

Shape and space

Higher Tier – Shape and space revision

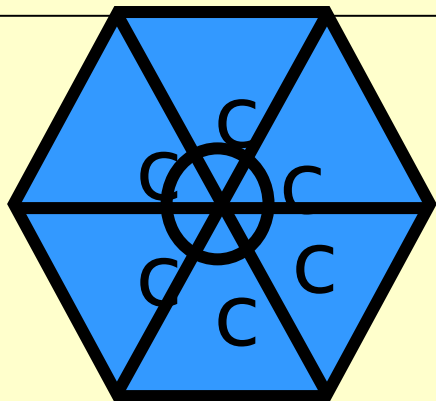
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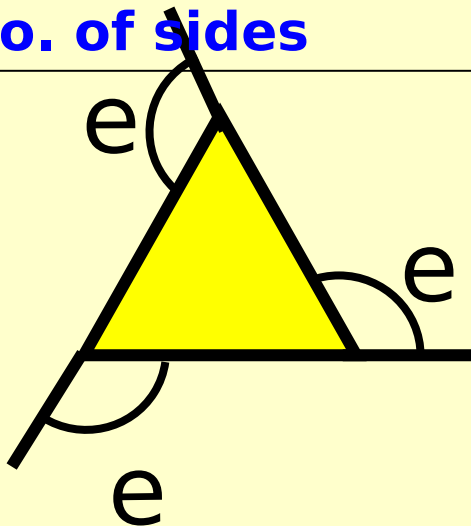
Angles and polygons

There are 3 types of angles in regular polygons

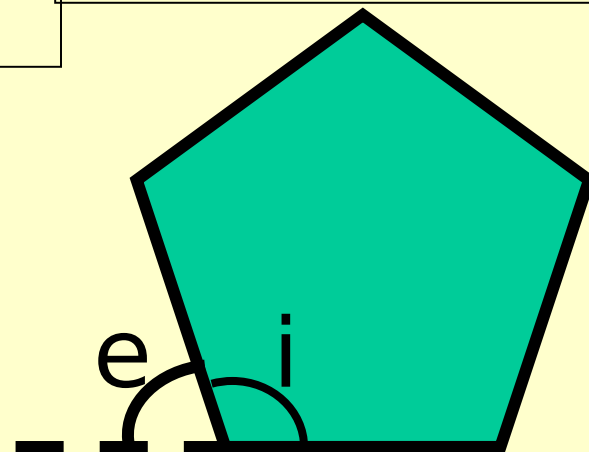
Angles at the centre = $\frac{360}{\text{No. of sides}}$



Exterior = $\frac{360}{\text{No. of sides}}$



Interior = $180 - e$ angles



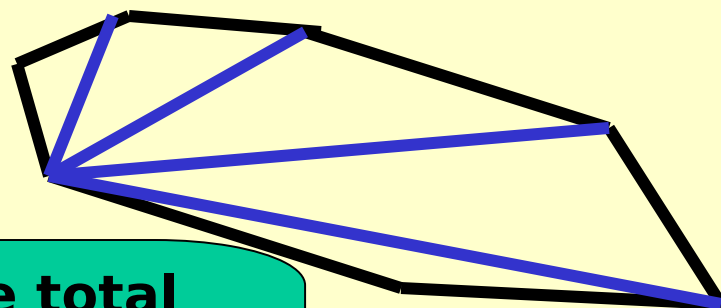
Calculate the value of c, e and i in regular polygons with 8, 9, 10 and 12 sides

Answers:

8 sides = $45^\circ, 45^\circ, 135^\circ$
 9 sides = $40^\circ, 40^\circ, 140^\circ$
 10 sides = $36^\circ, 36^\circ, 144^\circ$
 12 sides = $30^\circ, 30^\circ, 150^\circ$

To calculate the total interior angles of an irregular polygon divide it up into triangles from 1

corner. Then no. of x

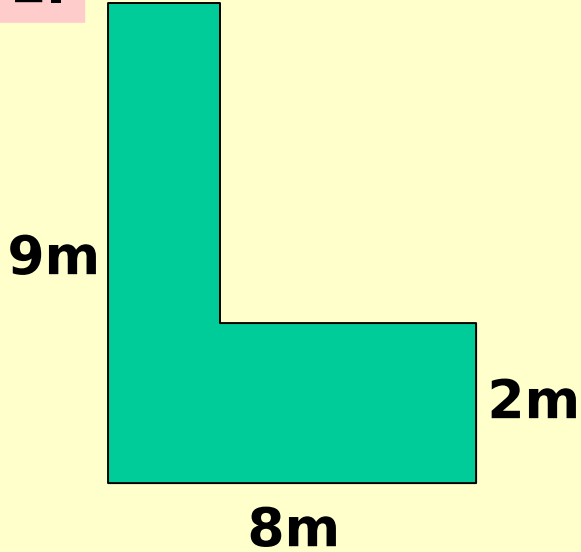


Total i
 = 5×180
 = 900°

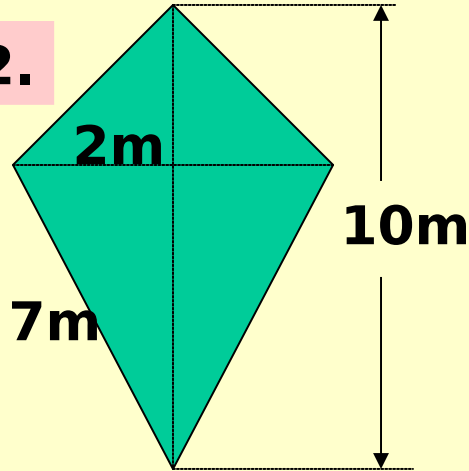
Area

What would you do to get the area of each of these shapes? **Do them step by step!**

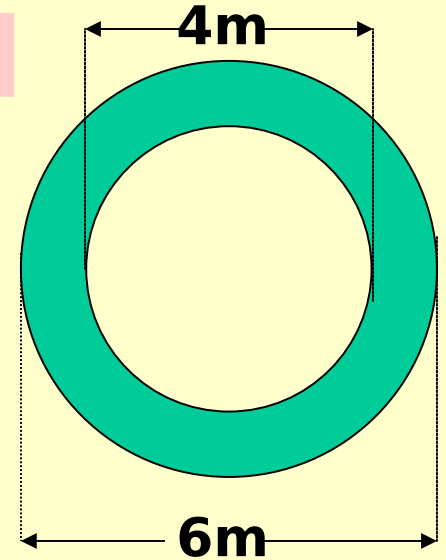
1. 1.5m



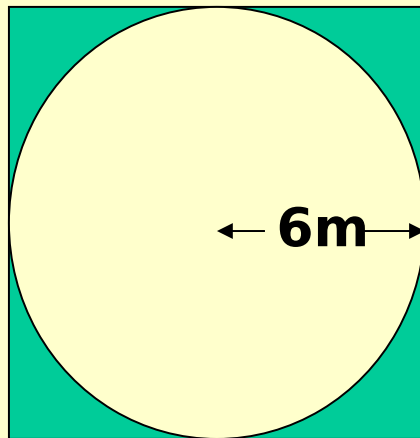
2.



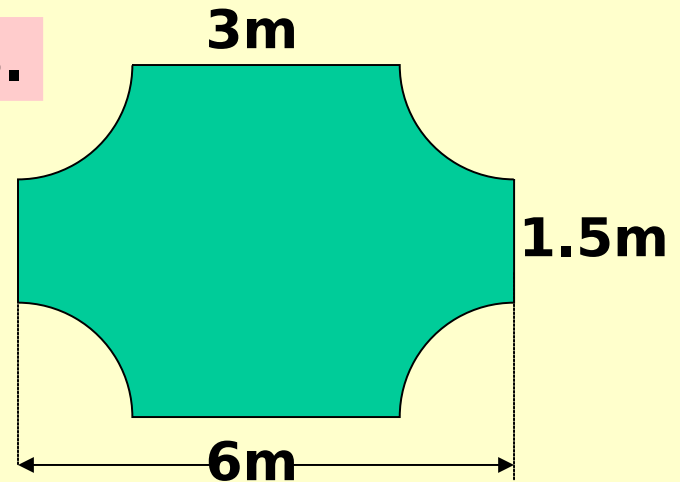
3.



4.



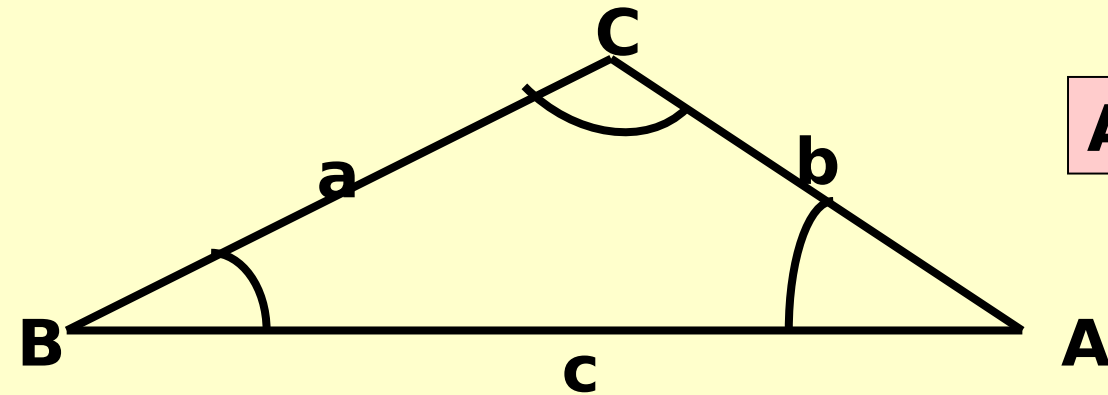
5.



Area of triangle

There is an alternative to the most common area of a triangle formula $A = (b \times h)/2$ and it's to be used when there are 2 sides and the included

you need to know how to label a triangle. Use capitals for angle case letters for the sides opposite to them.



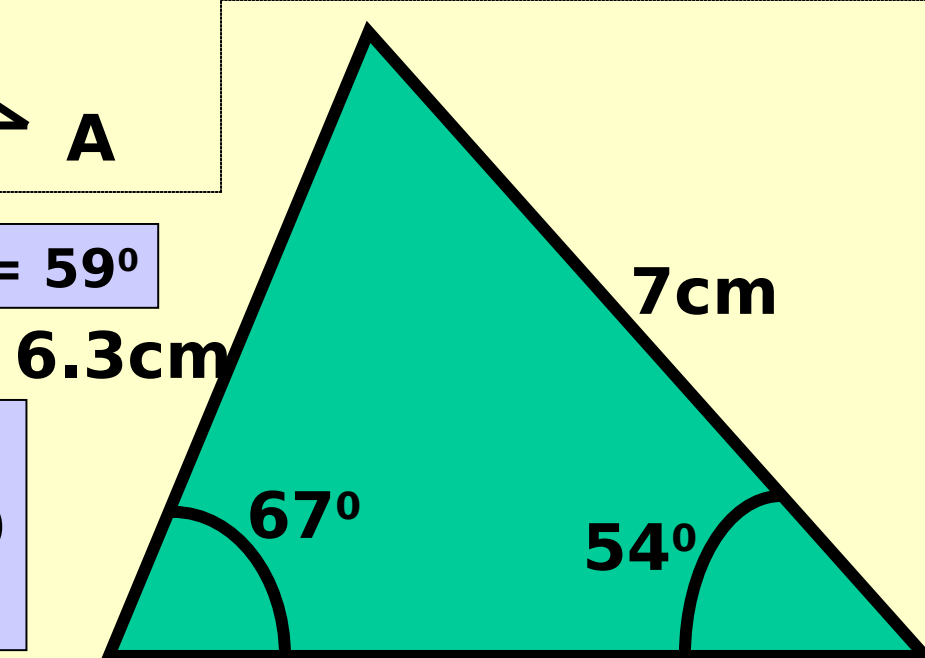
$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{The included angle} = 180 - 67 - 54 = 59^\circ$$

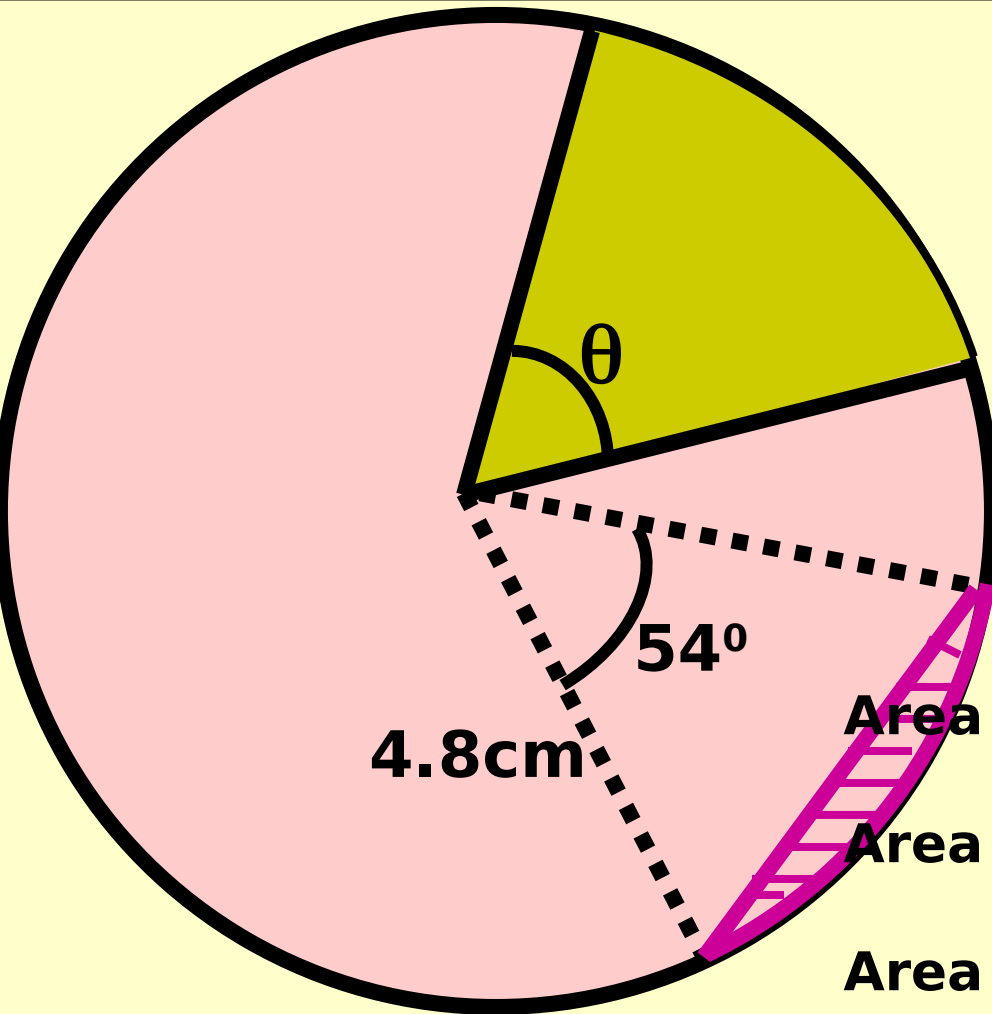
$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} = 0.5 \times 6.3 \times 7 \times \sin 59$$

$$\text{Area} = 18.9 \text{ cm}^2$$



Area and arc lengths of circles



Circle

$$\text{Area} = \pi \times r^2$$

$$\text{Circumference} = \pi \times D$$

Sector

$$\text{Area} = \frac{\theta}{360} \times \pi \times r^2$$

$$\text{Arc length} = \frac{\theta}{360} \times \pi \times D$$

$$\begin{aligned}\text{Area sector} &= 54/360 \times 3.14 \times 4.8 \times 4.8 \\ &= 10.85184\text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area triangle} &= 0.5 \times 4.8 \times 4.8 \times \sin 54^\circ \\ &= 9.31988\text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area segment} &= 10.85184 - 9.31988 \\ &= 1.54\text{cm}^2\end{aligned}$$

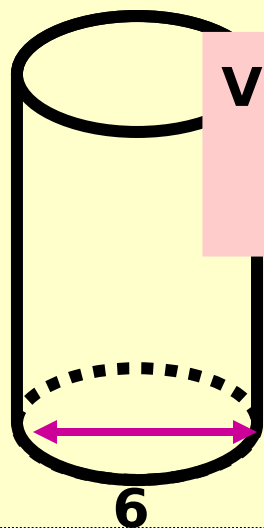
$$\begin{aligned}\text{Arc length} &= 54/360 \times 3.14 \times 9.6 \\ &= 4.52 \text{ cm}\end{aligned}$$

Segment

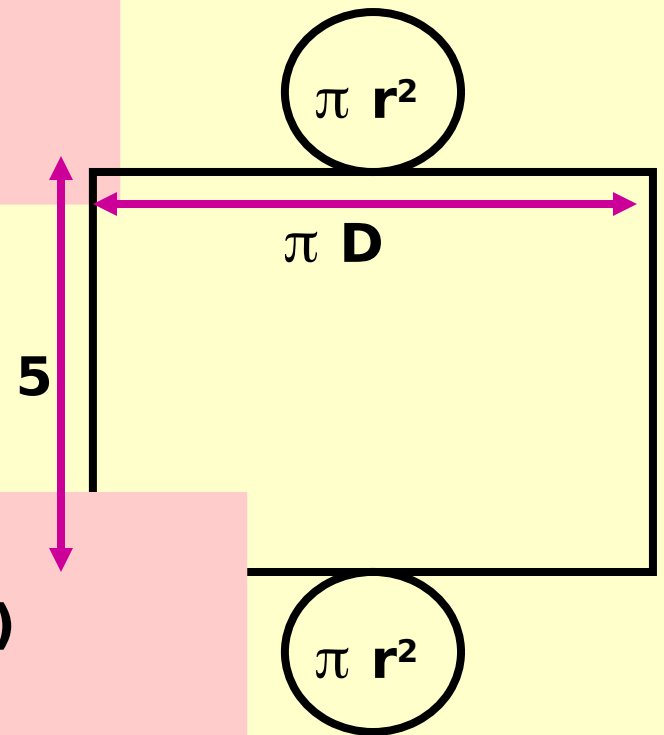
$$\text{Area} = \text{Area of sector} - \text{area of triangle}$$

Volume and surface area of solids

Calculate the volume and surface area of a cylinder with a height of 5cm and a diameter at the end of 6cm



$$\begin{aligned}\text{Volume} &= \pi \times r^2 \times h \\ &= 3.14 \times 3 \times 3 \times 5 \\ &= 141.3 \text{ cm}^3\end{aligned}$$

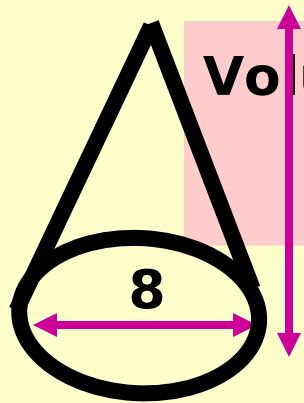


$$\begin{aligned}\text{Surface area} &= \pi r^2 + \pi r^2 + (\pi D \times h) \\ &= \pi \times 3^2 + \pi \times 3^2 + (\pi \times 6 \times 5) \\ &= 56.52 + 90.2 \\ &= 150.72 \text{ cm}^2\end{aligned}$$

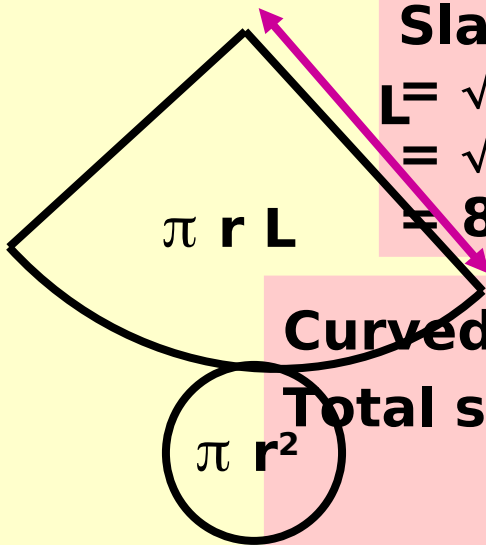
The formulae for spheres, pyramids (where used) and cones are given in the exam. However, you need to learn how to calculate the volume and

Volume and surface area of solids

Calculate the volume and surface area of a cone with a height of 7 cm and a diameter at the end of 8 cm



$$\begin{aligned}\text{Volume} &= \frac{1}{3} (\pi \times r^2 \times h) \\ &= \frac{1}{3} (3.14 \times 4 \times 4 \times 7) \\ &= 117.2 \text{ cm}^3\end{aligned}$$



Slant height (L)

$$\begin{aligned}L &= \sqrt{7^2 + 4^2} \\ &= \sqrt{65} \\ &= 8.06 \text{ cm}\end{aligned}$$

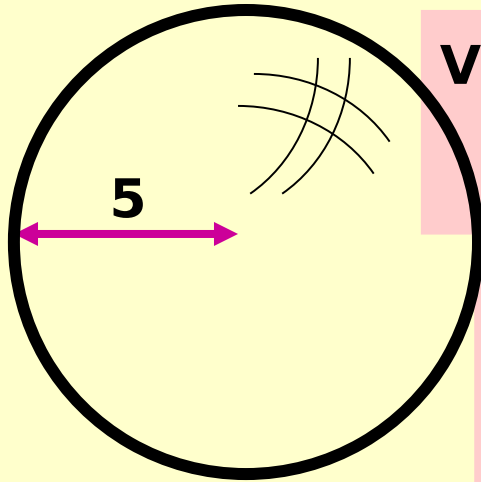
Curved surface area = $\pi r L$

Total surface area = $\pi r L + \pi r^2$

$$\begin{aligned}&= (3.14 \times 4 \times 8.06) + (3.14 \times 4 \times 4) \\ &= 101.2336 + 50.24 \\ &= 151.47 \text{ cm}^2\end{aligned}$$

Volume and surface area of solids

Calculate the volume and surface area of a sphere with a diameter of 10cm.



$$\begin{aligned}\text{Volume} &= \frac{4}{3} (\pi \times r^3) \\ &= \frac{4}{3} (3.14 \times 5 \times 5 \times 5) \\ &= 523.3 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Curved surface area} &= 4\pi r^2 \\ &= 4 \times 3.14 \times 5 \times 5 \\ &= 314 \text{ cm}^2\end{aligned}$$

Watch out for questions where the surface area or volume have been given and you are working backwards to find the radius.

Spotting P, A & V formulae

Which of the following expressions could be for:

- (a) Perimeter
- (b) Area
- (c) Volume

$$r(\pi + 3)$$

P

$$4\pi rl$$

A

$$\pi r(r + l)$$

A

$$\frac{1}{4}\pi d^2$$

A

$$\frac{4}{3}\pi r^2$$

A

$$\frac{4}{3}\pi r^3$$

V

$$\pi r + \frac{1}{2}r$$

P

$$4l^2h$$

V

$$\frac{1}{3}\pi rh$$

A

$$\frac{1}{3}\pi r$$

P

$$4\pi r^2h$$

V

$$\pi r + 4l$$

P

$$\frac{1}{3}\pi r^2h$$

V

$$3lh^2$$

V

$$\pi rl$$

A

Transformations

1. Reflection

Reflect the
triangle
using
the line:

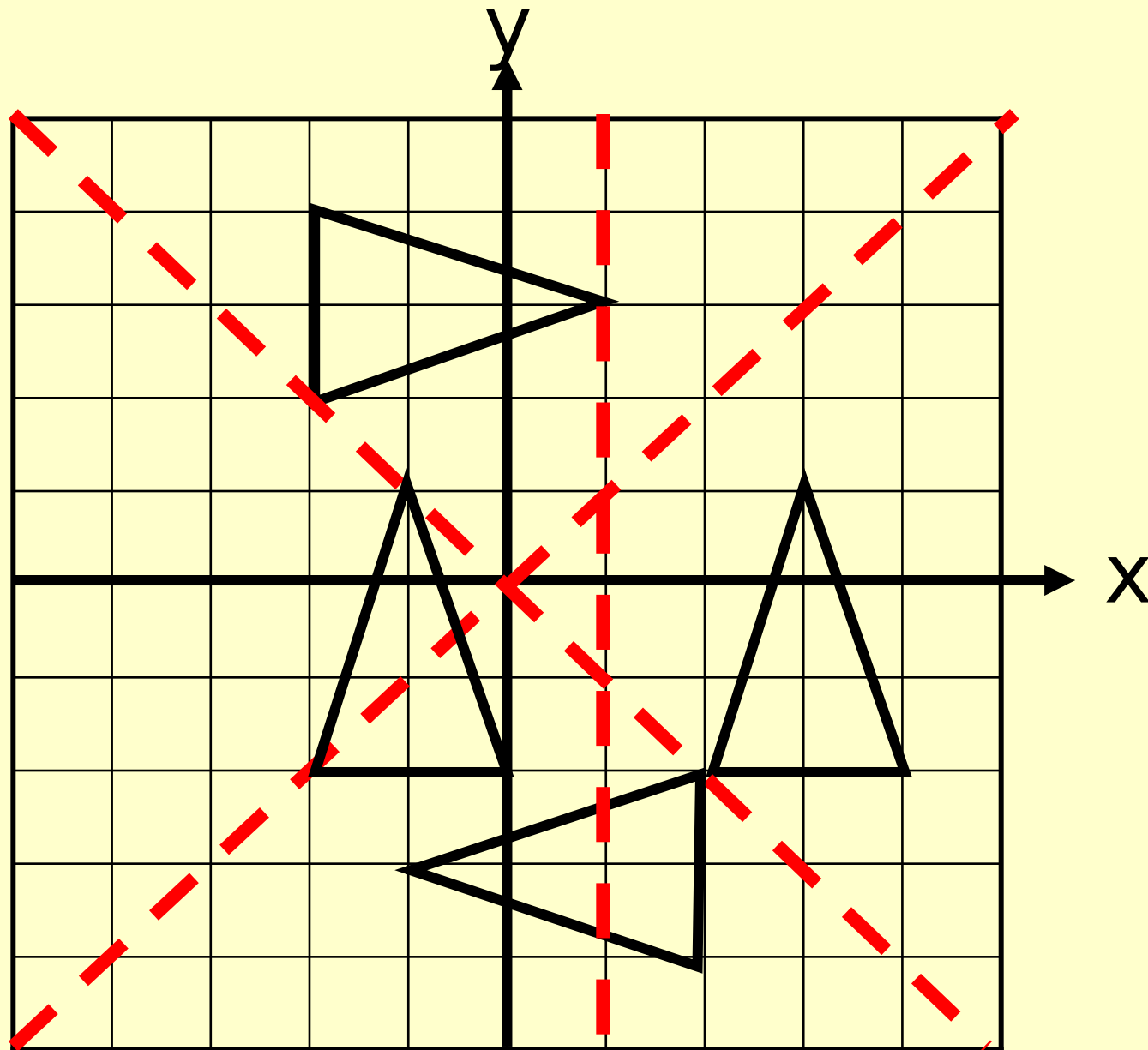
$$y = x$$

then the
line:

$$y = -x$$

then the
line:

$$x = 1$$



Transformations

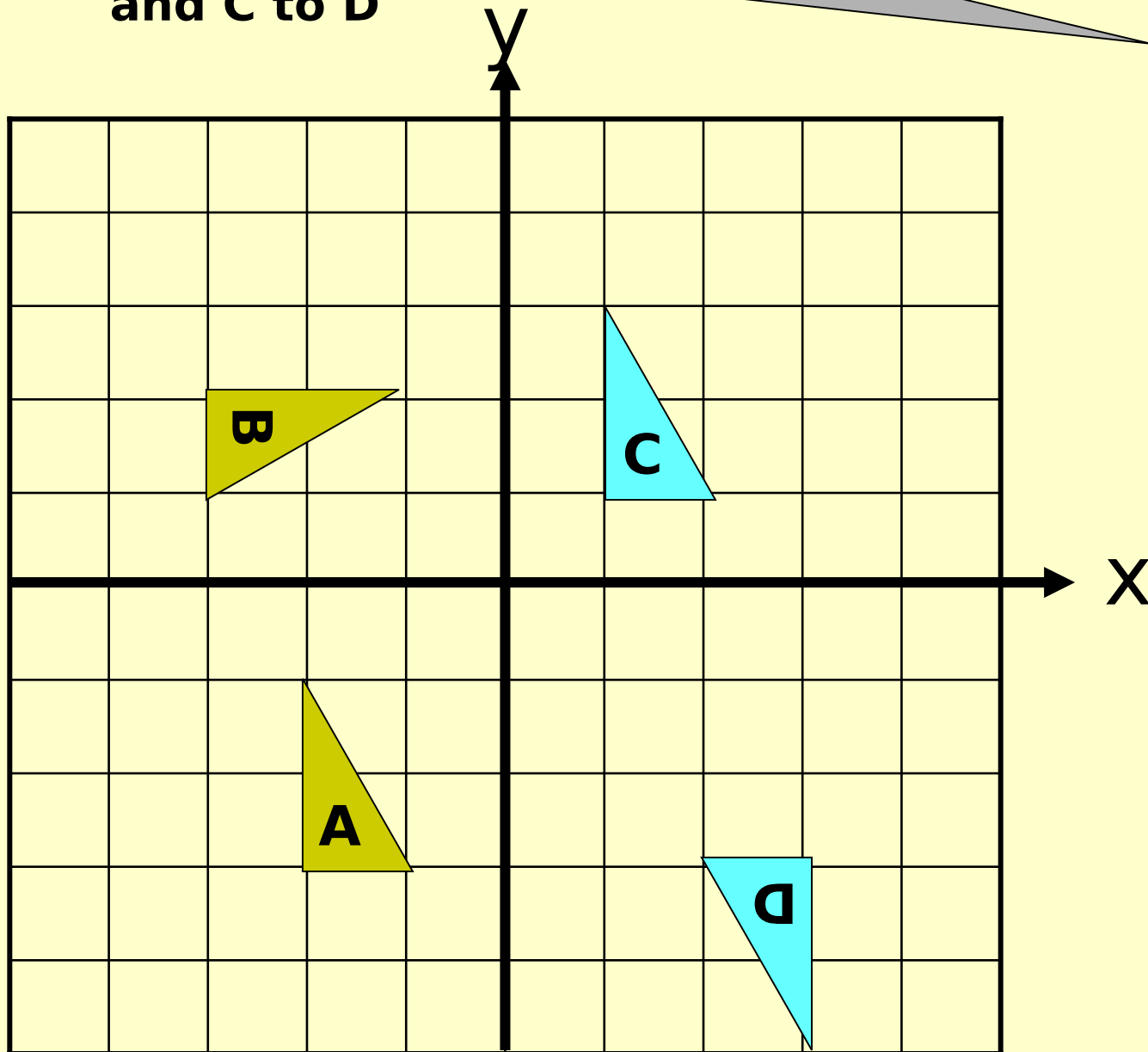
2. Rotation

When describing a rotation always state these 3 things:

- No. of degrees
- Direction
- Centre of rotation

e.g. a rotation of 90° anti-clockwise using a centre of (0, 1)

Describe the rotation of A to B and C to D



Transformations

3. Translation

What happens when we translate a shape ?

The shape remains the same size and shape and the same way up - it just.....

Horizontal translation

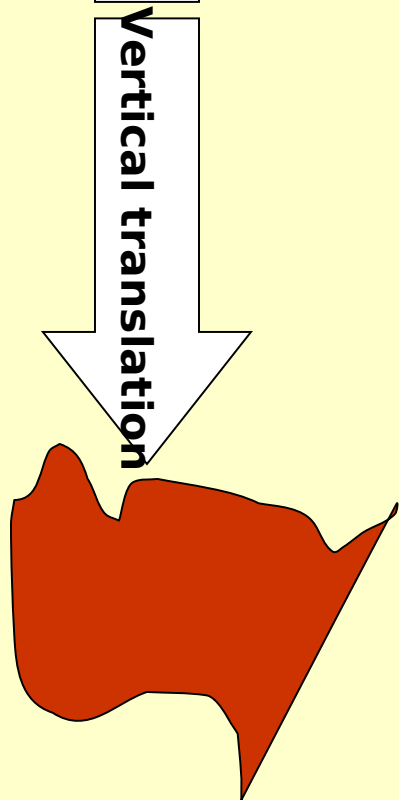
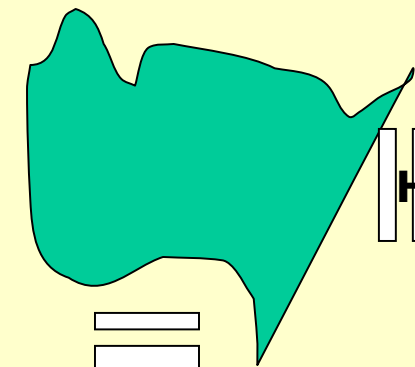
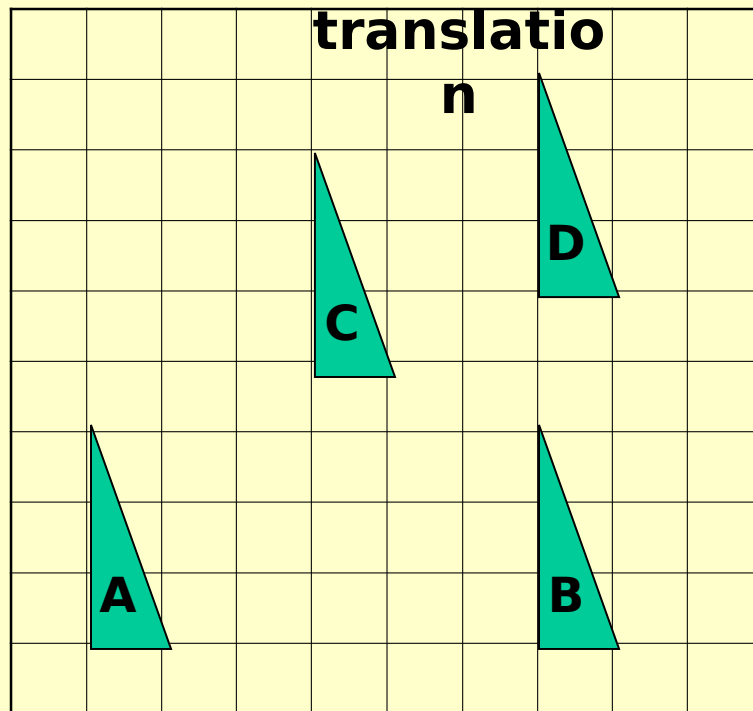
Use a vector to describe a translation

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

translation

Give the vector for the translation

- from.....
1. A to B $\rightarrow \begin{bmatrix} 6 \\ 0 \end{bmatrix}$
 2. A to D $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$
 3. B to C $\rightarrow \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
 4. D to C $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

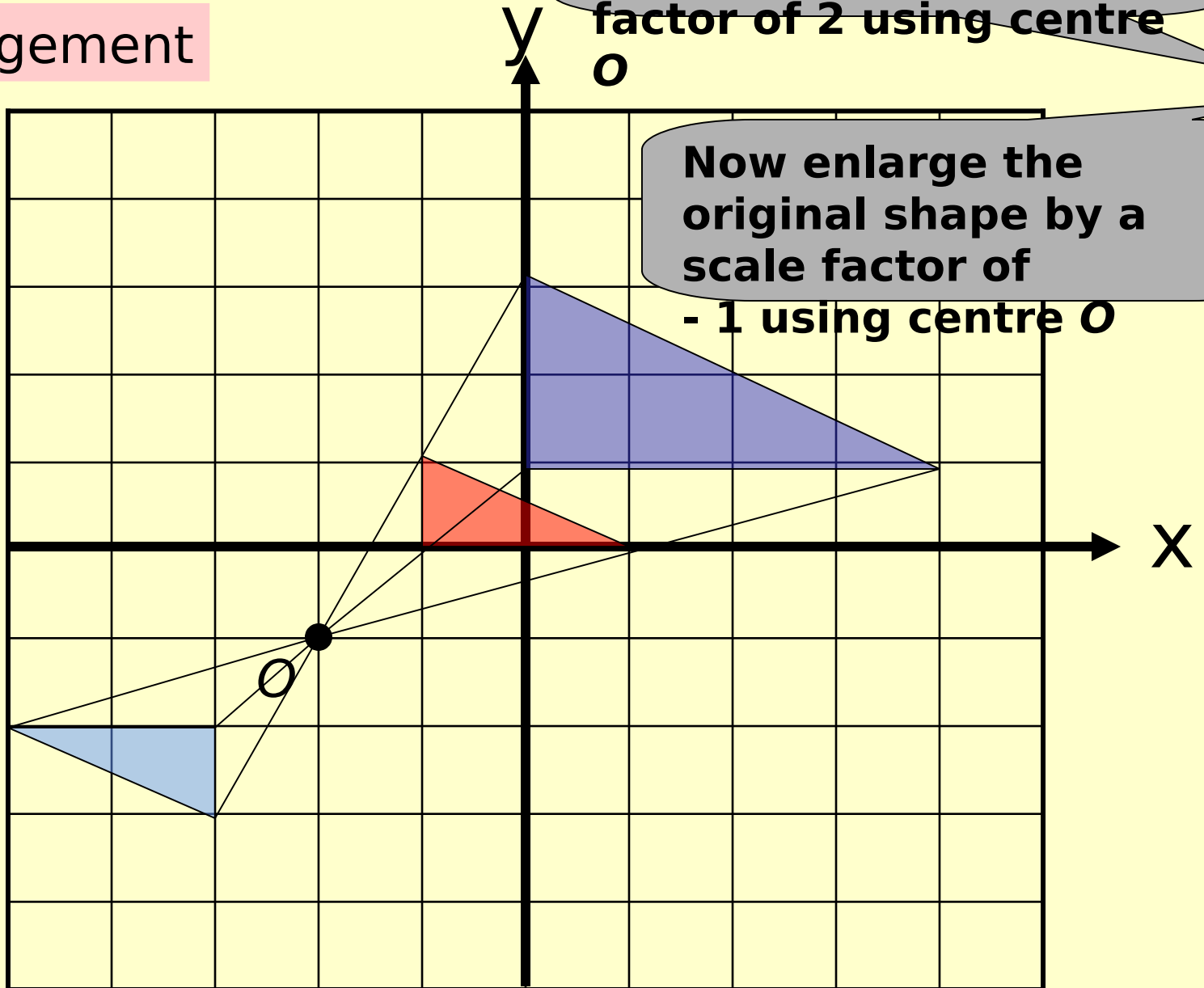


Transformations

4. Enlargement

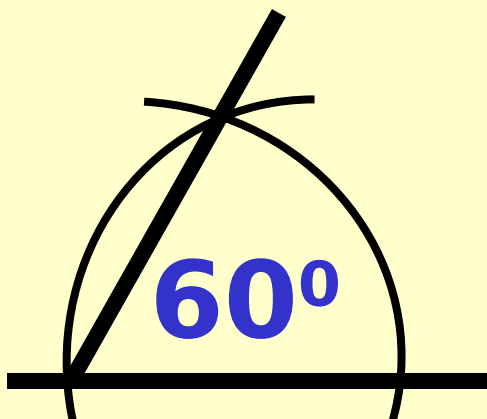
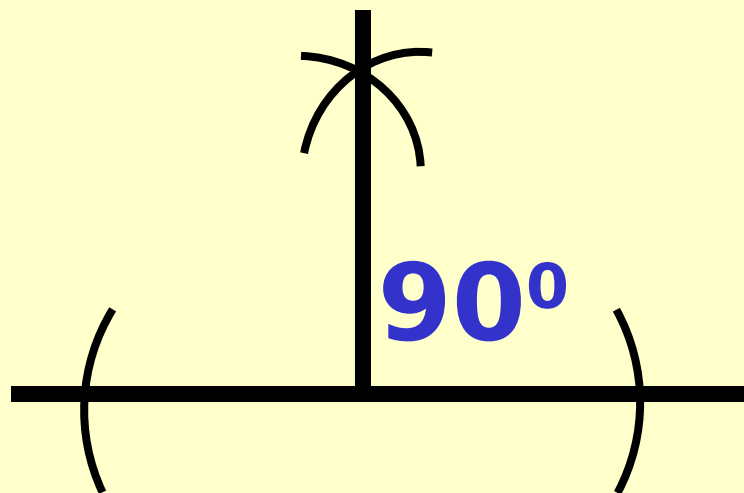
Enlarge this shape by a
scale
factor of 2 using centre
 O

Now enlarge the
original shape by a
scale factor of
- 1 using centre O

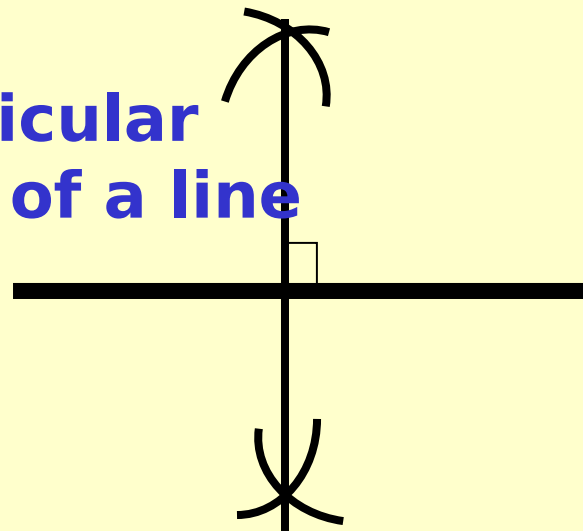


Constructions

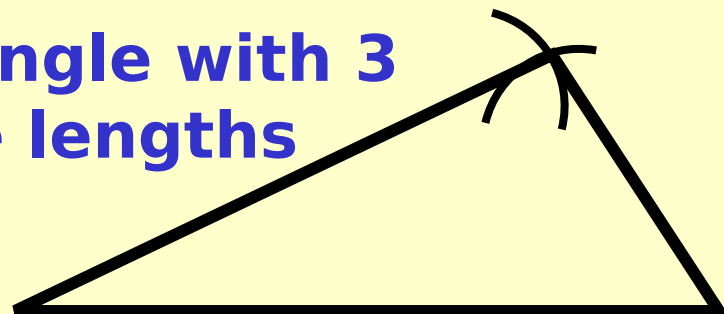
Have a look at these constructions and work out what has been done



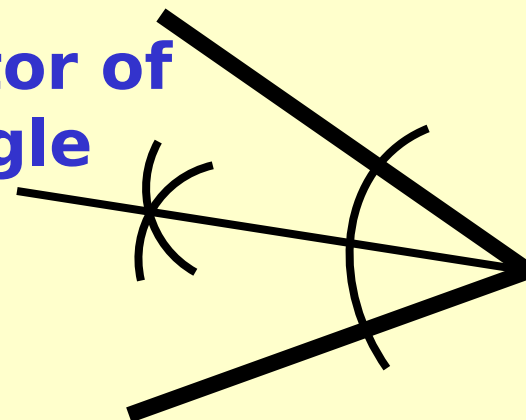
Perpendicular bisector of a line



Triangle with 3 side lengths



Bisector of an angle

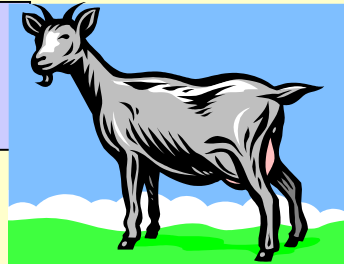


Loci

A **locus** is a drawing of all the points which satisfy a rule or a set of constraints. **Loci** is just the plural of locus.

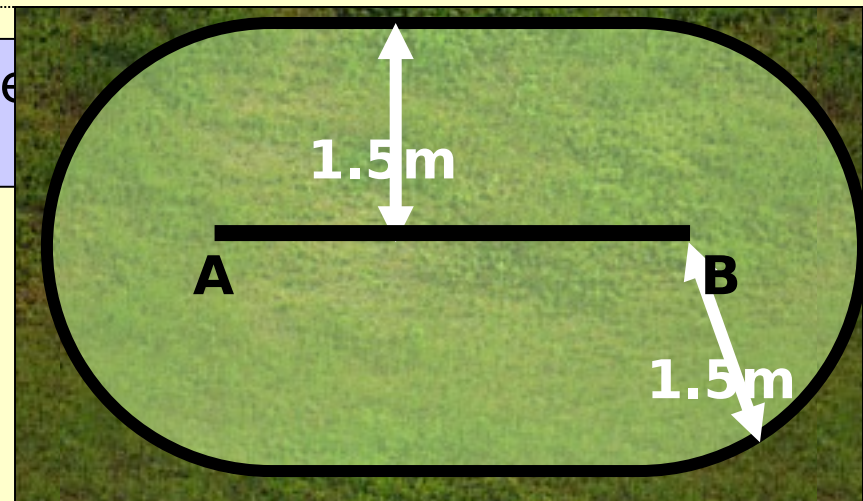
A goat is tethered to a peg in the ground at point A using a rope 1.5m long

1. Draw the locus to show all that grass he can eat



A goat is tethered to a rail AB using a rope (with a loop on) 1.5m long

2. Draw the locus to show all that grass he can eat



Similarity

Shapes are **congruent** if they are exactly the same shape

Shapes are **similar** if they are exactly the same shape

the same

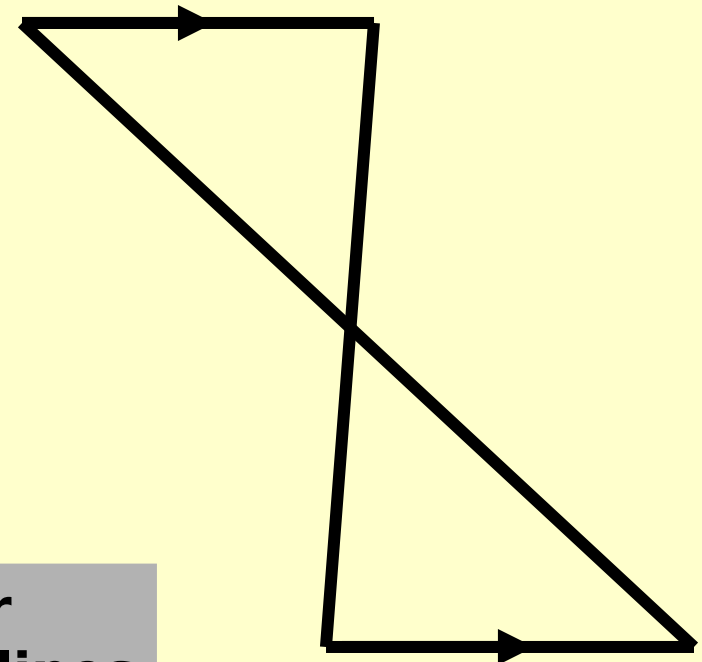
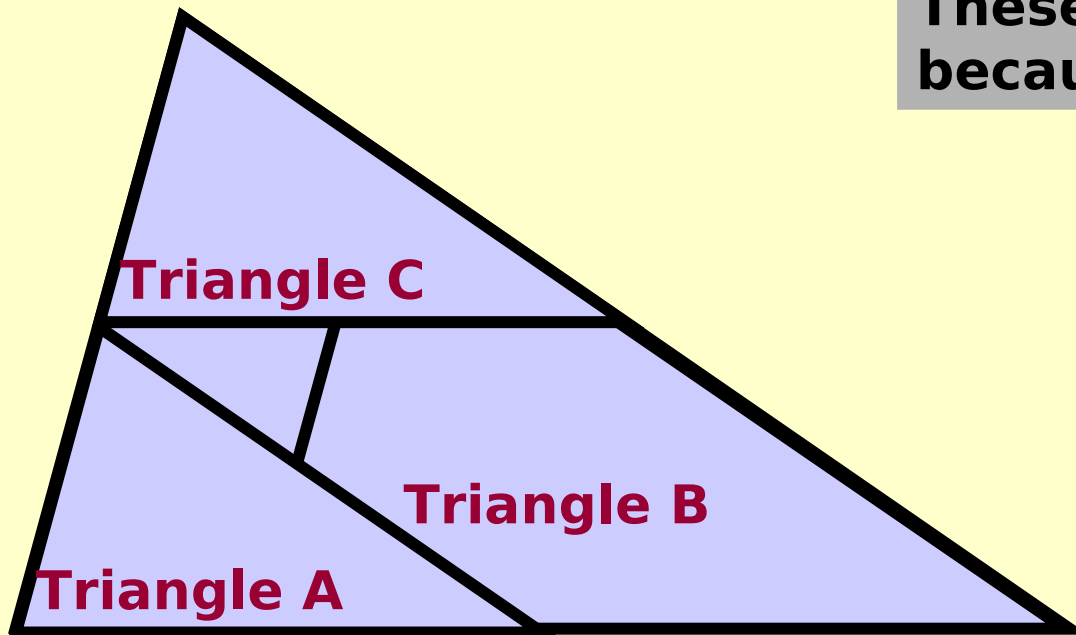


How can I spot similar triangles

triangles

but different sizes

These two triangles are similar because of the parallel lines



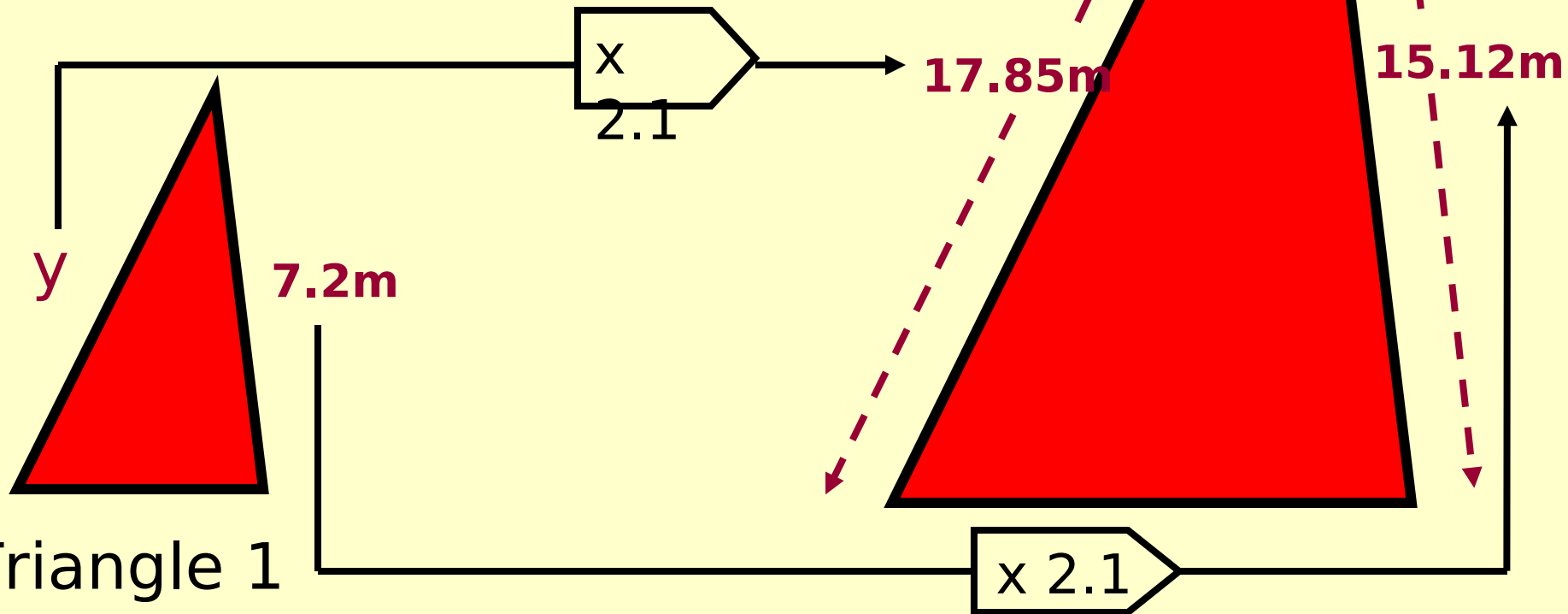
of these "internal" triangles are similar the big triangle because of the parallel lines

Similarity

These two triangles are similar. Calculate length y

$$y = 17.85 \div 2.1 =$$

8.5m Same multiplier



$$\text{Multiplier} = 15.12 \div 7.2 = 2.1$$

Similarity in 2D & 3D

Don't fall into the trap of thinking that the scale factor can be found by dividing one area by another area

These two cylinders are similar.

Write down all these equations immediately:

$$6.2 \times \text{scale factor} = L$$

$$\text{scale factor}^3 = 3343.75/214$$

$$\text{scale factor}^3 = 15.625$$

$$\text{scale factor} = 2.5$$

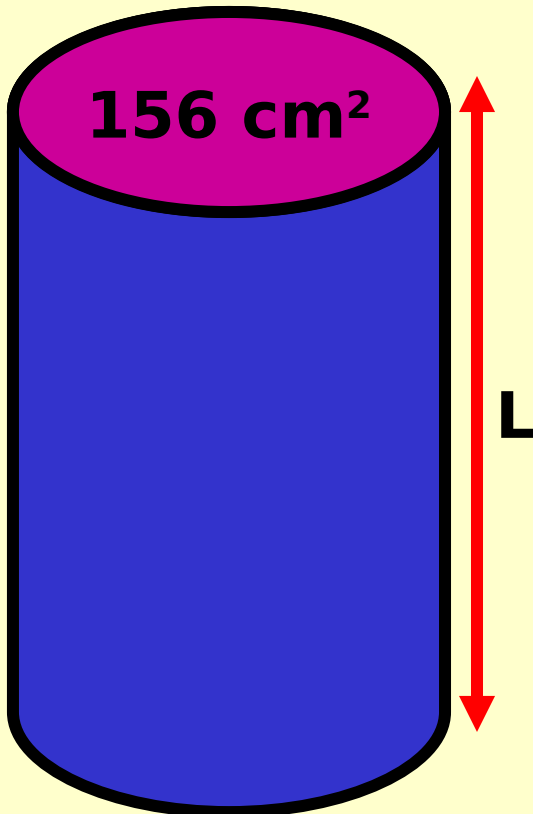
$$\text{So } 6.2 \times 2.5 = L \quad \text{and} \quad A \times$$

$$2.5^2 = 156$$

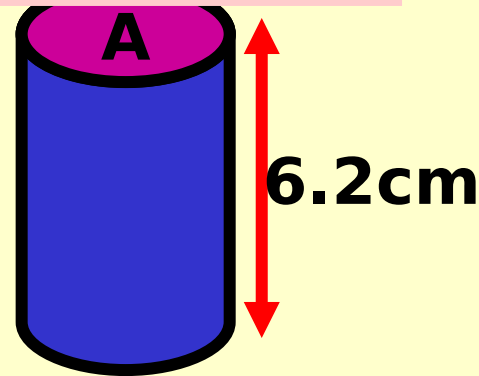
$$L = 15.5\text{cm}$$

$$24.96\text{cm}^2$$

$$A =$$



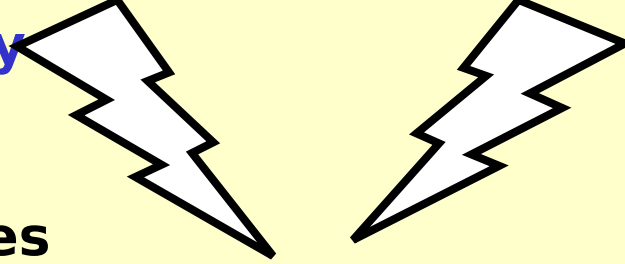
Volume =



Volume =

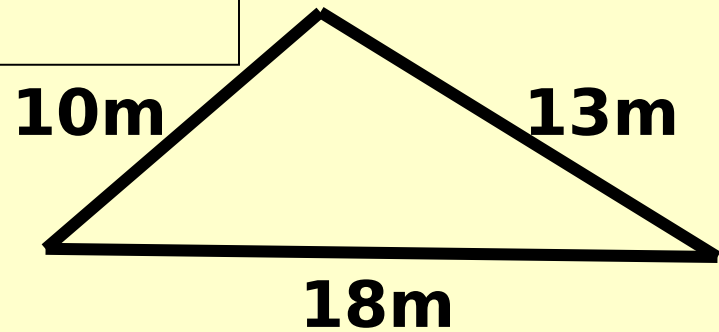
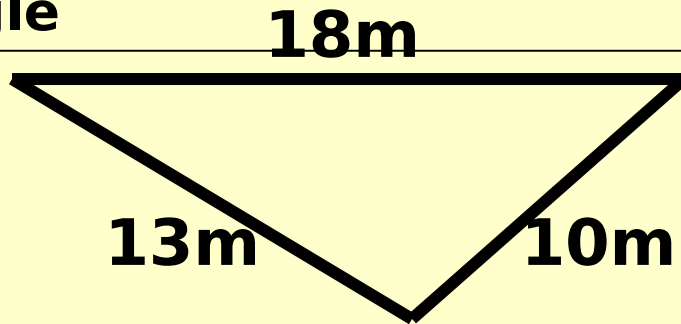
Congruence

Shapes are **congruent** if they are exactly the same shape and exactly the same size

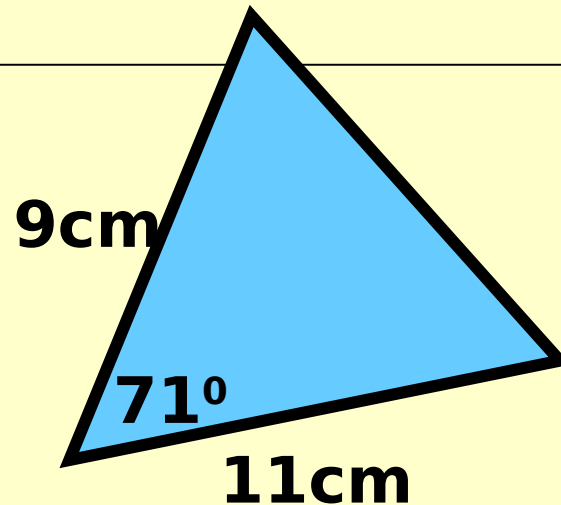
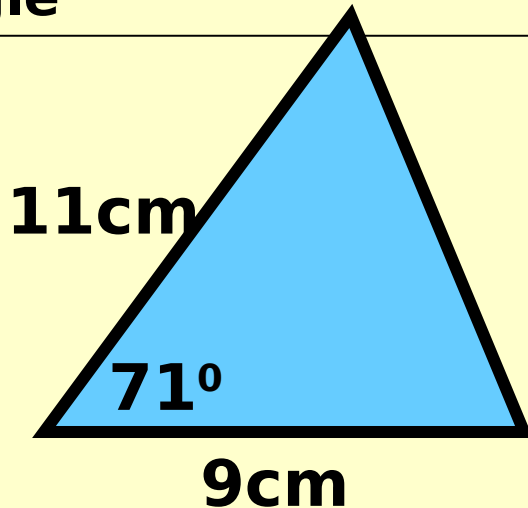


There are 4 conditions under which 2 triangles are congruent:

SSS - All 3 sides are the same in each triangle



SAS - 2 sides and the included angle are the same in each triangle



ASA - 2 angles and the included side are the same in each triangle

52°

RHS - T
same

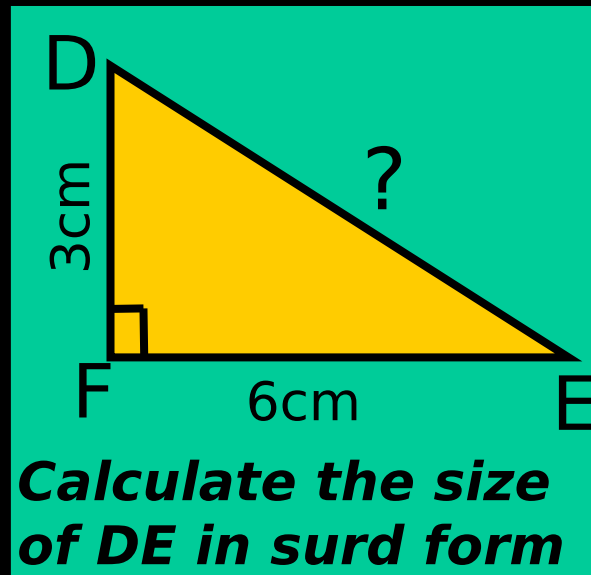
Be prepared to justify
these congruence rules
by **PROVING** that
they work

12

Pythagoras Theorem

Calculating the Hypotenuse

Be prepared to leave your answer in surd form (most likely in the non-calculator exam)



$$\begin{aligned} \text{Hyp}^2 &= a^2 + b^2 \\ DE^2 &= 3^2 + 6^2 \\ DE^2 &= 9 + 36 \\ DE^2 &= 45 \\ DE &= \sqrt{45} \\ DE &= \sqrt{9} \times \sqrt{5} \\ DE &= \underline{3\sqrt{5} \text{ cm}} \end{aligned}$$

$$\begin{aligned} \text{Hyp}^2 &= a^2 + b^2 \\ 21^2 + 45^2 \\ 441 + 2025 \\ 2466 \\ \sqrt{2466} \end{aligned}$$

$$\begin{aligned} \text{Hyp}^2 &= a^2 + b^2 \\ 59^2 + 11^2 \\ 3481 + 121 \\ 3602 \\ \sqrt{3602} \end{aligned}$$

$$AC^2$$

$$= AC$$

$$= AC$$

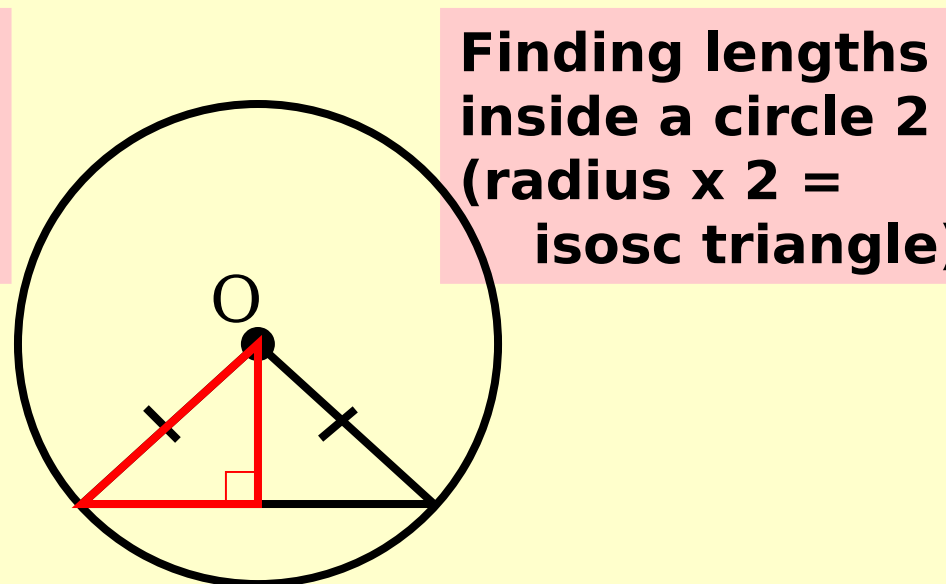
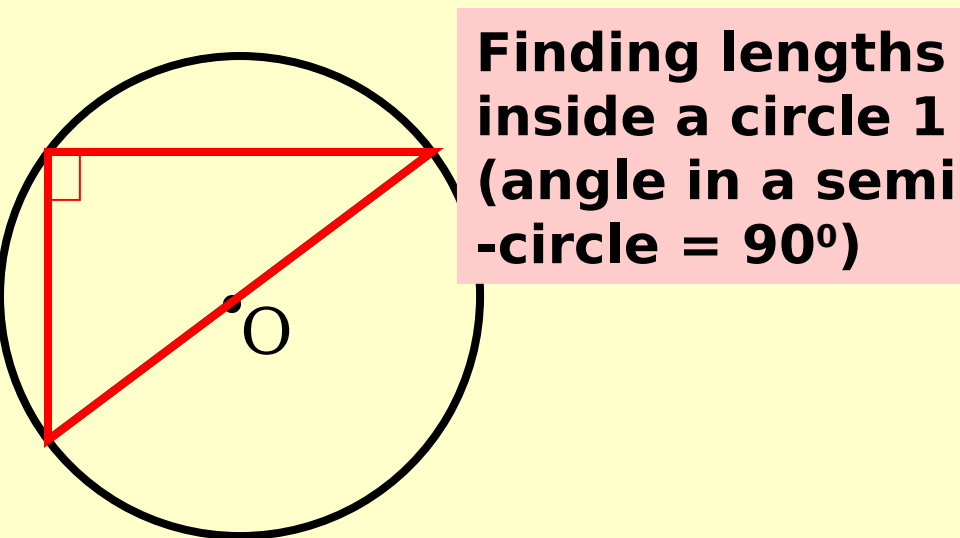
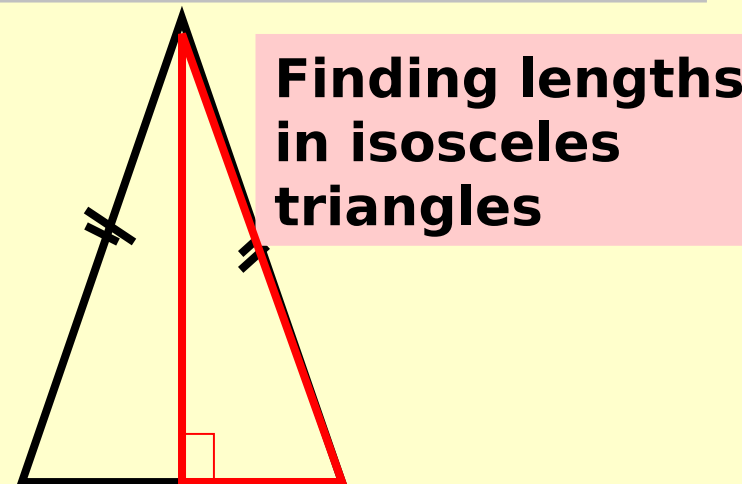
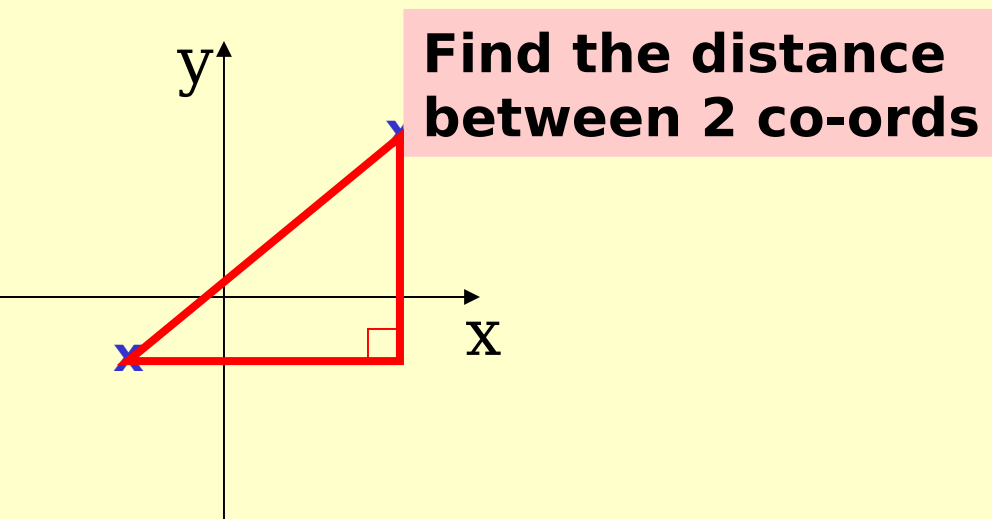
$$\underline{AC = 11.6\text{m}}$$

Longest side opposite $\square$

of AC is 1 up.

Pythagoras Questions

Look out for the following Pythagoras questions in disguise:



SOHCAHTOA

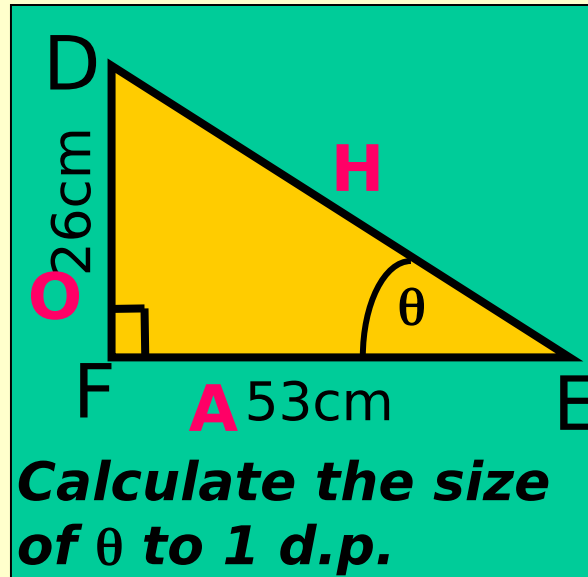
How to spot
a
Trigonometr

Right-angled
triangle

An angle
involved
in question

- Label sides **H, O, A**
- Write **SOHCAHTOA**
- Write out correct rule
- Substitute values in

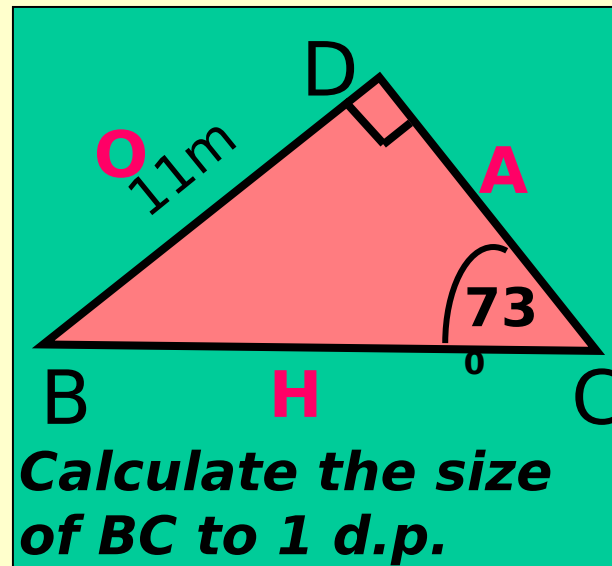
Calculating an angle



SOHCAHTOA

$$\begin{aligned}\text{Tan } \theta &= \text{O/A} \\ \text{Tan } \theta &= 26/53 \\ \text{Tan } \theta &= 0.491 \\ \theta &= \underline{26.1^\circ}\end{aligned}$$

Calculating a side



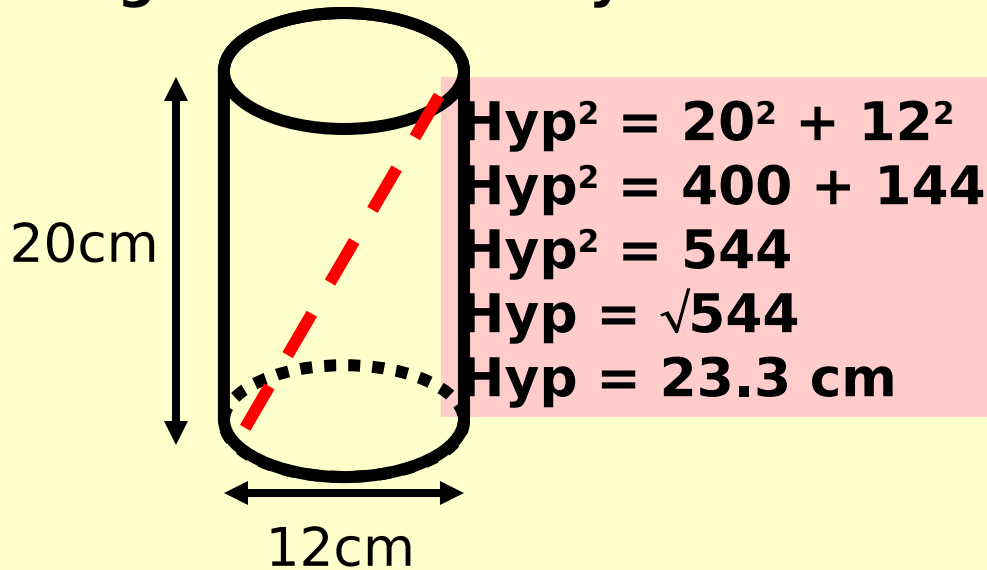
SOHCAHTOA

$$\begin{aligned}\text{Sin } \theta &= \text{O/H} \\ \text{Sin } 73 &= 11/H \\ H &= 11/\text{Sin } 73 \\ H &= \underline{11.5 \text{ m}}\end{aligned}$$

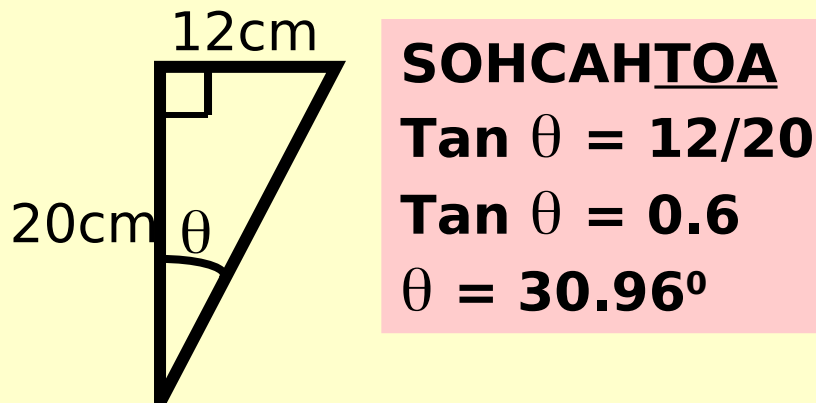
3D Pythag and Trig

Always work out a strategy first

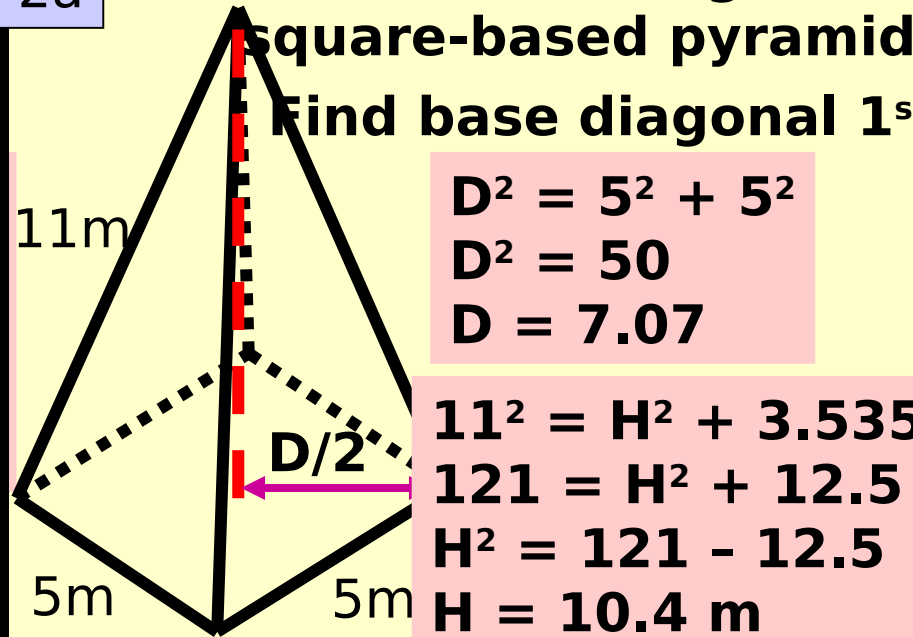
1a Calculate the length of the longest diagonal inside a cylinder



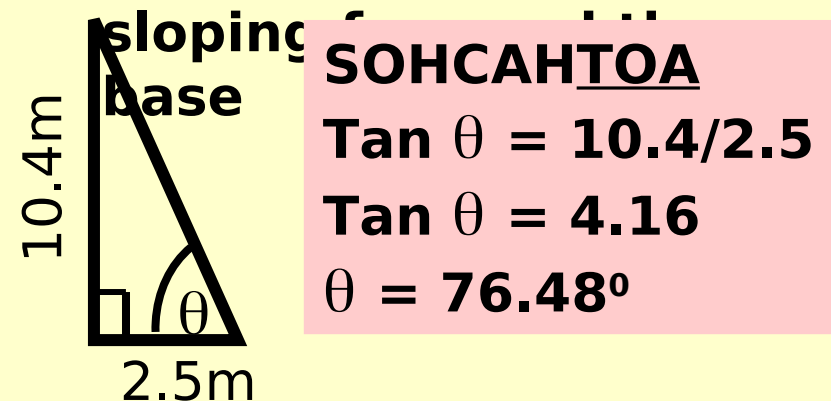
1b Calculate the angle this diagonal makes with the vertical



2a Calculate the height of a square-based pyramid. Find base diagonal 1st

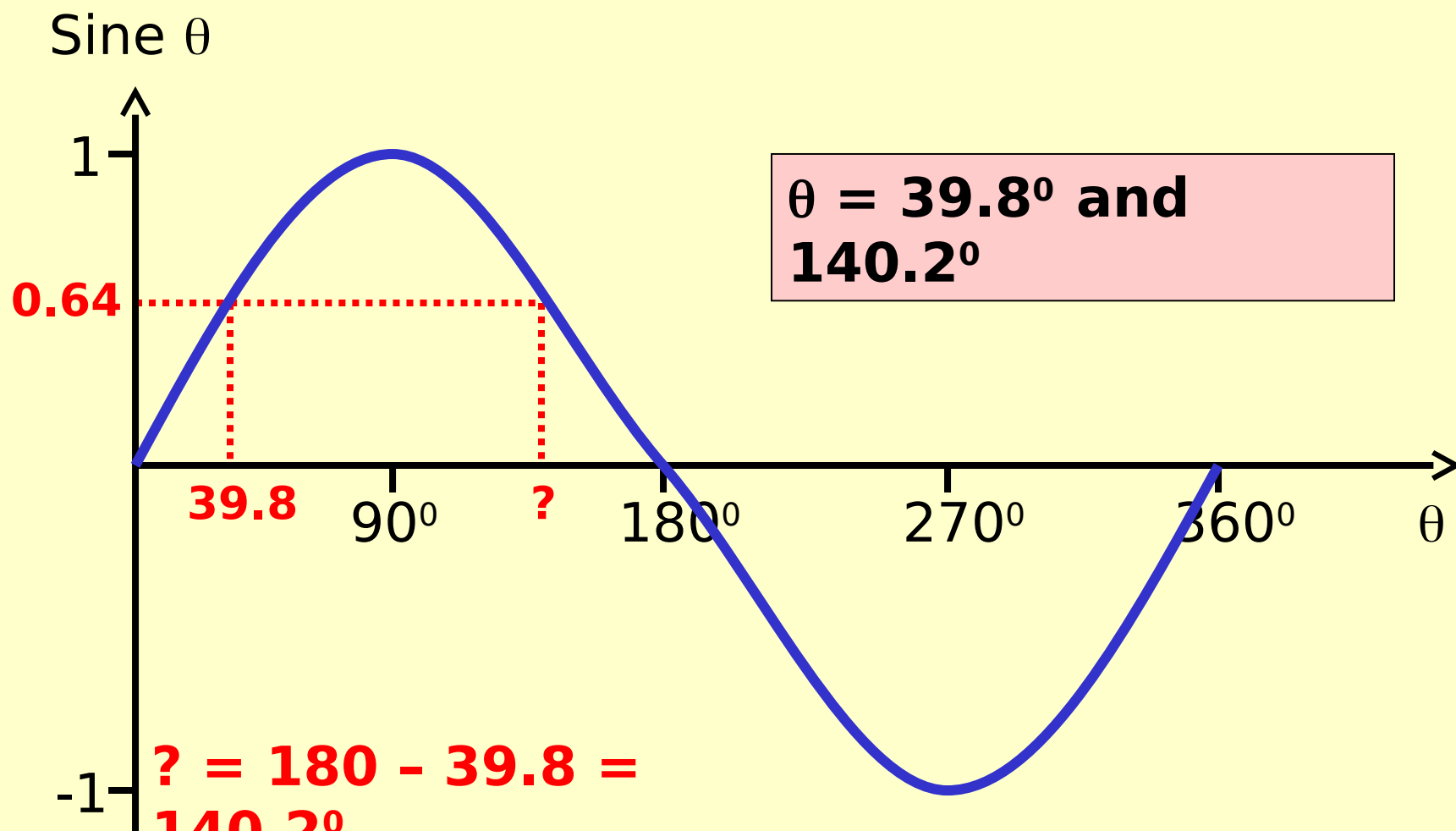


2b Calculate the angle between a sloping face and the base



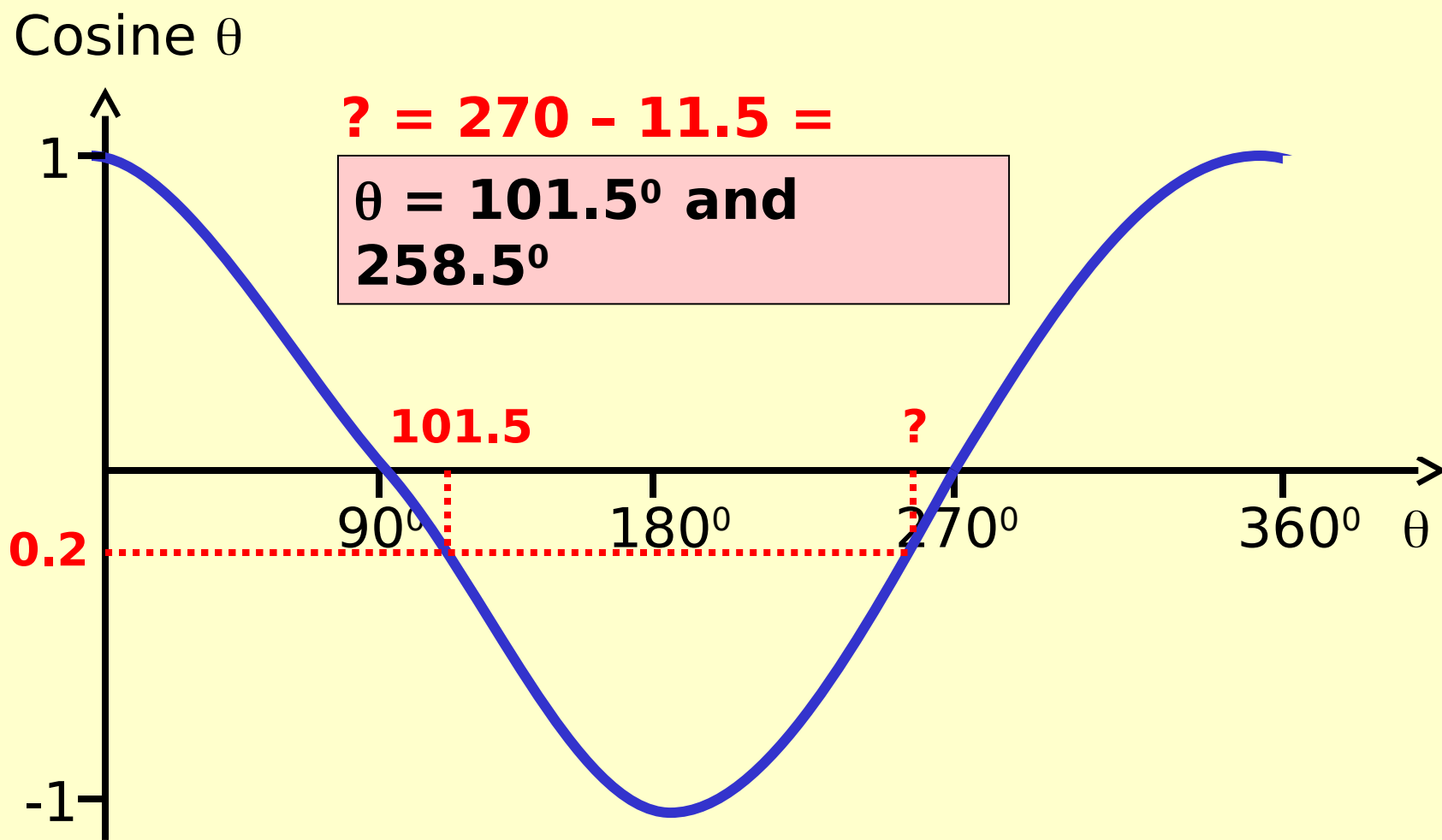
Trig of angles $> 90^\circ$ - The Sine Curve

We can use this graph to find all the angles (from 0 to 360) which satisfy the equation: **$\sin \theta = 0.64$**
The first angle is found on your calculator **$\text{INV}, \sin, 0.64$** **$\theta = 39.8^\circ$** .
You then use the symmetry of the graph to find any others.



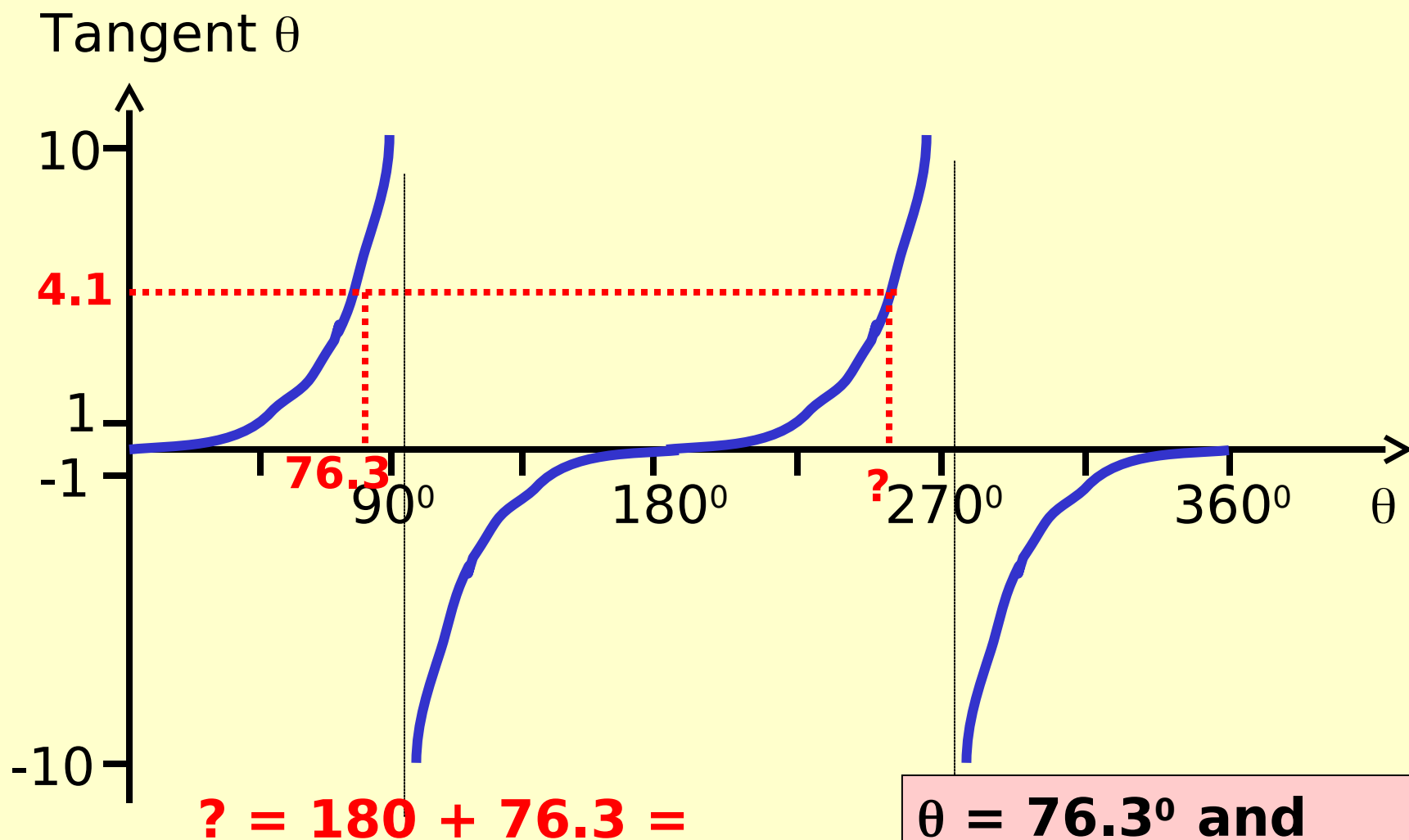
Trig of angles $> 90^\circ$ - The Cosine Curve

We can use this graph to find all the angles (from 0 to 360) which satisfy the equation: **$\cos \theta = -0.2$**
Use your calculator for the 1st angle **$\text{INV}, \cos, -0.2$** **$\theta = 101.5^\circ$**
You then use the symmetry of the graph to find any others.



Angles of angles $> 90^\circ$ - The Tangent Curve

You can use this graph to find all the angles (from 0 to 360) which satisfy the equation: **$\tan \theta = 4.1$**
Use your calculator for the 1st angle **$\text{INV}, \tan, 4.1$** **$\theta = 76.3^\circ$**
You then use the symmetry of the graph to find any others.



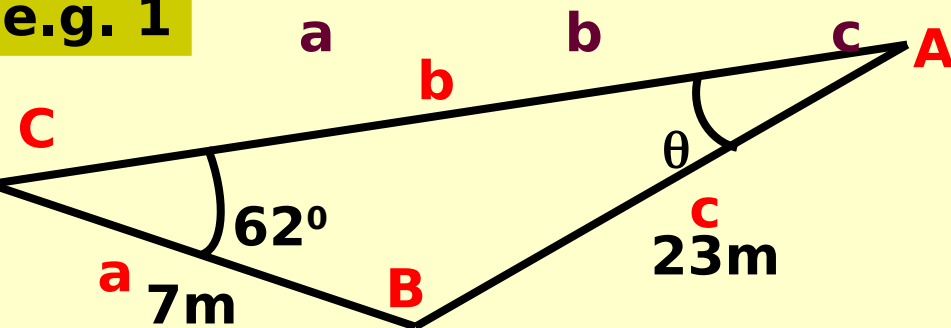
Sine rule

If there are **two angles** involved in the question it's a Sine rule question

Use this version of the rule to find angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

e.g. 1



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin \theta}{7} = \frac{\sin B}{b} = \frac{\sin 62}{23}$$

$$\sin \theta = \frac{\sin 62}{23} \times 7$$

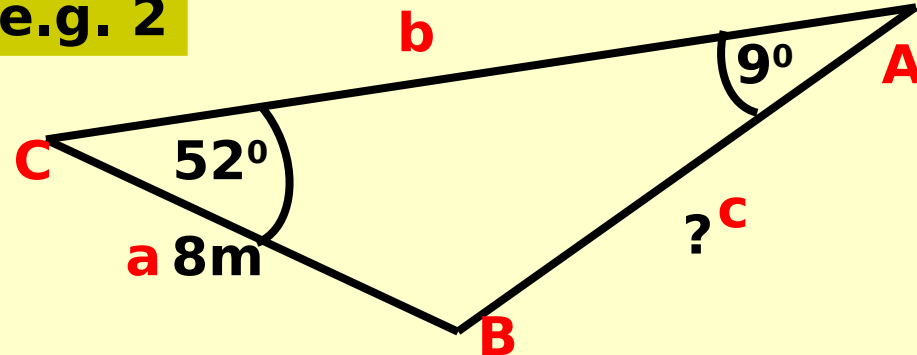
$$\sin \theta = 0.2687$$

$$\theta = 15.6^\circ$$

Use this version of the rule to find sides:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

e.g. 2



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{8}{\sin 9} = \frac{b}{\sin B} = \frac{?}{\sin 52}$$

$$? = \frac{8}{\sin 9} \times \sin 52$$

$$? = 40.3\text{m}$$

Cosine rule

If there is only one angle involved (and all 3 sides) it's a Cosine rule

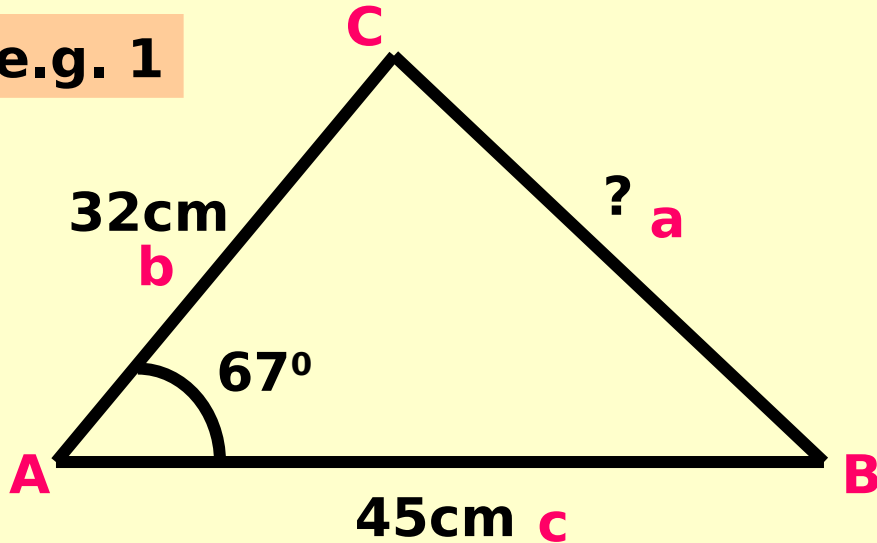
Use this version of the rule to find sides:

Use this version of the rule to find angles:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

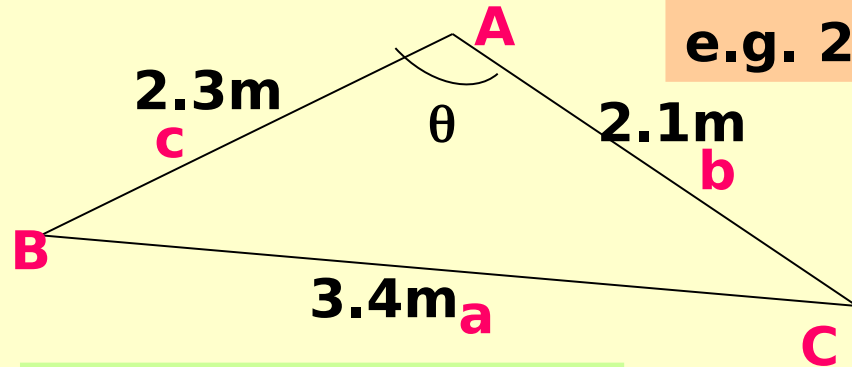
Always label the one angle involved - A

e.g. 1



$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\a^2 &= 32^2 + 45^2 - 2 \times 32 \times 45 \times \cos 67^\circ \\a^2 &= 3049 - 1125.3 \\a &= 43.86 \text{ cm}\end{aligned}$$

e.g. 2



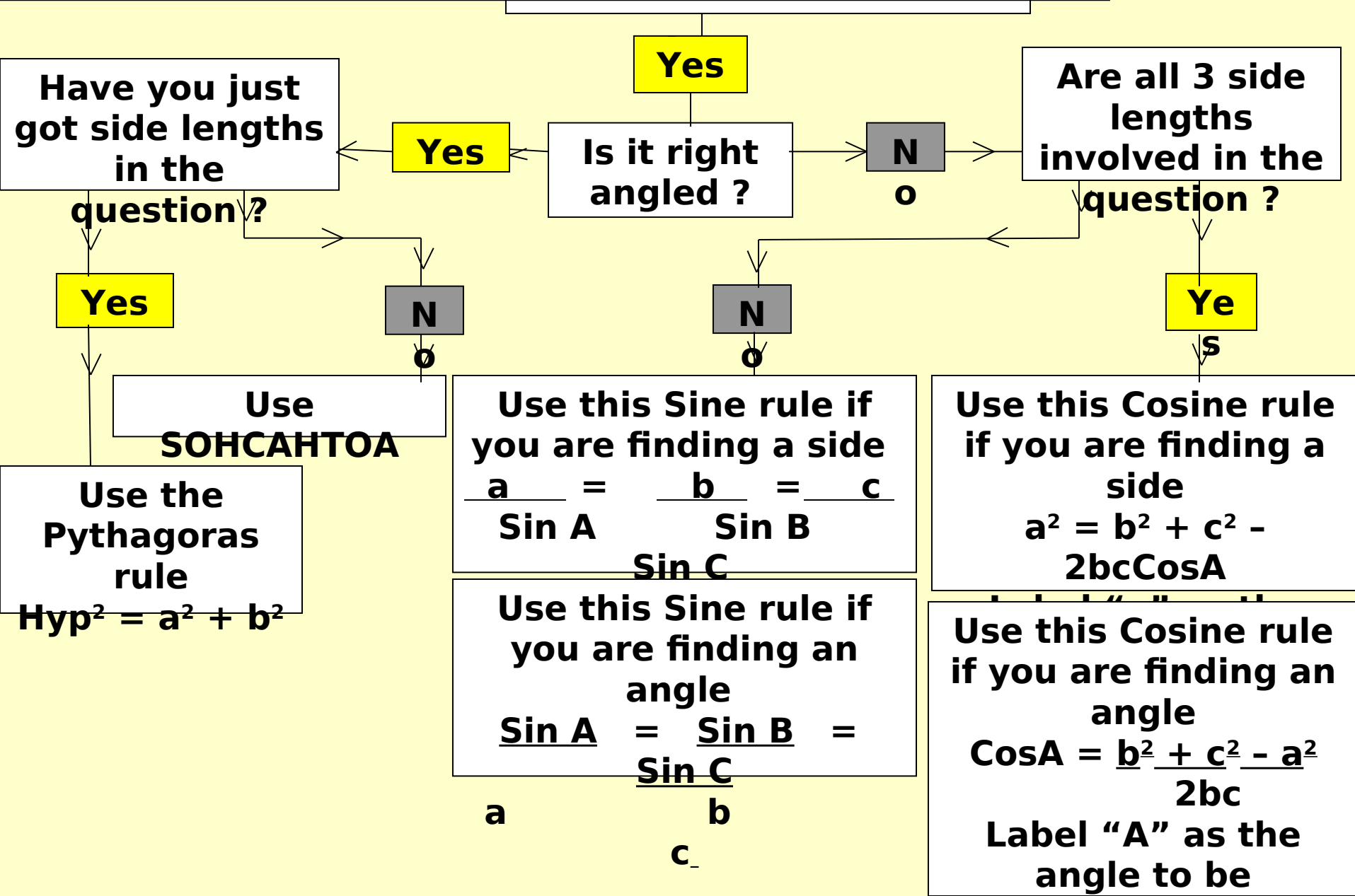
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \theta = \frac{2.1^2 + 2.3^2 - 3.4^2}{2 \times 2.1 \times 2.3}$$

$$\cos \theta = \frac{-1.86}{9.66}$$

$$\theta = 101.1^\circ$$

How to tackle Higher Tier trigonometry questions



**Extra tips
for trig
questions**

**Redraw triangles if
they are cluttered
with information or
they are in a 3D
diagram**

**Right angled triangles
can be easily found in
squares, rectangles
and isosceles
triangles**

**Remember to use
the**

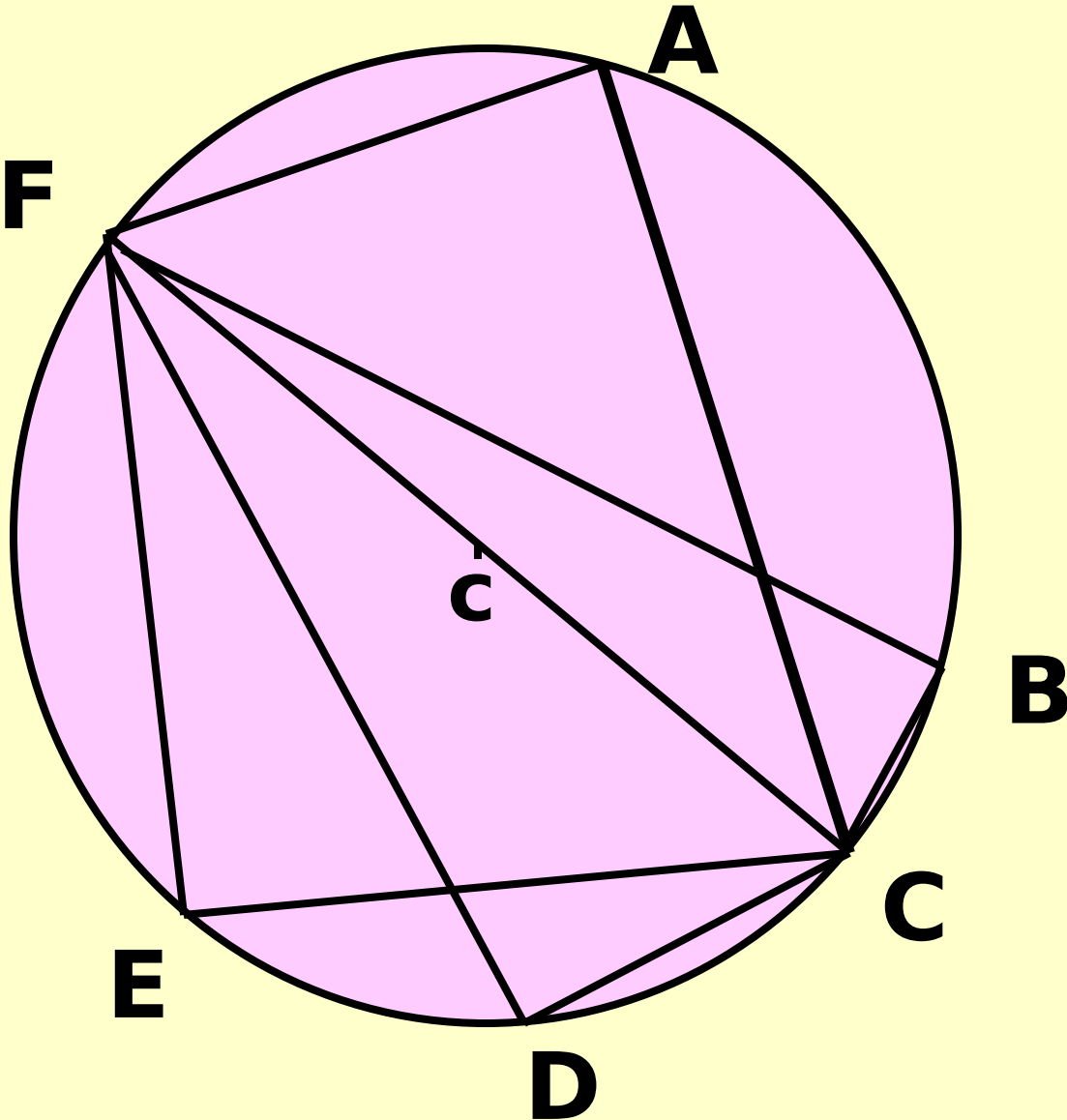
Shift

**Button when
calculating an**

**The ambiguous case
only occurs for sine
rule questions when
you are given the
following information
Angle Side Side in that
order (ASS) which
should be easy to
remember**

Circle angle theorems

Rule 1 - Any angle in a semi-circle is 90°

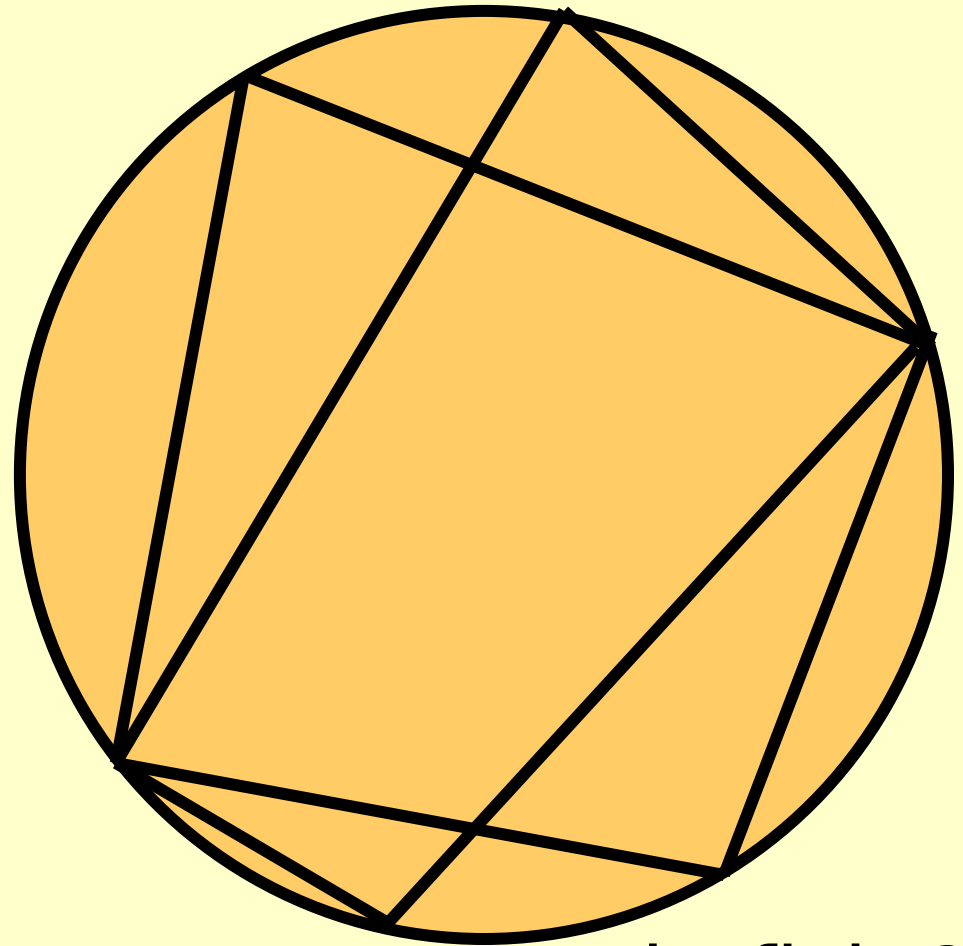


Which angles are equal to 90° ?

Circle angle theorems

Rule 2 - Angles in the same segment are equal

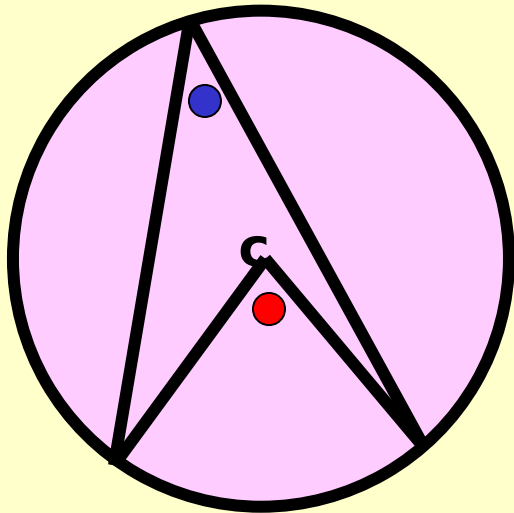
Which angles are equal here?



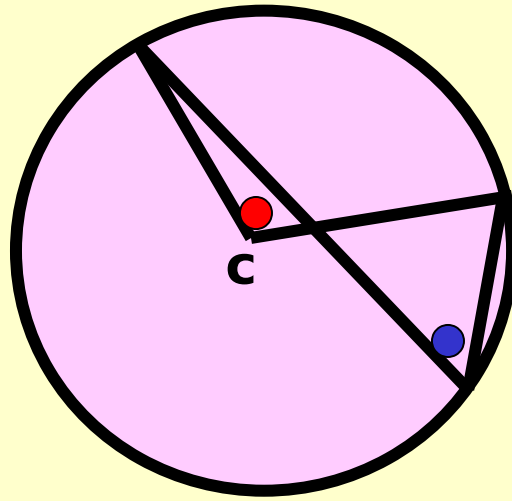
Big fish ?*!

Circle angle theorems

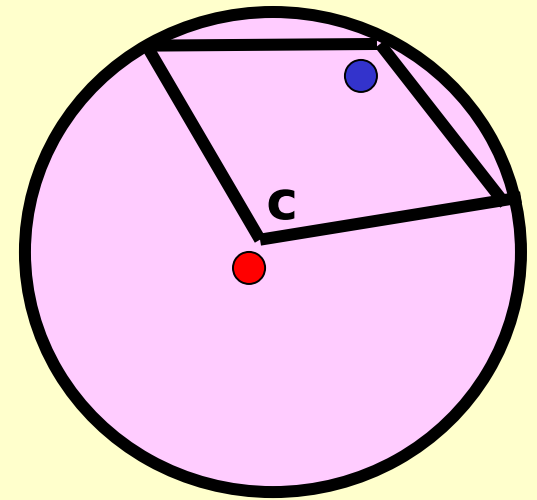
Rule 3 - The angle at the centre ●
is twice the angle at the
circumference ●



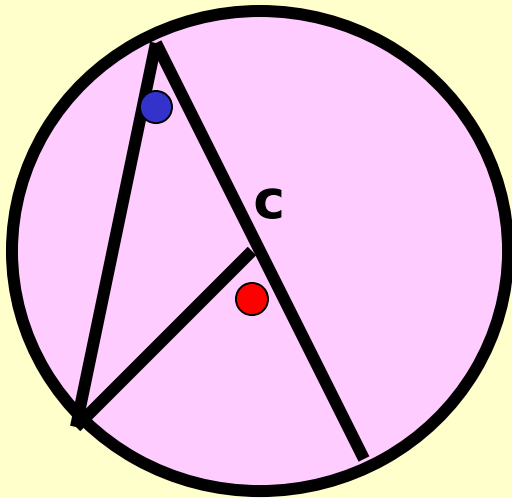
An arrowhead



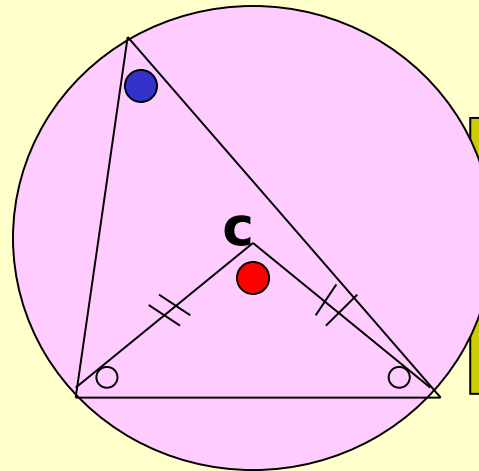
A little fish



A mini quadrilatera



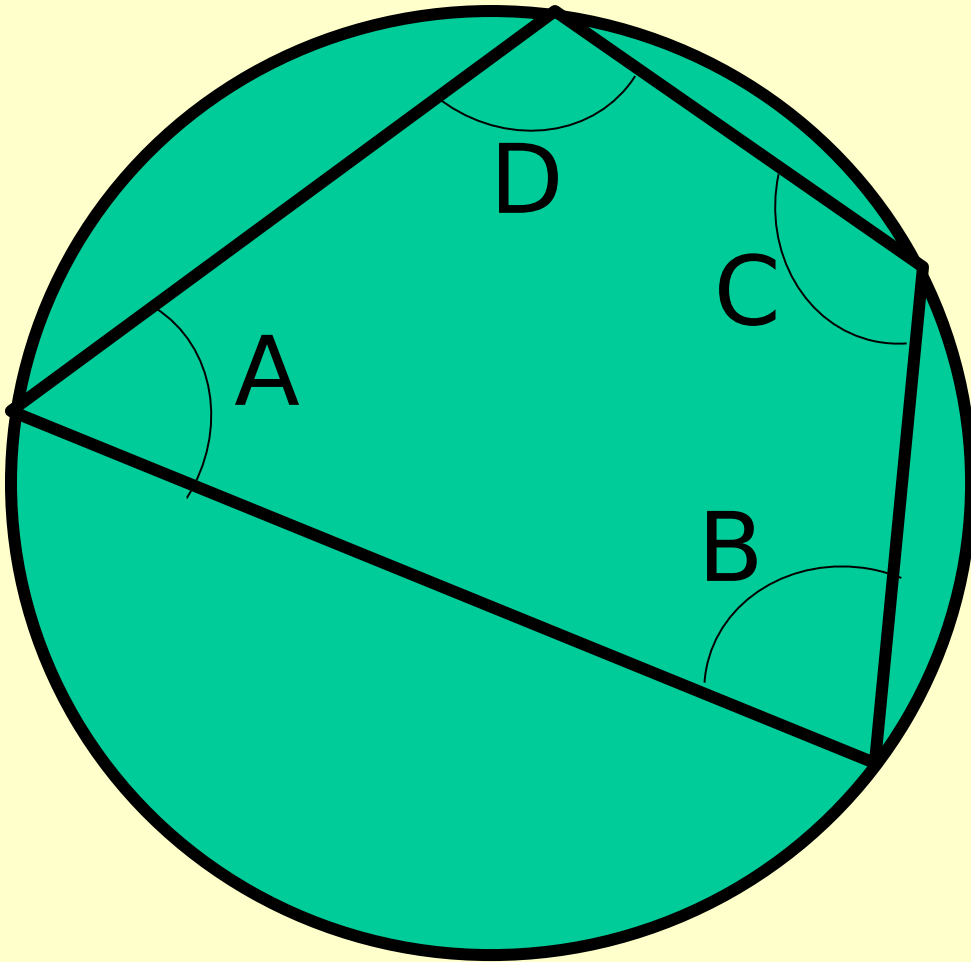
Three radii



Look out for the
angle at the centre
being part of a
isosceles triangle

Circle angle theorems

Rule 4 - Opposite angles in a cyclic quadrilateral add up to 180°



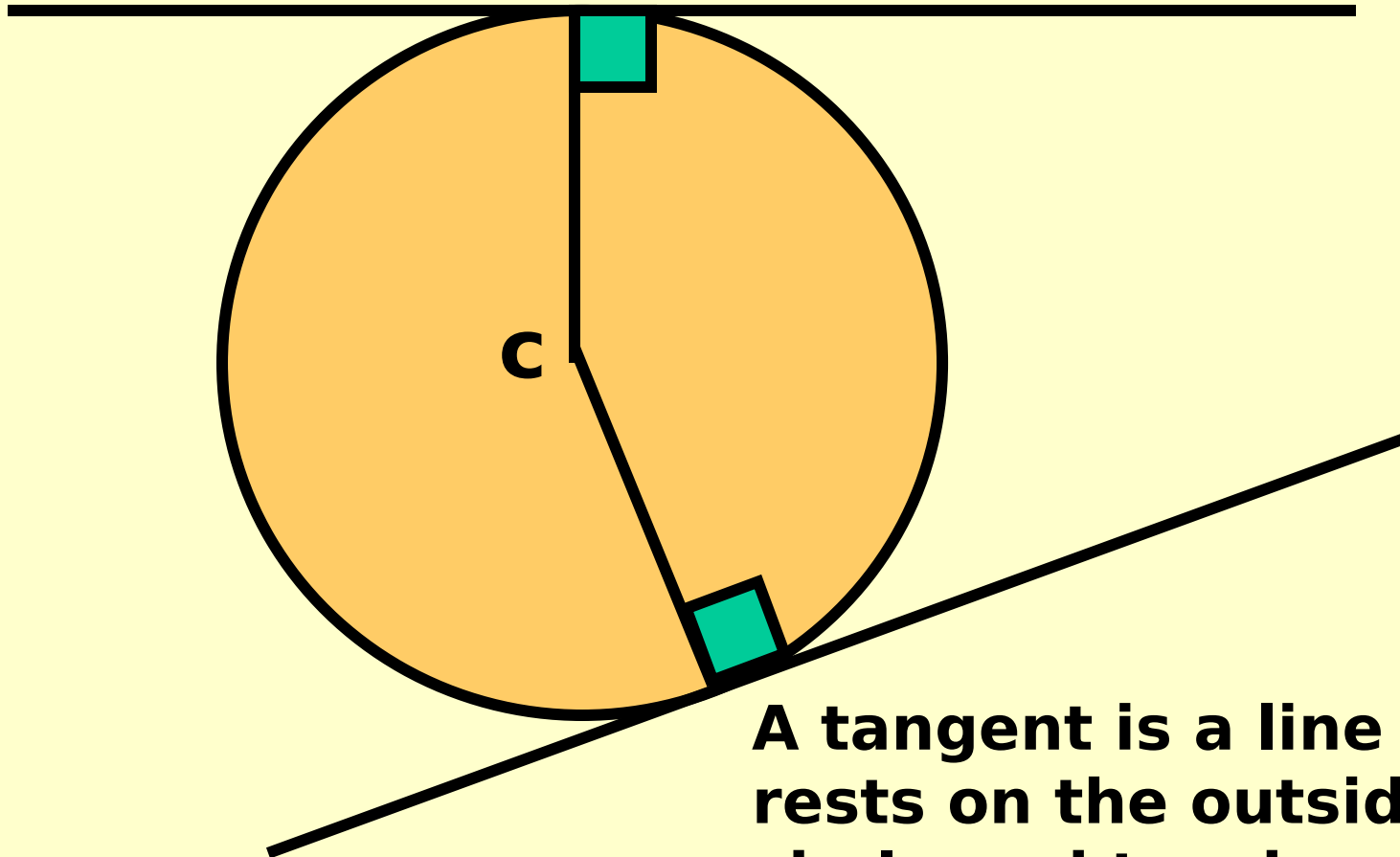
$$A + C = 180^\circ$$

and

$$B + D = 180^\circ$$

Circle angle theorems

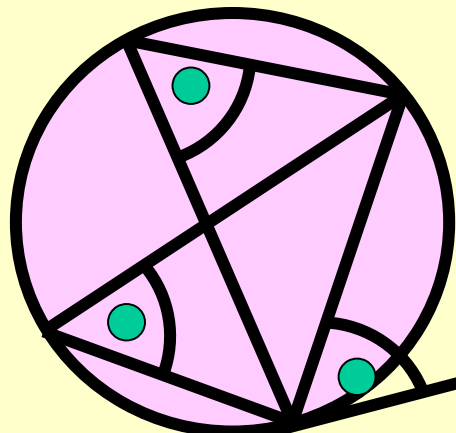
Rule 5 - The angle between the tangent and the radius is 90°



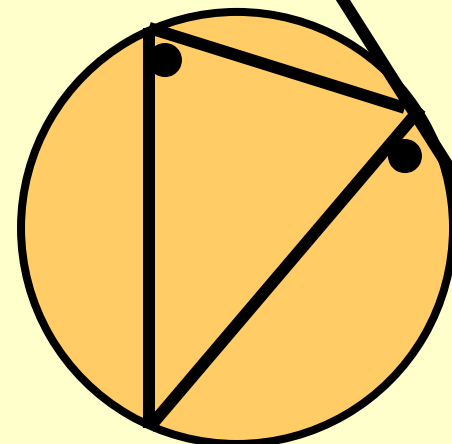
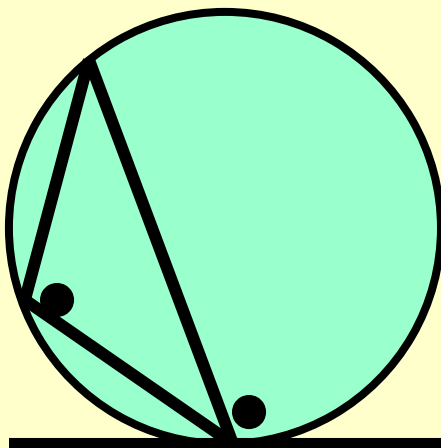
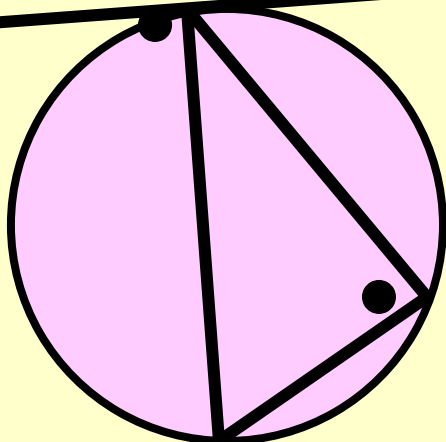
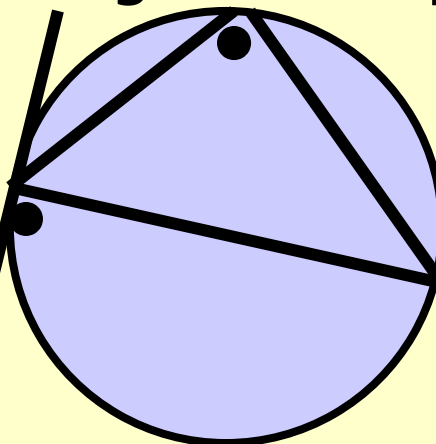
A tangent is a line which rests on the outside of the circle and touches it at one point only

Circle angle theorem

Rule 6 - The angle between the tangent and chord is equal to any angle in the alternate segment



Which angles are equal here?



Circle angle theorems

Rule 7 - Tangent lines and secant lines from an external point are congruent

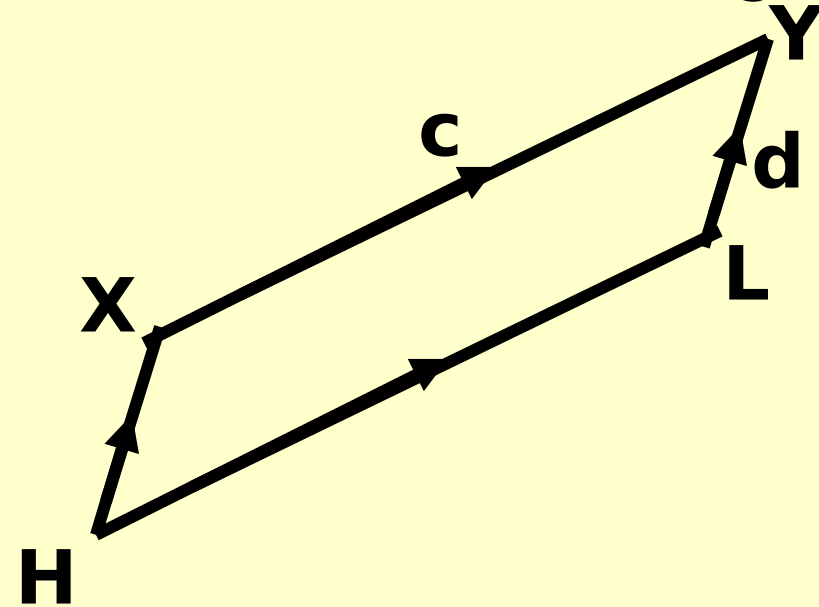
Be prepared to justify
these circle theorems
by **PROVING** that
they work

Vectors

Think of a vector as a “journey” from one place to another. A vector represents a “movement” and it has both magnitude (size) and direction

A vector is shown as a line with an arrow on it

It can be labelled in two ways:
Using a lower case bold letter (usually **a** or **b** - this is the vector's size)
Or using the starting point's letter followed by the destination point's letter with an arrow on top



Find in terms of c and d, the vectors XY, YX, HL, LH, LY, YL, HX, XH, HY, YH, LX, XL (e.g. **GE** - this shows the direction and size)

$$\overrightarrow{XY} =$$

c

$$\overrightarrow{HL} =$$

c

$$\overrightarrow{LY} =$$

d

$$\overrightarrow{HX} =$$

d

$$\overrightarrow{HY} = c +$$

d

$$\overrightarrow{YX} = -$$

c

$$\overrightarrow{LH} = -$$

c

$$\overrightarrow{YL} = -$$

d

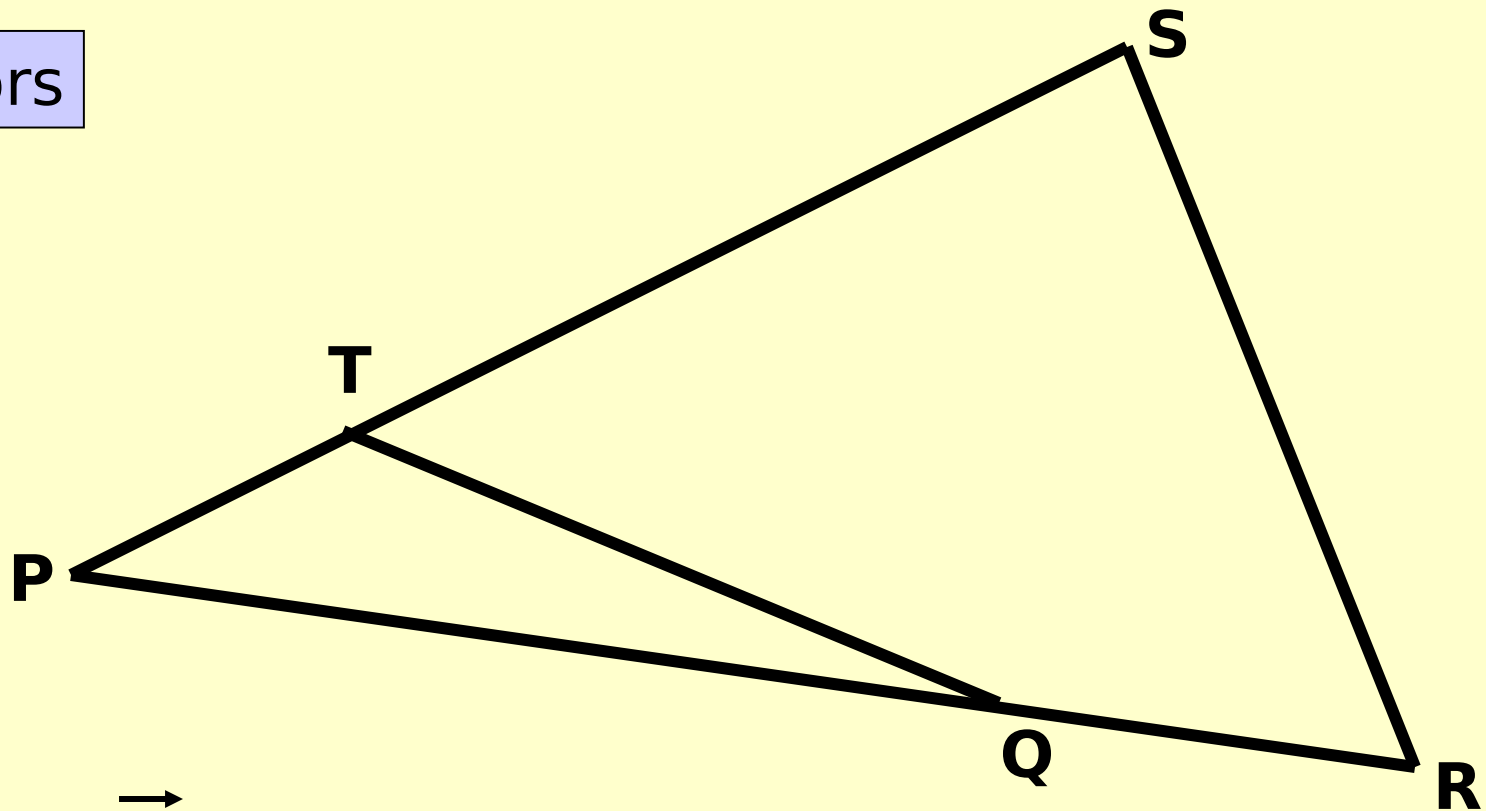
$$\overrightarrow{XH} = -$$

d

$$\overrightarrow{LX} = d -$$

c

Vectors



If $\vec{PS} = \mathbf{a}$, $\vec{PR} = \mathbf{b}$, Q cuts the line PR in the ratio 2:1 and T cuts the line PS in the ratio 1:3, find the value of :

(a) $\vec{PT}$

(b) $\vec{SR}$

(c) $\vec{PQ}$

(d) $\vec{QT}$

e) $\vec{QS} = \vec{QR} + \vec{RS}$ so $\vec{QS} = \frac{1}{3} \mathbf{b} - (-\mathbf{a} + \mathbf{b})$

Remember $\vec{SR} = -\mathbf{a} + \mathbf{b}$ $\vec{QS} = -\frac{2}{3} \mathbf{b} + \mathbf{a}$