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INTRODUCTION TO ECONOMICS

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CHAPTER FOUR

4. THEORY OF PRODUCTION AND COST

This chapter has **two major sections**. The first part introduces the basic concepts of production and other things related to it. The second part deals with theory of costs and the relationship between cost function and production function.

4.1 Theory of production in the short run

4.1.1 Definition of production

Production is the process of transforming inputs into outputs (goods and services)

4.1.2 Production function

Production function is a technical relationship between inputs and outputs. It shows the maximum output that can be produced with fixed amount of inputs and the existing technology. A general equation for production function can, for instance, be described as: $Q = f(X_1, X_2, X_3, \dots, X_n)$ where, Q is output, and $X_1, X_2, X_3, \dots, X_n$ are different types of inputs.

Inputs are commonly classified as fixed inputs or variable inputs. **Fixed inputs** are those inputs whose quantity cannot readily be changed when market conditions indicate that an immediate adjustment in output is required. **Capital (building, machineries) and land** are good example of fixed inputs **because** *their quantity cannot be manipulated easily in a short period of time. For instance, if the demand for Pepsi rises suddenly in a week, the Pepsi company cannot plant*

Variable inputs are those inputs whose quantity can be altered almost instantaneously in response to **desired changes in output**. That is, their quantities can easily be diminished when the market demand for the product decreases and vice versa. The best example of **variable inputs** in economics is **unskilled labor**.

kinds of time period which are short run and long run. **Short run** refers to a period of time in which the quantity of at least one input is fixed. In other words, short run is a time period which is not sufficient to change the quantities of all inputs so that at least one input remains fixed. Here it should be noted that short run periods of different firms have different durations. Some firms can change the quantity of all their inputs within a month while it takes more than a year for other types of firms.

Short run production

Consider a firm that uses two inputs: capital (fixed input) and labour (variable input). Given the assumptions of short run production, the firm can increase output only by increasing the amount of labour it uses. Hence, its production function can be given by: **$Q = f(L)$**

where, **Q** is output and **L** is the quantity of labour.
The production function shows different levels of output that the firm can produce by efficiently utilizing different units of labour and the fixed capital. In the above short run production function, the quantity of capital is fixed. Thus, output can change only when the amount of labour changes.

4.1.3 Total, average, and marginal product

In production, the contribution of a variable input can be described in terms of total, average and marginal product.

Total product (TP):

It is the total amount of output that can be produced by efficiently utilizing specific combinations of the variable input and fixed input. Increasing the variable input (while some other inputs are fixed) can increase the total product only up to a certain point. Initially, as we combine more and more units of the variable input with the fixed input, output continues to increase, but eventually if we employ more and more unit of the variable input beyond the carrying capacity of the fixed

In general, the TP function in the short-run follows a certain trend: it initially increases at an increasing rate, then increases at a decreasing rate, reaches a maximum point and eventually falls as the quantity of the variable input rises.

Marginal Product (MP):

It is the change in output attributed to the addition of one unit of the variable input to the production process, other inputs being constant. For instance, the change in total output resulting from employing additional worker (holding other inputs constant) is the marginal product of labour (MP_L). In other words, MP_L measures the slope of the total product curve at a given point.

$$MP_L = \frac{\Delta TP}{\Delta L} = \frac{\Delta Q}{\Delta L}$$

In the short run, the marginal product of the variable input first **increases**, reaches its **maximum** and then **decreases** to the extent of being **negative**. That is, as we continue to combine more and more of the variable input with **the fixed input**, the marginal product of the variable input increases initially and then declines.

Average Product (AP):

Average product of an input is the level of output that each unit of input produces, on the average. It tells us the mean contribution of each variable input to the total product. Mathematically, it is the ratio of total output to the number of the variable input.

$$AP_L = \frac{TP}{L}$$

Average product of labour first increases, reaches its maximum value and eventually declines. The AP curve can be measured by the slope of rays originating from the origin to a point on the TP curve.

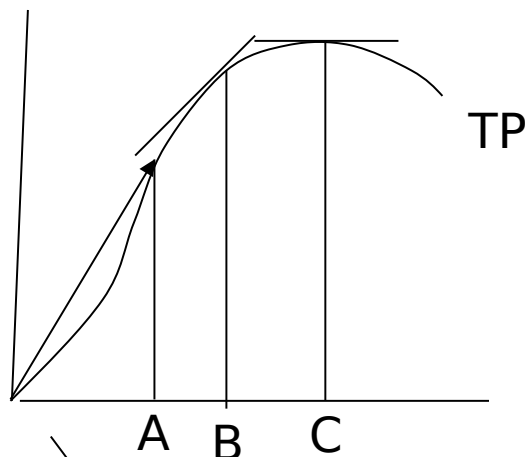
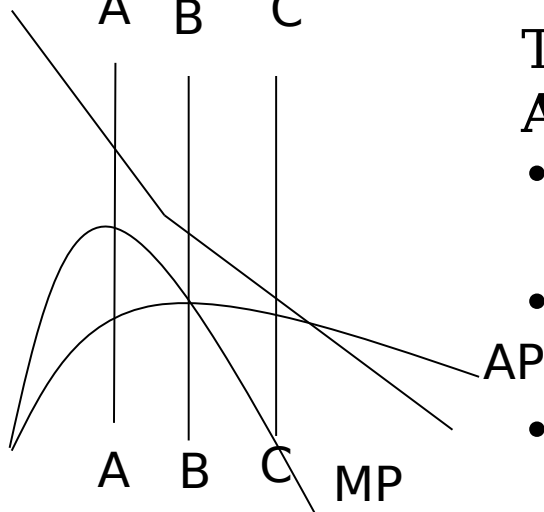


Figure 4.1: Total product, average product and marginal product curves



The relationship between **MPL** and **APL** can be stated as follows.

- When APL is increasing, **MPL > APL**.
- When APL is at its maximum, **MPL = APL**.
- When APL is decreasing, **MPL < APL**.

Example: Suppose that the short-run production function of certain cut-flower firm is given by: $Q = 4KL - 0.6K^2 - 0.1L^2$ where Q is quantity of cut-flower produced, L is labour input and K is fixed capital input ($K=5$).

- Determine the average product of labour (APL) function.
- At what level of labour does the total output of cut-flower reach the maximum?
- What will be the maximum achievable amount of cut-flower production?

$$\begin{aligned}
 \text{Solution: A) } AP_L &= \frac{TP}{L} = \frac{Q}{L} = \frac{4KL - 0.6K^2 - 0.1L^2}{L} \\
 &= 4K - \frac{0.6K^2}{L} - 0.1L = 20 - \frac{15}{L} - 0.1L \\
 &= \frac{20L - 15 - 0.1L^2}{L}
 \end{aligned}$$

B) When total product (Q) is maximum, MP will be zero.

$$MP_L = \frac{\partial Q}{\partial L} = \frac{\partial (4KL - 0.6K^2 - 0.1L^2)}{\partial L} = 4K - 0.2L = 0$$

$$20 - 0.2L = 0 \quad L = \frac{20}{0.2} = \mathbf{100}$$

Hence, total output will be the maximum when 100 workers are employed.

C) Substituting the optimal values of labor (L=100) and capital (K=5) into the original production function (Q):

$$Q_x = 4KL - 0.6K^2 - 0.1L^2 \\ = 4 * 5 * 100 - 0.6 * 5^2 - 0.1 * 100^2 = \mathbf{985}$$

4.1.4 The law of variable proportions

The law of variable proportions states that as successive units of a variable input(say, labour) are added to a fixed input (say, capital or land), beyond some point the extra, or marginal, product that can be attributed to each additional unit of the variable input will decline. **For example, if additional workers** are hired to work with a constant amount of **capital equipment**, the productivity of each additional worker is declining.

- This law assumes that technology is fixed and thus the techniques of production do not change.
- All units of labour are assumed to be of equal quality.

The law starts to operate after the marginal product curve reaches its maximum (this happens when the number of workers exceeds A in figure 4.1). This law is also called the ***law of diminishing returns***.

4.1.5 Stages of production

Economists have defined three stages of short run production.

Stage one:

It ranges from zero units of the variable input to where AP is maximized (where $MP=AP$). And **$MPL > APL$ and** both of them are rising initially and MP falls latter on. This stage is **not an efficient region** of production though the MP of variable input is positive. The reason is that the variable input (the number of workers) is too small to efficiently run the fixed input so that the

Stage two

- **fixed input is under-utilized** (not efficiently utilized).

Here, as the labour input increases by one unit, output still increases but at a decreasing rate. Due to this, the second stage of production is termed as the stage of ***diminishing marginal returns***.

This stage is the efficient region of production where the **marginal product of the variable input is declining but positive**. Thus, it is in the second stage that a rational firm operates.

Stage III:

This stage ranges from $MP_L=0$ onwards. This stage is also known as the stage of *negative marginal returns* to the variable input. The cause of negative marginal returns is the fact that the volume of the variable inputs is quite excessive relative to the fixed input; the fixed input is **over-utilized**.

4.2 Theory of costs in the short run

4.2.1 Definition and types of costs
Cost is, therefore, the monetary value of inputs used in the production of an item.

We can identify two types of cost of production: ***social cost*** and ***private cost***.

Social cost:

It is a cost which is incurred by the society due to the production process of firms. For instance, the production of **Pepsi** emits bad chemicals to the environment, which pollutes waters, air, etc. To control the understandable consequences of the production process on the environment and their property, the society incurs cost.

Private cost:

This refers to the cost of producing an item to the individual producer. It is the cost that the **Pepsi** factory incurs to produce the **Pepsi**.

Private cost of production can be measured
in two ways:
I) Economic cost

The cost of production to the individual producer for both purchased inputs (explicit cost) and non-purchased input (implicit cost).

Explicit cost: refers to the actual or the out-of-pocket expenditures that the firm incurs to purchase inputs like raw materials, workers, the necessary machines from the market.

Implicit cost: refers to costs which are not purchased from the market for the purpose of production. For instance, , the producer may use his/ her own building as a production place, he/she may also manage his firm by himself instead of hiring another manager, etc. since these inputs are used for the purpose production, their value has to be estimated and included in¹⁶

As to how to estimate the cost of these **non-purchased inputs is concerned**, we usually estimate their cost from what these inputs could earn in **their best alternative use**. Thus, in economics the cost of production includes the costs of all inputs used in the production process whether the inputs are purchased from the market or owned by the firm himself that is:

II) Accounting cost:

For accountant, the cost of production includes the cost of purchased inputs only. Accounting cost is the explicit cost of production only. Moreover, accountant's doesn't consider the cost of production from the opportunity cost of the resources point of view.

4.2.2 Total, average and marginal costs in the short run

Cost function shows the algebraically relation between the cost of production and various factors which determine it. Among others, the cost of production depends on **the level of output produced**, technology of production, prices of factors, etc. hence; **cost function** is a multivariable function. Symbolically,

$$C = f(x, t, p_i)$$

Where c- is total cost of production

x - is the amount of output

T - is the available technology of production.

P_i - is the price of input

In the short run, total cost (**TC**) can be broken down into two – total fixed cost (**TFC**) and total variable cost (**TVC**).

Fixed costs

It refers to costs which do not vary with the level of output. They are regarded as fixed because these costs are **unavoidable regardless of the level of output**. The firm can avoid fixed costs only if it stops operation.

The fixed costs may include salaries of administrative staff, expenses for building depreciation and repairs, expenses for land maintenance and the rent of building used for production.

Variable costs

On the other hand, variable costs include all costs which directly vary with the level of output. For example, if the firm produces zero output, the variable cost is zero. These costs may include the cost of raw materials, the cost of direct labour and the running expenses of fuel, water, electricity etc.

In general, the short run total cost is given by the sum of total fixed cost and total variable cost.

That is, $TC = TFC + TVC$
Based on the definition of the short run cost functions, let's see what their shapes look like;

Total fixed cost (TFC): Total fixed cost is denoted by a straight line parallel to the output axis. This is because such costs do not vary with the level of output.

Total variable cost (TVC): The total variable cost of a firm has an inverse S-shape. The shape indicates the law of variable proportions in production. At the initial stage of production with a given plant, as more of the variable factor is employed, its productivity increases. Hence, the TVC increases at a decreasing rate.

This continues until the optimal combination of the fixed and variable factor is reached. Beyond this point, as increased quantities of the variable factor are combined with the fixed factor, the productivity of the variable factor declines, and the TVC increases at an increasing rate.

Total Cost (TC): The total cost curve is obtained by vertically adding TFC and TVC at each level of output. The shape of the TC curve follows the shape of the TVC curve, i.e. the TC has also an inverse S-shape. It should be noted that when the level of output is zero, TVC is also zero which implies $TC = TFC$.

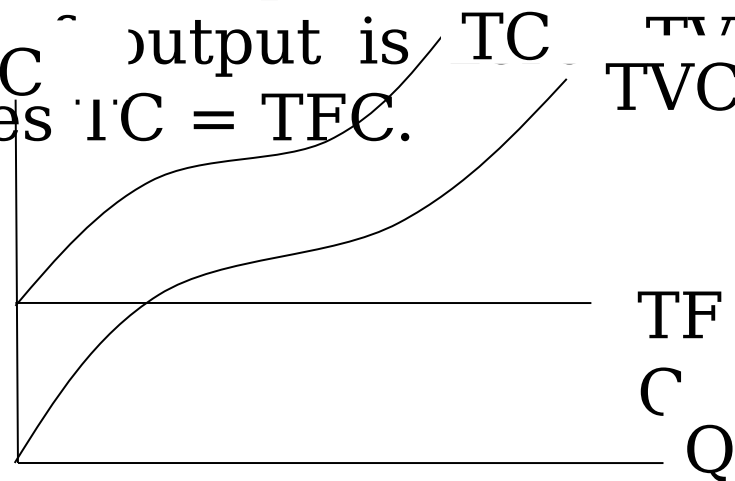


Figure 4.2: *Short run TC, TFC and TVC curves*

Per unit costs

From total costs functions we can derive per-unit costs. These are even more important in the short run analysis of the firm.

A) Average fixed cost (AFC) - Average fixed cost is total fixed cost per unit of output. It is calculated by dividing TFC by the corresponding level of output. The curve declines continuously and approaches both axes asymptotically.

$$AFC = \frac{TFC}{Q}$$

B) Average variable cost (AVC) - Average variable cost is total variable cost per unit of output. It is obtained by dividing total variable cost by the level of output.

$$AVC = \frac{TVC}{Q}$$

The short run **AVC falls** initially, reaches its minimum, and then starts to increase. Hence, the **AVC curve** has **U-shape** and the reason behind is **the law of variable proportions**.

C) Average total cost (ATC) or simply Average cost (AC)- Average total cost is the total cost per unit of output. It is calculated by dividing the total cost by the level of output.

$$AC = \frac{TC}{Q} \text{ OR } AC = \frac{TVC + TFC}{Q} \text{ OR } AC = AVC + AFC$$

Marginal Cost (MC)

Marginal cost is defined as the additional cost that a firm incurs to produce one extra unit of output. In other words, it is the change in total cost which results from a unit change in output. Graphically, **MC is the slope of TC function.**

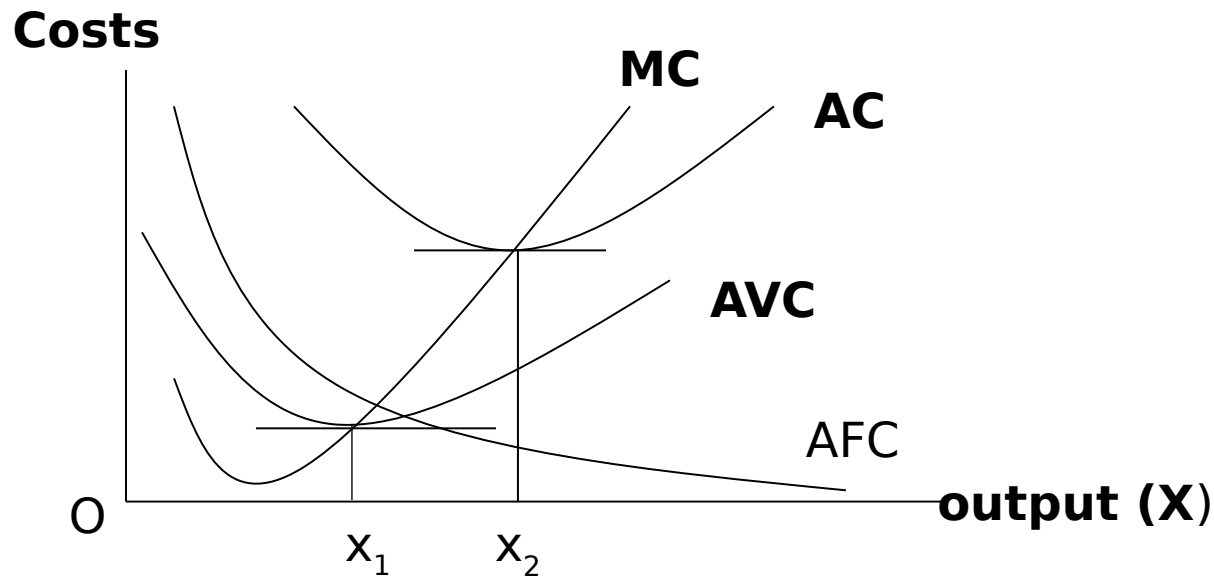
$$MC = \frac{dTC}{dQ}$$

In fact, **MC is also a change in TVC** with respect to a unit change in the level of output.

$$MC = \frac{dTVC + dTFC}{dQ} = \frac{dTVC}{dQ} \quad \text{Since, } \frac{dTFC}{dQ} = 0$$

Given inverse **S-shaped TC and TVC curves**, **MC** initially decreases, reaches its minimum and then starts to rise. From this, we can infer that the reason for the MC to exhibit U shape is also the law of variable proportions. **In summary**, AVC, AC and MC curves are all **U-shaped** due to **the law of variable proportions**.

Figure 4.3: *Short run AFC, AVC, AC and MC Curves*



The minimum point of the ATC (x_2) occurs to the right of the minimum point of the AVC (x_1). This is due to the fact that ATC includes AFC which falls continuously with increase in output. Initially the fall in the AFC offsets the rise in the AVC and thus the ATC declines.

But later on the rise in the AVC more than offsets the fall in the AFC and thus the ATC will start rising continuously. The AVC approaches the ATC asymptotically as X increases since the AFC declines continuously. It can also be noted that the MC curve passes through the minimum points of both AVC and AC curves.

Example: Suppose the short run cost function of a firm is given by: $TC = 2Q^3 - 2Q^2 + Q + 10$.

- A) Find the expression of TFC & TVC
- B) Derive the expressions of AFC, AVC, AC and MC
- C) Find the levels of output that minimize MC and AVC and then find the minimum values of MC and AVC

4.2.3 The relationship between short run production and cost curves

Suppose a firm in the short run uses labour as a variable input and capital as a fixed input. Let the price of labour be given by w , which is constant. Given these conditions, we can derive the relation between MC and MP_L as well as the relation between AVC and AP_L .

Unit products and unit cost curves are **mirror images** of each other, i.e.

when $AP(MP)$ is rising $AC(MC)$ is falling;

when $AP(MP)$ is falling $AC(MC)$ is rising; and

when $AP(MP)$ is maximum $AC(MC)$ is minimum

See below for illustration of these relations.

Let $TVC = \mathbf{w}L$, where $\mathbf{w}$ = given wage rate & L = labor input

Thus, $AVC = TV/Q = (\mathbf{w}L) / Q = \mathbf{w} (L / Q) = \mathbf{w} / (Q/L)$

But $Q/L = APL$,

$$\text{so, } \mathbf{AVC} = \mathbf{w/APL}$$

Similarly, $MC = \Delta TVC / \Delta Q$, since $TVC = \mathbf{w}L$

$$MC = \Delta \mathbf{w}L / \Delta Q = \mathbf{w} (\Delta L) / (\Delta Q) = \mathbf{w} / (\Delta Q / \Delta L)$$

But $\Delta Q / \Delta L = MPL$

$$\text{So, } \mathbf{MC} = \mathbf{w/MPL}$$

Graphically:

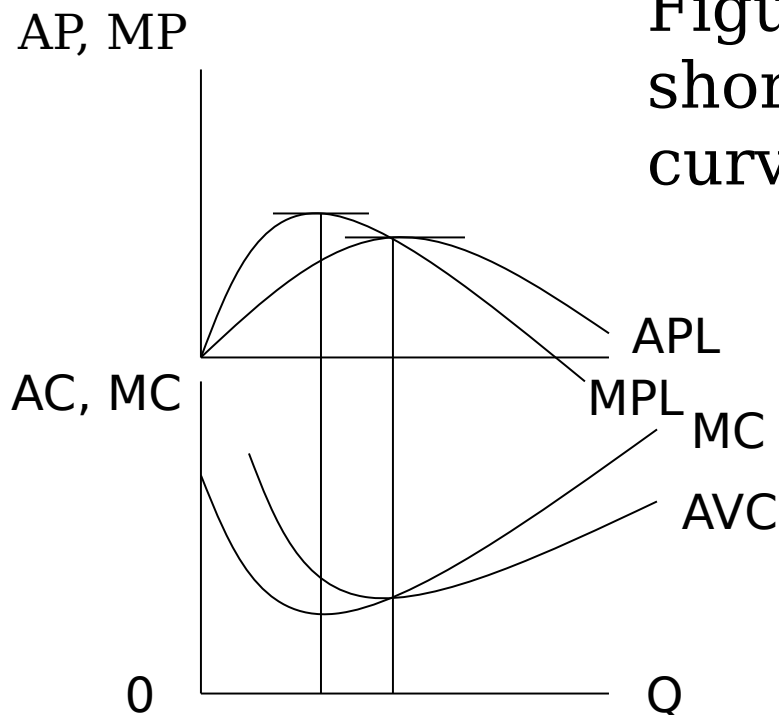


Figure 4.4: relationship between short run production and cost curves

- From this figure, we can conclude that the MC curve is **the mirror image of** MPL curve and AVC curve is **the mirror image** of APL curve