Unit 6
STATISTICS AND PROBABILITY
6.1 STATISTICAL DATA

Definition 6.1: Statistics is the science of collecting, organizing, presenting, analyzing_and interpreting data (quantitative information) in order to draw conclusions.

Basically, statistics is a procedural process performing five logical steps on numerical data. These are:

Collection of data Organizations of data Presentations of data Analysis of data Intrepretations of data

1. Collection of data

The first step of statistics is collection of data. This is the process of obtaining measurements or counts.

2. Organization of data

The second step of statistics refers to the organizing data. Collected data has to be organized in a suitable form to understand the information gathered. The collected data must be ***edited, classified and tabulated.***

3. Presentation of data

The main purpose of data presentation is to facilitate statistical analysis. This can be done by illustrating the data using graphs and diagrams like bar graph, histogram, pie-charts, and pictograms frequency polygons.

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4. Analysis of data

In order to meet the desired purpose of investigation, data has to be analyzed. The purpose of analyzing data is to highlight information useful for decision making.

5. Interpretation of data

Based on analyzed data, conclusions have to be drawn. This step usually involves decisions making about a large collection of objects (the population) based on information gathered from a small collection of similar objects (the sample).

The decision making processes used by the managers of modern businesses and industry is governed by statistical application. Statistical methods can be applied to any situation where numerical information is gathered with the objective of making rational decisions in the face of uncertainty.

Definition 6.2:

Descriptive statistics is a branch of statistics concerned with summarized and describing a large amount of data without drawing any conclusion about a particular bit of data.

Descriptive statistics describes information collected through numerical measurement, charts, graphs and tables. The main purpose of descriptive statistics is to provide an overview of the information collected.

Cont...

Definition 6.3:

Inferential statistics is a branch of statistics concerned with interpreting data and drawing conclusion.

We can classify data as **primary and secondary data.**

1. Primary data

Data is said to be primary, if it is obtained first hand for the particular purpose on which one is currently working. Primary data is original data, obtained personally from primary sources by observation, interview or direct measurement.

2. Secondary data

Data which has been collected previously (for similar or different purpose) is known as secondary data. Secondary data refers to that which is not originated by the researcher himself/herself, but which he/she obtains from someone else's records. Some source of secondary data are official data are official publications, journals, newspapers, different studies, national statistical abstracts, etc.

A statistical table is a systematical presentation or organization

Cont...

Example 13. An economist in the Labour Department of a Regional Social Affairs Bureau may classify the households in a certain locality by household in

TABLE 6.3: Monthly income of 300 households

Income (in Birr)	Number of households
Under 350	85
Between 350 and 650	72
Between 651 and 950	64
Between 951 and 1250	48
Between 1251 and 1550	21
above 1550	10
Total	300

Cont...

- A statistical table is a systematic presentation or organization of numerical data in columns and rows.
- Columns are vertical arrangements and rows are horizontal.
- The main purpose of a statistical table is to allow the reader to quickly access relevant information. A title and row and column headers are important.

Exercises:

1. Explain each of the following statistical terms by giving examples.
   a. qualitative data
   b. quantitative data
   c. population
   d. sample
2. Determine whether the following data is qualitative or quantitative.
   a. Gender **b.** Temperature **c.** Zip code
   d. Number of days **e.** Religions **f.** Occupations
   g. Ages **h.** Colors **i.** Nationality

6.1.2 Distributions and Histograms

Information (data) is obtained from a census, existing data sources, surveys or designed experiments. After data is collected, it must be organized into a manageable form. Data that is not organized is referred to as raw data.

Definition 6. 3:

A quantity that we measure from observation is called a variate or variable denoted by V. The distribution of a population function is the function which associates with each variate of the population function a corresponding frequency denoted by f.

Methods of organizing raw data include the drawing of tables or graphs, which allow quick overview of the information collected.

Example 1. Suppose there are 10 students in a group whose scores in a mathematics quiz were as follows:

13, 12, 14, 13, 12, 12, 13, 14, 15, 12

Cont...

Organize the data in tabular form. What are the variates? Give the frequency of each variate.

Solution: The data given above is raw data. We may now tabulate the given data in the form given below.

Score (V)	12	13	14	15
Number of students (f)	4	3	2	1

The table given above is called the frequency distribution table. The scores are the variate and the number of students getting a particular score is the frequency of the variate.

Definition 6.4:

A frequency distribution is a tabular or graphical representation of a data showing the frequency associated with each data value.

Example 2. Organize the data below into a frequency distribution table.

8, 9, 8, 7, 10, 9, 6, 4, 9, 8, 7, 8, 10, 9, 8, 6, 9, 7, 8, 8

Cont...

Solution: (Write the values in ascending order.)

Value(V)	4	5	6	7	8	9	10	Total
Frequency(f)	1	0	2	3	7	5	2	20

Quantitative data can also be represented graphically, through a histogram.

Definition 6.5:

A histogram *is a graphical representation of a frequency distribution in which the variate (V) is plotted on the x-axis (horizontal axis) and the frequency (f) is plotted on the y-axis (vertical axis).*

When we drawing a histogram

i. Construct a frequency distribution table of the given data.

ii. The x-axis

a. *Determine a suitable scale for the horizontal axis and determine the number of rectangle needed to represents each variate or group of variates as desired.*

b. Try not to break the x-axis

iii. The y-axis

a. *Display information about frequency on the vertical (y) axis.*

b. Determine the length of y-axis.

vi. Draw bars for each varaite (V).

v. Label the histogram with a title, and label the axis.

Cont...

Note:

1. *The height of each rectangle is the frequency.*
2. *The width of each rectangle should be the same.*

Example 1. The temperature in for the first 14 days of September in a certain town were recorded as

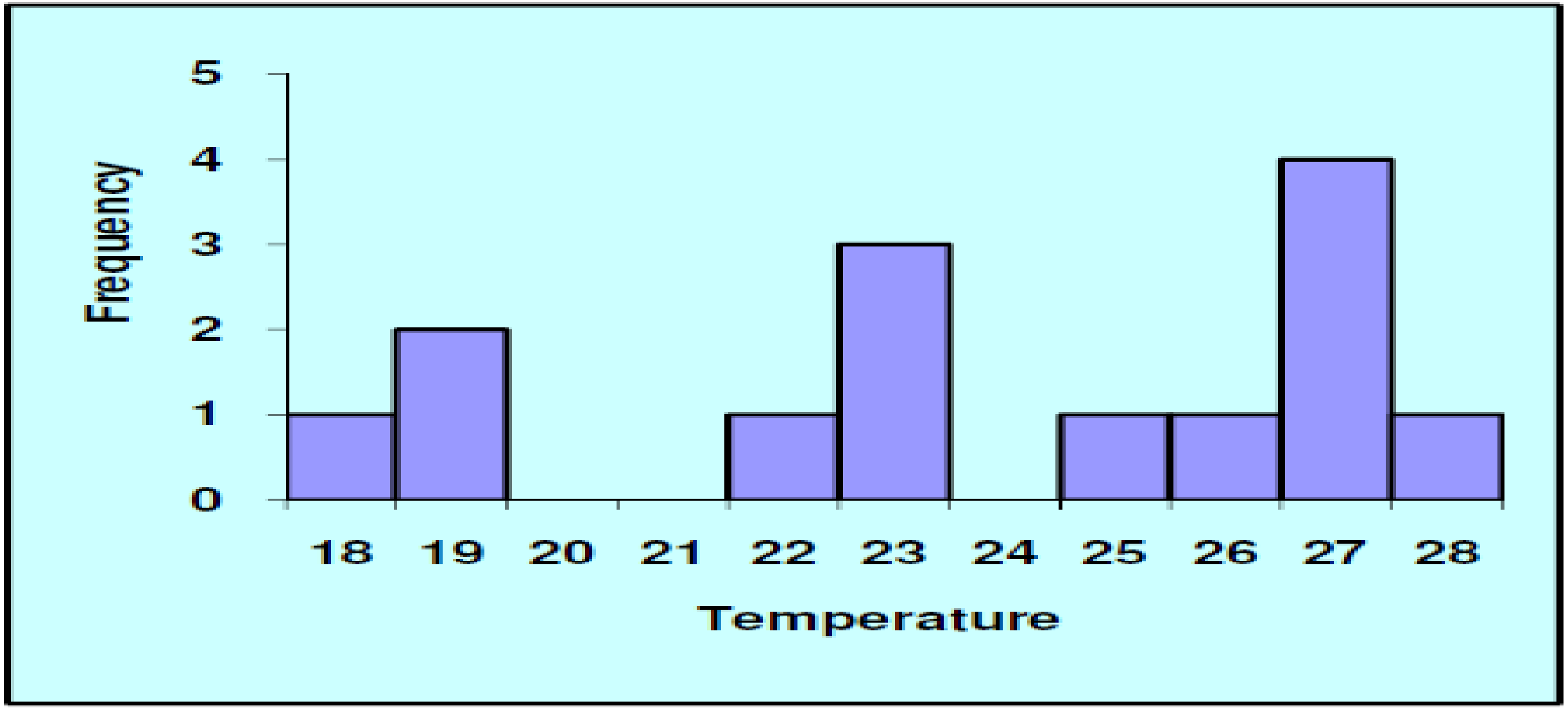
22, 27, 19, 23, 19, 18, 27, 27, 25, 23, 26, 27, 28, 23

Construct a frequency distribution table and a histogram for the given data.

Solution: Now we can draw the frequency distribution table from the raw

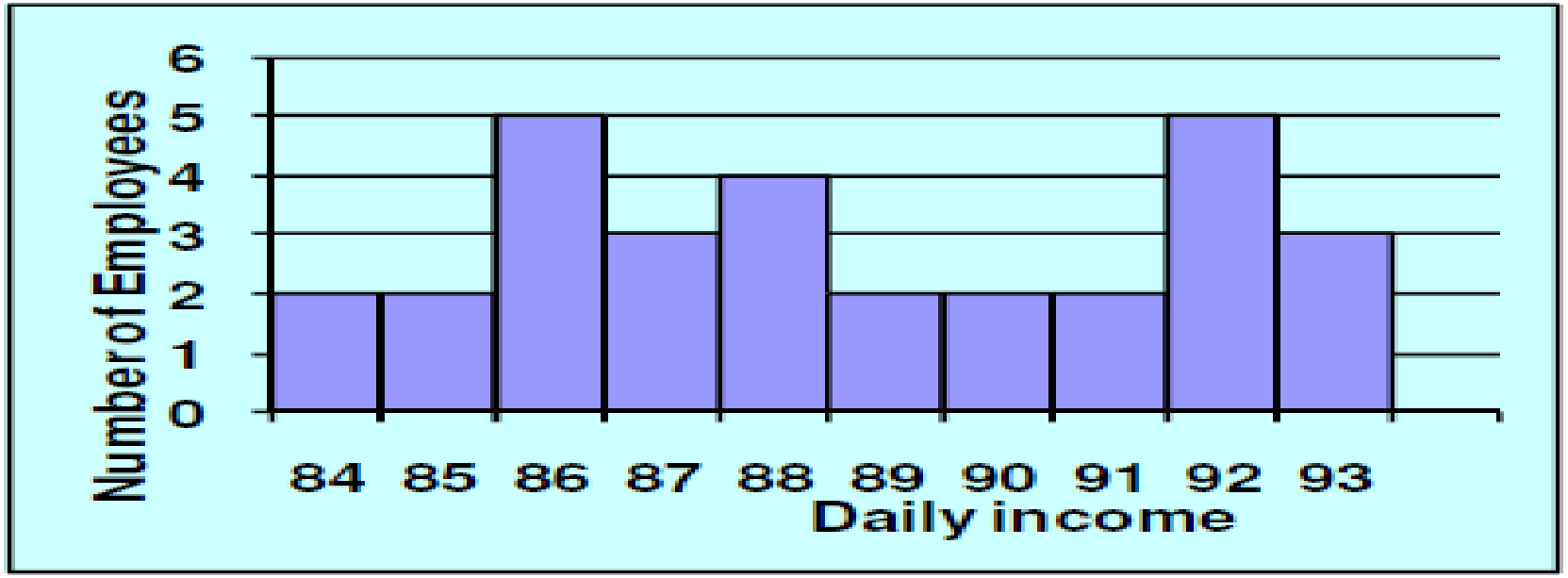
Temperature (in °C) (V)	18	19	20	21	22	23	24	25	26	27	28
frequency (f)	1	2	0	0	1	3	0	1	1	4	1

Cont...



Cont...

Example 2. The following histogram shows the daily income (in Birr) of 30 employees in a factory.



From the histogram, answer the following questions:

a. How many employees have a daily income of Birr 92?

b. How many employees collect a daily income of more than Birr 90?

c. What is the highest frequency?

d. What percent of the employees earn a daily income of more than Birr 89?

Cont...

Solution:

a. 5 employees have a daily income of Birr 92.

b. 10 employees earn a daily income of more than Birr 90.

c. The highest frequency is 5.

d. Percenta

$$= \frac{\text{Sum of the frequencies of employees earning more than 89}}{\text{Total Frequency}} \times 100\%$$

$$= \frac{2+2+5+3}{30} \times 100\% = \frac{12}{30} \times 100\% = 40\%$$

i.e., 40% of the employees earn a daily income of more than Birr 89.

6.1.3 Measures of Location (Mean, Median and Mode(s))

Quantitative variables contained in raw data or in frequency tables can also be summarized by means of a few numerical values. There are three commonly used measure of location or measure of average.

Namely;

1. The arithmetic mean (or the mean)
2. The mode (s) and
3. The median

1.Arithmetic Mean

Definition 6.6

- *The arithmetic mean (or the mean) of a variable is the sum of all the data values, divided by the total frequency (number of observations).*

If is given by

Cont...

Note: The mean of a population function can also be calculated from its frequency distribution. So, if the values occur times, respectively, the mean () is given by

Example 1. Find the mean of the following data:

7, 21, 2, 17, 3, 13, 7, 4, 9, 7, 9

Solution: $$\overline{x} = \frac{7+21+2+17+3+13+7+4+9+7+9}{11} = \frac{99}{11} = 9$$

Cont...

Example 2 The following table shows the age of 14 students in a certain class:

Age in years (V)	12	13	16	18
Number of students (f)	3	4	2	5

Compute the mean age of the students.

Solution:

$$\bar{x} = \frac{12\times3+13\times4+16\times2+18\times5}{3+4+2+5} = \frac{36+52+32+90}{14} = \frac{210}{14} = 15 \text{ years}$$

Cont...

Properties of the mean

1. *The sum of the deviation of individual observations from the mean (is zero. That is, let be n observations with mean (. Then the sum of the deviations of the observations from the mean is given by*

2. *If a constant k is added to (or subtracted from) each data value, then the new mean is the sum (or the difference) of the old mean and the constant k.*
3. *The mean of the sum or difference of two population functions (of equal numbers of observations) is equal to the sum or difference of the means of the two population functions.*
4. *The mean of a constant times a population function is equal to the constant times the mean of the population function. That is, if is the mean of population function and if* ***k*** *is a constant, then the mean of* ***is equal to .***
5. *The mean of any data is unique.*
6. *The mean is affected by extreme values.*

Cont...

Example 1. Let the ages of 5 children be 2, 3, 6, 9, 10. Then, the mean age

$$\overline{x} = \frac{2+3+6+9+10}{5} = \frac{30}{5} = 6$$

The sum of the deviations from the mean is:

$$(2 – 6) + (3 – 6) + (6 – 6) + (9 – 6) + (10 – 6) = –4 – 3 + 0 + 3 + 4 = 0$$

Example 2. The mean of 2, 4, 6, 8 is 5 and the mean of 5, 7, 9, 7 is 7. Then, the mean of the sum 7, 11, 15, 15 is 5+7 = 12.

Example 3. The mean of 8, 9, 6, 8, 4, is 7. If you multiply each of value by 5, you will obtain 40, 45, 30, 40, 20. Then the new mean is 5×7 = 35

2. Median

Definition 6.7:

*The **Median** is the value that lies in the middle of the data when it is arranged in ascending or descending order. So, half the data is below the median and half the data is above the median.*

Properties of the Median

1. The median can obtained even when some of the data values are not known.
2. It is not affected by extreme value.
3. It is unique for a given data set.
4. It is only used for quantitative data.

Example 1. Find the median of each of the following:

a. 6, 7, 9, 7, 11, 13, 15 **b.** 27, 23, 36, 38, 27, 40, 45, 39

Cont...

Solution:

a) First arrange the data in ascending order as 6, 7, 7, 9, 11, 13, 15

There are seven values (an odd number of values) and the middle value is the 4th element of the list which is 9.

Therefore 9 is the median of the data.

b) First, arrange the data in ascending order as 23, 27, 27, 36, 38, 39, 40, 45

There are eight values (an even number). The two middle values are the 4th and 5th elements of the list which are 36 and 38. The median is half the sum of 36 and 38. So, the median is

Example 2. Find t

V	1	2	3	4	5
f	2	3	2	4	2

g distribution

Cont...

Solution: There are 13 data values. So, the median is the 7th piece of data, which is 3.

Note that the median of a set of data with values arranged in ascending or descending order is:

i. the middle value of the list if there is an odd number of values.

ii. half of the sum of the two middle values if there is an even number of values.

3. Mode

Definition 6.8:

The value of the variable which occurs most frequently in a data set is called the mode.

Note:

a. The set of data may can have no mode.
b. The set of data may have two mode called bimodal
c. The set of data may have more than two modes called multimodal
d. If there is no observation that occurs with the highest frequency, we say the data has no mode.

Properties of the Mode

1. *The mode is not always unique.*
2. *It is not affected by extreme values*
3. *It can be used for both quantitative and qualitative data.*

Cont...

Example 1. Find the mode of each of the following data sets:

a. 4, 6, 12, 10, 7

b. 12, 10, 11, 13, 10, 14, 12, 18, 17

c. 9, 8, 7, 10, 6, 8

Solution:

a. It has no mode because each value occurs only once.

b. The values 10 and 12 both occur twice, while the others occur only once. It has two modes and the data is a bimodal.

c. 8 is the mode because it occurred twice (most frequently).

Example 2. Find the mean, median and mode of the following distribution of temperatures in a certain town for one month.

Temperature in $^{o}C(V)$	20	21	23	24	26	28
Number of days(f)	2	4	5	9	3	7

Cont...

Solution: Mean: $\bar{x} = \dfrac{(20\times 2)+(21\times 4)+(23\times 5)+(24\times 9)+(26\times 3)+(28\times 7)}{2+4+5+9+3+7}$

$$= \frac{40+84+115+216+78+196}{30} = \frac{729}{30} = 24.3$$

Therefore, the mean is 24.3°C.

The number of observations is an even number which is 30. So, the median is half the sum of the 15^{th} and 16^{th} values.

i.e., median = $\dfrac{15^{th} \text{ value} + 16^{th} \text{ value}}{2} = \dfrac{24+24}{2} = 24$

Therefore, the median is 24°C.

The value with highest frequency is the number 24. Therefore, the mode is 24°C.

6.1.4 Measures of Dispersion for Ungrouped Data

Dispersion or Variation is the scatter (or a spread) of data values from a measure of central tendency. ***The range, variance and standard deviation are some of measures of dispersion***.

1. Range.

Definition 6.9:

- *The range R of a set of numerical data is the difference between the highest and the lowest values. i.e.,*

  Range= Highest value – Lowest value

Example 1. The ages of six students are 24, 20, 18, 13, 16, 15 years, respectively. What is the range?

Solution: Range = highest value – lowest value = 24 – 13 = 11 years.

Example 2. Find the range of the distribution given in the table below.

V	2	8	9	13	15	18
f	3	4	2	1	5	4

Solution: The maximum value is 18 and the minimum value is 2.

Range = maximum value – minimum value = 18 – 2 = 16

Cont...

2. Variance

Definition 6.10

Variance, denoted by is defined as the mean of the squared deviations of each value from the arithmetic mean.

3. Standard deviation

Definition 6.11

- ***Standard deviation,*** *denoted by , is defined as the positive square root of the mean of the squared deviations of each value from the arithmetic mean.*

The actual method of calculating can be summarized in the following steps:

Step 1. Find the arithmetic mean of the distribution.

Step 2. Find the deviation of each data value from the mean

Step 3. Square each of these deviations,

Step 4. Find the mean of these squared deviations. This value is called the *variance* and is denoted by

Step 5. Take the principal square root of i.e.,

Standard deviation =

Cont...

Properties of variance and standard deviation

1. If a constant c is added to each value of a population function, then the new variance is the same as the old variance. The new standard deviation is also the same as the old standard deviation.

2. If each value of a population function is multiplied by a constant c, then

    i. The new variance is c^2 times the old variance.

    ii. The new standard deviation is $|c|$ times the old standard deviation.

Example 1. Find the variance and the standard deviation for the following data: 3, 5, 8, 11, 13

Solution:

x	$(x-\bar{x})$	$(x-\bar{x})^2$.
3	−5	25
5	−3	9
8	0	0
11	3	9
13	5	25
Total 40		68

$$\text{Variance} \quad (\sigma^2) = \frac{68}{5} = 13.6$$

$$\text{Standard deviation } (\sigma) = \sqrt{\sigma^2} = \sqrt{13.6} \approx 3.7$$

Example 4 Find the variance and standard deviation of the population function whose distribution is given in the following table.

V	2	3	5	6	8
f	3	4	4	5	4

Solution: First, the mean has to be calculated.

$$\bar{x} = \frac{3\times 2 + 4\times 3 + 4\times 5 + 5\times 6 + 4\times 8}{3+4+4+5+4} = \frac{100}{20} = 5$$

Cont...

x	f	xf	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
2	3	6	-3	9	27
3	4	12	-2	4	16
5	4	20	0	0	0
6	5	30	1	1	5
8	4	32	3	9	36
Total	20	100	0		84

$$\text{Variance } (\sigma^2) = \frac{84}{20} = 4.2$$

$$\text{Standard deviation } (\sigma) = \sqrt{4.2} \approx 2.05$$

Therefore, the population variance and standard deviation are 4.2 and 2.05 respectively.

6.2 Probability

Definition 6.12:

- *An experiment is a trial by which an observation is obtained but whose outcome cannot be predicted in advance.*

Experimental probability

- Probability determined using data collected from repeated experiments is called experimental probability.

Definition 6.13:

- *The probability set (or sample space) for an experiment is the set of all possible outcomes of the experiment.*

Definition 6.14:

- *An **event** is a subset of the probability set (sample space).*

Definition 6.15:

- *If an experiment has n equally likely outcomes and if m of these represent a particular event, then the probability of this event occurring is*

Theoretical Probability of an event

Definition 6.16:

- The theoretical probability of an event E, written as P(E) is defined as follows:

Definition 6.17:

- *Let S be the possibility set of an experiment and each element of S be equally likely to occur. Then the probability of the event E occurring, denoted by P(E), is defined as:*

Note: For any event E,

✓ .

✓ .

Example 1. The numbers 1 to 20 are each written on one of 20 identical cards. One

card is chosen at random.

a. List the set of all possible outcomes.

b. List the elements of the following events:

i. The number is less than 5.

ii. The number is greater than 15.

iii. The number is greater than 21.

iv. The number is divisible by 5.

v. The number is a prime.

Solution:

a. S = {1, 2, 3, …, 19, 20}

b. **i.** {1, 2, 3, 4}

ii. {16, 17, 18, 19, 20}

iii. { } or Æ, since no card has a number greater than 20.

iv. {5, 10, 15, 20}

v. {2, 3, 5, 7, 11, 13, 17, 19}

Example 2:

a. Give the sample space for tossing a coin.

b. What is the sample space for throwing a die?

Solution:

a. When we toss a coin there are only two possible outcomes: Heads (H) or

Tails (T). So S = {H,T}.

b. When we throw a die the score can be any of the six numbers 1, 2, 3, 4, 5, 6, so S = {1,2,3,4,5,6}.

Cont...

Example 3. In an experiment of tossing two coins,

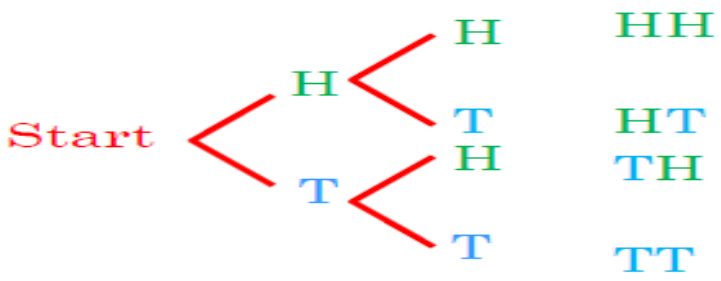
a. What are the possible outcomes?

b. How many different possible outcomes are there?

c. What is the probability of the coins landing with

i. two heads? **Ii.** two tails? **Iii.** one head?

Solution: Using a tree diagram, we get



a The set of possible outcomes is $S = \{HH, HT, TH, TT\}$.

b There are 4 possible outcomes.

c **i** The event two heads $= \{HH\}$ has one member, so

$$P(\text{two heads}) = \frac{1}{4}$$

ii $P(\text{two tails}) = \frac{1}{4}$

iii The event one head $= \{HT, TH\}$ has two members, so

$$P(\text{one head}) = \frac{2}{4} = \frac{1}{2}$$

UNIT 7

Scalar Quantities

Definition 7.1

- *Scalar quantities are those quantities of measures that have only magnitude and no direction. (Simply represented by a real number and a specified unit).*

Example:

1. The length of a side of a triangle is 4 cm. Since 4 is a real number with no direction on the length represents a scalar.

2. The daytime temperature of Mercury rises to 430. Since 430 is real number, this temperature represents a scalar.

Vector Quantities

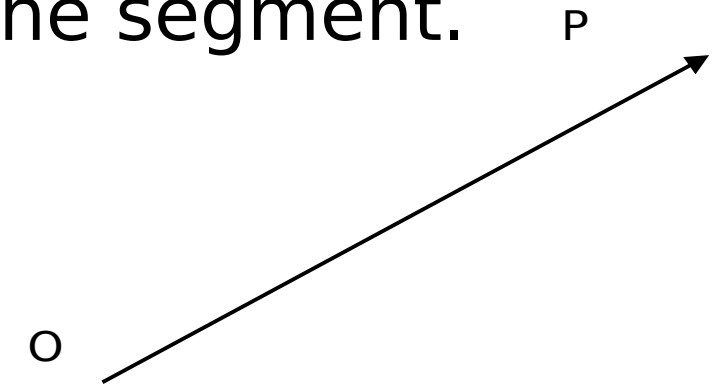
Definition 7. 2

- *Vector quantities are those quantities of measure that have both magnitude (length) and directions.*

Examples: The velocity of a car is 80 km/hr in the direction of north. This a vector.

7 .2 REPRESENTATION OF A VECTOR

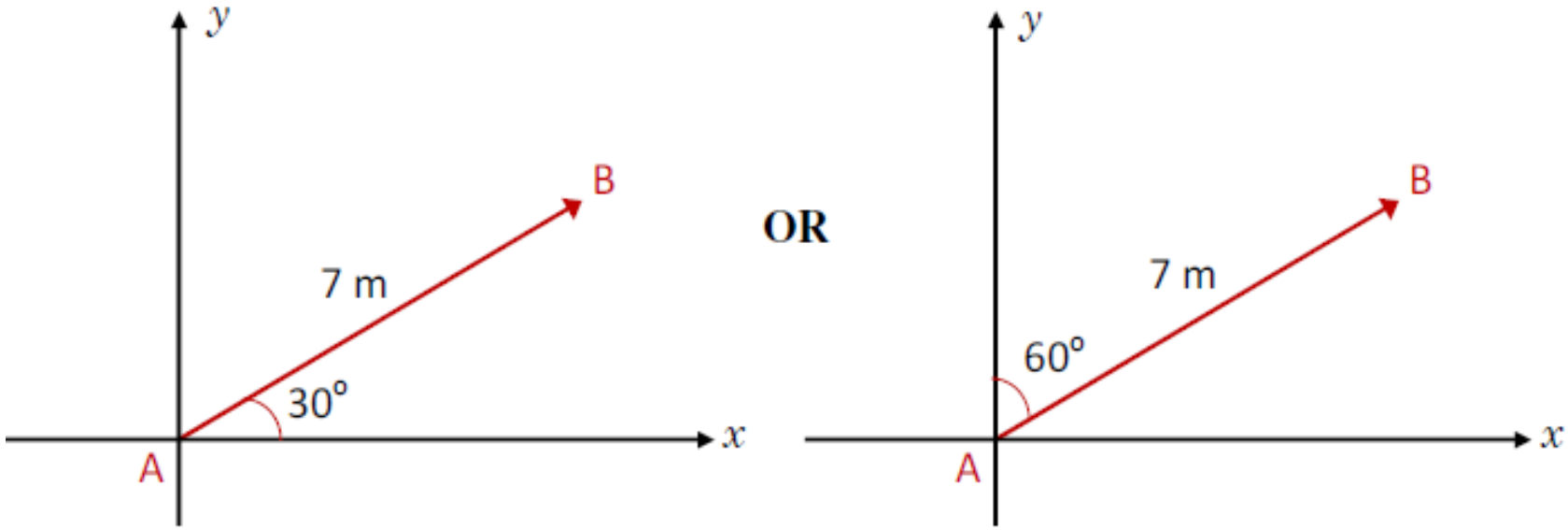
- A vector can be represented either algebraically or geometrically. Often, the most convenient way of representing vectors is geometrically, where a vector is represented by an arrow or directed line segment.



When a vector is represented by an arrow *(), the point O is called the* ***initial point*** and P is called ***the terminal point.*** Sometimes, vectors are represented using letters or a letter with a bar over it such as

Example 1. What does the vector in the following figure represent?

Cont...



Solution: The vector AB has a length of 7 m and direction of East 30o North [E30oN] (or a direction of North 60o East [N60oE]. Its initial point is A and its terminal point is B.

Magnitude (Length) Vectors

- The magnitude (length) of a vector or simply is the length of the line segment from the initial point O to the terminal point P, (the length of the directed line segment).

Notation: ***Magnitude of vector*** *is denoted as* ||.

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Example 2. Determine the length of the vector

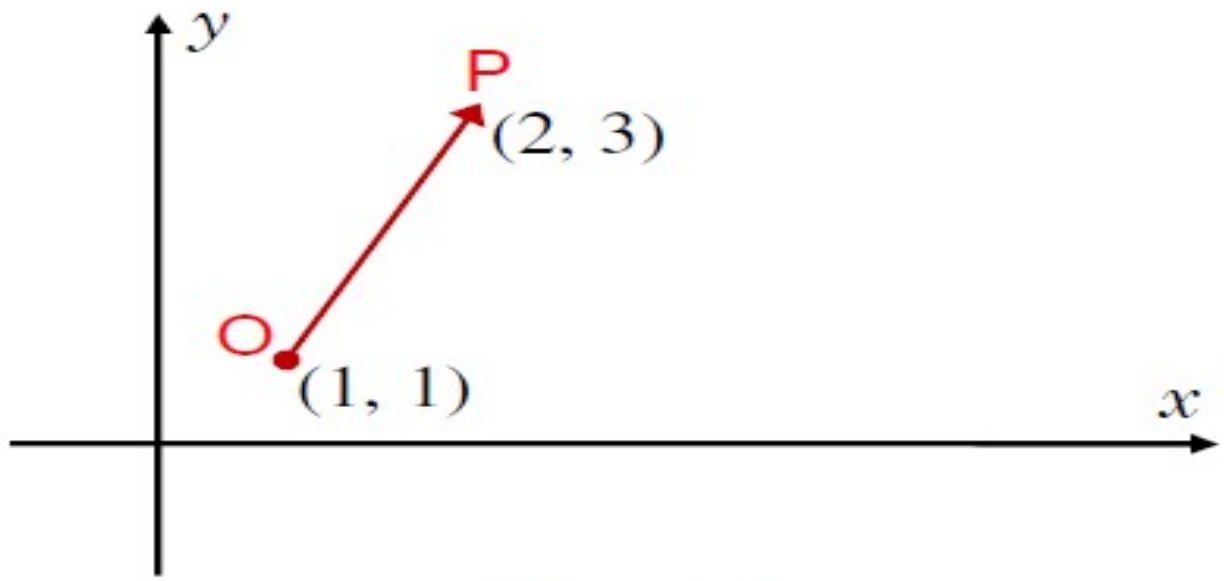


Figure 7.5

Solution: The magnitude of the vector $\overrightarrow{OP}$ is $|\overrightarrow{OP}| = \sqrt{5}$ (How?)

Direction of vectors

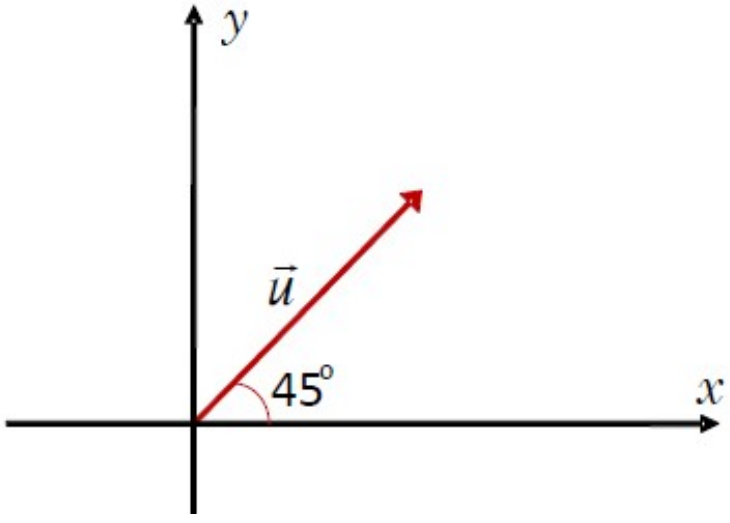
- The direction of a vector is the angle that is formed by the arrow (that represents the vector) with the horizontal line at its initial point (or with the vertical line in the case of compass directions).

Cont...

Note:

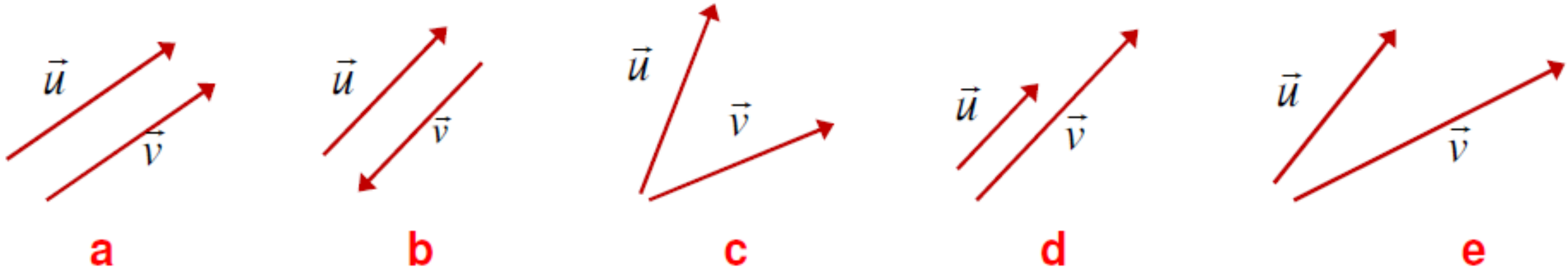
1. *If two vectors have opposite direction, they are called opposite vectors.*
2. *Vectors that have either the same or opposite directions are called parallel vectors.*
3. *Two vectors are said to be equal if they have the same length and the same direction.*

Example 6 The direction of the vector from the horizontal line at its initial point, , is . (or NE)



Cont - - -

Consider the following pair of vectors



What do you observe? Do they have the same length? Do they have the same direction?

- The two vectors in **a** have the same length and they have the same direction.
- The two vectors in **b** have the same length but they have opposite directions.
- The two vectors in **c** have the same length and different directions.
- The two vectors in **d** have different length, but they have same direction and
- The two vectors in **e** differ in both length and direction.

7.3 Addition and Subtraction of Vectors and Multiplication of a Vectors by a Scalar

7.3.1 Addition of Vectors.

Triangle law of addition of vectors

Consider the following (See figure below). Observe that

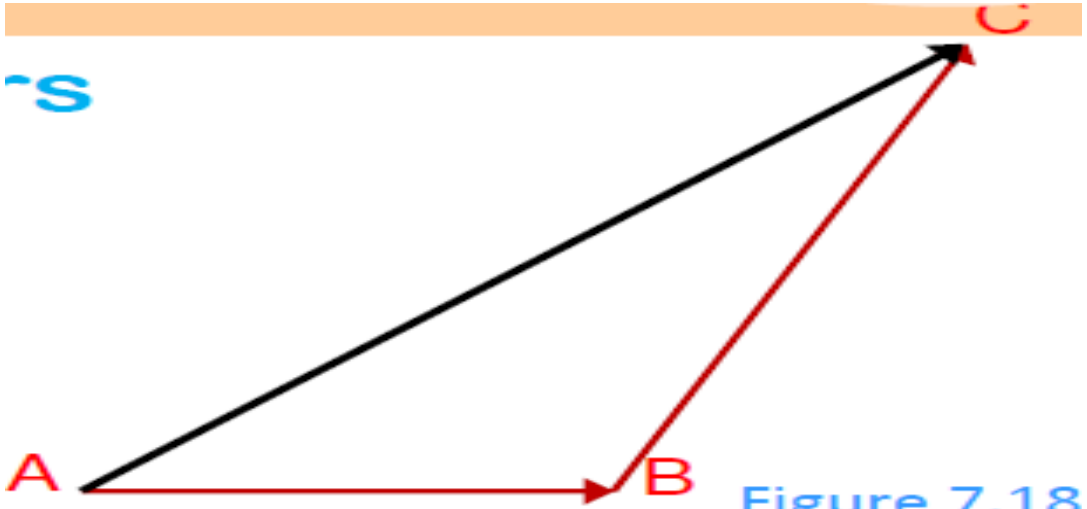


Figure 7.18

Definition: Triangle law of vector addition

- Let and be two vectors in a coordinate system. If = and = then their sum, is the vector represented by the directed line segment

Cont...

Example 1 A car travels 4 km to the North and then $4\sqrt{3}$ km to the East. What is the displacement of the car from A to the final position C?

Solution:

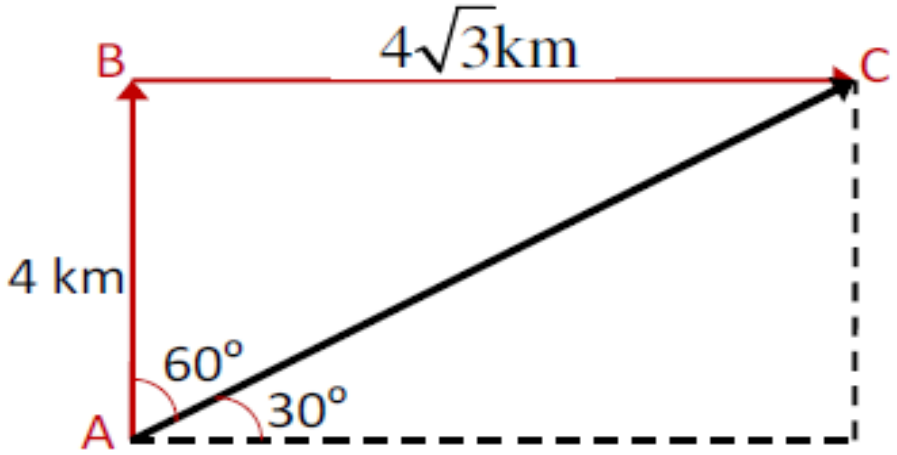


Figure 7.20

The magnitude $AC = \sqrt{4^2 + \left(4\sqrt{3}\right)^2} = \sqrt{16+48} = \sqrt{64} = 8$ and

$\tan(\angle BAC) = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{4\sqrt{3}}{4} = \sqrt{3}$. Therefore, $\angle BAC = 60^o$

So the displacement is the vector $\overrightarrow{AC}$, which is 8 km in the direction of North 60^o East.

Parallelogram law of addition of vectors

From previous discussions you notice that if two vectors have the same length and direction, then they are equal.

In Figure 7.25, $\overrightarrow{AD}$ and $\overrightarrow{CE}$ are equal. So $\overrightarrow{AC}+\overrightarrow{AD}$ and $\overrightarrow{AC}+\overrightarrow{CE}$ represent the same vector. But, from triangle law, $\overrightarrow{AC}+\overrightarrow{CE}=\overrightarrow{AE}$

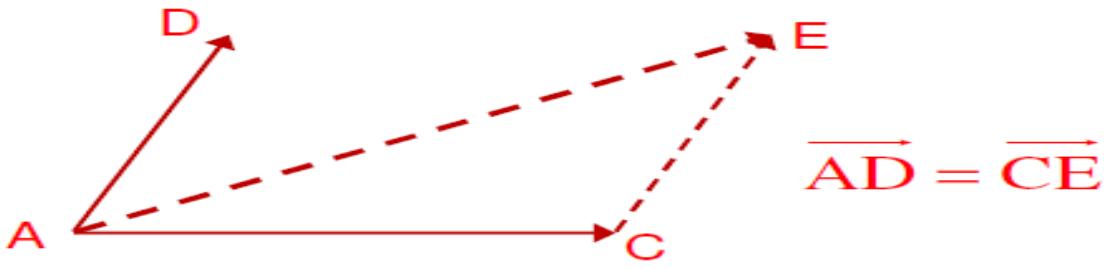


Figure 7.25

Therefore $\overrightarrow{AC}+\overrightarrow{AD}=\overrightarrow{AE}$.

Now let's see how we can construct a parallelogram and see the sum of two vectors (with the same initial point) as the diagonal of the constructed parallelogram.

Example 5 Given the vectors $\overrightarrow{AC}$, $\overrightarrow{AE}$, $\overrightarrow{AD}$, $\overrightarrow{AF}$, $\overrightarrow{AK}$ and $\overrightarrow{AG}$ described in Figure 7.26, determine the following vectors.

a $\overrightarrow{AC}+\overrightarrow{AE}$ **b** $\overrightarrow{AC}+\overrightarrow{AD}$

c $\overrightarrow{AE}+\overrightarrow{AD}$

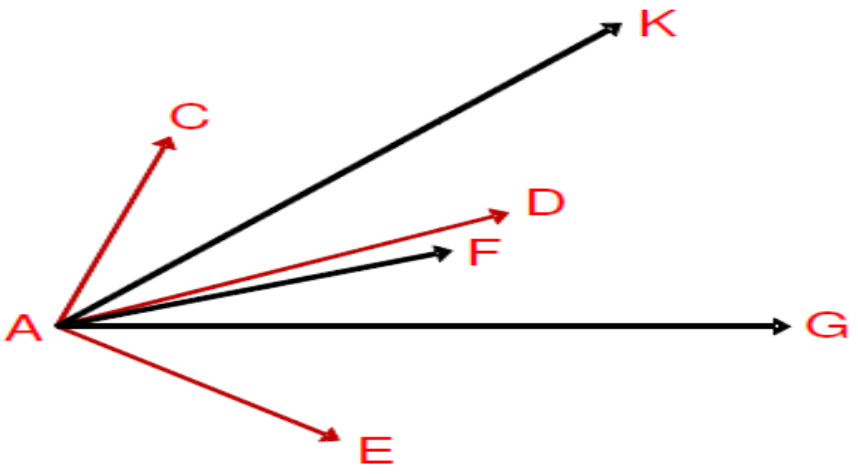


Figure 7.26

Solution: Construct a parallelogram and see that

a $\overrightarrow{AC}+\overrightarrow{AE}=\overrightarrow{AF}$

b $\overrightarrow{AC}+\overrightarrow{AD}=\overrightarrow{AK}$

c $\overrightarrow{AE}+\overrightarrow{AD}=\overrightarrow{AG}$

Subtraction of vectors

Figure 7.27

From addition of vectors, we recall that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, from which we can see that $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$.

Example 6 Describe the vectors $\overrightarrow{BC}$ and $\overrightarrow{BD}$ and determine $\overrightarrow{CD}$ in terms of $\overrightarrow{BC}$ and $\overrightarrow{BD}$.

Solution: $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ from Triangle Law.

Therefore, $\overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{BC}$.

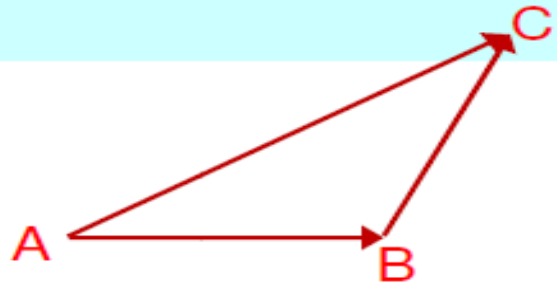


Figure 7.28

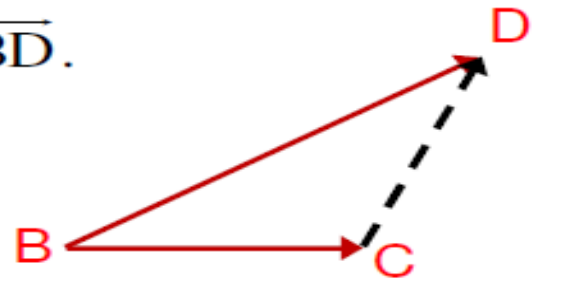


Figure 7.29

Multiplication of vectors by scalar

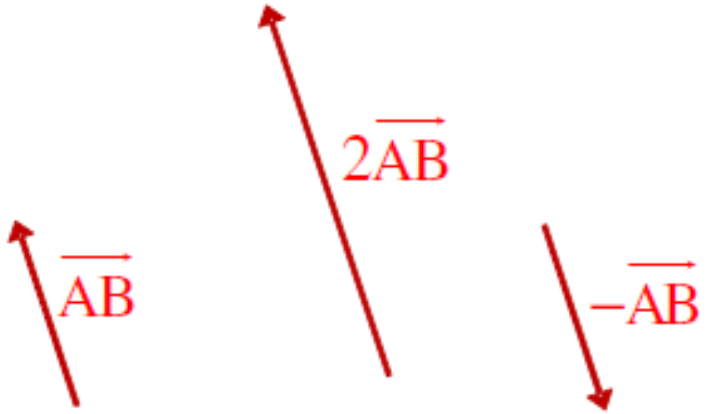
Scalar multiplication of a vector

Definition 7.4 Let be any given vector and *k* be any real number. The vector is the vector whose magnitude is *k* times the magnitude of and,

a. the direction of is the same as the direction of *if* $k > 0$.

b. the direction of is opposite to that of *if* $k < 0$.

Example 1. Figure below shows vector and the result of multiplying it by 2 and the result of multiplying it by −1.



Note: If , then any vector and are parallel vectors.

The end !

Thank You !!

Cont...

Arc Length

- The length of an arc of the circle of radius r that subtends an angle of at the center is given by

The Perimeters and Areas of a Sectors

- The area A of a sector of radius r and central angle is given by

- The perimeter P of the sector is the sum of the radii and the arc that bound it.

Cont - - -

The Perimeters and Areas of a Segment

- The area A and perimeter P of a segment of a circle of radius r, cut off by a chord subtending an angle at the center of a circle is given by

Example: A segment of a circle of a radius 12 cm is cut off by a chord subtending an angle 60 at the center of the circle. Find:

a. The area of the segment b. the perimeter of the sector

Solution:

- From the figure 5.106 shown on your text book page 230, area of the segment (the shaded part) = area of the sector OAB – area of triangle OAB;

Area of the sector OAB=

Area of triangle OAB=

Therefore, segment area =

Perimeter of the sector=2

5.5 Measurement

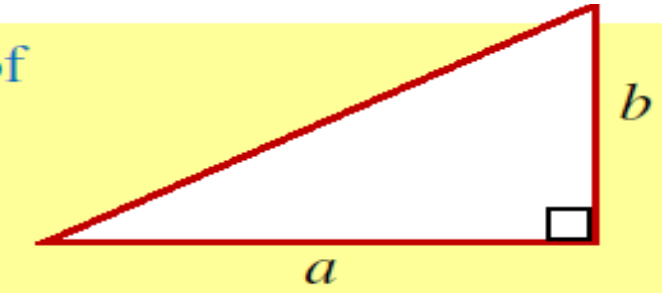
5.5.1 Areas of Triangles and Parallelograms

A. Areas of Triangles

Case i.

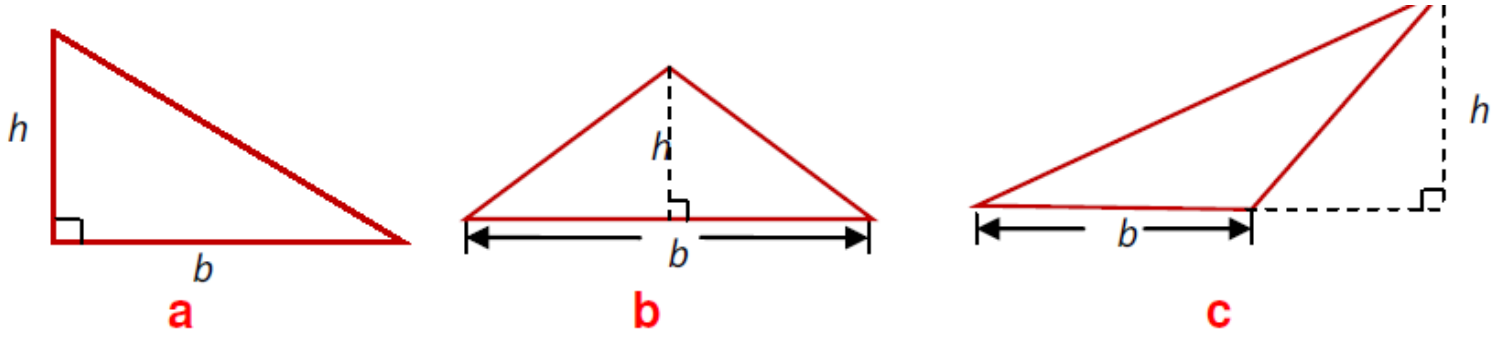
The area A of a right angle triangle with perpendicular sides of length a and b is given by

$$A = \frac{1}{2}ab$$



Case i.

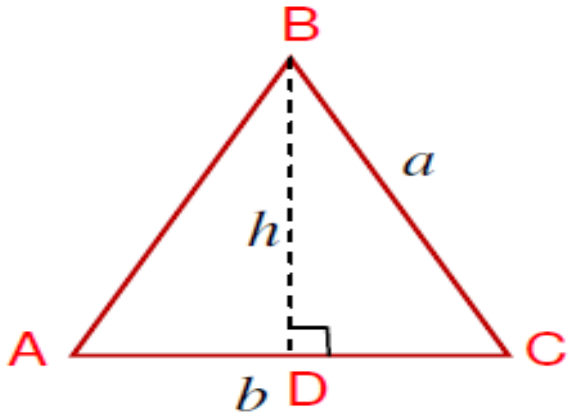
- The area A of any triangle with base b and the corresponding height h is given by



Cont...

Case iii.

- The area A of any triangle with side a and b units long and angle C () included between these sides is



Theorem 5.14: (Heron's Formula)

The area A of a triangle with sides a, b and c units long and semi-perimeter

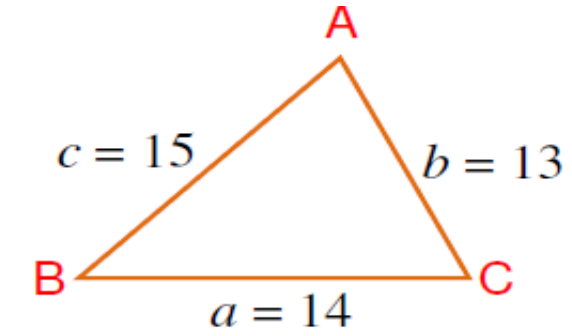
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Example: Given If AB=15 units, BC=14 units and AC=13 units, find

a. The area of

b. The length of the altitude from the vertex

c. The measure of .



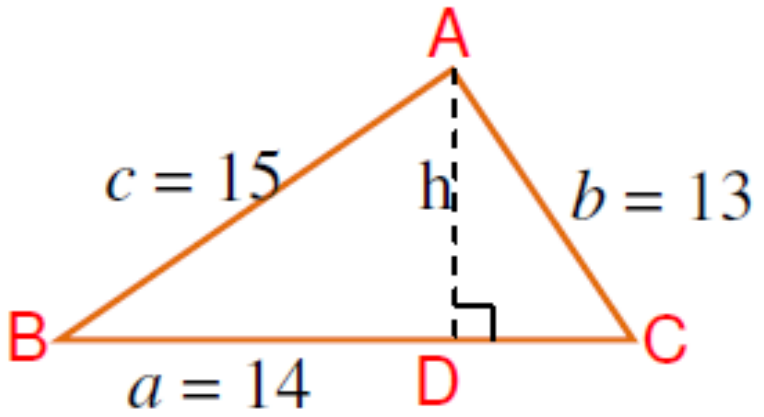
Solution:

a.

a=14	s-a=7
b=13	s-b=8
c=15	s-c=6

Therefore, area of

b. Let the altitude from the vertex A to the corresponding base BC be h, meeting BC at D as shown i



Then, area of

Therefore, 84=

Hence,

Therefore, the altitude of from the vertex A is 12 units long.

c. In the right angle triangle ABD shown in figure above, we see that

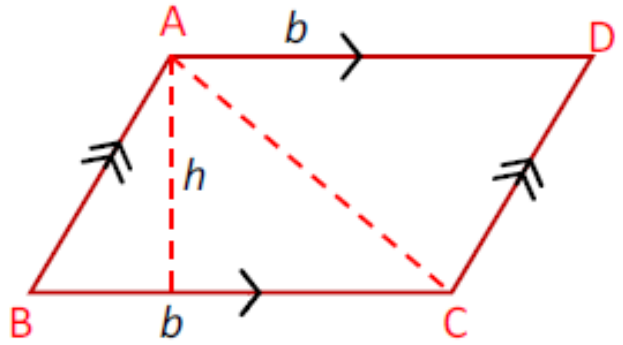
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Cont...

b. Areas of Parallelograms

Theorem 5.15

- *The area of a parallelogram with base b and perpendicular height h is*



Example: If one pair of opposite sides of a parallelogram have length 40 cm and the distance between them is 15 cm, find the area of the parallelogram.

Solution:

5.5.2 Further on Surface Areas and Volumes of Cylinders and Prisms

A. Prism

- A prism is a solid figure formed by two congruent polygonal regions in parallel planes, along with three or more parallelograms, joining the two polygons. The polygons in parallel planes are called **bases.**
- A prism is named by its base. Thus, a prism is called triangular, rectangular, pentagonal, etc., if its base is a triangle, a rectangle, a pentagon, etc., respectively.

In a prism

- ✓ The lateral edges are equal and parallel.
- ✓ The lateral faces are parallelograms.

- **A right prism** is a prism in which the base is perpendicular to a lateral edge. Otherwise it is an **oblique prism.**
- In a right prism

- ✓ All lateral edges are perpendicular to both bases.

Cont...

- ✓ The lateral faces are rectangles
- ✓ The altitude is equal to the length of each lateral edge.
- A regular prism is a right prism whose base is a regular polygon.

Surface area and volume of prisms

- The lateral surface area of a prism is the sum of the area of its lateral faces
- The total surface area of a prism is the sum the lateral areas and the area of the bases.
- The volume of any prism is equal to the product of its bases and its altitude.

Cont...

Example 1. The altitude of a rectangular prism is 4 units and the width and length of its base are 3 and 2 units respectively. Find:

a. the total surface area of the prism. **B.** the volume of the prism.

Solution:

a. To find AT, first we have to find the base area and the lateral surface area.

$AB = 2 \times 3 = 6$ sqr unit and $AL = Ph = (3 + 2 + 3 + 2) \times 4 = 40$ sqr unit.

$AT = 2AB + AL = 2 \times 6 + 40 = 52$ sqr. unit.

So, the total surface area is 52 sqr unit.

b. $V = AB\ h = 6 \times 4 = 24$ cubic unit

Cylinder

- **A circular cylinder** is a simple closed surface bounded on two ends by circular bases.
- A more general definition of a cylinder replaces the circle with any simple closed curve. Since not all cylinder is circular**.**

Surface area and volume of Circular Cylinders

- The lateral surface area of a right circular cylinder is the product of its height h and the circumference C of its base.

- The total surface area of a right circular cylinder is the sum of twice the area of the circular base and the area of the curved surface (lateral surface area).

- The volume V of the right circular cylinder is equal to the product of its base area and height

Example 3 If the height of a right circular cylinder is 8 cm and the radius of its base

is 5 cm find the following giving your answers in terms of .

a. its lateral surface area **b.** its total surface area

c. its volume

Solution:

a. The lateral surface area of the right circular cylinder is given by

b.

= 80 + 50 = 130 sqr. cm

c. The volume of the cylinder is