



Full Name: _____ **Time Allowed: 3:00 hrs.**

Grade: 12 Section:- _____ Subject: Mathematics 3rd Quarter Model Exam

Direction: Choose the correct answer from the given alternatives.

- Given $f(x) = \ln(x-1)$ and $g(x) = \sqrt{1-2x}$ the following is the domain of $f(g(x))$?
 - $\{X \in \mathbb{R} : x > 1\}$
 - $\{X \in \mathbb{R} : X > 0\}$
 - $\left\{\frac{1}{2}\right\}$
 - $\left\{\frac{1}{2}\right\}$
- Which one of the following is a one to one function?
 - $f : \mathbb{R} \rightarrow \mathbb{R} ; f(x) = \tan x$, where $\mathbb{R} : \mathbb{R}$
 - $g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = 2^x$
 - $h : [0, \infty) \rightarrow [0, \infty) h(x) = x^2$
 - $r : [0, \infty) \rightarrow [0, \infty), r(x) = x + 1$
- If $f(x) = \ln\left(\frac{x}{x-1} + 2\right)$, for $x > 1$, then which of the following is the inverse of f ?
 - $f^{-1}(x) = \frac{e^x}{x-1} - 2$
 - $f^{-1}(x) = \frac{e^x - 2}{e^x + 1}$
 - $\frac{e^x}{-1} - 2$
 - $\frac{x}{-1 - 2}$
- Which one of the following is has No vertical asymptote?
 - $f(x) = \ln(x+1)$
 - $f(x) = \frac{x^2+1}{x^3+8}$
 - $f(x) = \frac{9}{x-3}$
 - $f(x) = \frac{x-1}{x^2-x}$
- Which one of the following true?
 - A polynomial can have at least one vertical Asymptote.
 - The graph of a rational function can never cross its horizontal Asymptote.
 - The graph of $f(x) = \frac{3x-1}{x-1}$ has no horizontal Asymptote.
 - Oblique Asymptote may be crossed at by the graph of rational function.
- Which of the following is true about a conic section represented by the equation $\frac{x^2}{k} + \frac{y^2}{k-9} = 1$?
 - It is a circle whose center is at the origin for $K \in \mathbb{R}$.
 - It is an ellipse whose major axis is vertical when $K > 9$.
 - It is a hyperbola whose foci are $(-3, 0)$ and $(3, 0)$ when $0 < K < 9$

D. It is a hyperbola whose foci are $(-\sqrt{3}K, 0)$ and $(\sqrt{3}K, 0)$ when $0 < K < 9$

7. Suppose the following statements are the premises of an argument

“He was lazy or he did not like the class room.

If he was lazy, he could not pass the exam.

He passed the exam”

Which one of the following can be conclusion that makes the argument valid?

A. He did like the class room.

C. If he was not lazy, he did like the class room.

B. He did not like the class room.

D. He was not lazy and he did like the class room.

8. If $(P \vee Q) \implies (\neg R \wedge R)$ is true, then which one of the following is necessarily true?

A. $(P \vee Q) \implies Q$

B. $\neg R \wedge R$

C. $\neg P \iff R$

D. $\neg P \vee R$

9. For arbitrary propositions p and q. which one of the following is a valid equivalence?

A. $\neg(p \implies q) \equiv (q \implies p)$

C. $[p \vee \neg q] \equiv [p \implies q]$

B. $[\neg(p \implies q) \wedge p] \equiv (p \wedge \neg q)$

D. $[(p \vee q) \implies q] \equiv [p \implies \neg q]$

10. Which of the following is a rational function?

A. $f(x) = \frac{\sqrt{3}x^5 - 2x + \log^5}{x^4 + x^2 + \frac{\pi}{3}}$

C. $g(x) = 3x^4 - \sqrt{2x^3 + 4} + 5x$

B. $h(x) = \frac{x^2 - 2^x + 4x}{x^4 + 3x + 3}$

D. $r(x) = \frac{x^2 + \cos x + 5}{x + 4}$

11. If x and y non negative integers, which of the following is NOT true?

A. $(\forall x)(\exists y)(y > x^2 - 1)$

C. $(\exists x)(\forall y)(y > x^2 - 1)$

B. $(\exists y)(\forall x)(y \leq x^2 - 1)$

D. $(\exists y)(\exists x)(y \leq x^2 - 1)$

12. A parabola with focus at $(3, -1)$ has directrix $y = 3$ which one of the following is the equation of the parabola?

A. $(x - 3)^2 = -4(y + 1)$

C. $(x - 3)^2 = 4(y + 1)$

B. $(x - 3)^2 = -8(y - 1)$

D. $(x - 3)^2 = 8(y - 1)$

13. A satellite moves along a hyperbolic curve whose horizontal transverse axis is 8km and an asymptote $y = \frac{3x}{4} + 2$. Then what is the eccentricity of the hyperbola?

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{5}{4}$

D. $\frac{20}{4}$

14. A semi – elliptical arc over a tunnel for a road through a mountain has a major axis of 40 meters and height of 15 meters at the center. What is the equation of the semi elliptical arc over the tunnel, if the center is considered as the origin?

A. $\frac{x^2}{1600} + \frac{y^2}{215} = 1$

B. $\frac{x^2}{215} + \frac{y^2}{1600} = 1$

C. $\frac{x^2}{800} + \frac{y^2}{215} = 1$

D. $\frac{x^2}{1200} + \frac{y^2}{1600} = 1$

15. Suppose $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$. If x is a 2×2 matrix such that $Ax - A^T = 2A$. Then what is the value of x?

A. $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

16. Consider the system
$$\begin{cases} ax + y + z = 1 \\ x + 2y + 4z = 0 \\ 5x - y + z = 0 \end{cases}$$

If the determinant of the coefficient matrix is 2 then what is the solution of the system of equations?

A. $\left(3a, \frac{19a}{2}, \frac{-11a}{2}\right)$

C. $\left(3, \frac{19}{a}, \frac{-11}{2}\right)$

B. $\left(\frac{3}{a}, \frac{19}{2}, \frac{11}{2}\right)$

D. $\left(\frac{3}{2}, \frac{19}{2}, \frac{-9}{2}\right)$

17. What should be the value of K so that the system of equation?

$$\begin{cases} x - y + z = 1 \\ -x + 5y - 4z = 1 \\ 2x + 2y - z = K \end{cases} \text{ has a solution?}$$

A. 0

B. 1

C. -4

D. 4

18. If $A = \begin{bmatrix} 0 & x & 0 \\ 1 & -1 & 1 \\ 0 & y & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 11 & 1 \\ 30 & 0 \\ 20 & -1 \end{bmatrix}$, then what are the value of x and y?

A. $x = 3, y = -2$

C. $x = \frac{2}{3}, y = \frac{1}{3}$

B. $x = -3, y = 2$

D. $x = \frac{1}{3}, y = \frac{2}{3}$

19. In the set of complex numbers, the solution set of $x^2 - 2x + 5 = 0$ is

A. $\emptyset$

B. $\{2 + 1, 2 - i\}$

C. $\{1 + 2i, 1 - 2i\}$

D. $\{2 + 4i, 2 - 4i\}$

20. The following is a table of simple frequency distribution of a data with variable x

X	1	3	4	5	7	
frequency	2	5	6	5	2	

A. $\frac{3\sqrt{3}}{3}$

B. 3

C. $\sqrt{\frac{23}{10}}$

D. $\sqrt{\frac{36}{10}}$

21. A student needs to select 3 books from 3 different mathematics, 3 different physics and 1 chemistry book. What is the probability that one of them is physics and the other two are either mathematics or chemistry books?

A. 0.0857

B. 0.257

C. 0.429

D. 0.514

22. A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls what is the probability that at least one girl is selected?

A. $\frac{5}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{1}{6}$

23. Two machines A and B work independently. The probably that both machines A and B work is 0.4. If the conditional probability that machine B works given that machine A works is 0.5, then the conditional probability that machine A works given that machine B works is _____

A. 0.8

B. 0.3

C. 0.5

D. 0.7

24. Which one of the following is true about the derivative of $f(x) = x|x|$?

A. f is not differentiable at $x = 0$

C. $f'(x) = 2x$, for every $x \in (-\infty, \infty)$

B. $f'(x) = 2|x|$, for every $x \in (-\infty, \infty)$

D. $f'(x) = |x| + x$, for every $x \in (-\infty, \infty)$

25. if $F(x) = f(2x+2) \cdot g(1-x^2)$, with $f(2) = -3$, $f'(2) = 4$, $g(1) = -5$, and $g'(1) = 1$, then what is the actual value of $F'(0)$?

A. -40

B. -20

C. 0

D. 19

26. The ozone level (in ppb – parts per billion) on a sunny day in a metropolitan area is given by the formula $p(t) = 80 + 12t - t^2$ where t is time in hours and $t = 0$ corresponds to 9A.M. What is the rate of increase of the ozone level after 3 hours (i. e at 12A.M)?

A. 6ppb

B. 12ppb

C. 107ppb

D. 113ppb

27. If $f(x) = \ln(\sqrt{x^2-5})$, then one of the following is equal to $f'(x)$

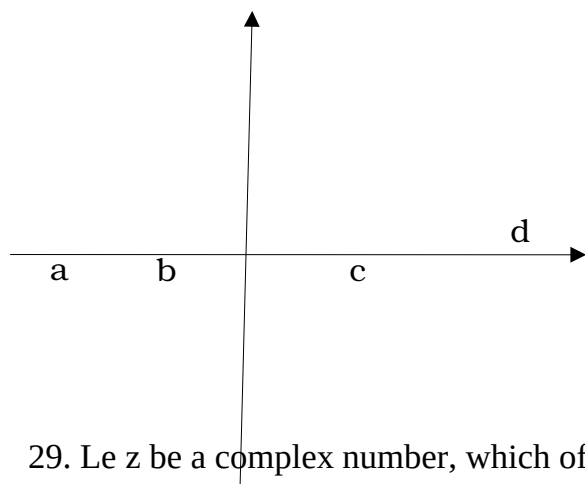
A. $\frac{x}{x^2-5}$

B. $\frac{-x}{\sqrt{x^2-5}}$

C. $\frac{2x}{\sqrt{x^2-5}}$

D. $\frac{-x}{x^2-5}$

28. Look at the following graph of $f'(x)$. Which of the following is true about f ?



A. f is increasing $[c, d]$

B. f is decreasing on $[b, c]$

C. f has a local minimum at d

D. f is a local extreme value at c

29. Let z be a complex number, which of the following is the solution set of $z^3 - iz = 0$?

A. $\left\{ \pm \frac{1}{\sqrt{2}}(1+i) \right\}$

B. $\left\{ \pm \frac{1}{\sqrt{2}}(1-i) \right\}$

C. $\{ \pm \sqrt{2}(1-i) \}$

D. $\{ 0, \pm \frac{1}{\sqrt{2}}(1+i) \}$

30. What is the principal argument $(5 + 5i)^{11}$?

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

31. The sequence $\left\{ \frac{(n-1)(2n+1)}{1-n^2} \right\}_{n=1}^{\infty}$ converges to

A. $-\infty$

B. -2

C. 0

D. 1

32. Let $f(x) = \begin{cases} a \frac{\sin x}{x-|x|}, & \text{if } x < 0 \\ e^{-x} + \cos x, & \text{if } x \geq 0 \end{cases}$

If f is continuous at $x = 0$, then what is the value of a ?

A. 4

B. 2

C. $\frac{1}{2}$

D. -4

33. Which one of the following is equal to $\lim_{x \rightarrow 0} \frac{2x + \tan x}{x \sec x}$?
 A. 2 B. 0 C. 1 D. 3
34. Which one of the following is necessary true about a function $f(x)$?
 A. If f is continuous at $x = a$, then f is differentiable at $x = a$
 B. If f is not differentiable of $x = 0$, then $\lim_{x \rightarrow 0} f(x) \neq \lim_{x \rightarrow 0} f(x)$
 C. If f is differentiable of $y = 0$, then $\lim_{x \rightarrow 0} f(x) = f(a) = \lim_{x \rightarrow 0} f(x)$
 D. If the derivative $f'(a) = 0$, then f attains its maximum value at $x = 0$
35. A rectangular field of length L and width W meters for $W < L$ has perimeter 400 meters. If is circular region of area W^2 is to be reserved for office purpose, what should be the length of the field so that the area of the remaining region is maximum?
 A. 50 meters B. 100 meters C. 120 meters D. 150 meters
36. The volume V of a melting ice cube after t seconds is $V = 2000 - 40t - 0.2t^2$ (cm^3). How fast is the volume changing when $t = 40$ seconds
 A. -24 B. 24 C. 16 D. none
37. A ladder 6m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate (speed) of $\frac{1}{2}$ m/se. How fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?
 A. $\frac{\sqrt{2}}{12}$ rad/sec B. $\frac{\sqrt{2}}{2}$ rad/sec C. $\frac{\sqrt{2}}{6}$ rad/sec D. $\frac{\sqrt{2}}{3}$ rad/sec
38. The graph of $y = 5x^4 - x^5$ has a point of inflection at _____
 A. (3, 162) only C. f has a relative minimum at $x = 0$
 B. (4, 256) only D. f has a relative maximum at $x = 0$
39. $\int \frac{e^{2x} - 4x}{xe^{2x}} dx$ is equal to
 A. $\ln |x| + 2e^{-2x} + c$ C. $\ln |x| + 4e^{-2x} + c$
 B. $\ln |x| - 4e^{2x} + c$ D. $\frac{-1}{x^2} + 2e^{-2x} + c$
40. Which one of the following is equal to $\int_0^{\pi/2} \frac{x - \sin x}{\sec x} dx$
 A. $\frac{\pi - 3}{2}$ B. $\frac{\pi - 1}{2}$ C. $\frac{3 - \pi}{2}$ D. $\frac{\pi + 3}{2}$
41. What is the area of the region between the graphs of $y = \sin x$ and x – axis where $0 \leq x \leq 2\lambda$?
 A. 4 B. 4π C. 2 D. 2π
42. The volume of the solid generated when the region bounded between the graph of $Y = \begin{cases} x^2, 0 \leq x \leq 2 \\ 4, 2 \leq x \leq 3 \end{cases}$ and x – axis is rotated about the x – axis is
 A. $\frac{32\pi}{5}$ B. $\frac{112\pi}{5}$ C. $\frac{112\pi}{3}$ D. $\frac{64\pi}{5}$
43. The volume of $\int_0^3 (x+1)e^{(x^2+2x)} dx$ is

A. $\frac{e^3}{2}$

B. $\frac{e^4 - e}{2}$

C. $\frac{e^3 - 1}{2}$

D. $e^3 - 1$

44. What is the radius of the longest possible circle that can be inscribed in the ellipse given by

$$5(x - 1)^2 + 3y^2 = 15?$$

A. $\sqrt{3}$

B. $\sqrt{5}$

C. 3

D. 5

45. Which of the following is a valid argument?

A. $\neg p \Rightarrow \neg q, p \vdash \neg p$

C. $P \Rightarrow \neg q, p, r \Rightarrow q \vdash \neg r$

B. $\neg p \vee q, r \Rightarrow P, r \vdash \neg q$

D. $\neg p, p \vee q, r \Rightarrow q \vdash \neg r$

46. Suppose a radioactive material loses one third of its mass per year. If its current mass is 81 gram then how much its mass will be just after 7 years

A. 27gm

B. 0.037gm

C. 4.74gm

D. 1.58gm

47. Which of the following is a divergent sequence?

A. i

B. $\left\{ \frac{1}{5^n} \right\}$

C. i

D. i

48. $\int (2 + \sin^2 x) \cot x \, dx$ is equal to _____

A. $2 \csc x - \sec^2 x + c$

C. $2 \ln |\sin| + \frac{1}{2} \sin^2 x + 2$

B. $2 \ln |\sin x| - \cos^2 x - c$

D. $2 \csc x + \sin x \cos x + c$

49. The radius of a right circular cylinder of height 10cm increases at a rate of 3cm/min. What is the rate of change of the volume of the cylinder when the radius is 8cm?

A. $80\pi \text{ cm}^3/\text{min}$

C. $480\pi \text{ cm}^3/\text{min}$

B. $320\pi \text{ cm}^3/\text{min}$

D. $640\pi \text{ cm}^3/\text{min}$

50. Let $s_n = 4n^2$, $n = 1, 2, 3 \dots$ be the sum of the first n terms of a sequence $\{a_n\}_{n=1}^{\infty}$, which of the following is the third term a_3 ?

A. 8

B. 12

C. 16

D. 20

Only for Natural Science

51. Line l passes through (0.5) and (-5, 5) what is the angle between y axis and l in radian measure?
- A. $\frac{\pi}{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{2}$ D. $\frac{\pi}{3}\pi$
52. Which of the following is a vector equation of the line tangent of the circle $x^2 + y^2 + 2x - 7 = 0$ at (1, 2)
- A. $(x, y) = (0, 3) + \lambda(-1, 2)$ C. $(x, y) = (0, 3) + \lambda(1, -1)$
B. $(x, y) = (1, 2) + \lambda(2, -1)$ D. $(x, y) = (1, 3) + \lambda(-1, 2)$
53. Let l be the line whose equation is $2x - y = 10$. Which one of the following is the equation of the image of l after a reflection in the line $y = 2x + 5$ followed by a rotation through the angle of 90° about the origin?
- A. $x + 2y = 0$ B. $2x + y = 0$ C. $x + 2y = 5$ D. $x - 2y = 5$
54. Suppose $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{B}$ is a vector in space such that $|\vec{A}| = |\vec{B}|$ if $\vec{U}$ is the unit vector in the direction of $\vec{B}$ then $\vec{A} \cdot \vec{U}$ is equal to _____
- A. 16 B. 12 C. 10 D. 14
55. If l is the line passing through (0, 2) and parallel to $\vec{v} = \vec{i} + 3\vec{j}$, which one of the following is true about the line l and the circle
- A. l is tangent to the circle at (0, 2)
B. l is tangent to the circle at some point P which $P \neq (0, 2)$
C. intersects the circle at two distinct points
D. the distance between l and the center of the circle is greater than $\sqrt{5}$
56. If a translation T takes the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is _____, then what is the image of the origin under T ?
- A. $(-3, 7)$ B. $(1, 2)$ C. $(1, -3)$ D. $(-2, 4)$
57. What is $\cot(\arcsin x)$, if $0 < x < 1$?

A. $\frac{x}{\sqrt{1-x^2}}$

B. $\frac{\sqrt{1-x^2}}{x}$

C. $\sqrt{1-x^2}$

D. $\frac{1}{\sqrt{1-x^2}}$

58. What is the possible value of x that solves this equation $\sin^{-1} x \cos^{-1} \left(\frac{3}{5} \right)$?

A. $\frac{\pi}{3}$

B. $\frac{3}{5}$

C. $\frac{5\pi}{2}$

D. $\frac{4}{5}$

59. What is the amplitude and period, respectively of the graph of $f(x) = 4 \sin \left(\frac{x}{3} \right) \cos \left(\frac{x}{3} \right)$?

A. $4, \frac{\pi}{3}$

B. $2, 3\pi$

C. $2, \frac{2\pi}{3}$

D. $4, 3\pi$

60. Let $\vec{a} = \vec{i} + 3\vec{k}$ and $\vec{b} = -\vec{i} + \vec{j}$ be vectors in space. Which of the following cosine of angle between $\vec{a}$ and $\vec{a} - \vec{b}$?

A. $\frac{9}{10}$

B. $\frac{\sqrt{3}}{3}$

C. $\frac{3}{\sqrt{10}}$

D. $\frac{-9}{10}$

61. Suppose P and Q are points in space such that the mid-point of $\overline{PQ}$ is on the negative Z-axis and the distance between P and Q is 6. If $P = (2, -1, 0)$ then what is the coordinate of Q?

A. $(-2, 1, 4)$

B. $(2, -1, 6)$

C. $(2, -1, -6)$

D. $(-2, 1, -4)$

62. What is the contra positive of “if $x \in \mathbb{N}$, then x is integer and $x < 0$ ”?

A. If x is not integer or $x < 0$, then $x \notin \mathbb{N}$

C. If x is not integer or $x \leq 0$, then $x \in \mathbb{N}$

B. If x is integer and $x > 0$, then $x \in \mathbb{N}$

D. If $x \in \mathbb{N}$, then x is not integer and $x \leq$

63. Which one of the following is valid assertion that can be proved by the principle of mathematical induction?

A. $2^n > 10n$ for every integer n such that $n \leq 6$

B. $r^2 > 0$ for every real number r such that $r \geq 1$

C. $2^n > 10n > 2n^n$ for every natural number $n \geq 1$

D. $2^n > 8n$ for every n such that $n \geq 3$

64. Which of the following is equivalent to $\neg$

A. $(\forall x) \neg$

C. $(\exists x) \neg \wedge Q(x) \neg$

B. $(\exists x) (\neg p(x) \implies \neg Q(x))$

D. $(\exists x) \neg \wedge Q(x) \neg$

61. Which of the following is the interest earned by investing Birr 500 at 4% per annum compounded quarterly in a year and half? (You can use: $(1.01)^4 = 1.041$, $(1.04)^4 = 1.170$, $(1.01)^6 = 1.062$ and $(1.04)^6 = 1.265$)
- A. Birr 850 B. Birr 325 C. Birr 310 D. Birr 205
62. For a skewed unimodal distribution, which of the following is true?
- A. Median is closer to mean than to mode.
B. For a positively skewed distribution median is greater than mean.
C. For a negatively skewed distribution mean is greater than mode.
D. Mode is between mean and median.
63. In an examination, 10% of the enrolled candidates were absent and 20% of those who took the examination failed. If 288 candidates passed the examination, how many were enrolled for the examination?
- A. 400 B. 420 C. 388 D. 360
64. In shop A, 20kg of sugar costs Birr 252 and in shop B, 40gm of sugar costs 50 cents What difference in cost is observed per kilo in Birr?
- A. 0.2 B. 0.15 C. 0.1 D. No difference
65. The payment for a part – time job of a person was Birr 4,500. After the income tax is reduced, the person received the net payment of Birr 3,150. What was the rate of taxation?
- A. 25% B. 30% C. 35% D. 15%