Chapter Two

KINEMATICS AND DYNAMICS OF PARTICLES

Projectile Motion

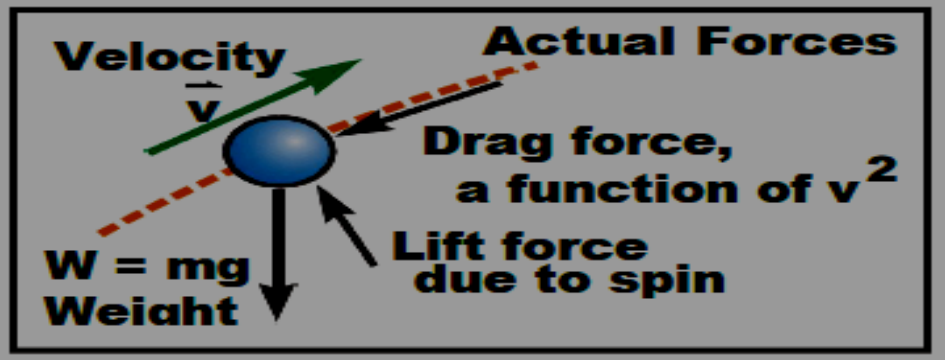
- The most common example of motion in two dimensions with constant acceleration is known as projectile motion.

1. Definition of a Projectile: An object that is "projected" or thrown, which has no capacity for self-propulsion.

2. Actual forces on a Projectile: Drag, lift due to spin, weight, wind.

3. The object projected is projectile.

4. The path followed by the projectile is trajectory.



Assumptions

- ✓ The X and Y axis motions are independent of each other.
- ✓ Air resistance neglected and time of flight for the first half and the second half becomes equal.

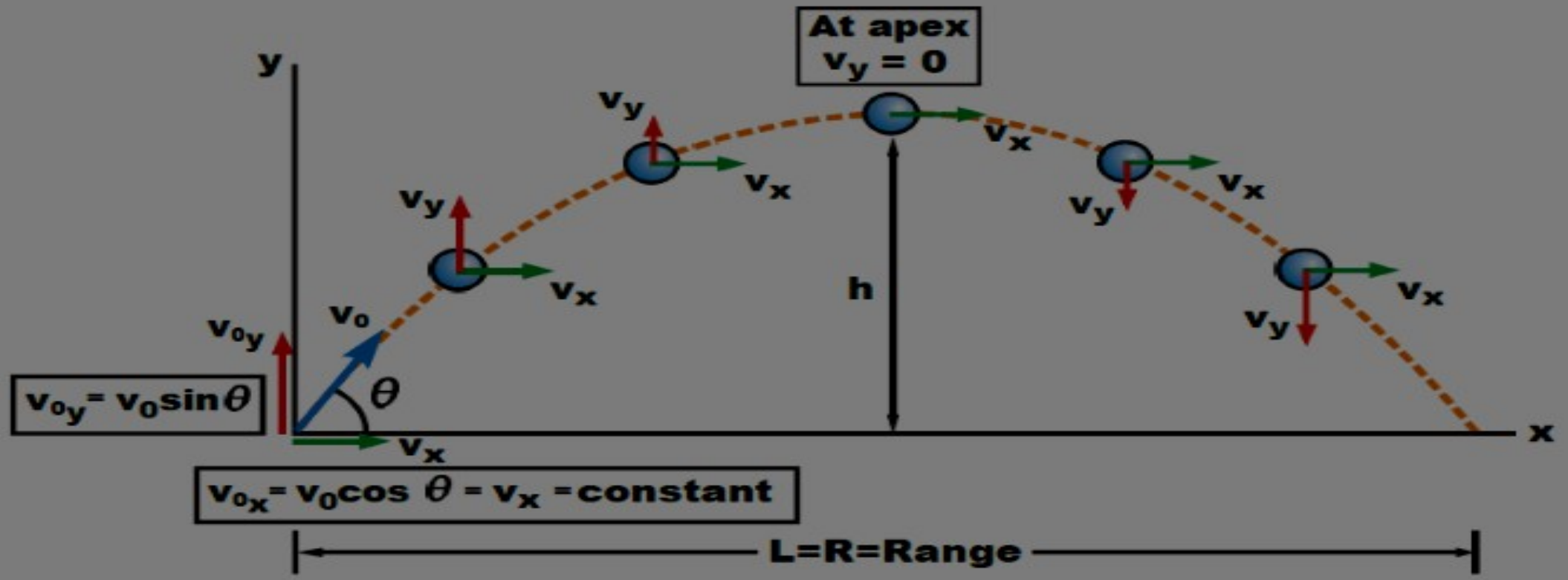
$$t_1 = t_2 = \frac{v_0 \sin\theta}{g}$$

$$\Rightarrow t_f = t_1 + t_2 = \frac{2v_0 \sin\theta}{g}$$

1

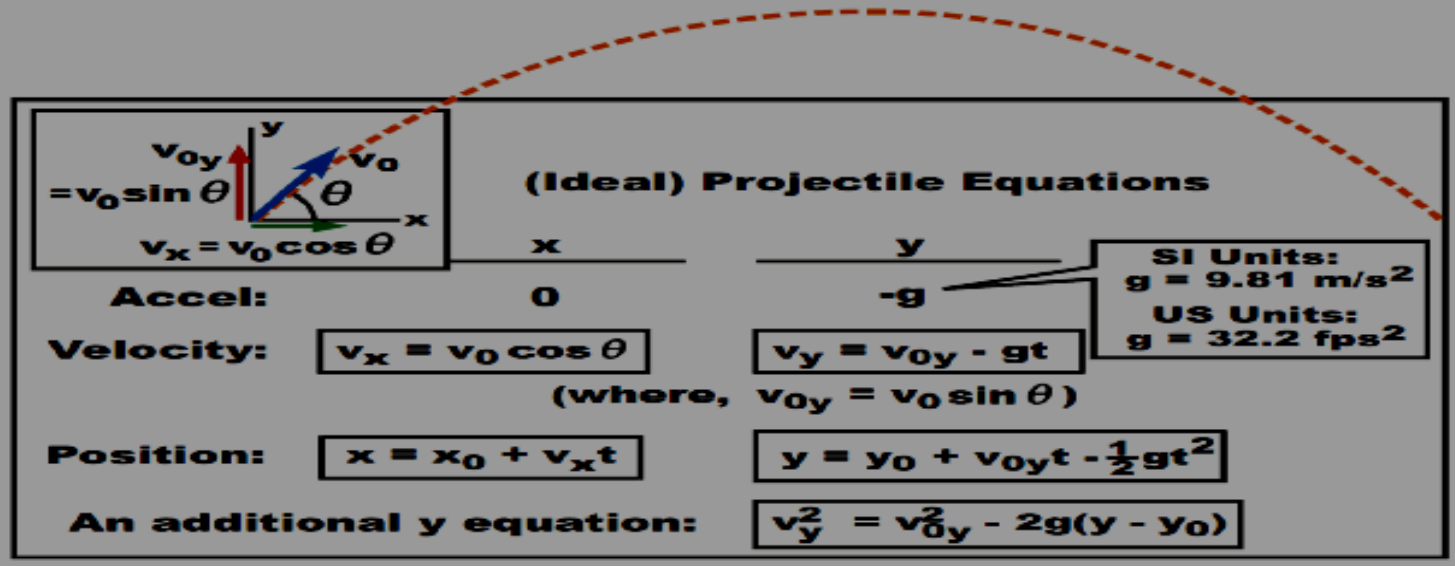
5. Ideal Projectile: The only force is weight. (This is what we will cover in this class.)

- ✓ If the only force is weight, then the x velocity stays constant. The y velocity changes with time and position.



2

6. Ideal Projectile Equations: If the only force is weight, then the x velocity stays constant ($a_x = 0$). The y velocity changes with time and position (y acceleration $a_y = -g$).



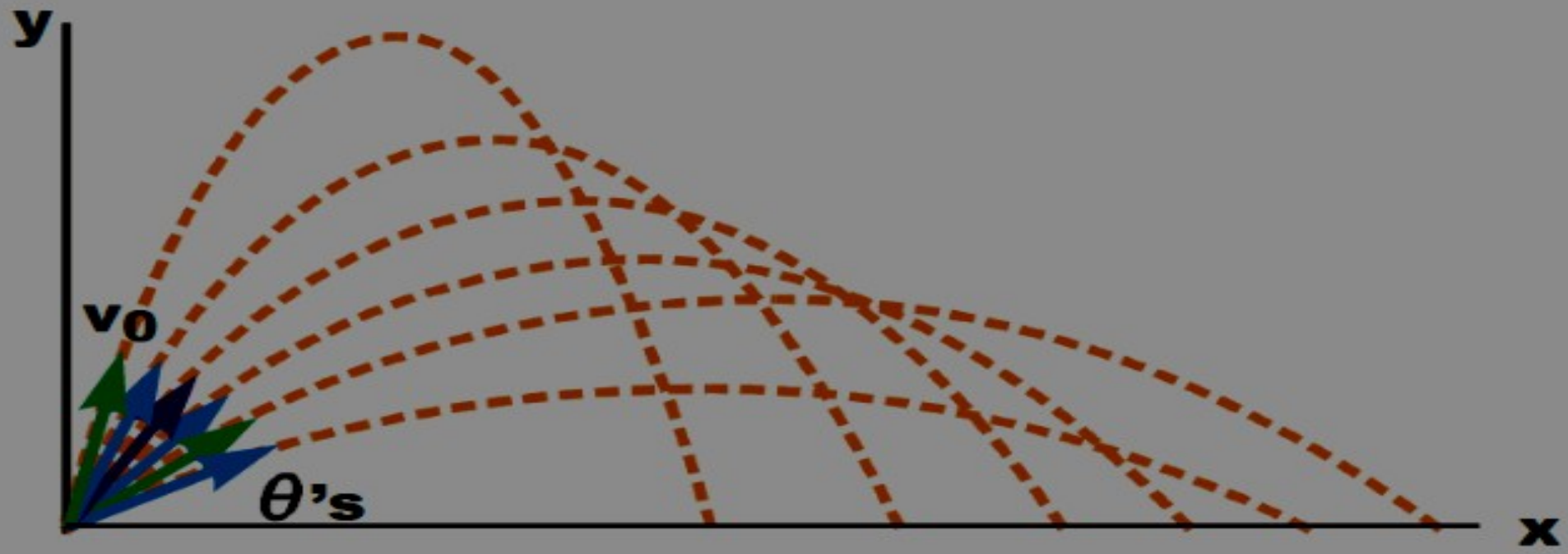
Accordingly

$$h_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

$$Range = \frac{v_0^2 \sin 2\theta}{g}$$

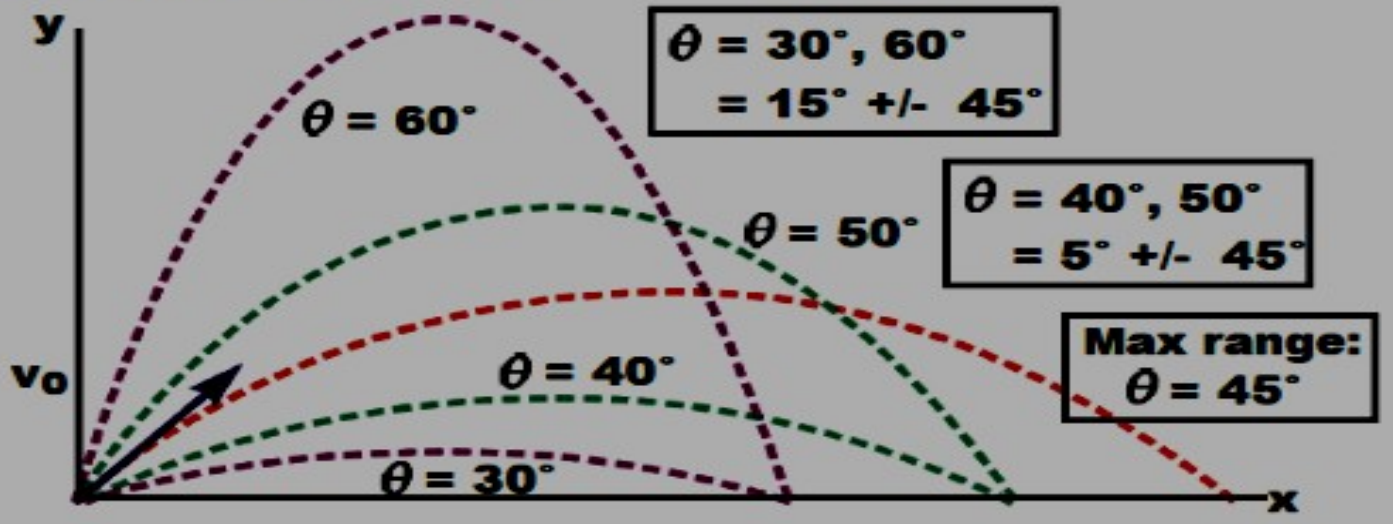
3

7. For each launch speed, v_0, and angle θ there is a different parabolic trajectory.



4

8. For a given launch speed, v_0, the max range is at $\theta = 45$. For the same v_0, launch angles at equal angular increments above and below 45 give (equal) ranges shorter than the max range.



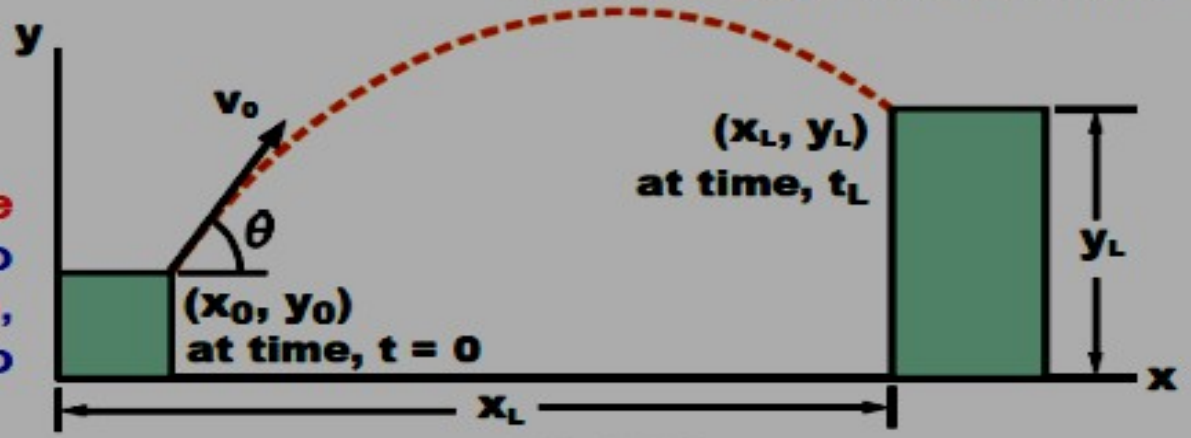
For a given v_o, max range at $\theta = 45^\circ$ (on level ground)

For a given v_o, launch angles at equal angular increments above and below 45° give (equal) ranges shorter than the max range.

5

9. A general projectile motion problem involves seven "pieces" of information [x_0, y_0, θ, x_L, y_L, and t_L] .

General Projectile Problem



Usually you are given five of these and asked to find the remaining two, usually applying the two position equations.

Projectile Problem Variables: (7 pieces of Info)

Launch Location: (x_0, y_0)

Launch Velocity and Angle: (v_0 at θ)

Landing Location and Time: (x_L, y_L) at time t_L

General Problem: Given 5 out of 7 of these "pieces" of info.
Use the two position equations to solve for the remaining two:

Position: $x = x_0 + v_x t$ $\qquad$ $y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

where, $v_x = v_0 \cos\theta$ $\qquad$ $v_{0y} = v_0 \sin\theta$

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2.2. **Particle Dynamics and Planetary Motion**

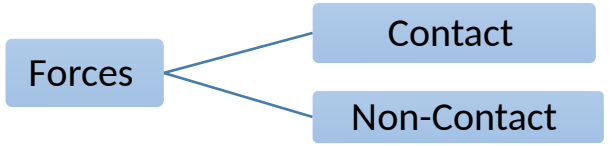
- In the previous section,
  - We have described motion in terms of
    - displacement,
    - velocity, and
    - acceleration without considering what might cause that motion.
- What cause particles to remain **at rest or accelerate**?
  - It is because of the mass of the object and forces acting on it.
- **Force**: any interaction that changes the motion of an object.
    - A force moves or tends to move, stops or tends to stop the motion of the object.
    - The force can also change the direction of motion of an object.
    - It can also change the shape or size of a body on which it acts.
- **Net force**: is defined as the vector sum of all the forces acting on the object. The object accelerates only if the net force () acting on it is not equal to zero.

The Concept of Force as A Measure of Interaction

- **In physics, any of the four basic fundamental forces**
  - Gravitational- *purely attractive force, which can act over long distances*
  - Electromagnetic- ***attractive and repulsive force, which acts on charged particles***
  - Weak ***forces-acts on the scale of the atomic nucleus***
  - Strong nuclear –***stronger than the electromagnetic force on the scale of the atomic nucleus keeps protons and neutrons bound together in the nucleus.***

  govern how particles interact.
- All other forces of nature can be traced to these fundamental interactions.
- The fundamental interactions are characterized on the basis of the following four criteria:
  - ❖The types of particles that experience the force,
  - ❖The relative strength of the force,
  - ❖The range over which the force is effective, and
  - ❖The nature of the particles that mediate the force

Type of Forces





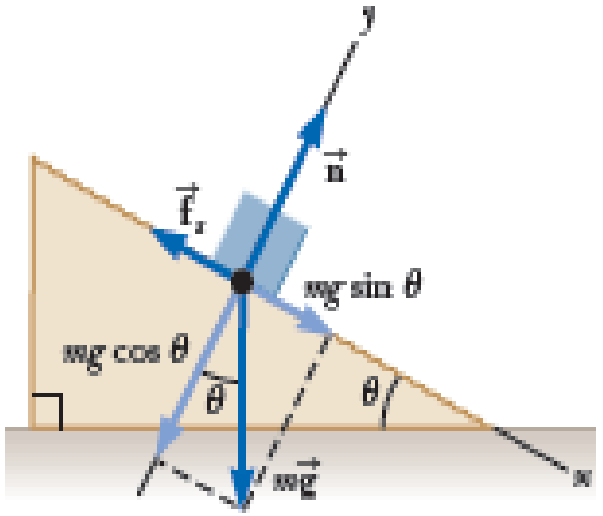
Contact Force

- It is a type of force that requires bodily contact with another object. And it is further divided into the following.

A. Muscular Forces

- Muscles function to produce a resulting force which is known as "***muscular force".***
- Muscular force exists only when it is in contact with an object.
- We apply muscular force during the basic day to day work of our life such as
  - breathing,
  - digestion, lifting a bucket,
  - pulling or pushing some object. Muscular force comes in handy to simply our work.

B. Frictional Forces

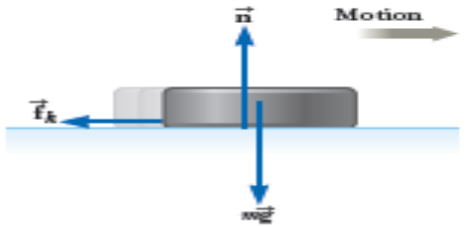


- When an object changes its state of motion, "frictional force" acts upon.
- It can be defined as the resisting force that exists when an object is moved or tries to move on a surface.
- The frictional force acts as a point of contact between two surfaces that is it arises due to contact between two surfaces.

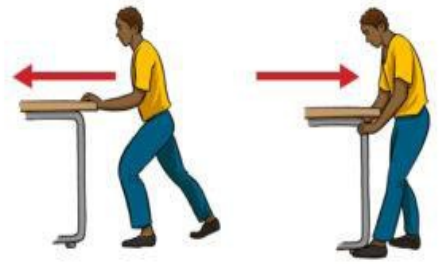
Examples; ***lighting a matchstick*** or ***stopping a moving ball come under frictional force***.

- **Normal Force**
- When a book is lying on the table, even though it seems that it‘s stationary, it‘s not.
- An opposing force is still acting on the book where in the force from gravity is pulling it towards the Earth. This force is the “normal force”.

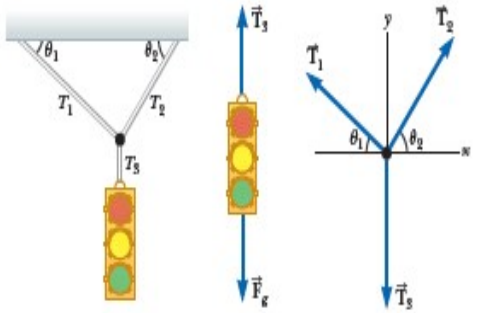
They always act perpendicular to the surface.



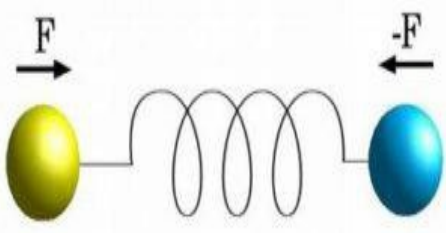
- 1. **Applied Force**
  - When you push a table across the room, you apply a force that acts when it comes in contact with another object. This is **“applied force”;** i.e. a force that is applied to a person or object.



- **Tension Force**
  - Tension is the ***force applied by a fully stretched cable*** or wire anchored on to an object. This causes a **“tension force”** that pulls equally in both directions and exerts equal pressure



- **Spring Force**
  - Force exerted by a compressed or stretched spring is **“spring force‘.** The force created could be a push or pull depending on how the spring is attached.
- **Air Resistance Force**
  - Air resisting forces are types of forces wherein objects experience a frictional force when moving through the air. These forces are resistive in nature



Non-Contact Force

- It is a type of force that does not require a physical contact with the other object. It is further divided into the following types of forces:

1. Gravitational force:-

- ***Gravitational force is*** an attractive force that can be *defined by Newton's law of gravity which states that*

"**gravitational forces between two bodies are directly proportional to the product of their masses and inversely proportional to the square of the distance between them**".

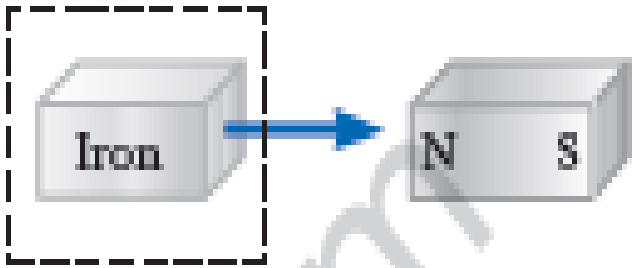
- It is a force exerted by large bodies such as planets and stars.

  E**xample**: water droplets falling down

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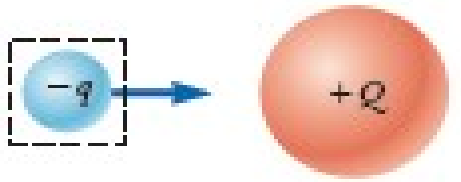
2. Magnetic Force

- The types of forces exerted by a magnet on magnetic objects are "magnetic forces".
- They exist without any contact between two objects.



3. Electrostatic Force

- The types of forces exerted by all electrically charged bodies on another charged bodies in the universe are "electrostatic forces".
- ***These forces*** can be both **attractive** and **repulsive** in nature based on the type of charge carried by the bodies.



2.2.3. *Newton's Laws of Motion and Applications*

- Laws of motions are formulated for the first time by English physicist ***Sir Isaac Newton in 1687***.
- Newton developed the three laws of motion in order to explain why the orbits of the planets are ellipses rather than circles, at which he succeeded.
- Newton‘s laws continue to give an accurate account of nature, except for very small bodies such as electrons or for bodies moving close to the speed of light.

Newton's First law of Motion:

States that

"Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it."

- This is sometimes called the *Law of Inertia*.
- Essentially, it makes the following two points:

  - ***An object that is not in motion will not move until a force acting upon it.***

  - ***An object in constant motion will not change its velocity until a force acts upon it.***

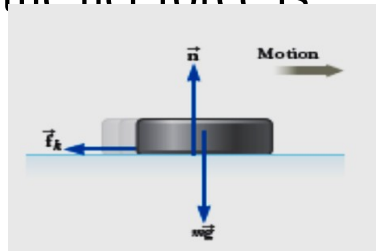
- ***Another way of stating Newton's First Law of motion:***

A body that is acted on by no net force moves at a constant velocity (which may be zero) and zero acceleration.

- So with no net force, the object just keeps doing what it is doing.
- ***net force*** means **the vector sum of total forces acting upon the object** must add up to zero.

- An object sitting on a floor has ***a gravitational force pulling it downward***, but there is also a ***normal force pushing upward*** from the floor, so the net force is zero.

- Therefore, it doesn't move.



Newton's Second law of Motion:

- ***States that***

*"The acceleration acquired by a point particle is **directly proportional** to the **net force acting on the particle** and **inversely proportional to its mass and the acceleration** is always in the direction of the net force".*

- *Mathematically*

- Where
  - $\Sigma \boldsymbol{F}$**=** is the net force acting on the particle,
  - m= is the mass of the particle and
  - $\boldsymbol{a}$**=**is the acceleration of the particle.
- when the ***net forces on an object sum up to zero***, we achieve the state defined in ***Newton's First Law***: the net ***acceleration must be zero***.

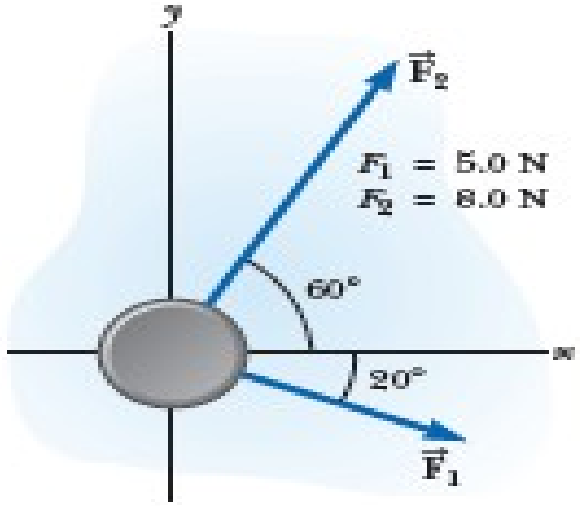
Cont……

- If the object is already moving, it will continue to move at a constant velocity,
  - but that **velocity will not change until a net force** is introduced.
- Obviously, an object at rest will not move at all without a net force.

Example

1. A box with a mass of 40 kg sits at rest on a frictionless tile floor. With your foot, you apply a 20 N force in a horizontal direction. What is the acceleration of the box?
2. A 3kg object undergoes an acceleration given by . Find the magnitude of the resultant force.

3. A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure below. The force has a magnitude of 5N, and is directed at 20° below the *x* axis. The force has a magnitude of 8N and its direction is 60° above the *x* axis. Determine both the magnitude ration



- Hint.(and ,,)

Newton's Third law of Motion

- ***States that*** —

  "For every action there is always an equal and opposite reaction".

- ***To understand this law***,
  - Consider two bodies ***A*** and ***B*** that are interacting and let F_{BA} is the force applied on body ***A*** by body ***B***, and F_{AB} is the force applied on body ***B*** by body ***A***.
  - These forces will be ***equal in magnitude*** *and* ***opposite in direction***.
  - In mathematical terms, it is expressed as:

- This is not the same thing as having a ***net force is zero***, however.
- Action and reaction forces are not treated the same as the forces acting on stationary object, normal force and weight of the object.
- **Notes that:-**
  - Action and reaction forces are always exist in pair
  - A single isolated force cannot exist
  - Action and reaction forces act on different objects

Exercise

1. Find the force needed to accelerate a mass of 40kg from velocity to in 10s.

2. If a man weighs 900N on earth, what is his weight on Jupiter where the acceleration due to gravity is ?

So/n

First

Then

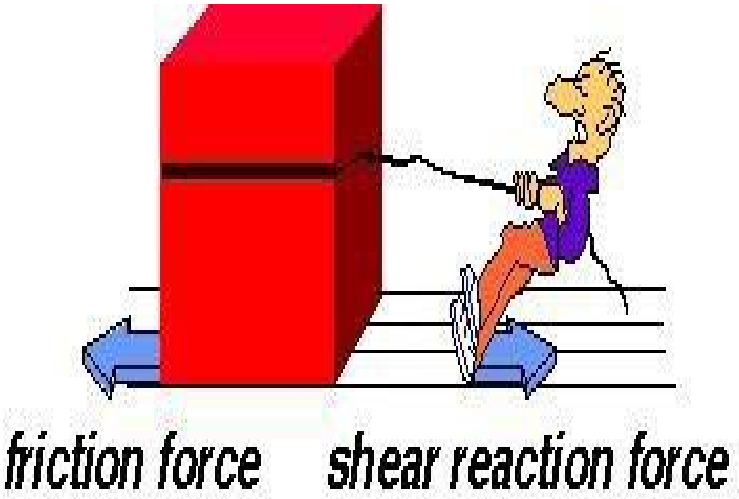
Now

Forces of Friction

- ***Frictional force refers to the*** force generated by two surfaces that are in contact and either at rest or slide against each other.
  - These forces are mainly affected by the surface texture and amount of force impelling them together.
- The ***angle*** and ***position*** of the object ***affect the amount of frictional force***.
  - *If an object is placed on a horizontal surface against another object, then the frictional force will be equal to the weight of the object.*
  - *If an object is pushed against the surface, then the frictional force will be increased and becomes more than the weight of the object.*
- Generally ***friction force is always proportional to the normal force between the two interacting surfaces.***
- ***Mathematically***
- Where the proportionality constant μ is the ***coefficient of friction***

Cont……

- *Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.*



- *Therefore, friction forces are categorized in the following manner:*

a) Static friction: exists between ***two stationary objects in contact to each other.*** Mathematically static friction is written as

Where is coefficient static friction

b) Kinetic friction: arises when the ***object is in motion on the surface***. The magnitude of the force of kinetic friction acting between two surfaces is

Where is the coefficient kinetic friction

Cont.…

- The values of and depend on the nature of the surfaces, but is generally less than $(\text{which implies that })$.

- Typical values range from around 0.03 to 1.0.
  - The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
  - The coefficients of friction are nearly independent of the area of contact between the surfaces.

Example

1. A 25-kg block is initially at rest on a horizontal surface. A horizontal force of 75N is required to set the block in motion. After it is in motion, a horizontal force of 60N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

Application of Newton's Laws of Motion

- We apply Newton's laws to objects that are either in equilibrium () ***Example (Tension force)*** or ***accelerating along a straight line*** under the action of constant external forces().Example
- when we apply Newton's laws to an object, we are interested only in external forces that act on the object.
- We assume that the objects can be modeled as particles so that we need not worry about rotational motion.
- We usually neglect the mass of any ropes, strings, or cables involved.
- procedure is recommended when dealing with problems involving Newton's laws:

Are a=constant and a=0 the same?

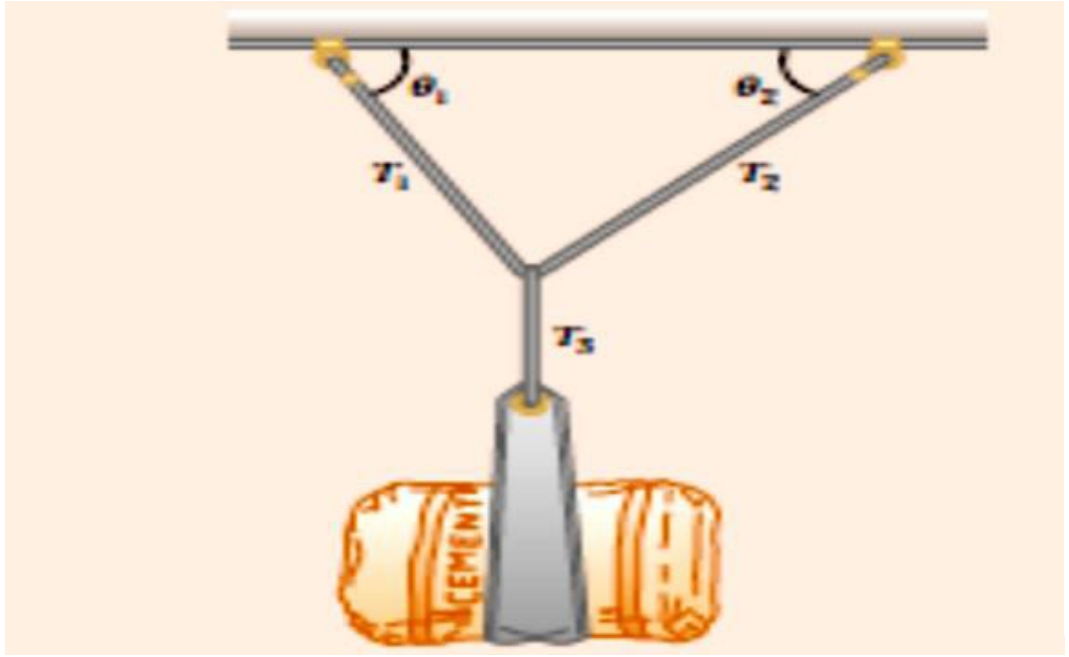
NO. Because **zero acceleration** means no change of **acceleration**. That is no increase or decrease of **acceleration** throughout the path. In the other hand **constant acceleration** means **constant** increase or decrease of **acceleration**.

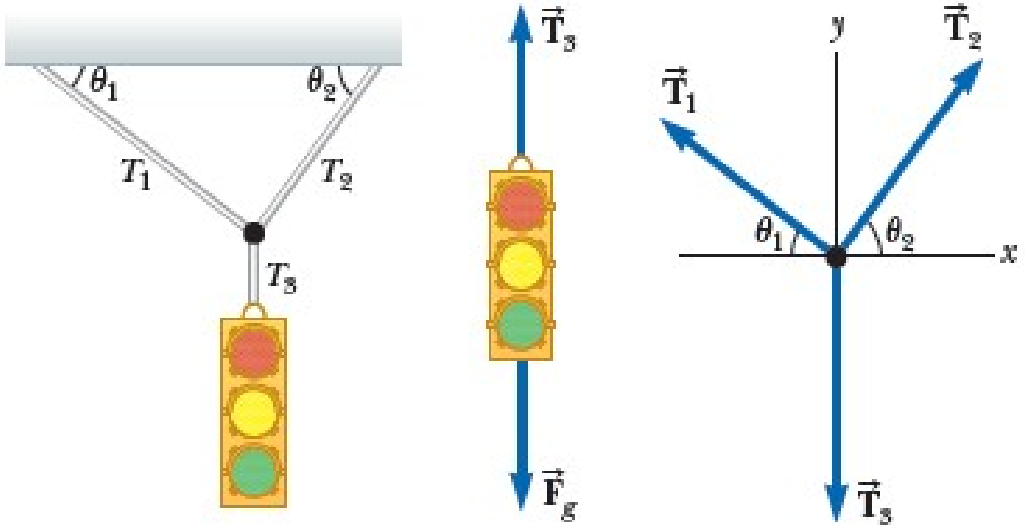
Procedure is recommended when dealing with problems involving Newton‘s laws:

1. Draw a sketch of the situation.
2. Consider only one object (at a time), and draw a free-body diagram for that body, showing all the forces acting on that body.
3. Newton's second law involves vectors, and it is usually important to resolve vectors into components. Choose an x and y axis in a way that simplifies the calculation.
4. For each body, Newton's second law can be applied to the x and y components separately. That is the x component of the net force on that body will be related to the x component of that body's acceleration: $F_x = ma_x$, and similarly for the y direction.
5. Solve the equation or equations for the unknown(s).

Example For System in Equilibrium

1. A bag of cement or a traffic light of weight 300 N hangs from three ropes as shown in the figure below. Two of the ropes make angles of 53.0° and 37.0° with the horizontal. If the system is in equilibrium, find the tensions , , and in the ropes.





Solution

Since the system is in equilibrium,

- From free body diagram (a)

- From free body diagram (b)
- Substituting For

But

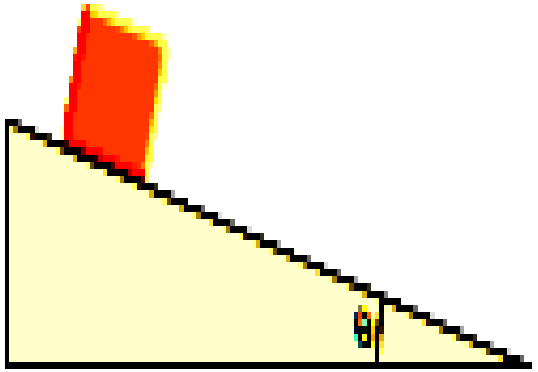
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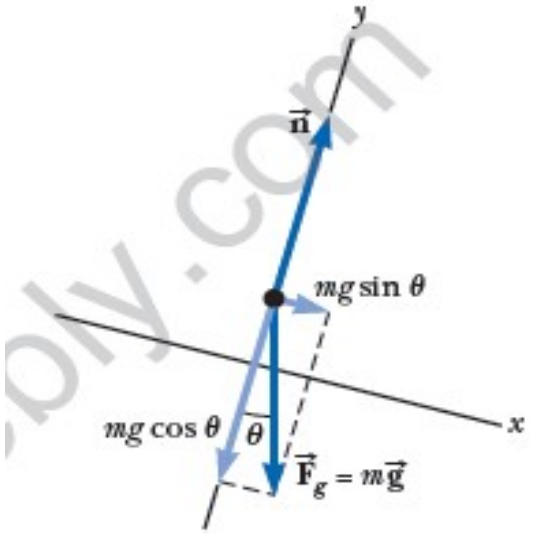
- For

So

Example 2 (For)

1. A block of mass m slides down an inclined plane as shown in the figure below. Find the expression for the acceleration of the block. (a)If the inclined plane is frictionless (b)If the inclined plane has coefficient of kinetic friction





Solution

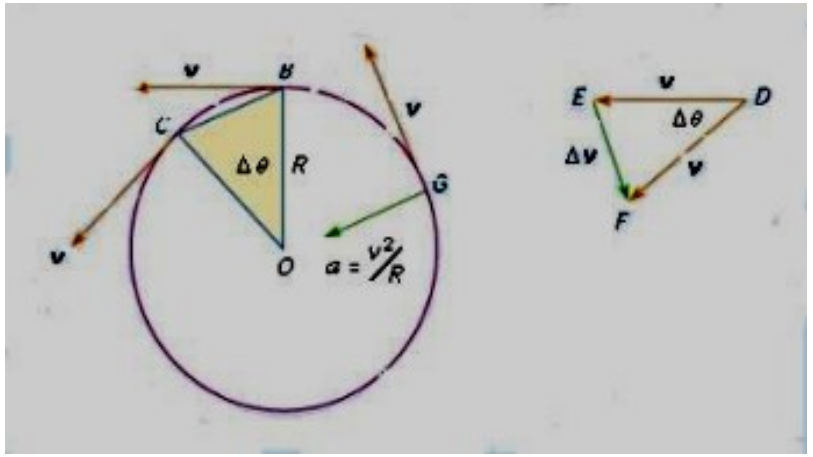
a) If the inclined plane is frictionless, the only forces acting on the block are the gravitational force (mg) downward and the normal force (n). *Resolving mg into parallel and perpendicular to the direction of motion of motion Considering the parallel component to be along x-axis*

b) If the inclined plane has coefficient of kinetic friction , the forces acting on the block are shown in free body diagram, from *the parallel component to the direction of motion*

from *the Perpendicular component to the direction of motion*

2.2.4. Uniform Circular Motion

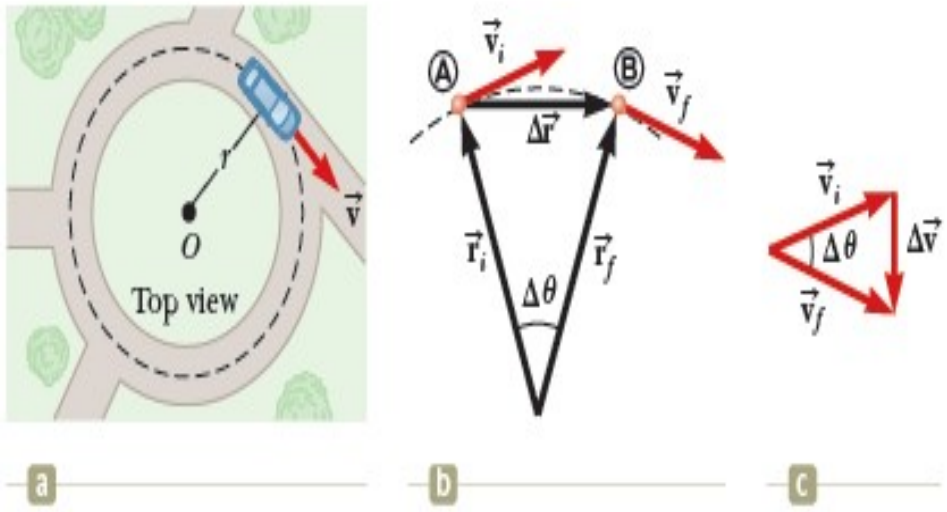
- Uniform Circular Motion is motion of objects in a circular path with a constant speed. Objects moving in a circular path with a constant speed can have acceleration.



- There are two ways in which the acceleration can occur due to:
  - change in magnitude of the velocity=constant
  - change in direction of the velocity

- For objects moving in a circular path with a constant speed, acceleration arises because of the ***change in direction of the velocity***.
- Hence, ***in case of uniform circular motion***:
  - ***Velocity is always tangent to the circular path*** and ***perpendicular to the radius of the circular path***.
  - Acceleration is always perpendicular to the circular path, and points towards the center of the circle. Such acceleration is called the ***centripetal acceleration***

Cont…



- ***Consider :***
- The particle follows a circular path of radius ***r***, part of which is shown by the dashed curve
- The particle is at **A** at time and its velocity at that time is it is at **B** at some later time , and its velocity at that time is
- Let us also assume and differ only in direction; their magnitudes are the same (that is, because it is *uniform* circular motion).
- From figure c **(**The vector connects the tips of the vectors**)**
- In both Figures b and c, we can identify triangles that help us analyze the motion.
- The angle in figure (b) and (c) are the same
- Because the velocity vector is always perpendicular to the position vector .

Therefore:

- ***The two triangles are similar.***

Cont….

- By SAS similarity (Two triangles ***are similar if the angle between any two sides is the same for both triangles*** and if the ratio of the ***lengths of these sides is the same***.)

- Now the r/ship b/n the lengths of the sides for the two triangle is

- The magnitude of the average acceleration by dividing both side by time interval(t)

(Centripetal acceleration)

- An acceleration of this nature is called a **centripetal acceleration** (*centripetal* means *center-seeking*).

Cont…

- **Period (T):-** Time required for one complete revolution For a particle moving in a circle of radius r with a constant speed

- **Where**
  - -is the Circumference of the particle's circular path.
- The ***period*** of a particle in uniform circular motion is a ***measure of the number of seconds for one revolution of the particle around the circle***.

- ***The inverse of the period is the rotation rate*** ***and is measured in revolutions per second.***

- Because ***one full revolution of the particle around the circle corresponds to an angle of radians***,
  - *The product* of and **the rotation rate** gives the **angular speed** of the particle, measured in radians/s or :

Cont…

Note :-

Where

,(and also 1,2,3 represents), ,or from the figure

,and ***there is a time(t)***

Now

If the radius(r) increases, the arc length also increases

if we divide both side by t

From this

show the radial acceleration

From , but we have

2.2.5. Newton’s Law of Universal Gravitation

- Gravity is the weakest of the four basic forces found in nature, and in some ways the least understood.

- Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions.

- But Newton was not the first person to suspect that the same force caused both our weight and the motion of planets.

- His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause.
- Some of Newton‘s contemporaries, such as
    - Robert Hooke,
    - Christopher Wren, and
    - Edmund Halley, had also made some progress toward understanding gravitation.
- But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections; circles, ellipses, parabolas, and hyperbolas.

- This theoretical prediction was a major triumph, it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

Cont..

- ***The gravitational force*** is always attractive, and **it depends only on the masses involved and the distance between them.**

- ***Newton's universal law of gravitation states that***

  "***every particle in the universe attracts every other particle with a force along a line joining them".***

- The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

- Gravitational attraction is along a line joining the centers of mass of these two bodies.
- The magnitude of the force is the same on each, consistent with Newton's third law.

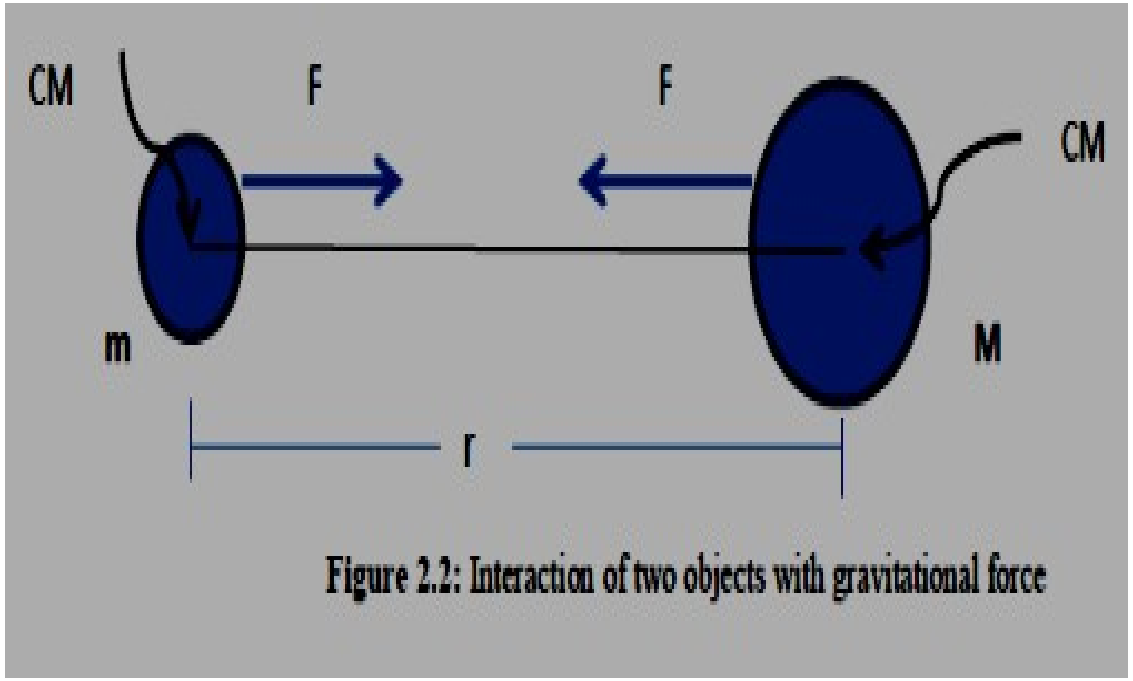


Figure 2.2: Interaction of two objects with gravitational force

- Here, r is the distance between the centers of mass of the bodies, ***G*** is the gravitational constant, whose value found by experiment is in SI units.
- The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us.
- In fact, our body weight is the force of attraction of the ***entire Earth*** on us with a mass of

- *As we now the acceleration due to gravity g is about on Earth . **why this is so***?

- The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton‘s universal law of gravitation gives:

- Where,
  - ***m*** is the mass of the object,
  - ***M*** is the mass of Earth, and
  - ***r*** is the distance to the center of Earth (the distance between the centers of mass of the object and Earth).
- The mass ***m*** of the object cancels, leaving an equation for ***g***:

- we obtain a value for the acceleration of a falling body:

Cont….

- This is the expected value ***and is independent of the body's mass***.

- ***Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further***, explaining the observation in terms of a force that causes objects to fall, in fact, in terms of a universally existing force of attraction between masses.

2.2.6. Kepler's Laws, Satellites Motion and Weightlessness

- The basic laws of planetary motion were established by Johannes Kepler (1571-1630) based on

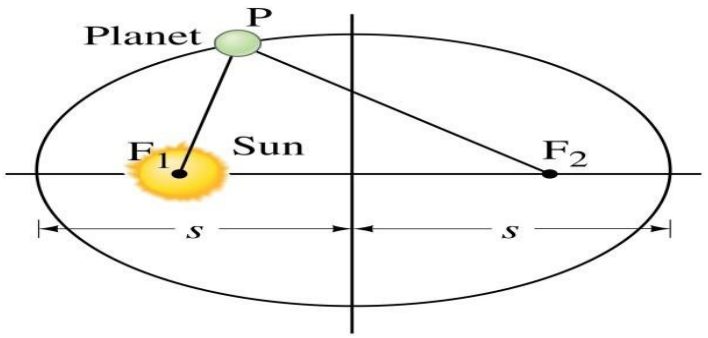
  the analysis of astronomical observations of Tycho Brahe (1546−1601).

- In 1609, Kepler formulated the first two laws.
- The third law was discovered in 1619.
- Later, in the late ***17th century, Isaac Newton proved mathematically that all three laws of Kepler are a consequence of the law of universal gravitation.***

- Kepler's complete analysis of planetary motion is summarized in three statements known as **Kepler's laws:**

1. ***All planets move in elliptical orbits with the Sun at one focus.***
2. ***The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.***
3. ***The square of the orbital period of any planet is proportional to the cube of the semi major axis of the elliptical orbit***.

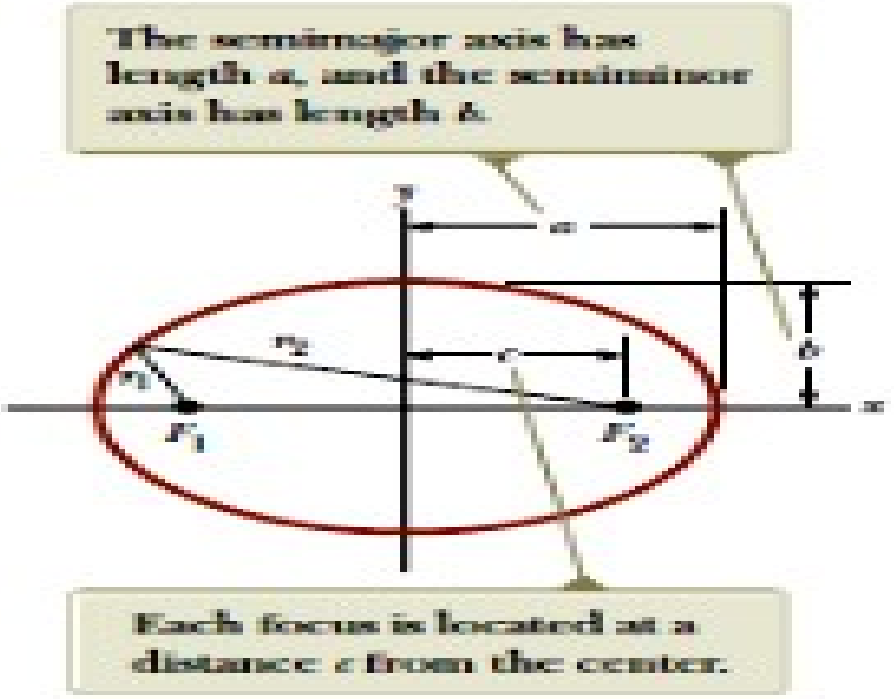
Kepler's First Law (Law of Orbits)



- ***States that***

  The orbit of each planet in the solar system is an ellipse, the Sun will be on one focus.

- The points F_1 and F_2 represented in figure 1 are known as the **foci** of the ellipse.
- The closer together that these points are, the more closely that the ellipse resembles the shape of a circle.
- In fact, a circle is the special case of an ellipse in which the two foci are at the same location.
- Kepler's first law is rather simple - all planets orbit the sun in a path that **resembles an ellipse**, with the sun being located at one of the foci of that ellipse.



Ellipse

- ***An ellipse*** is mathematically defined by choosing two points and each of which is a called a **focus,** and
- Then drawing a curve through points for which the sum of the distances and from and respectively, is a constant.
- **The longest** distance is called the **major axis,** and is drawn along the ***x*** **direction. This distance is** ***2a*** . the distance ***a*** is called the **semi major axis.**

- **Similarly**, **the shortest** distance through the center between points on the ellipse is called the **minor axis** of length ***2b,*** where the distance ***b*** is the **semi minor axis.**
- Either focus of the ellipse is located at a distance ***c*** from the center of the ellipse, where
- In **the elliptical orbit of a planet around the Sun**, the Sun is at one focus of the ellipse.

- ***Where Is the Sun?*** The ***Sun*** is located at ***one focus of the elliptical orbit of a planet***. It is *not* located at the center of the ellipse.

Eccentricity

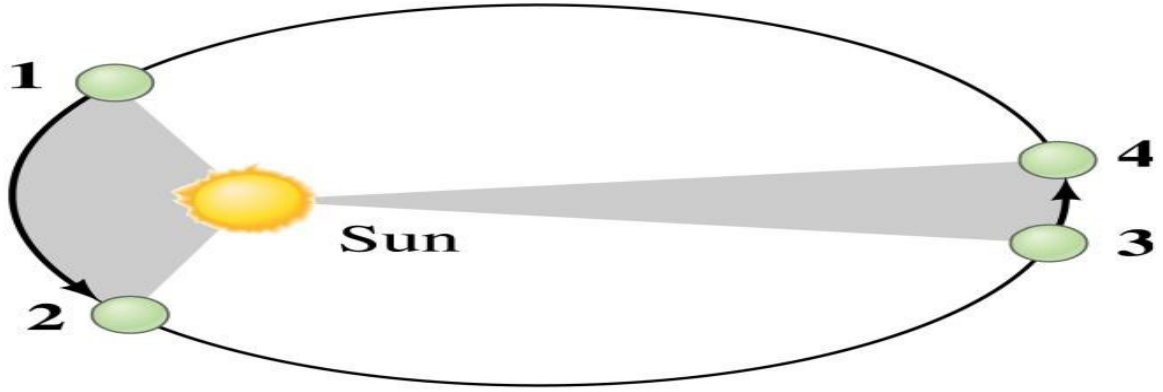
- The **eccentricity** of an ellipse is defined as
- and ***it describes the general shape of the ellipse***.
- Where
  - ***C*** —distance from the center of the ellipse,
  - a- is **semi major axis.**
- For a ***circle,*** and the eccentricity is therefore zero.
- The smaller ***b*** is compared with ***a***, the shorter the ellipse is along ***the y direction*** compared with its extent in the ***x* direction**.
- As ***b*** **decreases**, ***c*** **increases** and **the *eccentricity e* increases**.
- Therefore, higher values of eccentricity correspond to longer and thinner ellipses.
- The range of values of the eccentricity for an ellipse is .
- Eccentricities for planetary orbits vary widely in the solar system.
  - The eccentricity of the ***Earth's orbit is 0.017***, which makes it nearly circular.
  - On the other hand, the eccentricity of Mercury's orbit is 0.21, the highest of the eight planets

Kepler's Second Law (The Law of Areas)

- **States that —**

The radius vector connecting the centers of the Sun and the Planet sweeps out equal areas in equal intervals of time.

- The Figure below shows the two sectors of the ellipse having equal areas corresponding to the same time intervals.

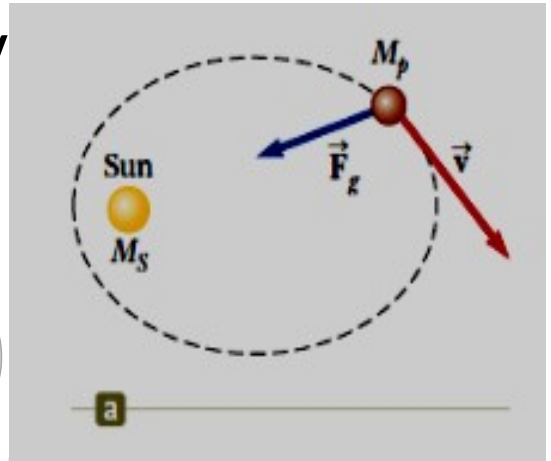


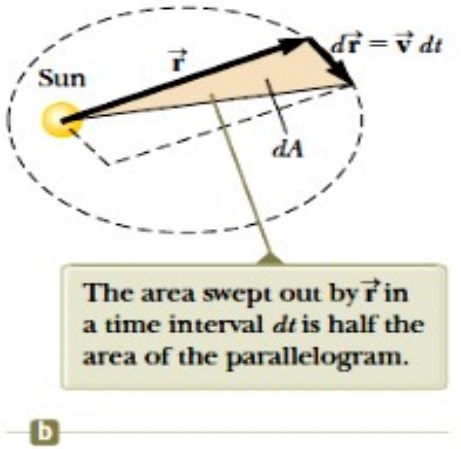
- ***Kepler's second law can be shown to be a consequence of angular momentum conservation***
- ***To proof***
- Consider a planet of mass moving about the Sun in an elliptical orbit (Fig.). Let's consider the planet as a system. **We model the Sun to be so much more massive** than the planet that the Sun does not move.
- ***The gravitational force*** exerted by the Sun on the planet is a central force, always along the **radius vector**, directed ***toward the Sun*** (Fig.a).
- The **torque on the planet due to this central force about an axis through the Sun is zero because is**

Torque :-is a quantity that measure the tendency of a force to rotate an object about some axis. Torque is a vector,

Recall that

- The ***external net torque*** on a system equals the time rate of change of angular momentum of the system; that is, ,
- Therefore, because the of the planet is a constant of the motion:
- Ev





Cont…

- In a time interval *dt*, the radius vector in Figure b sweeps out the area ***dA***, which equals half the area **of the parallelogram** formed by the vectors and .
- Because the displacement of the planet in the time interval *dt* is given by ***dt,***

By substituting for the absolute value

Divide both sides by *dt* to obtain

- where ***L*** and M_p are both constants.
- This result shows that that the derivative *dA/dt* is constant—

***the radius vector from the Sun to any planet sweeps out equal areas in equal time intervals as stated in Kepler's second law*.**

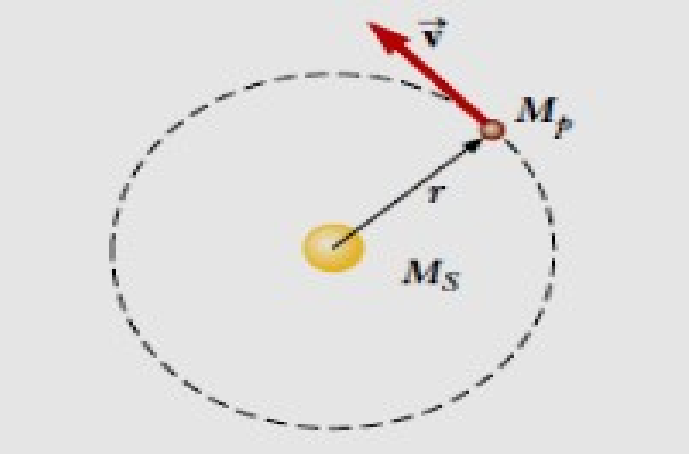
- A planet moves fastest when it is closest to the sun and slowest when it is furthest from the sun.

Kepler's Third Law (The Law of Harmony)

- **States that**

"the square of the orbital period of a planet is proportional to the cube of the average distance between the centers of the planet and the sun."

- Consider a ***planet of mass*** that is assumed to be moving about the ***Sun (mass)*** in a circular orbit as in Figure below,



- **-provides Centripetal acceleration of planet ,**
- We model the **planet** as **a particle under a net force** and as a particle in uniform circular motion and incorporate Newton's law of universal gravitation,

- The ***orbital speed*** of the planet is , where ***T*** is the period; therefore, the preceding expression becomes

- **Where -is constant**

- This equation is also valid for elliptical orbits if we replace ***r*** with the length ***a*** of the semimajor axis

- is Kepler's third law: the square of the period is proportional to the cube of the semimajor axis. Because the semimajor axis of a circular orbit is its radius, this equation is valid for both circular and elliptical orbits.

Generally Kepler's 3rd Law

- The third law makes a comparison between the motion characteristics of different planets.
- The comparison being made is that the ***ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets.***
- consider the orbital period and average distance from sun (orbital radius) for Earth and mars

Planet	Period(s)	Average Distance (m)	
Earth	3.156 x s	1.4957 x	2.977 x
Mars	5.93 x s	2.278 x	2.975x

- (**NOTE**: **The average distance value** is given in astronomical units where 1a.u. is equal to the distance from ***the earth to the sun (1AU-*** **1.4957 x** m).
- The ***orbital period*** is given in units of earth-years where 1 earth year is the time required for the earth to orbit the sun ***–(1Lyr= 3.156 x s .)***

- Observe that the ratio is the same for Earth as it is for mars.
- In fact, if the same ratio is computed for the other planets, it can be found that this ratio is nearly the same value for all the planets .
- ***Amazingly, every planet has the same ratio***.

Planets	Period(yr)	Average distance(au)	(yr2/au3)
Mercury	0.241	0.39	0.98
Venus	0.615	0.72	1.01
Earth	1.00	1.00	1.00
Mars	1.88	1.52	1.01
Jupiter	11.8	5.20	0.99
Saturn	29.5	9.54	1.00
Uranus	84.0	19.18	1.00
Neptune	165	30.06	1.00
Pluto	248	39.44	1.00

Satellite motion and Weightlessness

- Astronauts on the orbiting space station are *weightless* because:
  - There is no gravity in space and they do not weigh anything.
  - Space is a vacuum and there is no gravity in a vacuum.
  - Space is a vacuum and there is no air resistance in a vacuum.
  - The astronauts are far from Earth's surface at a location where gravitation has a minimal effect.
- Astronauts who are orbiting the Earth often experience sensations of weightlessness. These sensations experienced by orbiting astronauts are the same sensations experienced by anyone who has been temporarily suspended above the seat on an amusement park ride. Not only are the

- The cause of weightlessness is quite simple to understand.
  - ***Weightlessness*** is simply a sensation experienced by an individual when there are no external objects touching one's body and exerting a push or pull upon it.
- Weightless sensations exist when all contact forces are removed.
- These sensations are common to any situation in which you are momentarily (or perpetually) in a state of free fall.

- *When in free fall, the only force acting upon your body is the force of gravity, a non-contact force.*

- Since the force of gravity cannot be felt without any other opposing forces, you would have no sensation of it.
- You would feel weightless when in a state of free fall.

2.3. Work, Energy and Linear Momentum

Concept of Work…

- ***On a typical day,***
    - you probably wake up,
    - get dressed,
    - eat breakfast, and
    - head off to class or work.
  - After you spend all day at your job, you go home,
    - Eat dinner,
    - May be reading or studying, and
    - Then go to bed. In this sense, work can be just about anything
      - Construction ,
      - Typing on a keyboard,
      - Driving a bus,
      - Teaching a class,
      - Cooking food,
      - Treating patients, and so much more.

But in physics, work is more specific

- This is ***the displacement of an object due to force***.
  - **How much work is done** depends on the **distance the object is moved**.
  - **Work** can be defined as ***transfer of energy due to an applied force***.
  - **In physics** we say that ***work is done on an object when energy is transferred to that object***.
- ***If one object transfers (gives) energy to a second object, then the first object does work on the second object***.
- The energy of a moving object is called ***kinetic energy***.

Work and Energy

- In physics, ***work is done when a force acts on an object that undergoes a displacement from one position to*** another.
- Forces can vary as a function of ***position,*** and ***displacements*** can be along various paths between ***two points***.
- If ***no displacement takes place, no work is said to be done.*** Therefore for work to be done on an object, three essential conditions should be satisfied:
  - ***Force must be exerted on the object***
  - ***The force must cause a motion or displacement***
  - ***The force should have a component along the line of displacement***

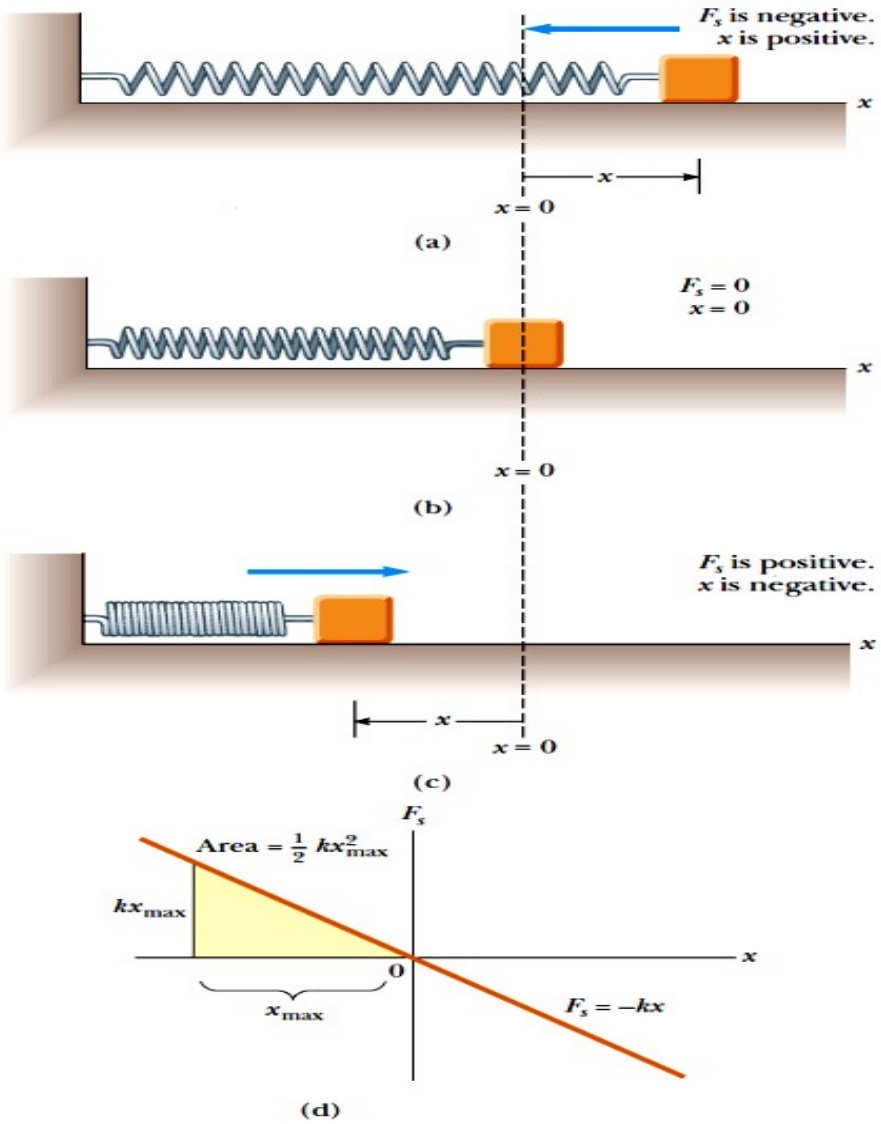
- If a particle subjected to a constant force undergoes a certain displacement, , the work done *W* by the force is given by:

- Where θ is the angle between .
- Work is a scalar quantity and its SI unit is Joule (J). Where,

- The sign of work depends on the direction of . Hence, the work done by the applied force is positive when the projection of onto is in the same direction as .

Work done by a Spring

- ***Work done by spring is a type of work done by varying Force***
- ***Consider***
  - ***the Mass-spring system*** where the force applied varies with ***position constantly***.
  - A block on a horizontal, frictionless surface is connected to a spring as shown in the figure below.
  - If the spring *is either stretched or compressed* a small distance from its ***un-stretched (equilibrium) configuration***, it exerts on the block a force that can be expressed as:



- From the figure on
  - fig. **a** (the Spring **is stretched spring**, is Negative, x-is positive)
  - fig. **b** (the Spring **is natural length of spring**, =0 , x=0)
  - fig. **c** (the Spring **is compresed spring**, is Positive, x-is negative)
- The negative sign in the above eqn. signifies that the force exerted by the spring is always directed ***opposite*** to the displacement from equilibrium.
- Because the ***spring force always acts toward the equilibrium*** position (x =0), it is sometimes called a ***restoring force.***

Work Done by

- The work done by the spring force as the block moves from to **0** is the area of the shaded triangle,
- The work done by the spring force on the block as the block moves from.

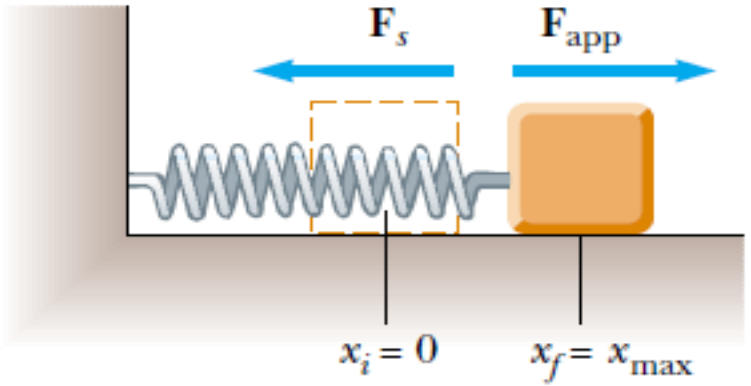
- The ***work done*** by the ***spring force is positive because the force is in the same direction as the displacement of the block (both are to the right).***
    - Because the block arrives at x=0 with some speed, it will continue moving, until it reaches a position)
- The work done by the spring force as the block moves from we find that

- Because for this part of the motion the displacement is to the right and the spring force is to the left.

- Therefore, the ***net*** **work done by the spring force as the block moves from** is *zero.*

- If the block undergoes an ***arbitrary displacement*** from **to ,** the work done by the spring force **on the block** is

- For **the work done on the spring** by an ***external agent*** that stretches the spring very slowly from as



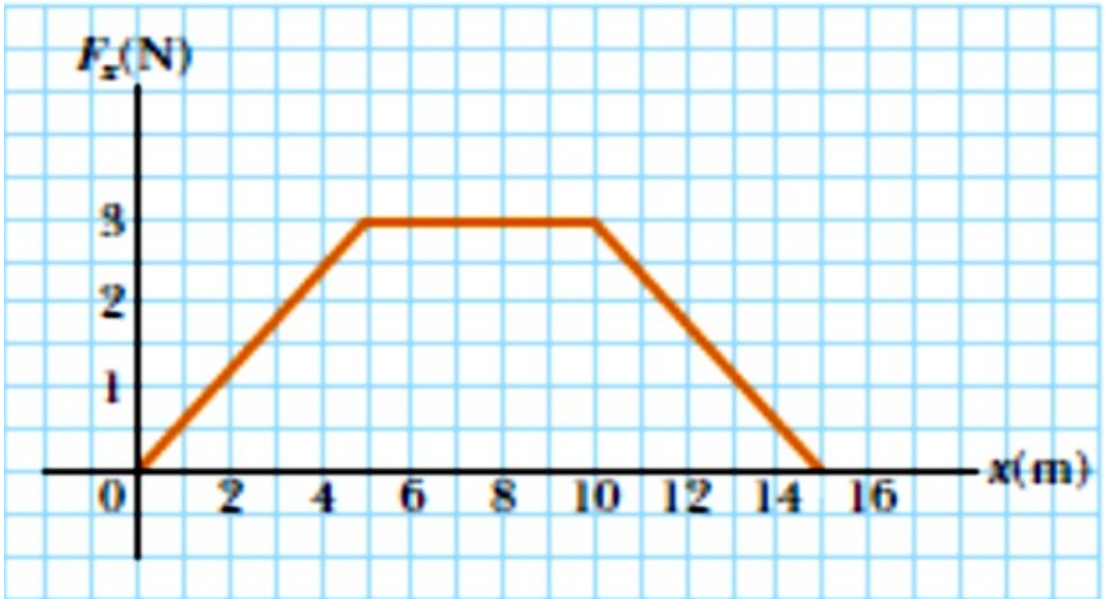
- At any value of the position, the *applied force* is **equal in magnitude** and **opposite in direction** to the spring force so that .
- Therefore, the work done by this applied force ***(the external agent)*** on the block–spring system is

(work by External force on block-spring system)

- The work done by an ***applied force on a block–spring*** system between **arbitrary positions of the block** is

Example

1. A particle is subject to a force that varies with position as in figure below. What is the total work done by the force over the distance, x= 0 to x = 15.0 m?



Solution

- *The total work done is equal to the area under the versus x curve*

1st For Triangle

2nd For Rectangle

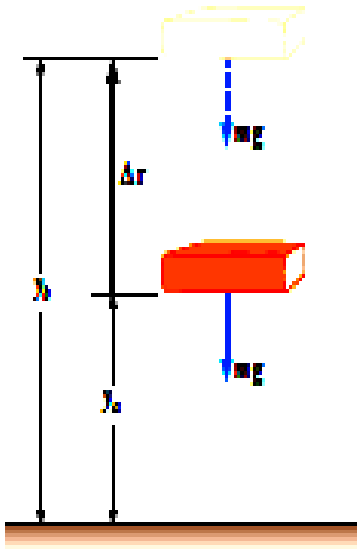
3rd For Triangle

Then the Total work done is the sum of the above

Energy

- Energy is the crown for physics. It is found in every branch of physics.
- It is defined as the capacity of a physical system to perform work.
- And it exists in several forms such
  - kinetic,
  - potential,
  - thermal,
  - chemical and other forms.
- And its SI unit is joule (J).
- ***Kinetic energy (KE)*** is the capacity of an object to do work by virtue of its motion. For an object of mass *m* and moving with speed *v*, the kinetic energy is calculated as:

- ***Potential Energy (PE)*** is the energy that is stored in an object due to its position relative to some zero position.
  - An object possesses gravitational potential energy if it is positioned at a height above (or below) the zero height.
  - The potential energy concept can be used only when dealing with a sp... forces called ***Conservative F***
- Mathematically the potenti... given by
- *This equation is valid only for objects near the surface of the Earth, where g is approximately constant*



Work- Energy theorem

- One of the possible outcomes of doing work on a system is that the system changes its speed.
- We have discussed how to find the work done on a particle by the forces that act on it, but how is that work manifested in the motion of the particle?
- According to Newton‘s second law of motion, the sum of all the forces acting on a particle, or the net force, determines the product of the mass and the acceleration of the particle, or its motion.
- Therefore, we should consider the work done by all the forces acting on a particle, or the network, to see what effect it has on the particle‘s motion.
- Let a force ‘*F*’ is applied on an object initially moving with velocity “*u*”. If it is displaced to a **displacement “”** and changes its *velocity into “v”*, then its motion will be expressed by:

- Multiplying this equation by “m” and dividing throughout by 2, we get:

- Hence, ; where F is the force that caused the havo!
- Therefore, we can write, ; where ***W = Fx*** is the work done by this force.
- So what just happened? We just proved that, is the work done by the force!
- In other words, ***the work done is equal to the change in K.E. of the object! This is the Work-Energy theorem or the relation between Kinetic energy and Work done***.
- In other words, the work done on an object is the change in its kinetic energy.

- ***The work-kinetic energy theorem states that:***

In the case in which work is done on a system and the only change in the system is in its speed.

Example

1. A 0.6kg particle has a speed of 2m/s at point *A* and kinetic energy of 7.5J at point *B*. What is
   a) Its kinetic energy at *A*?
   b) Its speed at *B*?
   c) The total work done on the particle as it moves from *A* to *B*?

Solution

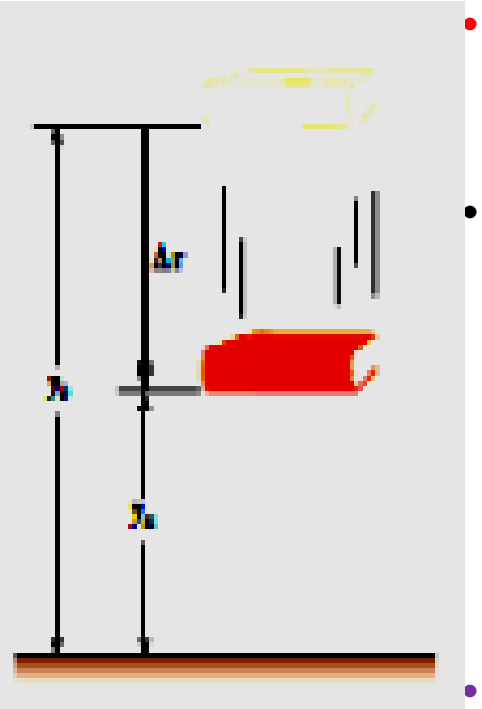
Given

Mass=0.6Kg, Speed=2m/s at point A, K.E =7.5J at Point B

(a) From work energy theorem we have

Mechanical Energy (ME):

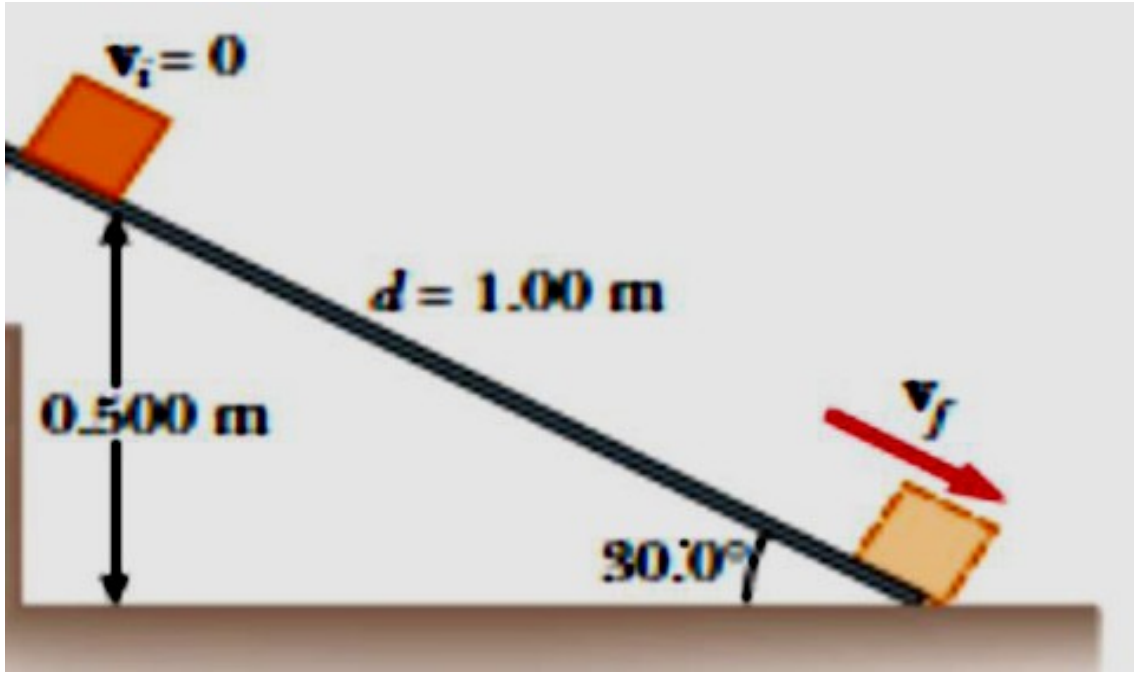
- is defined as the sum of kinetic energy and potential energy.
- Consider the ***book-earth system*** shown in the figure below. As the book falls from to , the ***work done by the gravitational force*** on the book is



- ***From the work–kinetic energy theorem,*** the work done on the book is equal to the change in the kinetic energy of the book:
- As the book falls from initial height to a final height ,
- ***The mechanical energy of an isolated, friction-free system is conserved*****.**
- An ***isolated system*** is one for which there are ***no energy transfers across the boundary.*** For the object of mass m falling in a gravitational field

Example

1. A 3.00-kg crate slides down a ramp. The ramp is 1.00m in length and inclined at an angle of 30.0°, as shown in the figure below. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.



Solution

- *Since the non-conservative force (friction force) exists in the system,*

- But

2.3.2. Power

- *Power is defined as the time rate of energy transfer.*
- If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval is *W*,
- Then the average power during this interval is defined as

- The ***instantaneous power*** P is defined as the limiting value of the average power as approaches zero:

- But

Cont.…

- In general, *power is defined for any type of energy transfer*. Therefore, the most general expression for power is

- The *SI unit of power is joules per second (J/s), also called the watt (W) (after James Watt*):

- A **unit of power in the U.S. customary system is the horsepower** (hp):

- A unit of energy (or work) can now be defined in terms of the unit of power. One kilowatt-hour (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s. The amount of energy represented by 1 kWh is:

Example

1. An older model car accelerates from rest to speed v in 10 seconds. A newer, more powerful car accelerates from rest to $2v$ in the same time period. What is the ratio of the power of the newer car to that of the older car?

Solution

$$P_{av} = \frac{W}{\Delta t}$$

$$W_{old} = \Delta K = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2 = \frac{1}{2}mv^2$$

$$W_{new} = \Delta K = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2 = \frac{1}{2}m(2v)^2 = 2mv^2$$

$$P_{old} = \frac{W_{old}}{\Delta t} = \frac{mv^2}{2(\Delta t)} \quad and\ P_{new} = \frac{W_{new}}{\Delta t} = \frac{2mv^2}{\Delta t}$$

$$\frac{P_{new}}{P_{old}} = \frac{(\frac{2mv^2}{\Delta t})}{(\frac{mv^2}{2(\Delta t)})} = 4$$

Exercise

1. The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0s. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration ?

Solution

Linear Momentum

- Momentum is defined as ***the quality of a moving object to exert a force on anything that tries to stop it***.
- The linear momentum of a particle or an object that can be modeled as a particle of ***mass m*** moving with a ***velocity*** is defined to be the product of its mass and velocity:

- Momentum () is a ***vector quantity in the direction of the velocity*** with SI unit *kgm/s*.
- Using Newton‘s second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle.

- ***Therefore, the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.***

- The impulse of the net force acting on the particle is equal to the change in momentum of the particle.

Conservation of Linear Momentum

Recall Newton's third law: When two objects of masses and interact (meaning that they apply forces on each other), the force that object 2 applies to object 1 is equal in magnitude and opposite in direction to the force that object 1 applies on object 2

- Let:
  - = the force on from
  - = the force on from

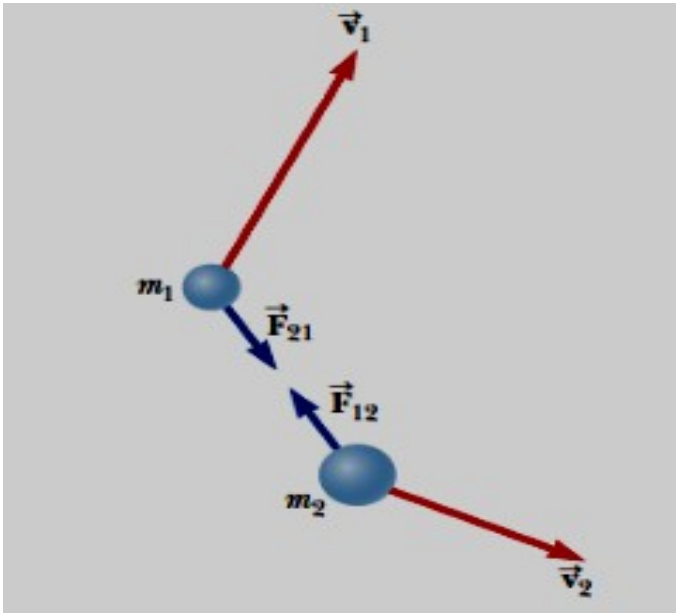


Fig:- Two particles interact with each other. According to Newton's third law, we must have

- Then,

Then from the definition of , we have

- Notice that the derivative of the sum with respect to time is zero. Consequently, this sum must be constant.
- Then

Cont..

- Where are the initial values and the final values of the momenta for the two particles for the time interval during which the particles interact.
- This result, known as the law of conservation of linear momentum, can be extended to any number of particles in an ***isolated system***.

- We can state it as follows:

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant".

Example

1. A 60-kg archer stands at rest on frictionless ice and fires a 0.030-kg arrow horizontally at 85 m/s (Fig.below). With what velocity does the archer move across the ice after firing the arrow?



Solution

- We choose the direction of firing of

the arrow as the positive *x* direction.

Identifying the archer as particle 1 and

the arrow as particle 2, we have

- Using the isolated system (momentum) model,

- Solve this equation for and substitute numerical values:

2.3.4. Collisions

- **Consider**
  - *The* ***collision between two particles of masses*** *and shown in the figure below.*
- If ***the two particles form an isolated system***, the momentum of the system must be conserved.

Therefore, the total momentum of an isolated system before a collision equals the total momentum of the system after the collision.

- But, the total ***K.E*** of the system of particles may or may not be conserved, depending on the type of collision.
- Based on Conservation ***kinetic energy*** collisions classify as
  - ***elastic* and**
  - ***inelastic*.**

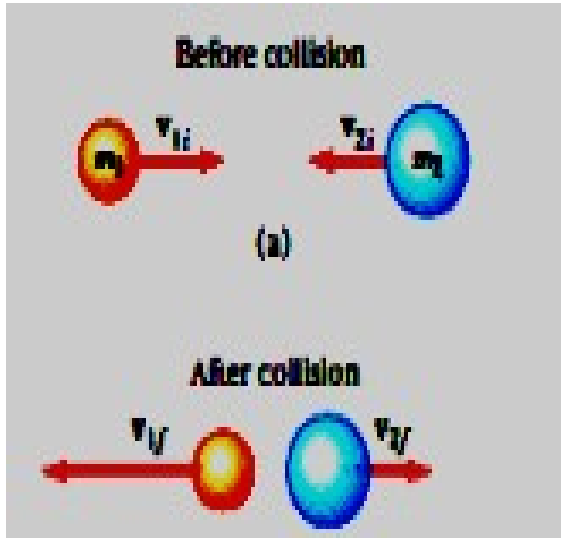
***A. Elastic collision*: -** An elastic collision between two objects is one in which the total kinetic energy as well as total momentum of the system is conserved.

Perfectly* elastic** collisions ***occur between atomic and subatomic particles.

- **Consider**
  - two particles of masses *and* moving with initial velocities *and* along the same straight line, as shown in the figure below.
  - The two particles collide head-on and then leave the collision site with different velocities, *and* .
  - If the collision is elastic, both the momentum and kinetic energy of the system are conserved. Therefore, considering velocities along the horizontal direction we have:

……1

……2



Because all velocities in the figure are either to the left or the right, they can be represented by the corresponding speeds along with algebraic signs indicating direction. We shall indicate *v* as positive if a particle moves to the right and negative if it moves to the left.

From conservation of kinetic energy,

- We can cancel the factor in the equation, and rewrite as:
- Then, factor both sides
- Separate the terms containing and
- Dividing equation (a) by equation (b)

.....3

- Equation (3) along with equation (1) can be used to solve problems dealing with elastic collisions.

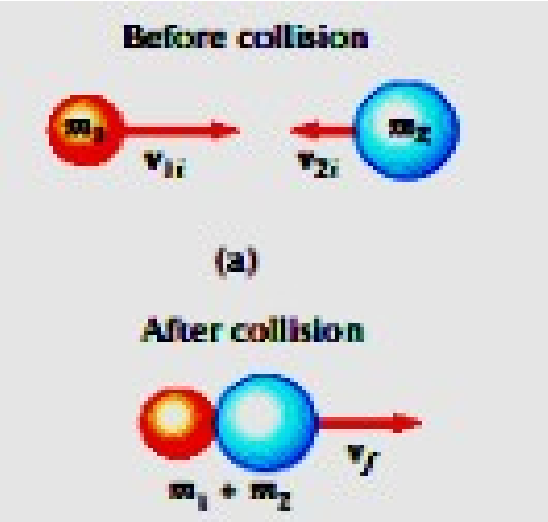
B. Inelastic Collision: -

- An ***inelastic collision is one in which the total kinetic energy*** of the system is ***not conserved***. But the ***momentum of the system is conserved***. Therefore, for inelastic collision of two particles:

$$\frac{1}{2}m_1 v_{1i}^2+\frac{1}{2}m_2 v_{2i}^2 \neq \frac{1}{2}m_1 v_{1f}^2+\frac{1}{2}m_2 v_{2f}^2$$

C. Perfectly Inelastic Collision: -

- When the colliding objects ***stick together after the collision***, the collision is called ***perfectly inelastic***. Consider two particles of masses *and* moving with initial velocities *and* along the same straight line, as shown in the figure below.
- The two particles ***collide head-on, stick together***, and then move with ***some common velocity*** after the collision.

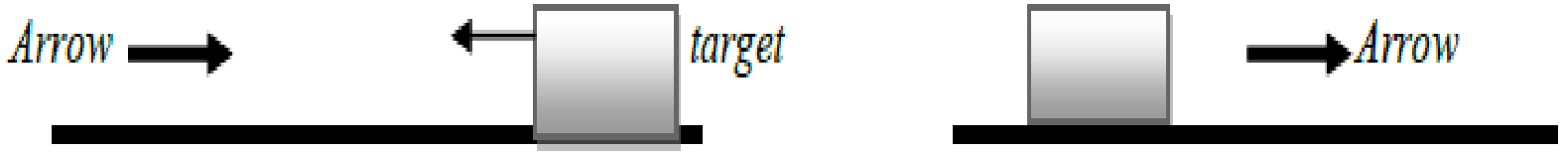


The total momentum **before the collision equals** the total momentum of the composite system ***after the collision***

This gives

Example 1

1. An archer shoots an arrow toward a target that is sliding toward her with a speed of 2.50 m/s on a smooth surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the 300-g target, which is stopped by the [illegible] ugh the target?



Solution

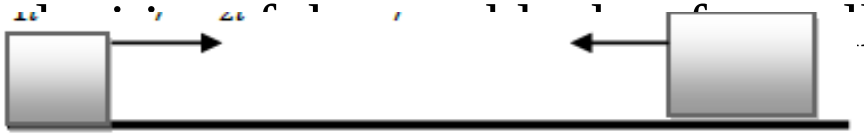
$= 22.5\text{-g}$ and $= 300\text{-g}$

=?

- *From conservation of linear momentum*

Example 2

1. A block of mass initially moving to the right with a speed of 4m/s on a horizontal frictionless track collides with a second block of mass initially moving to the left with a speed of 2.5m/s. If the collision is elastic, find the v[illegible] lision. (?,)



Before Collision

Solution

Since the collision is elastic both momentum and kinetic energy are conserved

a) From conservation of momentum

$$(1.6Kg)(4m/s)+(2.1Kg)(-2.5m/s)=(1.6Kg)+(2.1Kg)$$

$$1.6+2.1Kg=1.15 \qquad (a)$$

b) ***From conservation of kinetic energy***

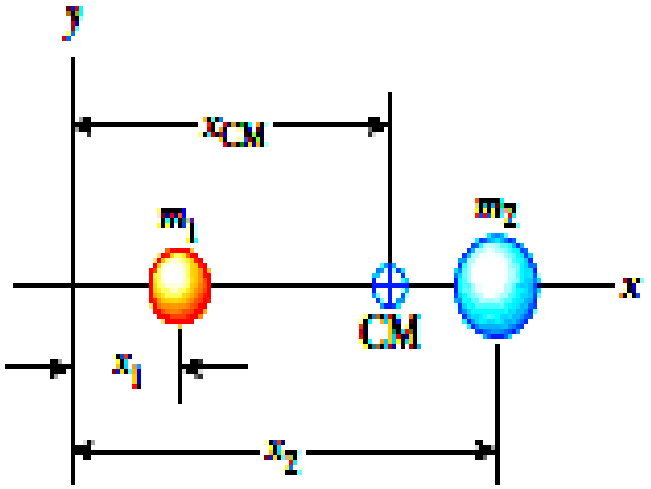
We have

(b)

• *Solving equation (a) and (b) simultaneously*

2.3.5. Center of Mass

- The ***center of mass is*** the point at which *all the mass can be considered to be "concentrated".*
- The center of mass of the system is located somewhere on the line joining the particles and is ***closer to the particle having the larger mass***.
- The ***center of mass of the pair of particles located on the x-axis*** as shown in the figure below lies somewhere between the particles, it is given by:



- We can extend this concept to a system of many particles with masses ***in three dimensions***.
- ***The x coordinate*** of the center of mass of *n* particles is defined to be

- ***Similarly, the y and z coordinates***

Therefore,

$$\vec{r}_{CM} = \frac{\sum_i m_i \vec{r}_i}{M}$$

Example

Chapter Summery

- **Kinematics** is the study of motion of objects without considering the agents that cause the motion, and **Dynamics** is the study of motion of objects with the cause of the motion. For motion with constant acceleration,

- **Projectile motion** is a motion in a curved path in a gravitational field. The maximum height of the projectile is given by

- The Range(R) is the horizontal displacement of the projectile covered in a total time of flight (,)
- **Force**: any interaction that changes the motion an object.
- **Friction force** is always proportional to the normal force between the two interacting surfaces.

- There are two types of frictional force: *Static friction*, *Kinetic friction*:
- **Uniform Circular Motion** is motion of objects in a circular path with a constant speed. For objects moving in a circular path with a constant speed, acceleration arises because of the change in direction of the velocity, and such acceleration is called the **centripetal acceleration**

Summary……

Newton’s universal law of gravitation

states that the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

- **Weightlessness** is simply a sensation experienced by an individual when there are no external objects touching one's body and exerting a push or pull upon it.
- **The work (W) done** on a system by an agent exerting a constant force ()and moving the system through the displacement () is
- *Energy*
- **Kinetic energy** is represents the energy associated with the motion of the particle:
- **Potential energy** is the energy associated with the configuration of a system of objects that exert forces on each other. The potential energy concept can be used only when dealing with a special class of forces called ***conservative forces.***
- **Power** is defined as the time rate of energy transfer:
- **Momentum** is defined as the quality of a moving object to exert a force on anything that tries to stop it.
- The **center of mass** is the point at which all the mass can be considered to be "concentrated".

Chapter Three

Fluid Mechanics

Fluid mechanics

- **Fluid mechanics** is the branch of **physics** concerned with the **mechanics** of **fluids** in motion (fluid *dynamics*) or at rest (fluid *statics*) and the forces on them.

- This study area deals with many and diversified problems such as surface tension, fluid statics, **flow in enclose bodies,** or flow round bodies (solid or otherwise), flow stability, etc.

- The ***applications of fluid mechanics*** are enormous:
  - breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few.

- In **physics**, a **fluid** is a substance that continually ***deforms (flows)*** under an applied shear stress, or external force.

- **Fluids** are a phase of matter and include **liquids** and gases.

Properties of Bulk Matter

States of Matter

Solid

- Has a definite volume and shape

Liquid

- Has a definite volume but not a definite shape

Gas – unconfined

- Has neither a definite volume nor shape

Fluids

- A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container.
- Both liquids and gases are fluids.
- Fluids do not sustain shearing stresses or tensile stresses.

The only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides.

The force exerted by a static fluid on an object is always perpendicular to the surfaces of the object.

Bulk matter…..

- A sufficiently large force will permanently deform or break an object, but otherwise, when the external forces are removed, the object tends to return to its original shape and size. This is called **elastic behavior**.
- **Definitions:**
- **Elastic materials** are materials that regain their original shape and size when the deforming force is removed.
- **Elastic deformation** is a reversible deformation by a force applied within the elastic limit. Beyond elastic limit, a force applied on an object causes permanent and irreversible deformation called **plastic deformation**.

- ***Plastics materials:*** do not regain their original shape and size when the deforming force is removed.
- The *elastic properties of solid materials are described in terms* of **stress** and **strain**.
- ***Stress is*** **the force per unit area that is causing some deformation** on an object.
    - *It has SI unit N/m_2 called the Pascal (Pa), the same as the unit of pressure*

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

- **Strain:-** *measures the amount of deformation by the applied stress and defined as the change in configuration of a body divided by its initial configuration. Strain is unit less quantity.*

$$\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

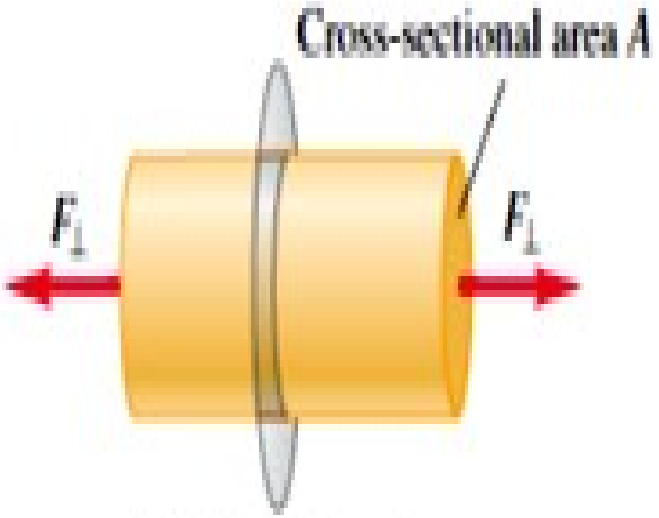
There are three kinds of strains:

1. ***Tensile Strain:*** When the ends of a bar(rod or wire) of uniform cross-sectional area A are pulled with equal and opposite forces of magnitude $F_{\perp}$ (Figure 1(a)), the bar will undergo a stretch by the tensile stress defined as the ratio of the force magnitude $F_{\perp}$ to the cross-sectional area A:

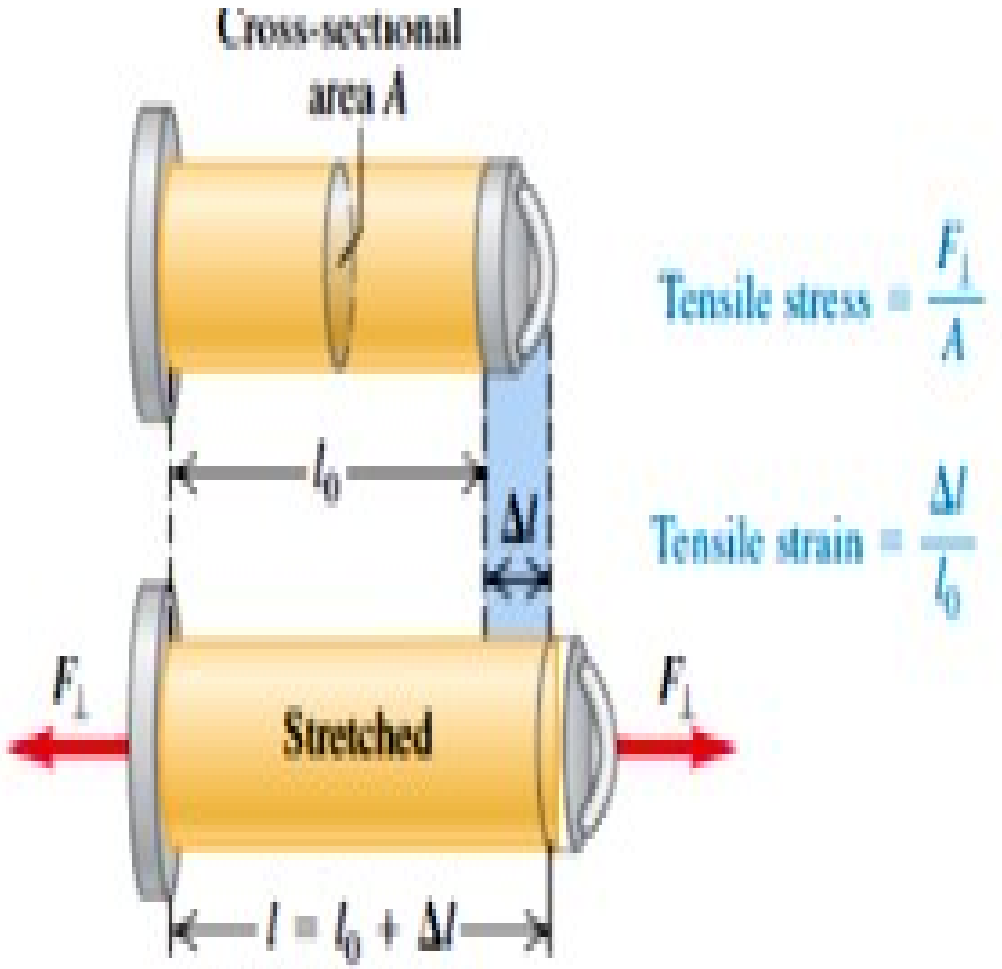
$$\text{……..3.3}$$

Tensile stress :-The fractional change in length of an object under a tensile stress is called the tensile strain (Figure 1b)

$$\text{……..3.4}$$



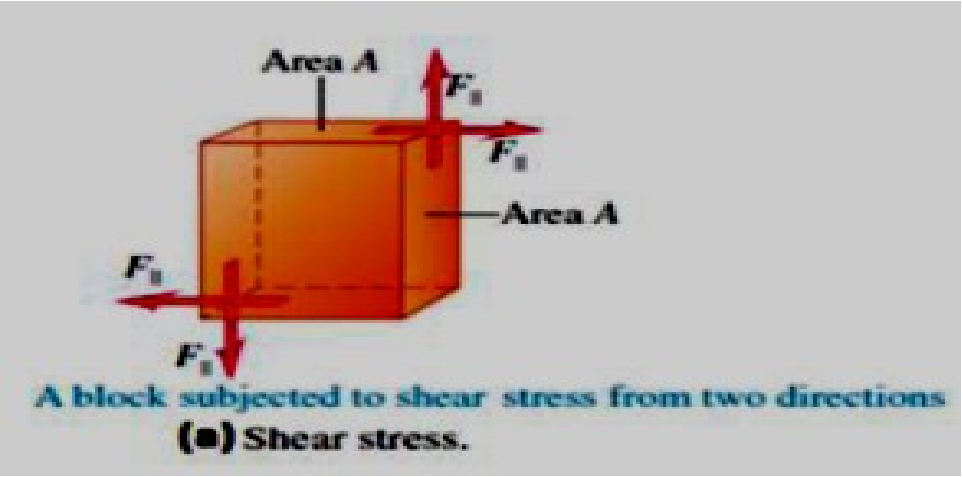
(a) A bar in tension



(b) Tensile strain

2. Shear Stress and Strain

- Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force (Figure (a) below).

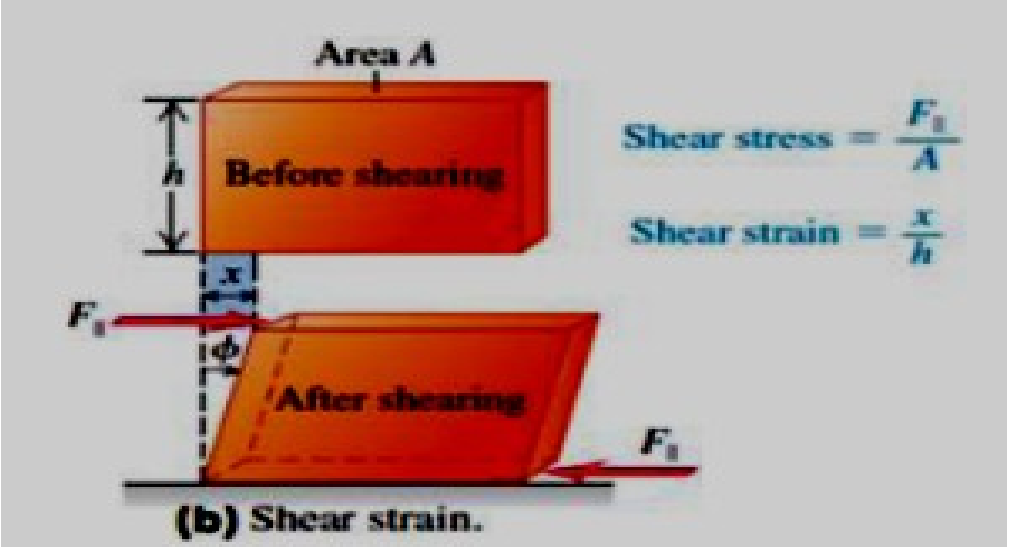


A block subjected to shear stress from two directions
(a) Shear stress.

- *The stress in this case is called a shear stress*. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram.

.....3.5

- A book pushed sideways, as shown in Figure (b) below, is an example of an object subjected to ***a shear stress***.
- To a first approximation (for small distortions), no change in volume occurs with this deformation.
- We define the shear stress as F/A, the ratio of the tangential force to the area A of the face being sheared.



- ***The shear strain*** *is defined as the ratio x/h, where x is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the shear modulus is*

.....3.6

3. Volume Stress and Strain

- **Volume Stress** is a stress which *causes volume deformation on an object* and defined as the ratio of the change in the magnitude of the applied force ΔF to the surface area A.

- ***Volume strain*** is the fractional change in volume (Figure 3) that is - the change in volume, ΔV , divided by the original volume V_0:

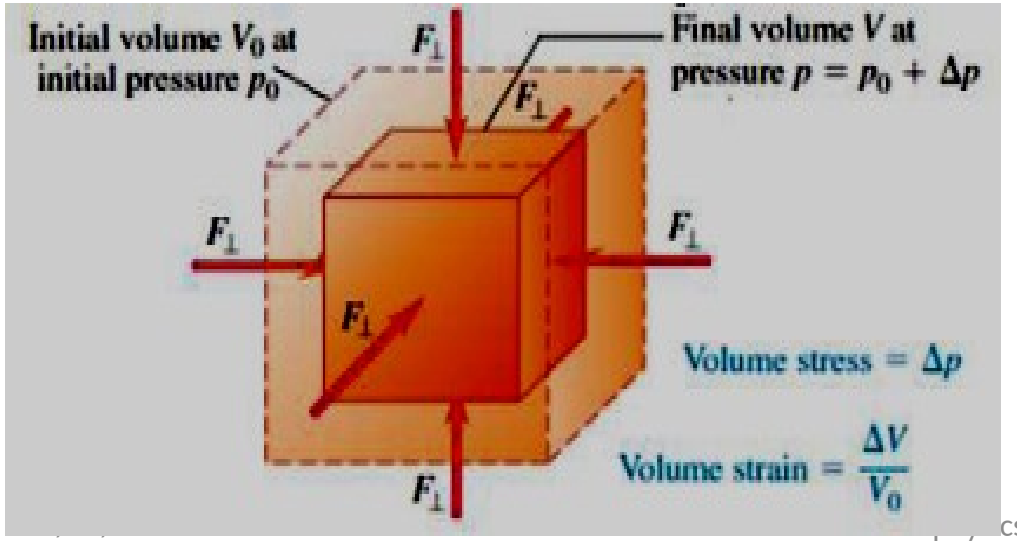


Fig. A block undergoing volume strain by the applied volume stress

Elasticity Moduli

- ***The stress will be proportional*** to ***the strain*** if the stress is sufficiently small.
- In this regard, the proportionality constant known as **elastic modulus** depends on the ***material being deformed*** and ***on the nature of the deformation.***

- This relationship between stress and strain is similar to ***Hooke‘s law ( )***, relationship between force and extension of a spring.
- The elastic modulus is similar to a spring constant.
- **Generally as stress increase, strain increase, as spring force increase change in displacement increase.**

Corresponding to the three types of strains, there are three types of elastic module.

1. ***Young's Modulus(Y):*** is the *ratio of the tensile stress to the tensile strain*.
   - It *measures the resistance of a solid to a change in its length and typically used to characterize a rod or wire stressed under either tension or compression.*
   - Note that because strain is a dimensionless quantity, Y has units of force per unit area.
   - Typical values are given in Table 3.1. Experiments show (a) that for a fixed applied force, the change in length is proportional to the original length and (b) that the force necessary to produce a given strain is proportional to the cross-sectional area.

.....3.9

2. Shear Modulus (S): with units of Pascal, ***is the ratio of shear stress to shear strain***.

- It *is the measure of the resistance to motion of the planes within a solid parallel to each other.*
- A material having a large shear modulus is difficult to bend.
- Like Young‘s modulus, the unit of shear modulus is the ratio of that for force to that for area.

…….3.10

3. Bulk Modulus: its SI unit is Pascal, ***is the ratio of the volume stress to the volume strain.***

- Bulk modulus measures the resistance of solids or liquids to changes in their volume.
- A material having a large bulk modulus doesn‘t compress easily.
- Note that a negative sign is included in this defining equation so that B is always positive.
- An ***increase in pressure (positive ΔP)*** causes a ***decrease in volume negative ΔV) and vice versa***

.....3.10

- A negative sign is inserted in this defining equation so that B is a ***positive number***. This operation is necessary because an increase in pressure (positive) causes a decrease in volume (negative) and vice versa.

- The reciprocal of the bulk modulus is called the compressibility of the material.
  - ***Note:-*** both solids and liquids have a bulk modulus.(*see from the table*)
- However, ***no shear modulus and no Young's modulus are given for liquids***
  - because a liquid does not sustain a shearing stress or a tensile stress.

- If a ***shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.***
- ***Strain Energy is energy stored in a stretched wire. If x is the stretch due to applied force F,***

$$\text{.....} \quad 3.12$$

Table : Typical Values for Elastic Moduli

Typical Values for Elastic Moduli			
Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example:

1. Suppose that the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel wire have if we do not want it to stretch more than 0.5 cm under these conditions?

Solution:

- *From the definition of Young‘s modulus, we can solve for the required cross-sectional area. Assuming that the cross section is circular, we can determine the diameter of the wire. From Equation:*

Because of the radius of the wire can be found from

Activity:

1. A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is . The volume of the sphere in air is 0.50 . By how much does this volume change once the sphere is submerged?

Solution

Then

3.2. Density and Pressure in Static Fluids

- **Density** (ρ) is the quantity of mass (m) per unit volume (V) of a body with SI unit and given by:

- **Specific gravity** (SG): also called relative density (it relates density of two substances)
- is the ratio of the density of the substance to the density of another substance which is taken as a standard.
- Equivalently it is the ratio of the mass of a substance to the mass of a reference substance for the same given volume
- The reference Substance **for liquids is nearly always Water at its densest (at or 39.2)** the density of pure water at is usually taken as the standard and this has been defined to be exactly .
- For gases it is ***air*** at room temperature (or 68)

- Nonetheless, the temperature and pressure must be specified for both the sample and reference or standard. Pressure is nearly always 1 atm (101.325kPa).

- Then generally Specific gravity(SG)

- Specific gravity is a dimensionless quantity and the same in any system of measurement.

For example,

- the density of aluminum is ; therefore,
- The SG of aluminum is

Example:
A solid sphere made of wood has a radius of 0.1 m. The mass of the sphere is 1.0 kg. Determine a) density and b) specific gravity of the wood.

Solution:

- The volume of the sphere wood,

a) The density of the wood,

b) Specific gravity of the wood,

Pressure

- Is the ratio of the force acting perpendicular to s surface to the surface area (A) on which the force acts.
- SI unit of pressure is , called ***Pascal (Pa).***
- Another commonly used pressure unit is ***atmosphere (atm)*** equal to ***101.3 kPa,*** which is the average pressure, exerted by the Earth‘s atmosphere at sea level.

- The pressure produced by the column of fluid of height h and density is given by:

- The density of liquids and solids is considered to be constant.
- In reality, ***the density of a liquid will increase slightly with increasing depth*** (Why?). The variation in density is usually negligible and can be ignored.
- **Note that:**
    - All points at same level in a fluid have same pressure.
    - Fluid pressure increases with increase in the depth of the fluid.
    - Fluid pressure does not depend on the shape of the container.

- **Atmospheric Pressure**: is the pressure due to the weight of the atmosphere exerted on the surface of the Earth. Atmospheric pressure decreases with increase in altitude as a result of decrease in the density of the air.
- **Gauge pressure:** is the difference in pressure between a system and the surrounding atmosphere.

- Because gauge pressure is the pressure relative to atmospheric pressure, therefore, it is positive for pressures above atmospheric pressure, and negative for pressures below it.

- **Absolute Pressure:** In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal‘s principle. The total pressure, or absolute pressure, is thus the sum of gauge pressure and atmospheric pressure:

- In most cases the absolute pressure in fluids cannot be negative.
- Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is

Example

1. A submerged wreck is located 18.3 m beneath the surface of the ocean off the coast of South Florida. Determine the a) gauge pressure and b) absolute pressure on a scuba diver who is exploring the wreck. Note: the density of sea water is .

Solution

a) ()(18.3m)=

3.3. Buoyant Force and Archimedes' Principles

- **Pascal's Principle –**
  - sates that pressure applied to a confined fluid in a container is transmitted equally to all regions of the fluid and to the walls of the container.
- An important application of Pascal's principle is the hydraulic press (Figure 4). A downward force applied to a small piston of area causes a pressure of .
- The pressure is transmitted throughout the fluid and reaches the larger piston of area without any change.

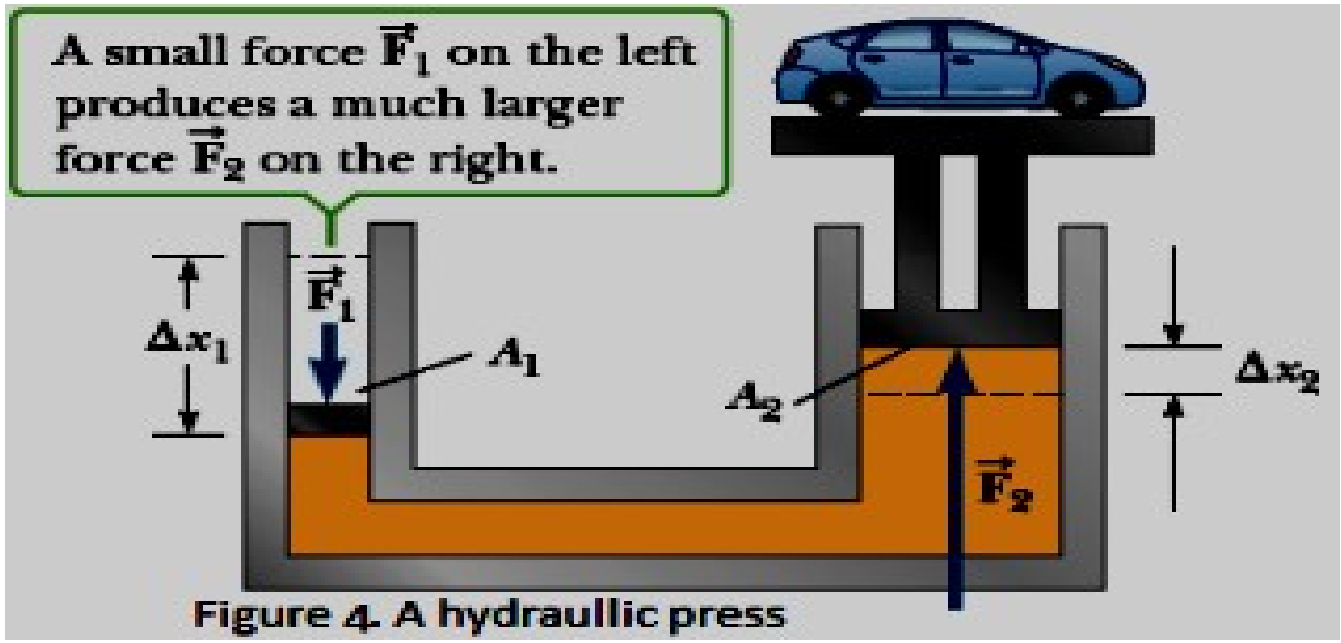


Figure 4. A hydraullic press

- As the fluid moves it pushes the larger piston with and exerts a pressure of .
- According to Pascal‘s principle, these two pressures are equal implying:

…….(3.18)

Example

1. The small piston of a hydraulic lift has an area of . A car weighing sits on a rack mounted on the large piston. The large piston has an area of . How large force must be applied to the small piston to support the car?

Solution

.

.

.

.

Then

3.3.1. Archimedes' principle

- Any object which is partially or totally submerged in a liquid has buoyant force acting on it which pushes the object up.
- That is why a rock appears to weigh less when it is submerged in liquid, or why it is very difficult to push a beach ball under water.
- The famous Greek mathematician, Archimedes‘ developed a principle which describes this around 250 B.C.
- Archimedes‘ principle can be ***stated as***

Anybody completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.

- Where, is the magnitude of the buoyant force and is the weight of the displaced fluid.

- The reason for this is that the pressure of the fluid is dependent on the depth of the fluid.

- So the pressure at the top of an object is less than the pressure at the bottom of the object which creates a net force.

Example:

1. A sample of an unknown material *weighs 300 N* in air and 200 N when submerged in an alcohol solution with a density of . What is the density of the material?

Given:

$W_{air} = 300\ N$

$W_{alcohol} = 200\ N$

$\rho_{alcohol} = 0.70 \times 10^3\ kg/m^3$

Unknown:

$\rho_{material}$ or ρ_o =?

Solution:

$$F_{buo} = W_{air} - W_{alcohol} = 300N - 200N = 100\ N$$

$$\frac{W_{air}}{F_{buoy}} = \frac{\rho_o}{\rho_{alcohol}} \Longrightarrow \rho_o = \rho_{alcohol}\left(\frac{W_{air}}{F_{buoy}}\right)$$

$$= (0.70 \times 10^3\ kg/m^3)\left(\frac{300N}{100N}\right)$$

$$= 2.1 \times 10^3\ kg/m^3$$

3.4. Moving Fluids and Bernoulli Equations (Fluid Dynamics)

- Up until this point, we have discussed fluids which are ***static. That is, they are not in motion.***
- We now turn our attention to ***fluids in motion***, or **hydro dynamics**.
- There are many categories of fluids in motion, categorized by whether
  - the fluid flow is steady, or not steady,
  - compressible or incompressible,
  - viscous or non-viscous.
- In ***steady flow, the velocity of the fluid particles at any point is constant*** as time goes by. Different parts of the fluid may be flowing at different rates, but the fluid in one location is always flowing at the same rate.
- An in compressible flow is the flow of a fluid which cannot be compressed. Most liquids are nearly incompressible.
- A viscous fluid is one which does not flow easily, like honey, while a non-viscous fluid is one which flows more easily, like water.
- We will mostly be concerned with the steady flow of in compressible, non-viscous fluids.

- If *the flow is steady*, then the velocity of the fluid particles at any point is a constant with time. The various layers of the fluid slide smoothly past each other. This is called ***streamline or laminar flow***. It assumes that as each particle in the fluid passes a certain point it follows the same path as the particles that preceded it. There is no loss of energy due to internal friction in the fluid, which is the fluid, is assumed to travel smoothly in regular layers; the velocity and pressure remain constant at every point in the fluid.

- Above some certain velocity, the flow is not smooth and becomes turbulent. Illustrations of turbulent and laminar flow are shown in Figure 3.5. Turbulent flow is the irregular movement of particles in a fluid and results in loss of energy due to internal friction between neighboring layers of the fluid, called viscosity. There is disruption to the layers of fluid; the speed of the fluid at any point is continuously changing both in magnitude and direction.

- **Factors affecting laminar flow** are
  - density,
  - compressibility,
  - temperature and
  - viscosity of the fluid.
- Assumptions made in the ideal fluid flow to understand the complex motions of real fluids:
  - The fluid is non-viscous, i.e there is no internal friction between adjacent layers.
  - The flow is steady; the velocity of the fluid at each point remains constant.
  - The fluid is incompressible; density of the fluid is constant.
  - The flow is irrotational; the fluid has no angular momentum about any point.

Equation of Continuity

- Equation of continuity ***expresses conservation of mass for an incompressible fluid flowing in a tube***. It says:

"the amount (either mass or volume) of fluid flowing through a cross section of the tube in a given time interval must be the same for all cross sections",

Or

"the product of the area and the fluid speed at all points along a tube is constant for an incompressible fluid"

- We see that if the cross sectional area is decreased, and then the flow rate increases. This is demonstrated when you hold your finger over part of the outlet of a garden hose. Because you decrease the cross sectional area, the water velocity increases. The product Av, which has the dimensions of volume per unit time, is called the flow rate. The condition Av = constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present

3.4.1. Bernoulli’s Equation

- In the 18th century, the Swiss physicists Daniel Bernoulli derived a relationship between the velocity of a fluid and the pressure it exerts.
- Qualitatively, Bernoulli‘s principle states that swiftly moving fluids exert less pressure than slowly moving fluids.
- Bernoulli‘s principle is extremely important in our everyday life.
- It is the primary principle which leads to lift on an airplane wing and allows the plane to fly. It is the primary reason a sailboat can sail into the wind. It is the primary reason a baseball can curve. It is an important reason that smoke is drawn up a chimney.
- Bernoulli‘s equation is really a consequence of a fundamental principle of physics: the conservation of energy.
- It can be derived using energy principles.

- Consider a fluid moving through a pipe. The pipe‘s cross sectional area changes, and the pipe changes elevation. At one point the pipe has a cross sectional area of , a height of , a pressure of , a velocity of and moves a distance of in a time of . At another point along the pipe these quantities are given by and .
- Conservation of energy gives the following equation, called Bernoulli‘s equation,

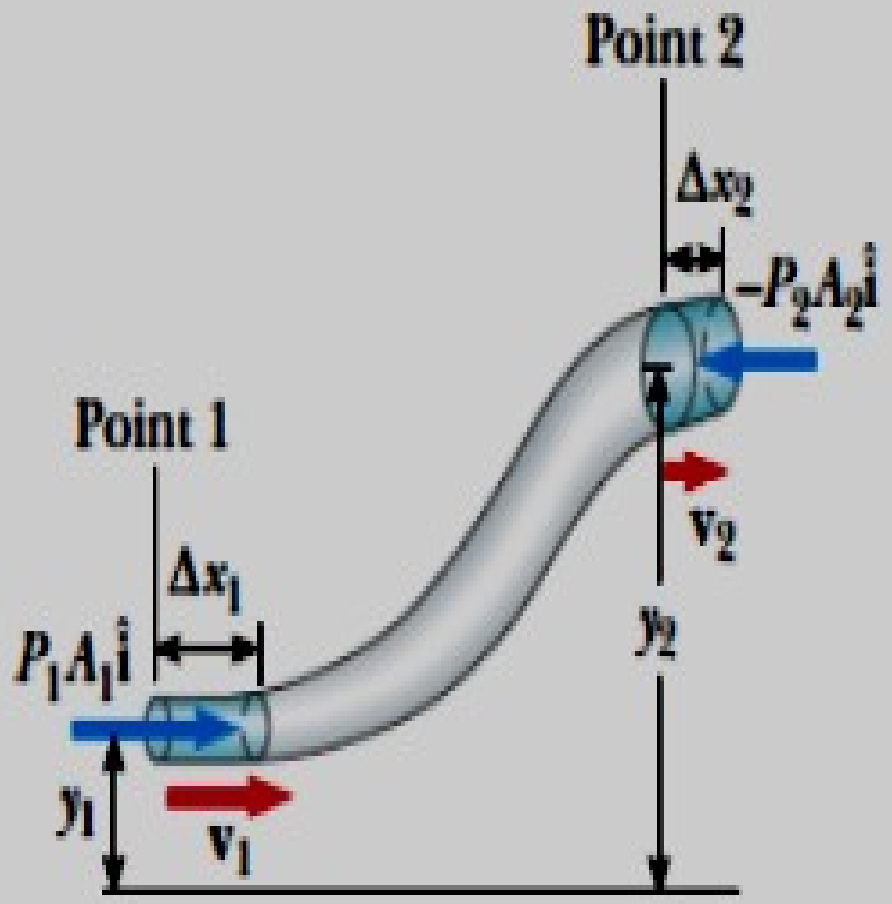


Figure 3.5: A fluid moving with steady flow through a pipe with varying cross-sectional area.

1. Water circulates throughout a house in a hot water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of , what will be the velocity and pressure in a 2.6 cm diameter pipe on the second floor 5.0m above?

Given:

$$v_1 = 0.50\ m/s v_2 = ?$$

$$h_1 = 0\ m\ (basement) h_2 = 5.0\ m$$

$$d_1 = 0.04\ m d_2 = 0.026\ m$$

$$A_1 = \pi r_1^2 = \pi(^{d_1}/_2)^2 = 0.0004\pi A_2 = \pi r_2^2 = \pi(^{d_2}/_2)^2 = 1.69 \times 10^{-4}\pi$$

$$P_1 = 3.03 \times 10^5 Pa P_2 = ?$$

Unknown:

$$v_2$$
$$P_2$$

Solution:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{(0.0004\pi \times 0.50 \frac{m}{s})}{1.69 \times 10^{-4}\pi} = 11.83\ m/s$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Longrightarrow P_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 - \frac{1}{2}\rho v_2^2 - \rho g h_2$$

$$= P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

$$= 3.03 \times 10^5 Pa + \frac{1}{2}(1.0 \times 10^3\ kg/m^3)(0.50^2 - 11.83^2)(m/s)^2 + (1.0 \times 10^3\ kg/m^3)\left(9.80 \frac{m}{s^2}\right)(0 - 5.0)m$$

$$= 1.84 \times 10^5 Pa.$$