

INTEGRATION

Integration is the reverse of differentiation.

Since integration is the reverse process of differentiation, the **standard integrals** listed in table 1 may be deduced and readily checked by differentiation.

Table 1. Standard integrals

$$(i) \int ax^n dx = \frac{ax^{n+1}}{n+1} + c \text{ except where } n = -1$$

$$(ii) \int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$(iii) \int \sin ax dx = -\frac{1}{a} \cos ax + c$$

$$(iv) \int \sec^2 ax dx = \frac{1}{a} \tan ax + c$$

$$(v) \int \csc^2 ax dx = -\frac{1}{a} \cot ax + c$$

$$(vi) \int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$(vii) \int \frac{1}{x} dx = \ln x + c$$

QUESTIONS

1. Determine (a) $\int 5x^2 dx$

(b) $\int 2t^3 dx$

Solutions

$$(a) \int 5x^2 dx = \frac{5x^{2+1}}{2+1} + c$$

$$(b) \int 2t^3 dx = \frac{2t^{3+1}}{3+1} + c$$

$$= \frac{5x^3}{3} + c \quad \text{ans.}$$

$$\frac{2t^4}{4} + c \text{ ans.}$$

2. Determine (a) $\int \frac{3}{x^2} dx$ (b) $\int 3\sqrt{x} dx$

Solutions

$$(a) \int \frac{3}{x^2} dx = \int 3x^{-2} dx = \frac{3x^{-2+1}}{-2+1} + c \quad (b) \int 3\sqrt{x} dx = \int 3x^{\frac{1}{2}} dx = \frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$\therefore -3x^{-1} + c = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\therefore \frac{-3}{x} + c \text{ ans.} = 2x^{\frac{3}{2}+c}$$

$$\therefore 2\sqrt{x^3} + c$$

3. Determine (a) $\int 4\cos 3x dx$

(b) $\int 5\sin 2\theta d\theta$

Solutions

$$(a) \int 4\cos 3x dx = (4) \left(\frac{1}{3} \right) \sin 3x + c$$

$$(b) \int 5\sin 2\theta d\theta =$$

$$5 \left(\frac{-1}{2} \right) \cos 2\theta + c$$

$$\therefore \frac{4}{3} \sin 3x + c = \frac{-5}{2} \cos 2\theta + c$$

4. Determine (a) $\int 7\sec^2 4t dt$

(b) $\int 3\operatorname{cosec}^2 2x dx$

Solutions

$$(a) \int 7\sec^2 4t dt = (7) \left(\frac{1}{4} \right) \tan 4t + c \quad (b)$$

$$\int 3\operatorname{cosec}^2 2x dx = (3) \left(\frac{-1}{2} \right) \cot 2x + c$$

$$\therefore \frac{7}{4} \tan 4t + c = \frac{-3}{2} \cot 2x + c$$

5. Determine (a) $\int 5e^{3x} dx$

(b) $\int \frac{2}{3e^{4t}} dt$

Solutions

(a) $\int 5e^{3x} dx = 5\left(\frac{1}{3}\right)e^{3x} + c$ (b)

$\int \frac{2}{3e^{4t}} dt = \int \frac{2}{3} e^{-4t} dt = \left(\frac{2}{3}\right)\left(\frac{-1}{4}\right)e^{-4t} + c$

$\frac{5}{3}e^{3x} + c = -\frac{1}{6}e^{-4t} + c$

$-\frac{1}{6e^{4t}} + c$

6. Determine $\int \frac{3}{5x} dx$

Solution

$\int \frac{3}{5x} dx = \int \left(\frac{3}{5}\right)\left(\frac{1}{x}\right) dx$

$= \frac{3}{5} \ln x + c$ **ans.**

Application of Integration

7. Find y given that $\frac{dy}{dx} = 2x - 3$ that $y = -4$ when $x = 1$.

Solution

If $\frac{dy}{dx} = 2x - 3$, then $y = \int (2x - 3) dx = x^2 - 3x + c$

when $x = 1$, $y = 1 - 3 + c = -4$ so $c = -2$

Hence $y = x^2 - 3x - 2$

8. The gradient of the tangent at a point on a curve is given by $x^2 + x - 2$. Find the equation of the curve if it passes through (2,1).

Solution

Gradient $\frac{dy}{dx} = x^2 + x - 2$

Then $y = \int (x^2 + x - 2) dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

$$\text{when } x=2, y=\frac{8}{3}+\frac{4}{2}-4+c=1$$

$$\text{Hence } c=\frac{1}{3}.$$

$$\text{The equation of the curve is } y=\frac{x^3}{3}+\frac{x^2}{2}-2x+\frac{1}{3} \vee 6y=2x^3+3x^2-12x+2.$$