

"Teach A Level Maths"

**Vol. 1: AS Core
Modules**

**20: The Mid-Ordinate
Rule**

Module

C3

AQA

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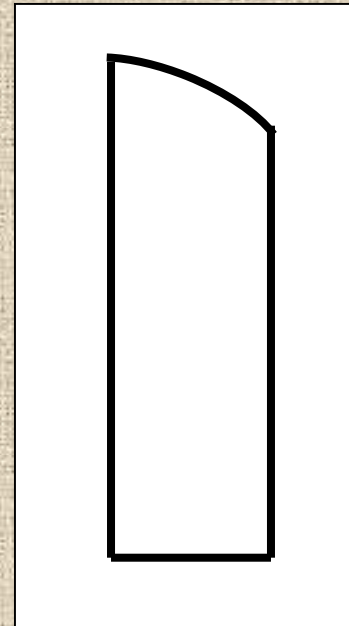
To find an area bounded by a curve, we need to evaluate a definite integral.

If the integral cannot be evaluated, we can use an approximate method.

You have already met the Trapezium rule for doing this. This presentation uses another method, the **mid-ordinate rule.**

As before, the area under the curve is divided into a number of strips of equal width. However, this time, the top edge of each strip

'is' replaced by a straight line so the strips become rectangles.



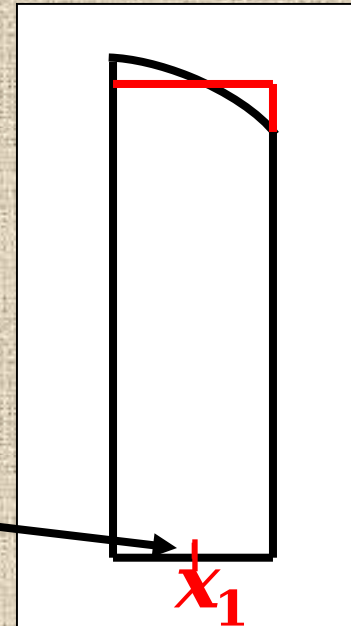


As before, the area under the curve is divided into a number of strips of equal width. However, this time, the top edge of each strip

is replaced by a straight line so the strips become rectangles.

The top of the rectangle is drawn at the point on the curve whose x -value is at the middle of the strip.

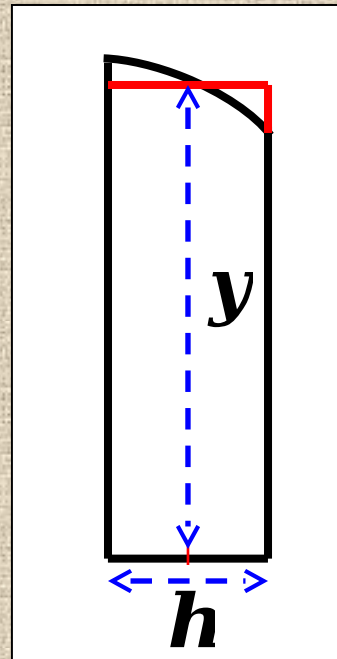
The total area of the rectangles gives an approximation to the area under the curve.





To find the area of a strip we need

width $\times$ height



$$\text{Area} = h \times y$$

The width is h (as in the Trapezium rule)

The height is y (the value of the function at the **mid-point** of the base)

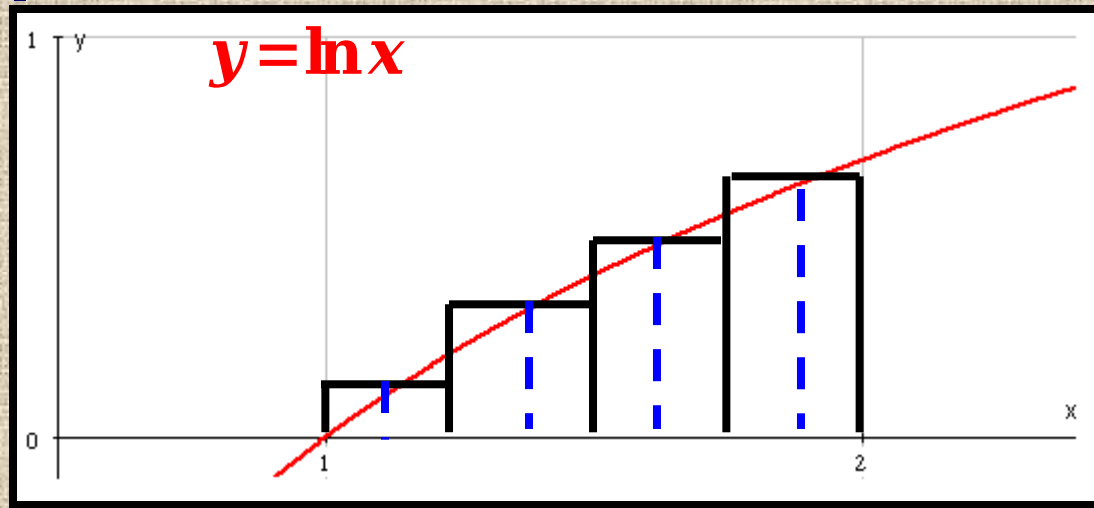
The total area is $h(y_1 + y_2 + \dots + y_n)$



e.g.1 Use 4 strips with the mid-ordinate rule to estimate the value of

$$\int_1^2 \ln x \, dx$$

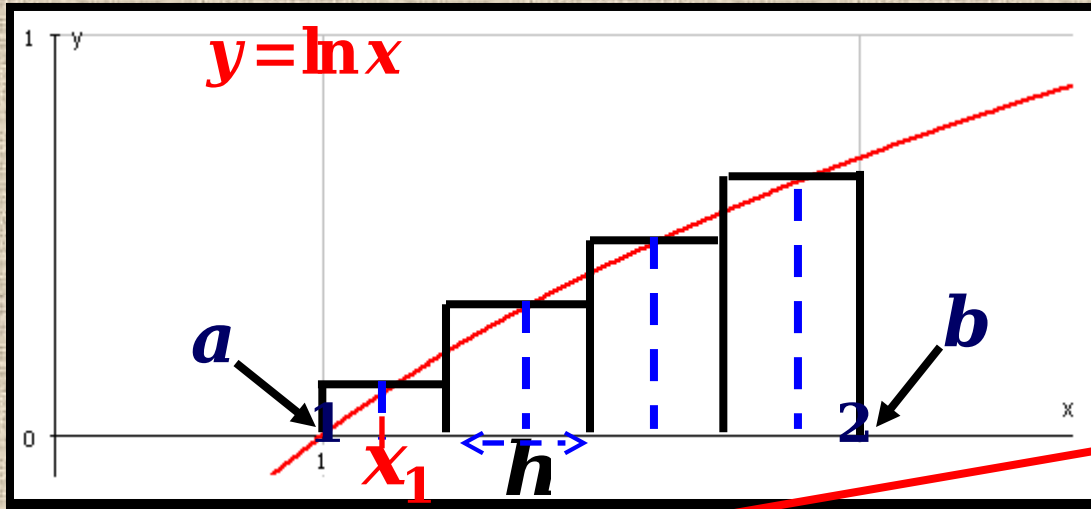
Solution:
Give the answer to 4 d.p.



Notice that the number of x-values is the same as the number of strips.



$$\text{Area} \approx h(y_1 + y_2 + \dots + y_n); \quad n=4$$



$$h = \frac{b - a}{4} = 0.25$$

N.B.

$$x_1 = a + \frac{h}{2}$$

$$\Rightarrow x_1 = 1.125$$

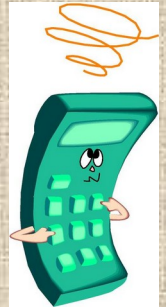
So,

x	1.125	1.375	1.625	1.875
y	0.11778	0.31845	0.48551	0.62861

2

$$\int_1^2 \ln x \, dx \approx 0.25 (y_1 + y_2 + y_3 + y_4)$$

$$\approx 0.3876 \text{ (4 d.p.)}$$





SUMMAR

- The mid-ordinate rule for estimating an area is

$$\int_a^b y dx = h (y_1 + y_2 + y_3 + \dots + y_n)$$

where n is the number of strips.

- The width, h , of each strip is given by $h = \frac{b - a}{n}$ (but should be checked on a sketch)

- The 1st x -value is at the mid-point of the width of the 1st rectangle

$$x_1 = a + \frac{h}{2}$$

- The number of ordinates is the same as the number of strips.

- The accuracy can be improved by increasing n .



Exercise

S



1. Estimate $\int_0^2 \frac{1}{1+x^2} dx$ using the mid-ordinate rule with 4 strips, giving your answer to 3 d.p. How can your answer be improved?

2. Estimate $\int_0^\pi \sin^2 x dx$ using the mid-ordinate rule with 3 strips. Give your answer to 3 s.f.

**N.B.
Radians !**



Solution

S

$$1. \int_0^2 \frac{1}{1+x^2} dx$$

$$n=4,$$

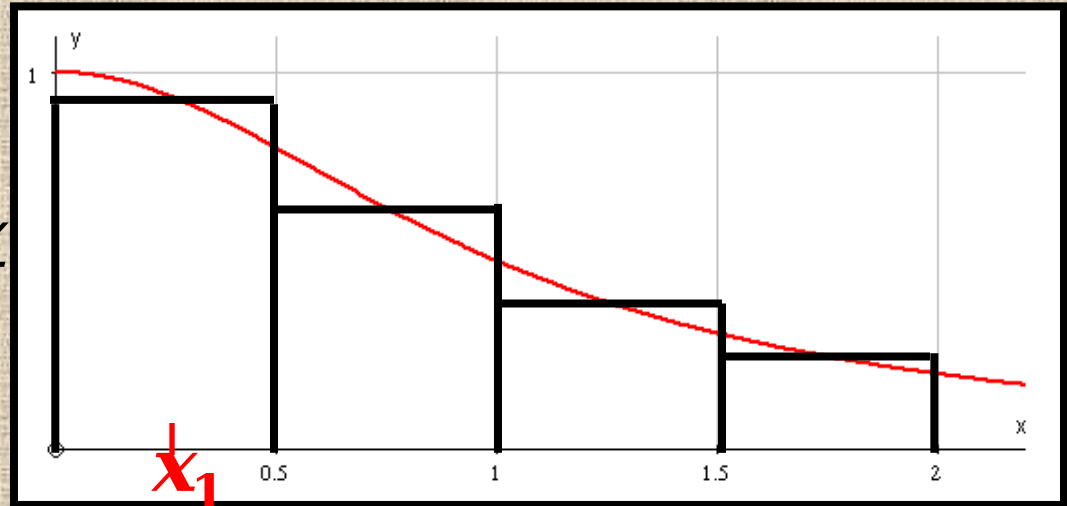
$$h=0.5$$

$$A \approx h(y_1 + y_2 + y_3 + y_4)$$

x	0.25	0.75	1.25	1.75
y	0.9412	0.64	0.3902	0.2462

$$A \approx 0.5(y_1 + y_2 + y_3 + y_4) \approx 1.109 \text{ (3 d. p.)}$$

The answer can be improved by using more strips.





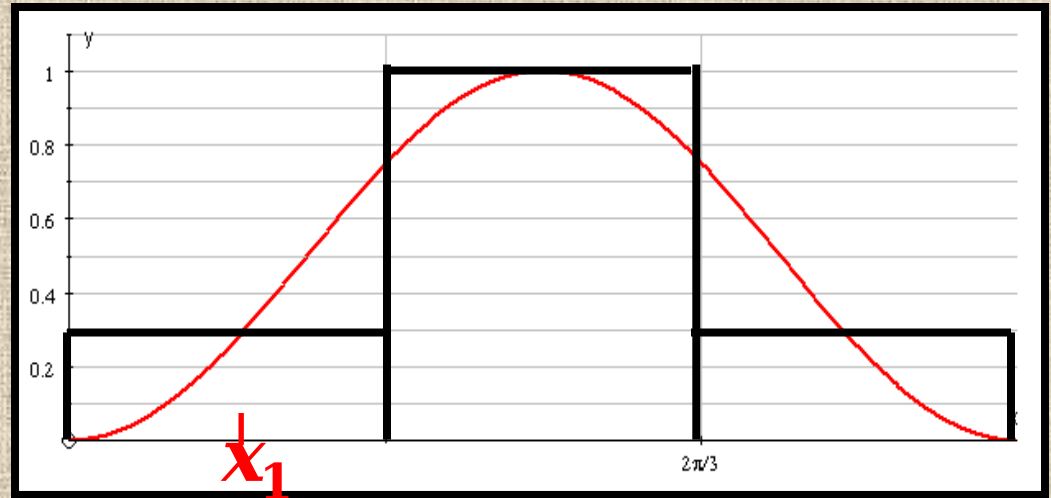
Solution

s

$$\int_0^{\pi} \sin^2 x \, dx$$

$$n=3,$$

$$h=\frac{\pi}{3}$$

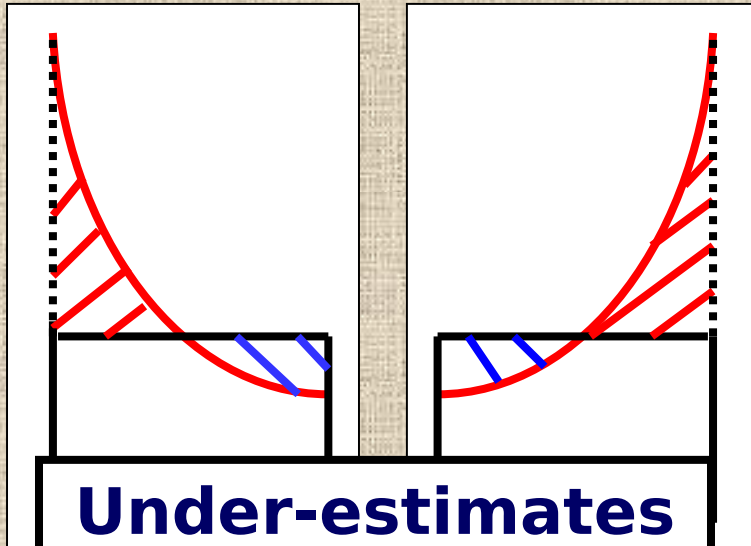


x	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$
y	0.25	1	0.25

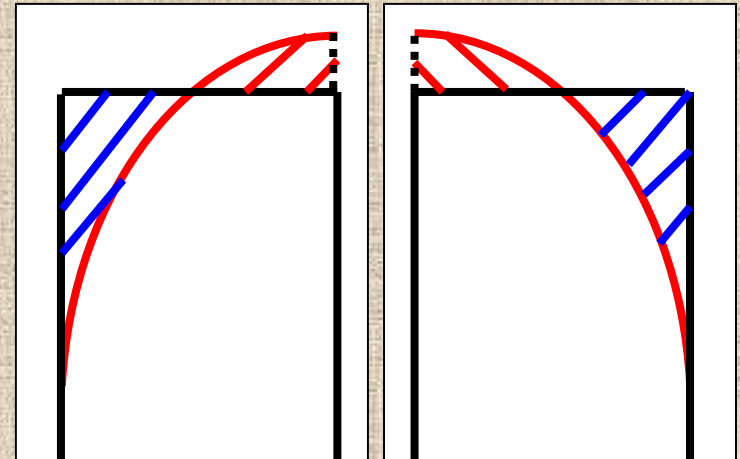
$$A \approx h (y_1 + y_2 + y_3) \approx 1.57 \quad (3 \text{ s. f.})$$



The following sketches show sample rectangles where the mid-ordinate rule under- and over-estimates the area.



**Under-estimates
(concave
upwards)**



**Over-estimates
(concave
downwards)**

The **blue** shaded areas are not under the curve but are included in the rectangle.

The **red** shaded areas should be included but are not.

The following slides contain repeats of information on earlier slides, shown without colour, so that they can be printed and photocopied. For most purposes the slides can be printed as “Handouts” with up to 6 slides per sheet.

SUMMAR

- The mid-ordinate law for estimating an area is

$$\int_a^b y dx = h (y_1 + y_2 + y_3 + \dots + y_n)$$

where n is the number of strips.

- The width, h , of each strip is given by $h = \frac{b - a}{n}$ (but should be checked on a sketch)

- The 1st x -value is at the mid-point of the width of the 1st rectangle

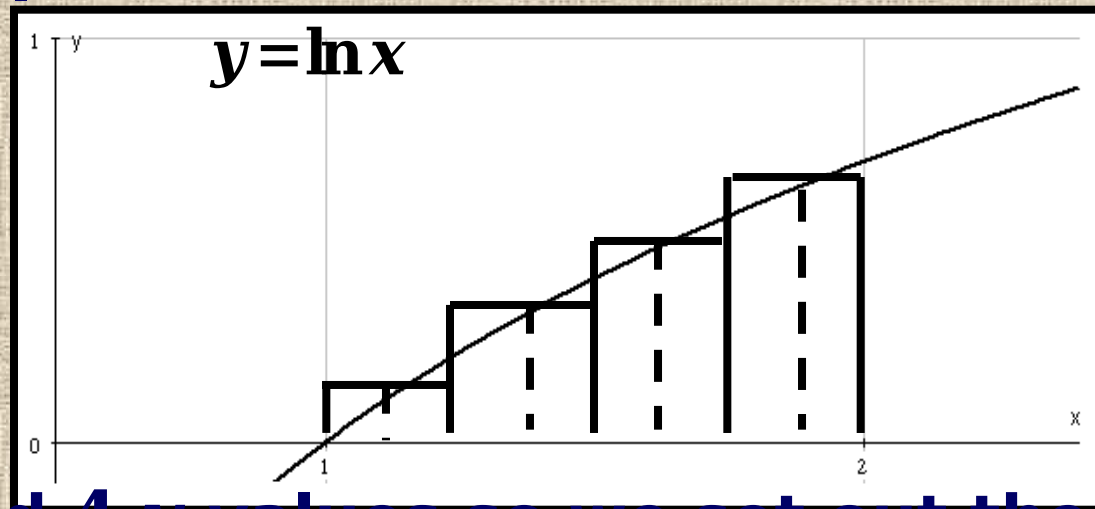
$$x_1 = a + \frac{h}{2}$$

- The number of ordinates is the same as the number of strips.
- The accuracy can be improved by increasing n .

e.g.1 Use 4 strips with the mid-ordinate rule to estimate the value of

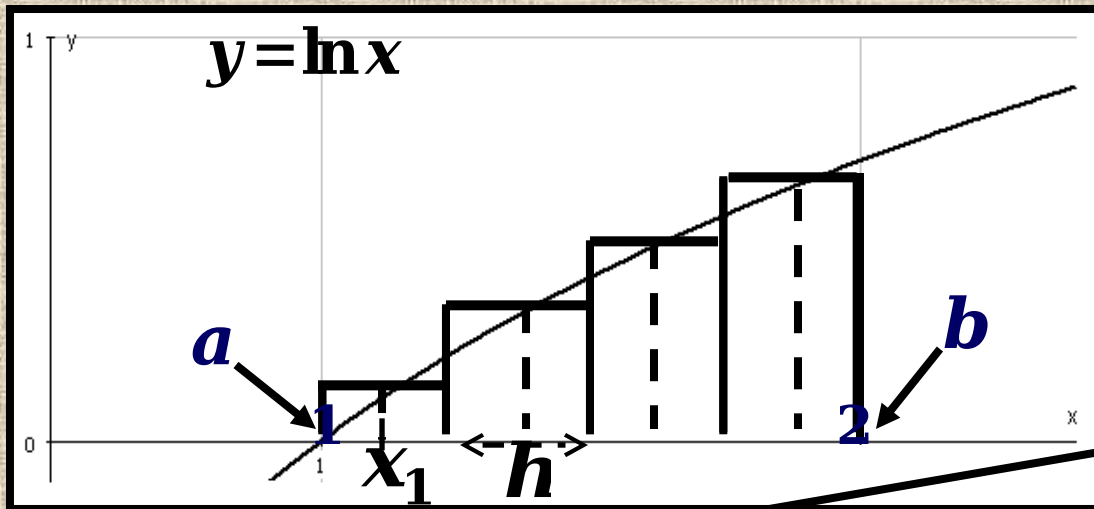
$$\int_1^2 \ln x \, dx$$

Solution
Give the answer to 4 d.p.
:



We need 4 y -values so we set out the calculation in a table as for the Trapezium rule.

$$\text{Area} \approx h(y_1 + y_2 + \dots + y_n); \quad n=4$$



$$h = \frac{b - a}{4} = 0.25$$

N.B.

$$x_1 = a + \frac{h}{2}$$

$$\Rightarrow x_1 = 1.125$$

So,

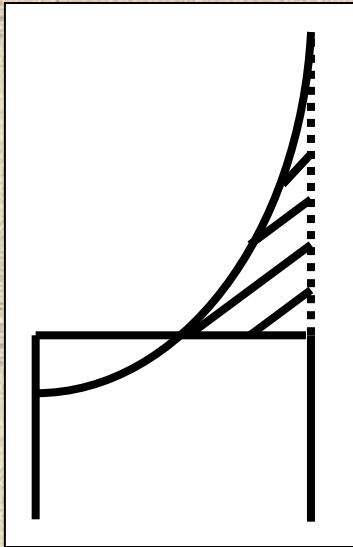
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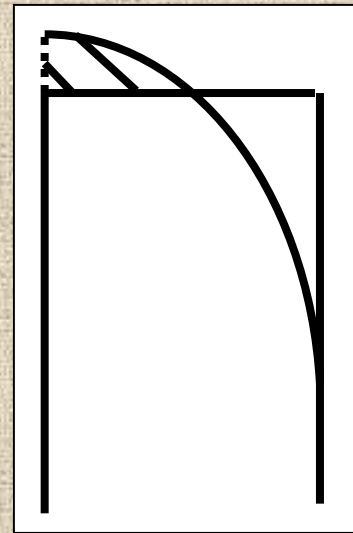
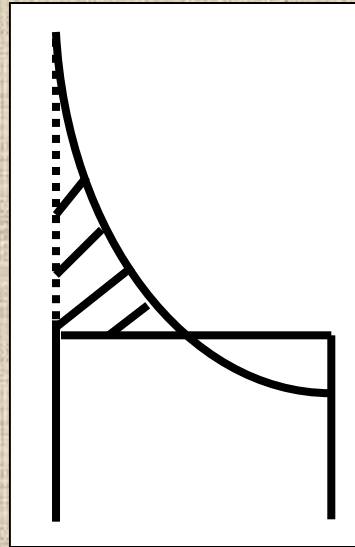
$$\int_1^2 \ln x \, dx \approx 0.25 (y_1 + y_2 + y_3 + y_4)$$

$$\approx 0.3876 \text{ (4 d.p.)}$$

The following sketches show sample rectangles where the mid-ordinate rule under- and over estimates the area.



**Underestimates
(concave
upwards)**



**Overestimates
(concave
downwards)**