec{a}=2\vec{i}-\vec{j}+2\vec{k}$ and $\vec{b}=\vec{i}-\vec{j}$ is

A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{4}$

65. Suppose $r(x):x\,is\,even, p(X):x\,is\,prime, \wedge q(x):x\,is\,a\,multiple\,of\,2.$ Which one of the following has the truth value True on the set of natural numbers:

A. $(\exists x)(r(x)\wedge p(x))$ C. $(\exists x)(r(x)\wedge\neg q(x))$

B. $(\forall x)(r(x)\wedge p(x))$ D. $(\forall x)(r(x)\Longrightarrow p(x))$