Don Bosco secondary and preparatory school

Grade 12 Physics handout (5)

Alternating current (a.c) circuit

A.c generator is the source of alternating current

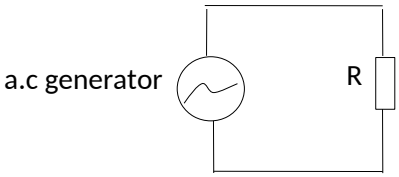
Alternating current (a.c) :- oscillation of current i.e the current that moves back and forth.

Alternating current is generated by the induced electromotive force,ε in the generator. The induced electromotive(voltage) and current are expressed in sinusoidal function. Based on Lenz's law, the induced voltage and current in the generator are in phase and expressed in the same manner as

$V=V_0\sin\omega t$ and $I=I_0\sin\omega t$ where V and I are instant voltage and current respectively.

V_0, I_0 --- maximum (pick) values of voltage and current.

Expressions of the generated voltage and current by the a.c generator are the same in a resistor connected to the a.c circuit.



Like any other forms of energies, electric energy is also expressed in terms of average work done to induce voltage as well as current. Then the voltage and current expressed as average values.

The values are named as root mean square (rms) values. The rms value is the square root of square of average value i.e $X_{rms}=\sqrt{\overline{X}^2}$. They are derived from average electric power, $\overline{P}=\frac{\overline{V}^2}{R}\vee\overline{P}=R\overline{I}^2$ $\overline{V}^2=\frac{V_1{}^2+V_2{}^2+\ldots+V_n{}^2}{n},\overline{I}^2=\frac{I_1{}^2+I_2{}^2+\ldots+I_n{}^n}{n}$

$$\rightarrow\overline{V}^2=\frac{V_0{}^2}{2},\wedge\overline{I}^2=\frac{I_0{}^2}{2}\,as\,n\cong\infty\in each\,case.$$

$$V_{rms}=\sqrt{\overline{V}^2}\wedge I_{rms}=\sqrt{\overline{V}^2}$$

$V_{rms}=\frac{V_0}{\sqrt{2}}\wedge I_{rms}=\frac{I_0}{\sqrt{2}}$ Where V_{rms} ---rms of voltage

I_{rms} ---rms of current

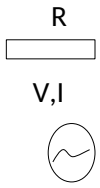
A.C circuit :-path of electric current connected to a.c source.

The a.c circuit contains different electric elements separately or commonly according to the applications. In the circuit, there can be energy dissipative devices or storage devices.

The a.c circuit is named as oscillatory circuit. Property of voltage and current in each electrical device by sinusoidal function and phase diagras.

Resistive circuit (R—circuit) and alternating current (a.c)

Voltage and current are in phase.



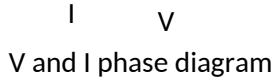
$V=V_0 \sin\omega t$ and $I=I_0 \sin\omega t$

Value of the resistance the same at any instant of V and I.

$$R=\frac{V}{I}=\frac{V_0 \sin\omega t}{I_0 \sin\omega t} \rightarrow \therefore R=\frac{V_0}{I_0}$$

The sine function and phase diagrams of V and I are as follow.





V and I phase diagram

The resistor resists current by producing resisting voltage. Then the dissipative power known as real power in the a.c circuit is expressed as :

$P=V_{rms}I_{rms}$ $\text{------} \rightarrow P=\frac{V_0 I_0}{\sqrt{2}\sqrt{2}}=\frac{V_0 I_0}{2}$

eg. Pick values of voltage and current of a.c source in a resistive circuit are 220V and 30A . Find dissipative power and the resistance.

Solution $i¿ P=\frac{V_{rms} I_{rms}}{\sqrt{2}\sqrt{2}}=\frac{V_0 I_0}{2} \rightarrow P=\frac{220\times 30}{2} \rightarrow \therefore P=3300\, watt$

$$ii¿\ P=\frac{V_{rms}^2}{R}\vee P=RI_{rms}^2$$

$$R=\frac{V_{rms}^2}{P}=\frac{V_0^2}{¿¿}$$

Capacitive circuit (C—circuit) and alternating current

Capacitor is energy storage device. It is charged and discharged. Then the current flows in the capacitive circuit with change of polarities on the two parallel plates of the capacitor. Energy (voltage) is stored on the capacitor, after current reaches the capacitor. Then current comes to the capacitor before the voltage. The process shows current and voltage are out of phase by $\frac{\pi}{2}$. The phase is called

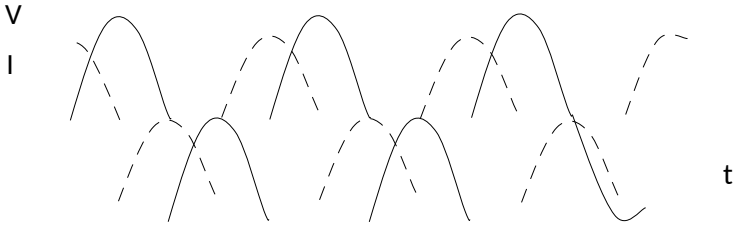


phase lead since current leads voltage. Sinusoidal functions of voltage (V) and current (I) are expressed as follow.

$I=I_0\sin\omega t$

$V=V_0\sin(\omega t+\frac{\pi}{2})$

* $V=V_0\sin\omega t$



Sinusoidal graphs of current and voltage

Phaser diagram of current and voltage are shown as follows.



The capacitor opposes change of current in the a.c circuit to keep stored charge as it is or fluctuation of stored energy in the capacitor. This opposition against change of current by the capacitor is known as capacitive reactance (X_c). It is not fixed like resistance. Its value is determined on the bases of complex number (Z=a+bi). Since it has no real, it is expressed in terms of imaginary value i.e ($Z=X_ci$)

The capacitive reactance (X_c) is derived as follows starting with definition of current.

$$I=\frac{\Delta Q}{\Delta t},\Delta Q=C\Delta V\rightarrow I=\frac{C\Delta V}{\Delta t}$$

$$I=\frac{CdV}{dt},V=-V_0\cos\omega t\rightarrow I=\frac{-Cd\left(V_0\cos\omega t\right)}{dt}=-CV_0\omega\left(-\sin\omega t\right).\ but\ I=I_0\sin\omega t$$

$$I_0\sin\omega t=C\omega V_0\sin\omega t$$ ---------- $I_0=C\omega V_0$ ---------- $\frac{V_0}{I_0}=\frac{1}{C\omega}$

In terms of Ohm's law, $\frac{V_0}{I_0}$ is the resistance,R . Now $\frac{1}{C\omega}$ is equivalent to the resistance and named as capacitive reactance,X_c. Therefore, $\mathbf{X_c=}\frac{1}{C\omega}$. This is definition of capacitive reactance.

eg. 400μF capacitor is connected an a.c source with 50H_z frequency. What will be the capacitive reactance? ($\pi=3.14$)

Solution:- $X_c=\frac{1}{C\omega}$, where $\omega=2\pi f$

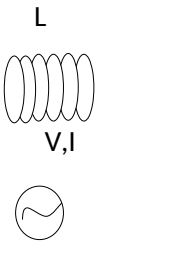
$$X_c=\frac{1}{400\mu\times2\pi50}=\frac{1}{400\times10^{-6}\times2\pi\times50}=\frac{10^6}{4000\pi}=\frac{100}{4\times3.14}$$

$$\therefore X_c=7.96\,\Omega$$

NB – Unit of reactance is also ohm (Ω) similar to a resistance.

Inductive circuit (L-circuit) and alternating current (a.c)

In the inductor voltage is produced first to store energy. Then voltage leads current by a phase difference of $\frac{\pi}{2}$. The phase difference in the inductor is said to be phase lag since the current lags in the inductive circuit.



voltage and current in the inductive circuit are expressed as

$$V=V_0\sin\omega t\wedge I=I_0\sin\left(\omega t+\frac{\pi}{2}\right)\rightarrow I=I_0\cos\omega t$$

The sinusoidal functions of voltage and current and their phase diagrams are shown as follow.



a) sinusoidal graphs of voltage & current



b) phase diagram of V and I

In the a.c circuit the inductor is magnetized and demagnetized. This is done due to opposing of change of current by the inductor. Opposing of current the inductor is called inductive reactance,X_L. Like capacitive reactance it is expressed a complex number (Z=a+bi). The real part is zero. Then it is stated as **$Z= X_L i$.**

The value of X_L is derived using $V=-L\frac{\Delta I}{\Delta t}$.

$$\rightarrow V=-L\frac{dI}{dt}, I=I_0 cos\omega t$$

$$\rightarrow V=-L\frac{d(I_0 cos\omega t)}{dt}=-L\omega I_0(-sin\omega t)$$

$$\rightarrow V_0 sin\omega t=L\omega I_0 sin\omega t \rightarrow \frac{V_0}{I_0}=L\omega$$

$L\omega$ is equivalent to the resistance,R since $\frac{V_0}{I_0}=R$. Then $L\omega$ defines the inductive reactance,X_L.

Therefore**, $X_L=L\omega$ or $X_L=2\pi fL$ since $\omega=2\pi f$** . The is expression of the inductive reactance.

eg. A 0.6H of an inductor is connected to an a.c source with 50Hz frequency. What is the inductive reactance?

Solution:- $X_L=L\omega=2L\pi f$

$X_L=2\times 0.6\times \pi \times 50$

$\therefore X_L = 60\,\pi \mathbf{Hz}$