

WAVE FUNCTIONS



What is a Wave Function



Connection with Trig Identities



**Earlier
Maximum and Minimum Values**



**Solving Equations involving the Wave
Function**



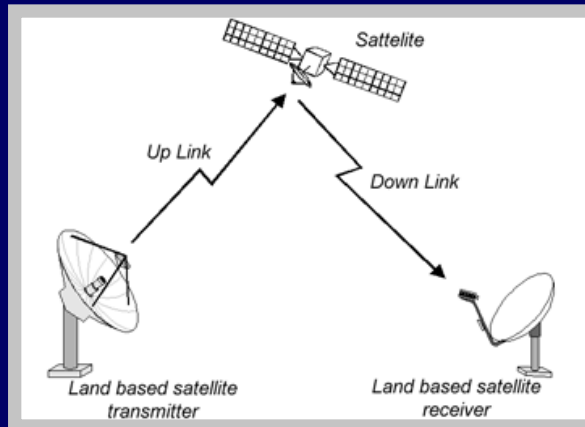
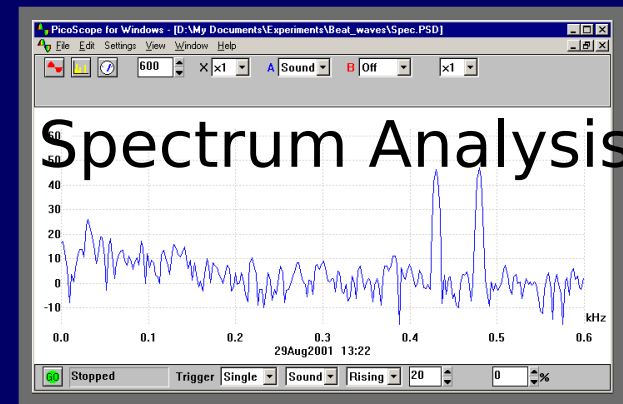
Exam Type Questions



The Wave Function

Expressing $a\cos x + b\sin x$ in the form $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$.

Many wave shapes, whether occurring as sound, light, water or electrical waves, can be described mathematically as a combination of sine and cosine waves.

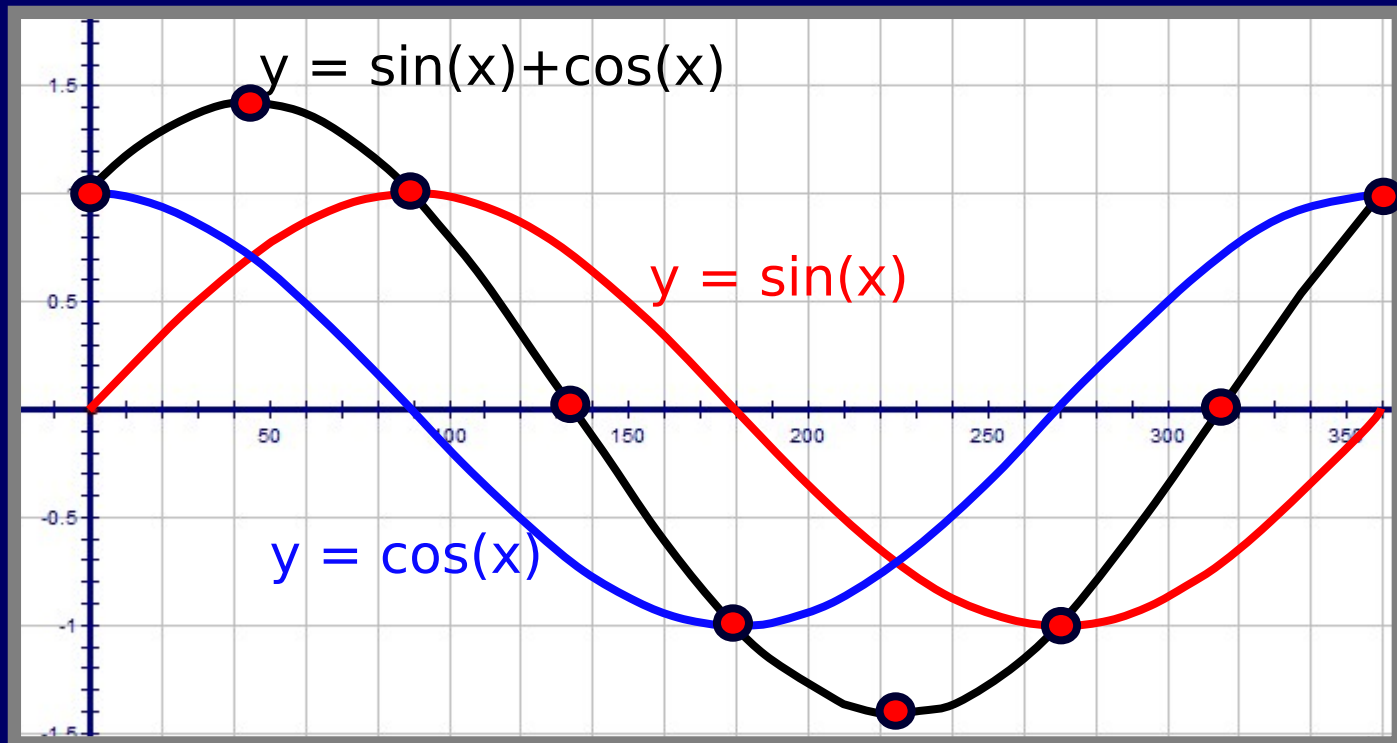


General shape for $y = \sin x + \cos x$

1. Like $y = \sin(x)$ shifted **left**

2. Like $y = \cos x$ shifted **right**

3. Vertical height different



The Wave Function

Whenever a function is formed by adding cosine and sine functions the result can be expressed as a related cosine or sine function. In general:

$$a \cos x + b \sin x = k \cos(x \pm \alpha)$$

or

$$a \cos x + b \sin x = k \sin(x \pm \alpha)$$

a, b, k and α are constants

Given a and b we can calculate k and α

With these constants the expressions on

the right hand sides $=$ those on the left hand side

FOR ALL VALUES OF

The Wave Function

Worked Example:

Write $4\cos x + 3\sin x$ in the form $k\cos(x - \alpha)^\circ$, where $0 \leq \alpha \leq 360^\circ$

$$4\cos x + 3\sin x = k\cos(x - \alpha)^\circ$$

Remember !!!!!

$$\cos(x - \alpha) = \cos x \cos \alpha + \sin x \sin \alpha$$

$$= k(\cos x \cos \alpha + \sin x \sin \alpha)$$

Re-arrange

$$4\cos x + 3\sin x = k\cos \alpha \cos x + k\sin \alpha \sin x$$

The left and right hand sides must be equal
for all values of x .

So, the coefficients of $\cos x$ and $\sin x$ must be equal:

$$k\cos \alpha = 4$$

$$k\sin \alpha = 3$$

A pair of simultaneous equations to be solved

The Wave Function

$$\begin{aligned}k \cos \alpha &= 4 \\k \sin \alpha &= 3\end{aligned}$$



Square and add
 $\sin(+)$ and
 $\cos(+)$

$$k^2 \cos^2 \alpha + k^2 \sin^2 \alpha = 4^2 + 3^2$$

$$k^2 (\cos^2 \alpha + \sin^2 \alpha) = 16 + 9$$

$$k^2 = 25$$

$$k = 5$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \tan^{-1} \left(\frac{3}{4} \right) = 36.9^\circ$$

The Wave Function

$$k \cos \alpha = 4$$
$$k \sin \alpha = 3$$

$$k = 5$$

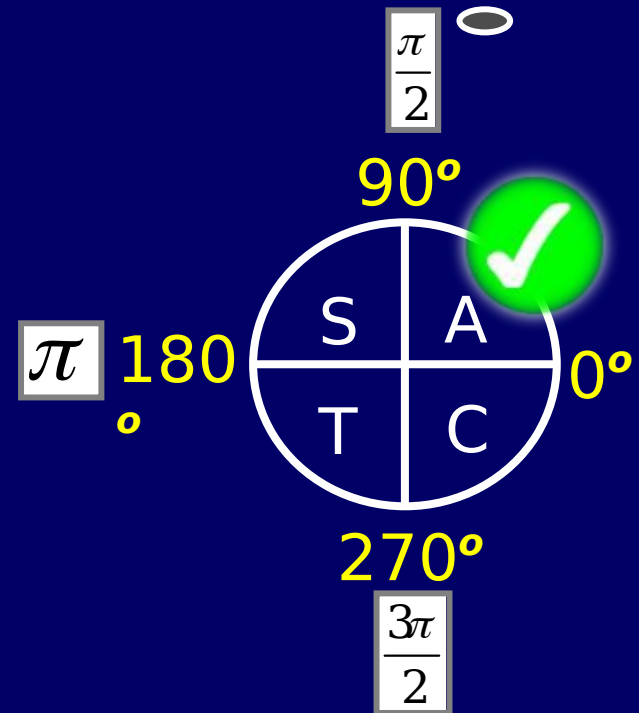
$$\alpha = 36.9^\circ$$

Note:
 $\sin(+)$ and
 $\cos(+)$

$\Rightarrow \alpha$ is in the first quadrant

$$4 \cos x + 3 \sin x = k \cos(x - \alpha)^\circ$$

$$4 \cos x + 3 \sin x = 5 \cos(x - 36.9)^\circ$$



The Wave Function

Expand and
equate
coefficients

Example

Write $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$, where $0 \leq \alpha \leq 2\pi$

$$\cos x - \sqrt{3}\sin x = R\cos x \cos \alpha - R\sin x \sin \alpha$$

$$R\cos \alpha = 1$$

$$R\sin \alpha = \sqrt{3}$$

Square and add

$$R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 1 + 3 \quad R^2 = 4 \quad R = 2$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \left(\frac{\sqrt{3}}{1} \right) = \tan^{-1} \left(\frac{\sqrt{3}}{1} \right) = 60^\circ$$

$\Rightarrow \alpha$ is in the first quadrant

Find tan ratio

note:

$\sin(+)$ and $\frac{\pi}{2}$

$\cos(+90^\circ)$

π

180°



270°

$\frac{3\pi}{2}$

The Wave Function

$$\alpha = \tan^{-1} \left(\frac{\sqrt{3}}{1} \right) = 60^\circ$$

α is in the first quadrant

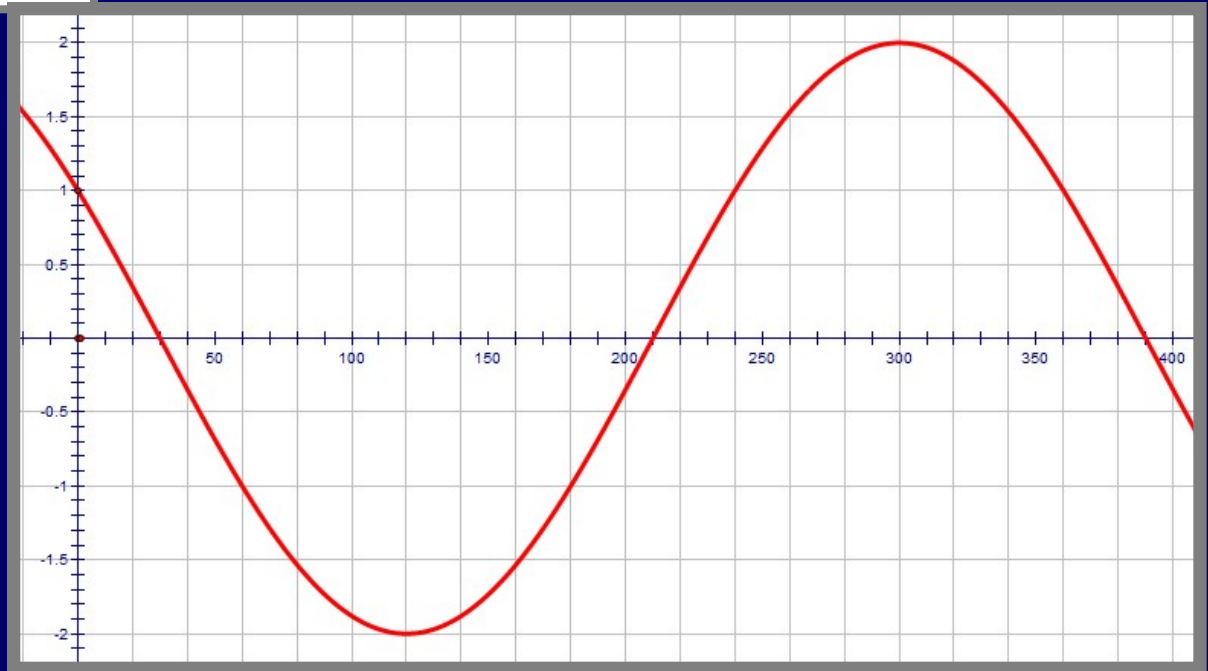
$$180^\circ = \pi$$

$$60^\circ = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

Finally:

$$\cos x + \sqrt{3} \sin x = 2 \cos \left(x + \frac{\pi}{3} \right)$$



The Wave Function

Expand and
equate
coefficients

Example

Write $5\cos 2x + 12\sin 2x$ in the form $k\sin(2x + \alpha)$, where $0 \leq \alpha \leq 360^\circ$

$$5\cos 2x + 12\sin 2x = k\sin 2x \cos \alpha + k\cos 2x \sin \alpha$$

$$k\cos \alpha = 12$$

$$k\sin \alpha = 5$$

Square and add

$$k^2 \cos^2 \alpha + k^2 \sin^2 \alpha = 12^2 + 5^2$$

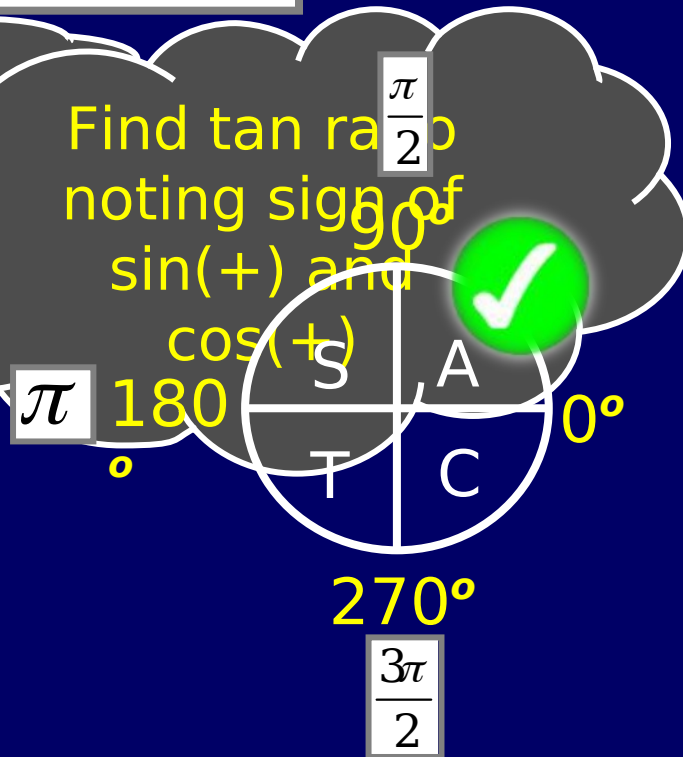
$$k^2 = 13^2$$

$$k = 13$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \tan^{-1} \left(\frac{5}{12} \right) = 22.6^\circ$$

$\Rightarrow \alpha$ is in the first quadrant

Find tan ratio
noting sign of
sin(+) and
cos(+)

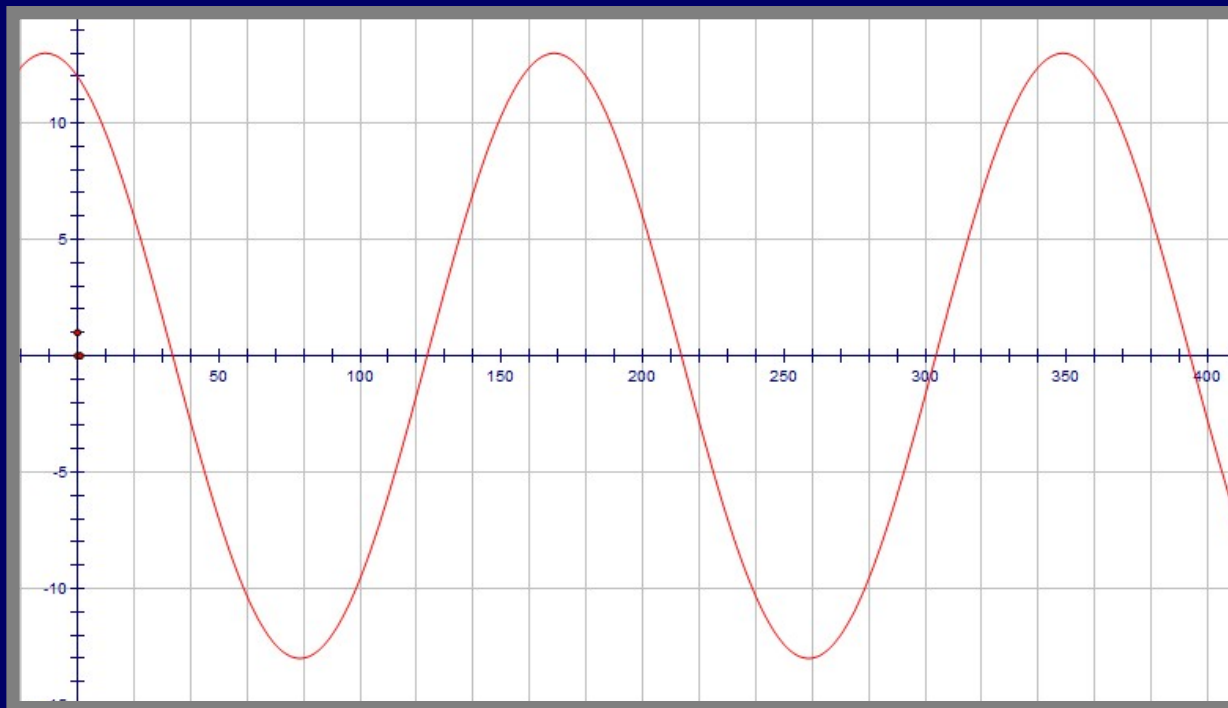


The Wave Function

$$\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$$

α is in the first quadrant

Finally: $5\cos 2x + 12\sin 2x = 13\sin(2x + 22.6)^\circ$



Maximum and Minimum Values

Worked Example:

a) Write $\sin x - \cos x$ in the form $k \cos(x - \alpha)$

b) Hence find:

- i) Its maximum value and the value of x at which this maximum occurs.
- ii) Its minimum value and the value of x at which this minimum occurs.

Maximum and Minir

Expand and
equate
coefficients

$$\sin x - \cos x = k \cos(x - \alpha) = k \cos x \cos \alpha + k \sin x \sin \alpha$$

$$k \cos \alpha = -1$$

$$k \sin \alpha = 1$$

$$k^2 (\cos^2 \alpha + \sin^2 \alpha) = 1 + 1 = 2$$

$$\therefore k = \sqrt{2}$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{1}{-1} \Rightarrow \alpha = \tan^{-1}(-1) = 45^\circ$$

$\Rightarrow \alpha$ is in the 2nd quadrant

$$\therefore \alpha = 135^\circ$$

Square and add

Find tan ratio
note: 

sin and

cos

π

$\frac{\pi}{2}$

90°

0°

270°

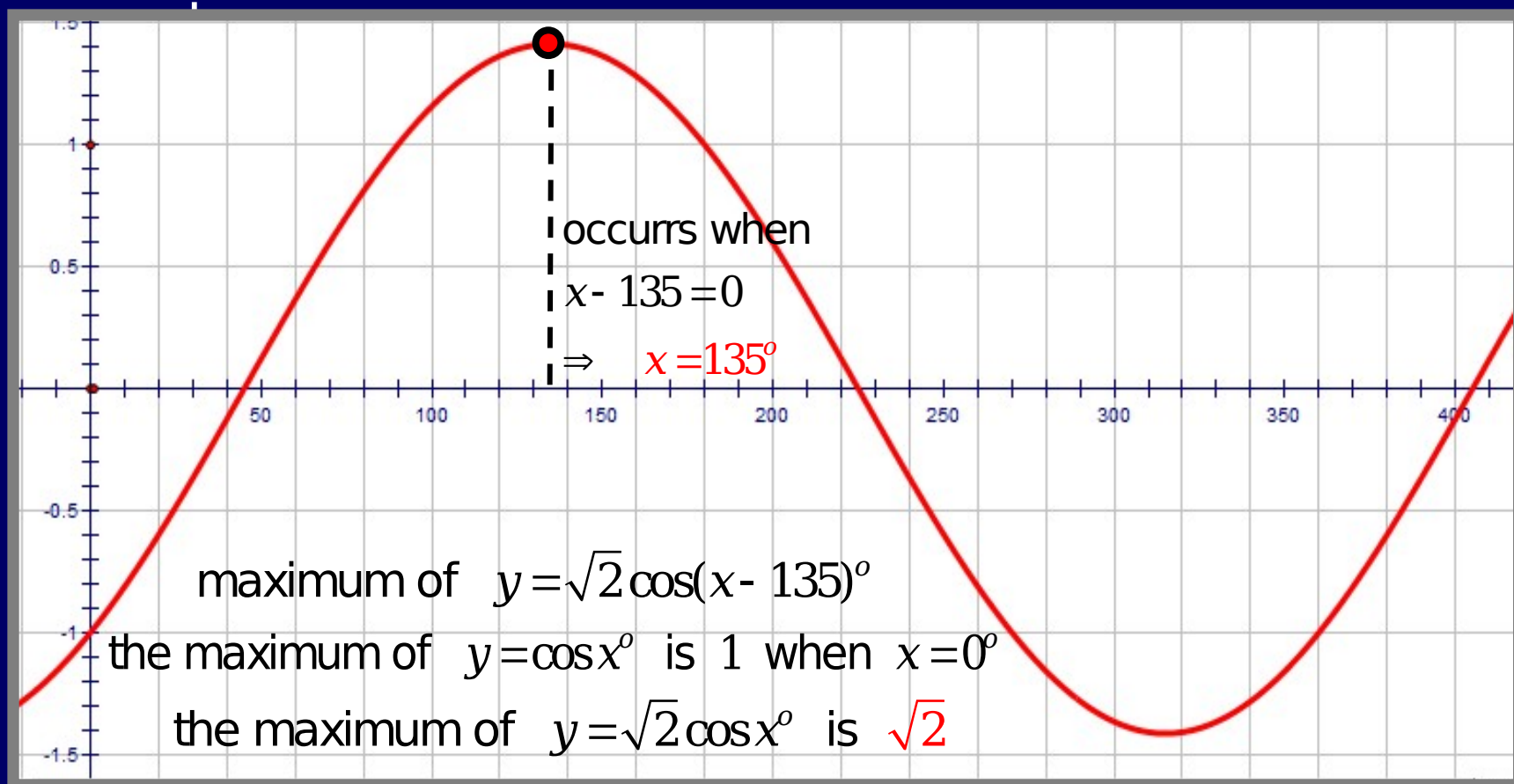
$\frac{3\pi}{2}$



Maximum and Minimum Values

Maximum, we

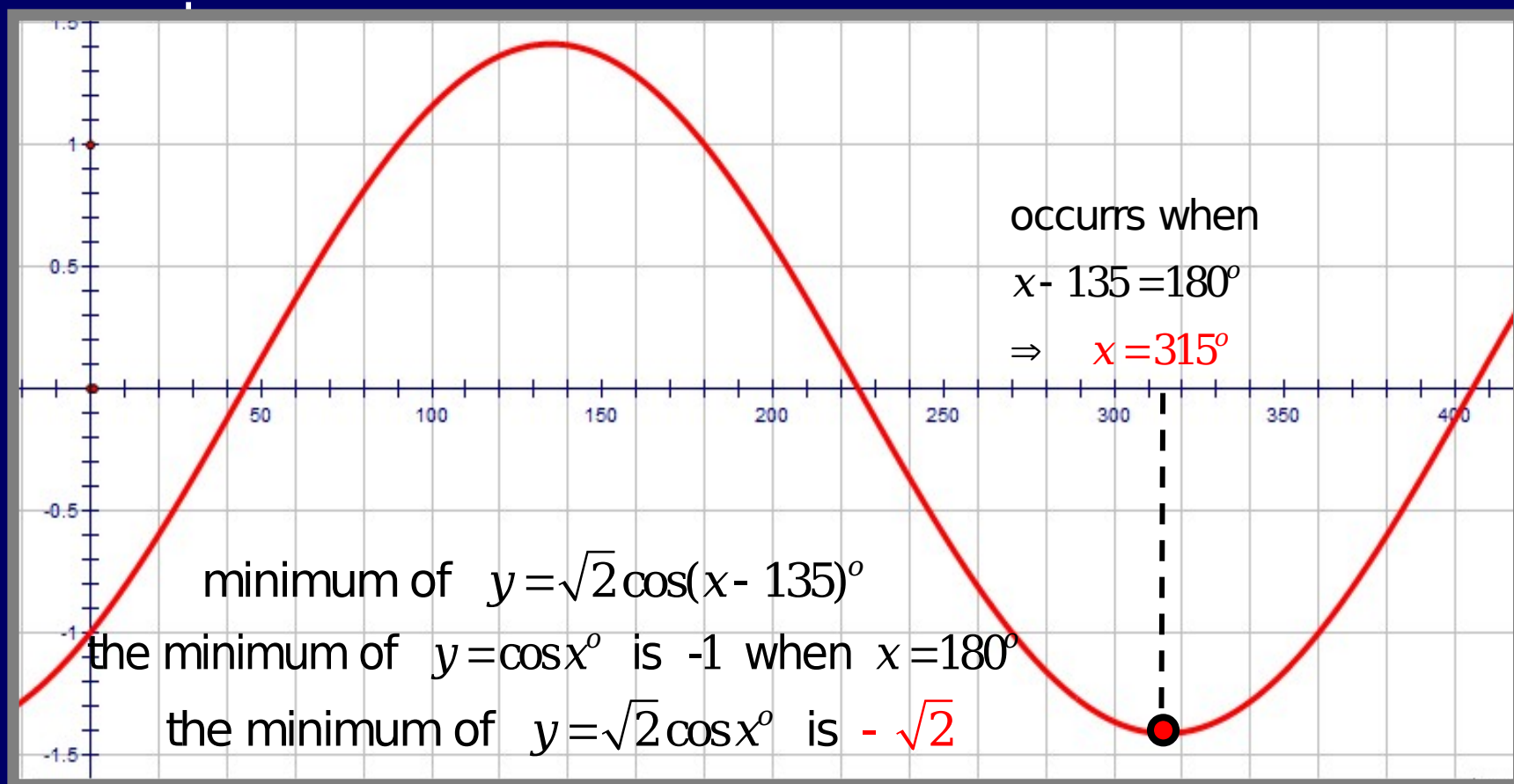
$$\sin x - \cos x = \sqrt{2} \cos(x - 135)^\circ$$



Maximum and Minimum Values

Minimum, we

$$\sin x - \cos x = \sqrt{2} \cos(x - 135)^\circ$$



Maximum and Minimum Values

Example

A synthesiser adds two sound waves together to make a new sound. The first wave is described by $V = 75\sin t^\circ$ and the second by $V = 100\cos t^\circ$, where V is the amplitude in decibels and t is the time in milliseconds.

Find the minimum value of the resultant wave and the

value of t at which it occurs.

a) Express the resultant wave in the form $k\sin(t - \alpha)^\circ$

$$V_{\text{result}} = 75\sin t + 100\cos t = K\sin(t - \alpha)$$

$$V_{\text{result}} = 25(3\sin t + 4\cos t) = 25k\sin(t - \alpha)$$

$$3\sin t + 4\cos t = k\sin(t - \alpha)$$

For later,
remember $K = 25k$

Maximum and Minimum

Expand and
equate
coefficients

$$3\sin t + 4\cos t = k\sin(t - \alpha)$$

$$3\sin t + 4\cos t = k\sin t \cos \alpha - k\cos t \sin \alpha$$

$$k\cos \alpha = 3$$

$$k\sin \alpha = -4$$

Square and add

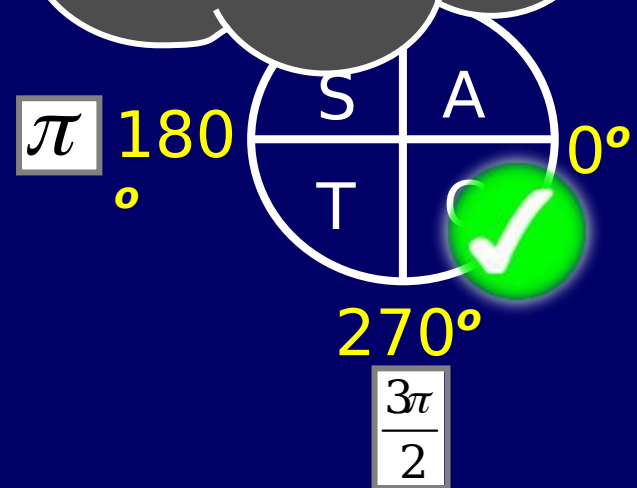
$$k^2(\cos^2 \alpha + \sin^2 \alpha) = 3^2 + 4^2 \quad k = 5$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

$\Rightarrow \alpha$ is in the 4th quadrant

$$\alpha = 360 - 53.1 = 306.9^\circ$$

Find tan ratio
note:
sin(-) and
cos(+)



Maximum and Minimum Values

$$V_{result} = 125 \sin(t - 306.9)$$

remember $K = 25k = 25 \times 5 = 125$

The minimum value of sin is -1 and it occurs where the angle is 270°

Therefore, the minimum value of V_{result} is -125

The minimum occurs where $t - 306.9 = 270^\circ$

$$t = 270 + 306.9 = 576.9^\circ$$

$$t = 576.9 - 360 = 216.9^\circ$$

$$t = 216.9^\circ$$

Adding or subtracting 360° leaves the sin unchanged

Maximum and Minimum Values

Minimum, we
have:

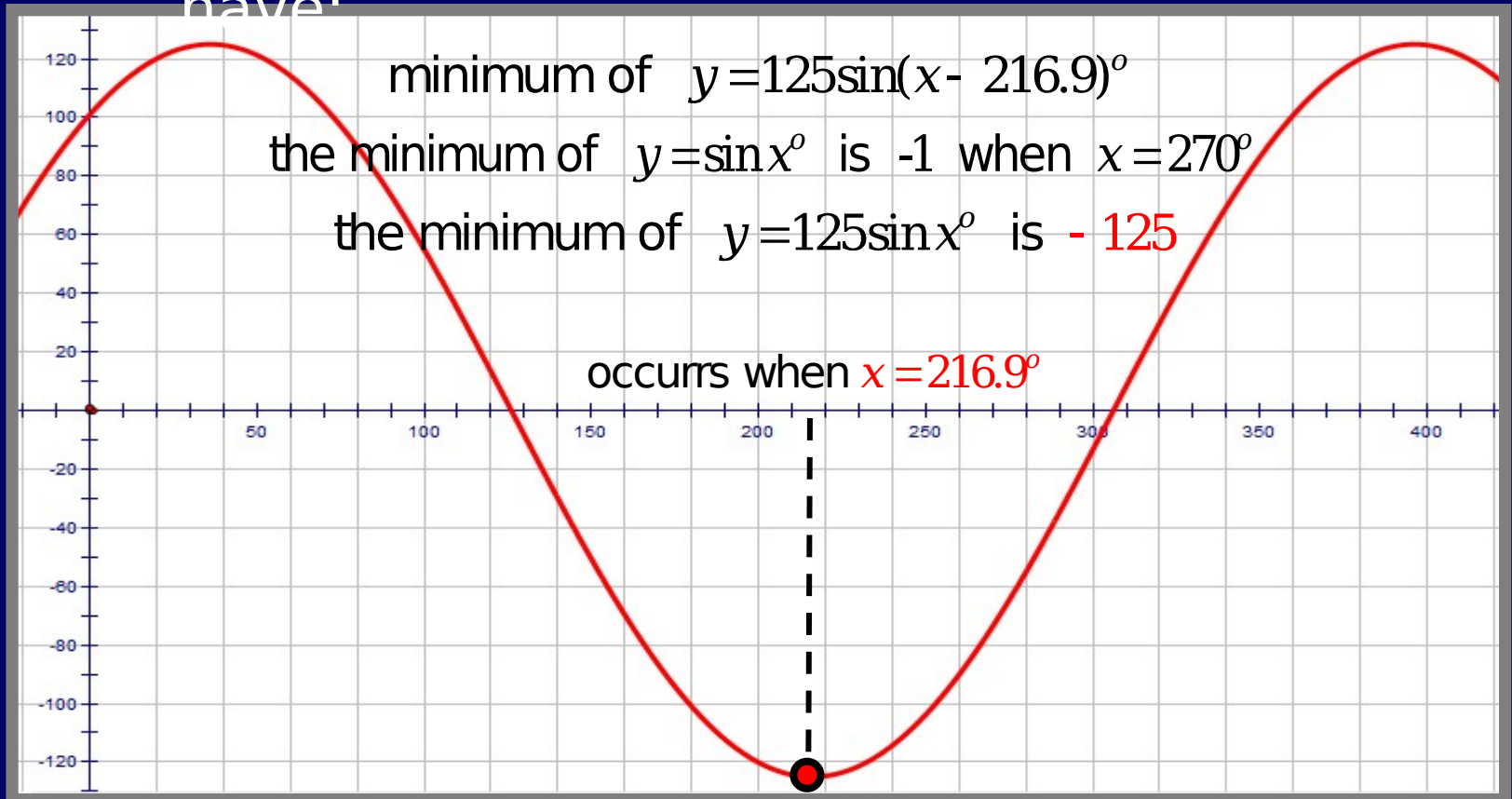
$$V_{result} = 75\sin t + 100\cos t = 125\sin(t - 216.9^\circ)$$

minimum of $y = 125\sin(x - 216.9^\circ)$

the minimum of $y = \sin x^\circ$ is -1 when $x = 270^\circ$

the minimum of $y = 125\sin x^\circ$ is -125

occurs when $x = 216.9^\circ$



Solving Trig Equations

True for ALL
x means
coefficients
equal.

Worked
Example

$$\text{Solve } \sqrt{3}\cos x + \sin x = \sqrt{2} \text{ for } 0 \leq x \leq 2\pi$$

Step 1: Write $\sqrt{3}\cos x + \sin x$ in the form $k\cos(x - \alpha)$

$$\sqrt{3}\cos x + \sin x = k\cos(x - \alpha).$$

$$\sqrt{3}\cos x + \sin x = k\cos x \cos \alpha + k\sin x \sin \alpha$$

Compare
Coefficients
Square & Add

$$k\cos \alpha = \sqrt{3}$$

$$k\sin \alpha = 1$$

$$k^2 \cos^2 \alpha + k^2 \sin^2 \alpha = 3 + 1$$

$$k^2 = 4$$

$$k = 2$$

Solving Trig Equa

Find tan ratio
note:
sin(+) and
cos(+)

$$k \cos \alpha = \sqrt{3}$$

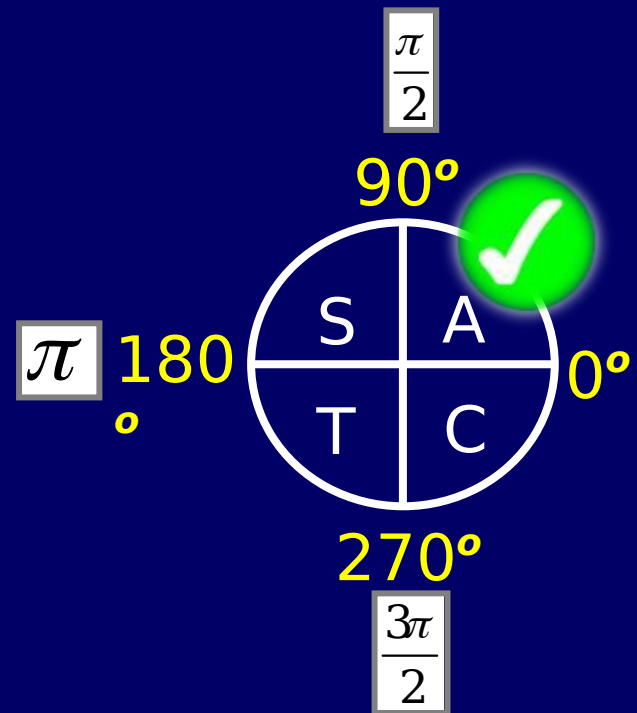
$$k \sin \alpha = 1$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$180^\circ = \pi$$

$$\alpha^\circ = 30^\circ = \frac{\pi}{6}$$

$$\sqrt{3} \cos x + \sin x = 2 \cos \left(x - \frac{\pi}{6} \right)$$



Solving Trig Equations

Step 2:

Re-write the trig. equation using your result from step 1, then solve.

$$\sqrt{3}\cos x + \sin x = \sqrt{2}$$

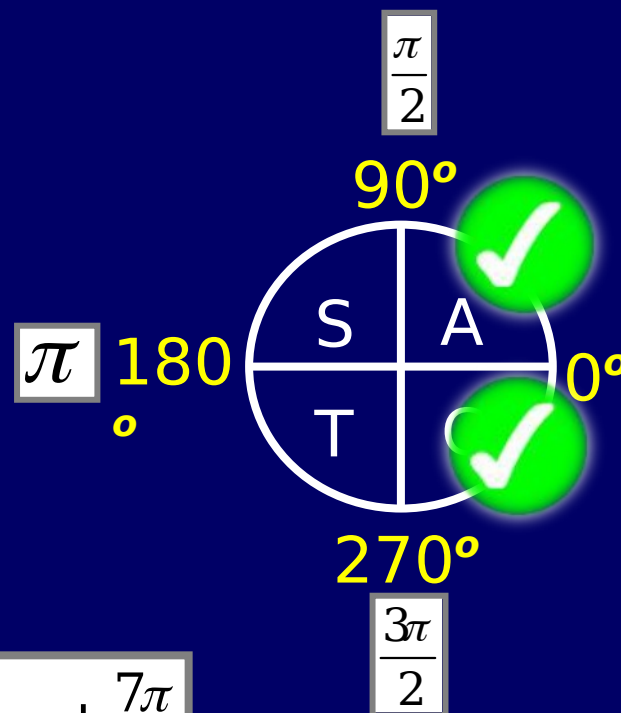
$$2\cos\left(x - \frac{\pi}{6}\right) = \sqrt{2}$$

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$$

$$\left(x - \frac{\pi}{6}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\left(x - \frac{\pi}{6}\right) = 45^\circ \text{ and } 315^\circ$$

$$\left(x - \frac{\pi}{6}\right) = \frac{\pi}{4} \text{ and } \frac{7\pi}{4}$$



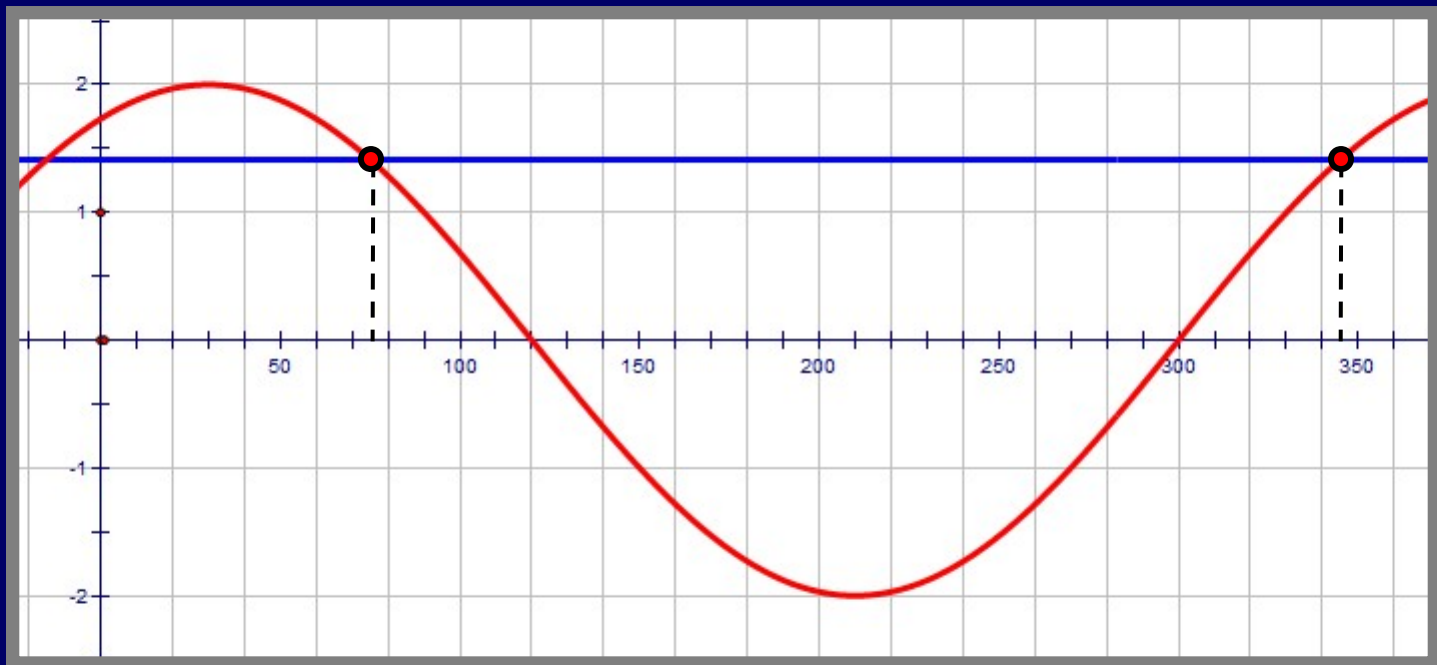
Solving Trig Equations

Step 2:

$$x - \frac{\pi}{6} = \frac{\pi}{4} \quad \text{or} \quad x - \frac{\pi}{6} = \frac{7\pi}{4}$$

$$x = \frac{\pi}{4} + \frac{\pi}{6} \quad \text{or} \quad x = \frac{7\pi}{4} + \frac{\pi}{6}$$

$$x = \frac{5\pi}{12} \quad (75^\circ) \quad \text{or} \quad x = \frac{23\pi}{12} \quad (345^\circ)$$



Solving Trig Equ

Expand and
equate
coefficients

Example

a) Express $2\cos 2x - 3\sin 2x$ in the form $k\sin(2x - \alpha)$

b) Hence solve $2\cos 2x - 3\sin 2x = 1$ for $0 \leq x \leq 360^\circ$

$$-3\sin 2x + 2\cos 2x = k\sin(2x - \alpha)$$

$$= k\sin 2x \cos \alpha - k\cos 2x \sin \alpha$$

$$k\sin \alpha = -2$$

$$k\cos \alpha = -3$$

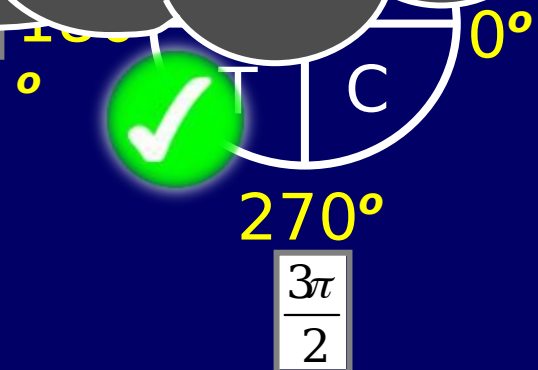
Square and add

$$k = \sqrt{13}$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \tan^{-1} \left(\frac{2}{3} \right) = 33.7^\circ$$

$$\alpha = 180^\circ + 33.7^\circ = 213.7^\circ$$

Find tan ratio
note:
 $\sin(-)$ and
 $\cos(-)$



Solving Trig Equations

b) We now have $2\cos 2x - 3\sin 2x = \sqrt{13}\sin(2x - 213.7)^\circ$

We solve $2\cos 2x - 3\sin 2x = 1$
by solving $\sqrt{13}\sin(2x - 213.7) = 1$

$$\sin(2x - 213.7) = \frac{1}{\sqrt{13}}$$

$$\text{In the 1}^{\text{st}} \text{ quadrant } \sin^{-1}\left(\frac{1}{\sqrt{13}}\right) = 16.1^\circ$$

$$2x - 213.7 = 16.1^\circ, (180 - 16.1^\circ), (360 + 16.1^\circ), (360 + 180 - 16.1^\circ)$$

$$2x - 213.7 = 16.1^\circ, 163.9^\circ, 376.1^\circ, 523.9^\circ,$$

$$\dots \quad 2x = 229.8^\circ, 310.2^\circ, 589.9^\circ, 670.2^\circ, \dots$$

$$x = 114.9^\circ, 155.1^\circ, 294.9^\circ, 335.1^\circ$$

Solving Trig Equations

(From a past paper)

Exempl

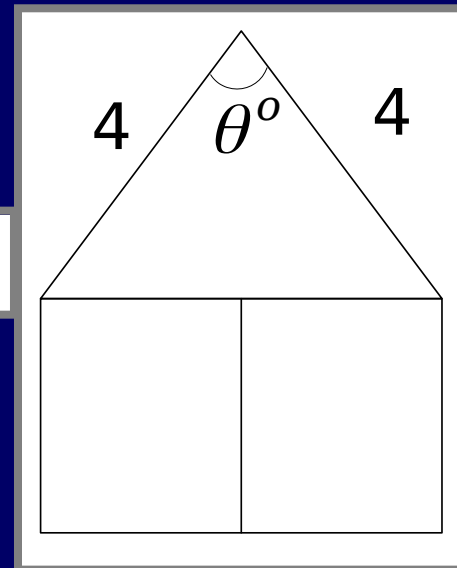
A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.

- a) If θ° is the angle shown in the diagram and A is the area m^2 of the gable end, show that

$$A = 8(2 + \sin\theta^\circ - 2\cos\theta^\circ)$$

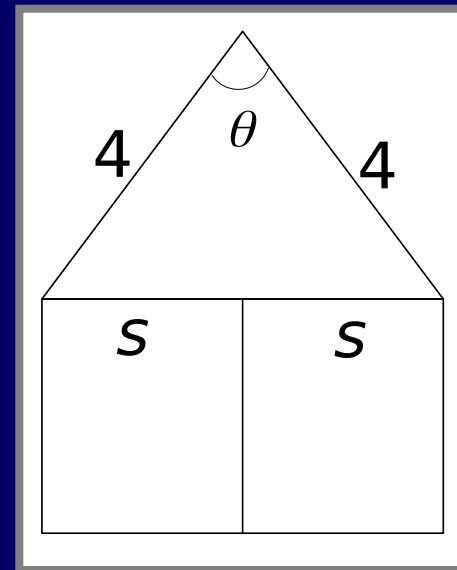
- b) Express $8(2 + \sin\theta^\circ - 2\cos\theta^\circ)$ in the form $k\sin(\theta - \alpha)^\circ$

- c) Find algebraically the value of θ° for which the area of the gable end is 30m^2 .



Solving Trig Equations

(From a past paper)



Part

(a)

Let the side of the square frames be s .

Use the cosine rule in the isosceles triangle

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(2s)^2 = 4^2 + 4^2 - 2 \times 4 \times 4 \cos \theta$$

$$4s^2 = 32(1 - \cos \theta)$$

$$s^2 = 8(1 - \cos \theta)$$

This is the area of one of the

squares.

The formula for the area of a triangle

$$= \frac{1}{2} ab \sin C$$

$$\text{Area of Triangle} = \frac{1}{2} \times 4 \times 4 \times \sin \theta = 8 \sin \theta$$

Total area = Triangle + 2 $\times$
square:

$$A = 8 \sin \theta + 2 \times 8(1 - \cos \theta) = 8(2 + \sin \theta - 2 \cos \theta)$$

Solving Trig Equations

(From a past paper)

Part (b)

Express $8\sin\theta - 16\cos\theta$ in the form $k\sin(\theta - \alpha)$

Consider $8(\sin\theta - 2\cos\theta) = 8t\sin(\theta - \alpha)$

Solve $\sin\theta - 2\cos\theta = t\sin(\theta - \alpha)$ and rem

$$\sin\theta - 2\cos\theta = t\sin\theta \cos\alpha - t\cos\theta \sin\alpha$$

Square and

$$\left. \begin{aligned} t\cos\alpha &= 1 \\ t\sin\alpha &= 2 \end{aligned} \right\}$$

$$t^2 = 5$$

$$t = \sqrt{5}$$

$$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \tan^{-1}\left(\frac{2}{1}\right)$$

$$\alpha = 63.4^\circ$$

Find tan ratio

note:

$\sin(+)$ at $\frac{\pi}{2}$

$\cos(+)$ at 0°

π

180°



270°

$\frac{3\pi}{2}$

Finally:

$$8\sin\theta - 16\cos\theta = 8\sqrt{5}\sin(\theta - 63.4^\circ)$$

Solving Trig Equations

(From a past paper)

Part (c)

Find algebraically the value of θ if the area is the 30m^2

$$A = 8(2 + \sin\theta - 2\cos\theta) = 16$$

$$A = 16 + 8\sqrt{5}\sin(\theta - 63.4)^\circ$$

$$30 = 16 + 8\sqrt{5}\sin(\theta - 63.4)^\circ$$

$$14 = 8\sqrt{5}\sin(\theta - 63.4)^\circ$$

$$\sin(\theta - 63.4) = \frac{7}{4\sqrt{5}}$$

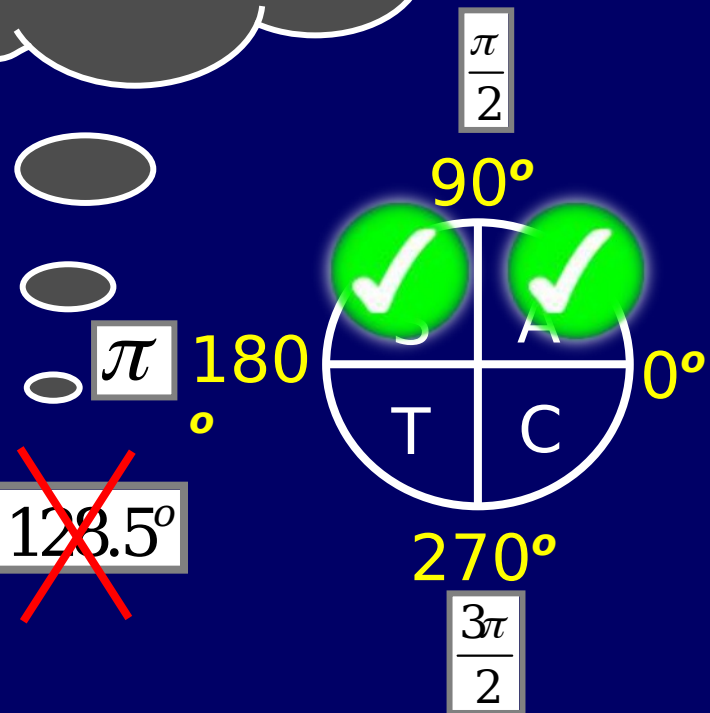
$$\theta - 63.4 = \sin^{-1}\left(\frac{7\sqrt{5}}{20}\right)$$

$$\therefore \theta = 114.9^\circ$$

From diagram

$$\theta^\circ < 90^\circ$$

ignore 2nd quad

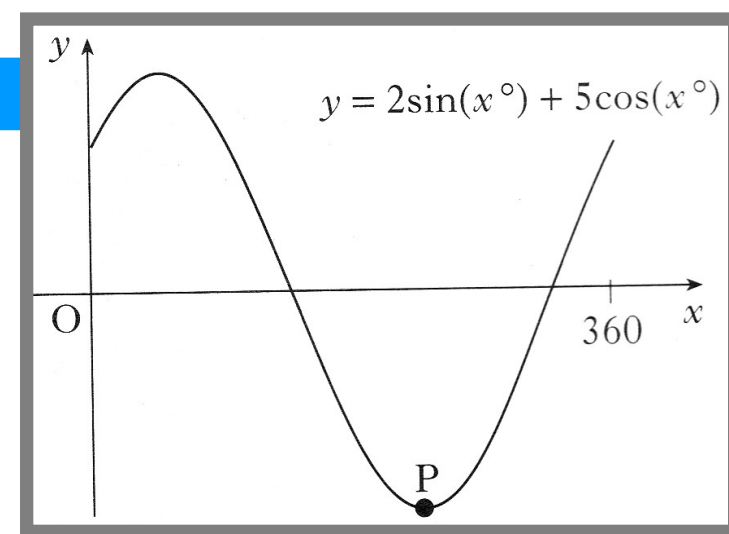


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Higher Maths







a)

Expand $k\sin(x + a)$: $k\sin(x + a) = k\sin x \cos a + k\cos x \sin a$

Equate coefficients: $k\cos a = 2$ $k\sin a = 5$

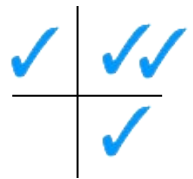
Square and add: $k^2 = 2^2 + 5^2$ $k = \sqrt{29}$

Dividing: $\tan a = \frac{5}{2}$ acute $a = 68^\circ$

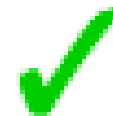
Put together: $2\sin x + 5\cos x = \sqrt{29}\sin(x + 68^\circ)$

Minimum when: $(x + 68^\circ) = 270^\circ \rightarrow x = 202^\circ$

P has coords. $(202^\circ, -\sqrt{29})$



$a = 68^\circ$



Expand $k \sin(x - a)$: $k \sin(x - a) = k \sin x \cos a - k \cos x \sin a$

Equate coefficients: $k \cos a = 1$ $k \sin a = 1$

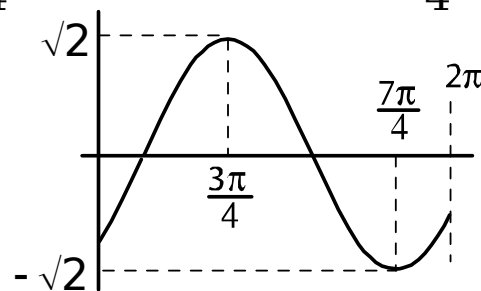
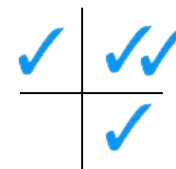
Square and add $k^2 = 1^2 + 1^2$ $k = \sqrt{2}$

Dividing: $\tan a = 1$ acute $a = \frac{\pi}{4}$

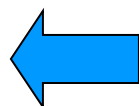
Put together: $\sin x - \cos x = \sqrt{2} \sin(x - \frac{\pi}{4})$ $k = \sqrt{2}$ $a = \frac{\pi}{4}$

Sketch Graph $\max = \sqrt{2}$ $\min = -\sqrt{2}$

$\max$ at $x = \frac{3\pi}{4}$ $\min$ at $x = \frac{7\pi}{4}$



$$a = \frac{\pi}{4}$$



Previous

Table of exact values



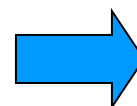
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Next



$$8\cos x - 6\sin x$$

$$k \cos(x + a) \text{ where } k > 0 \text{ and } 0 < a < 360$$

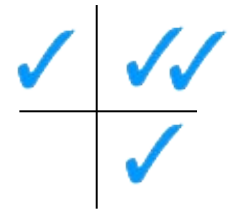
Expand $k \cos(x + a)$: $k \cos(x + a) = k \cos x \cos a - k \sin x \sin a$

Equate coefficients: $k \cos a = 8$ $k \sin a = 6$

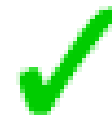
Square and add $k^2 = 8^2 + 6^2$ $k = 10$

Dividing: $\tan a = \frac{6}{8}$ acute $a = 37^\circ$

Put together: $8\cos x - 6\sin x = 10\cos(x + 37^\circ)$



$$a = 37^\circ$$



$$\cos x - \sin x$$

Express as $R\cos(x - a)$: $R\cos(x - a) = R\cos x \cos a + R\sin x \sin a$

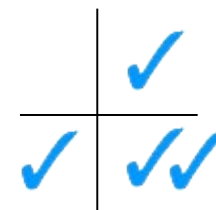
Equate coefficients: $R\cos a = 1$ $R\sin a = -1$

Square and add $R^2 = 1^2 + 1^2$ $R = \sqrt{2}$

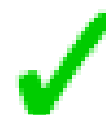
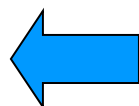
Dividing: $\tan a = -1$ acute $a = \frac{\pi}{4}$

Put together: $\cos x - \sin x = \sqrt{2} \cos\left(x - \frac{7\pi}{4}\right)$

Max value: $\sqrt{2}$ when $\left(x - \frac{7\pi}{4}\right) = 0$, $x = \frac{7\pi}{4}$



$$a = \frac{7\pi}{4}$$



Previous

Table of exact values



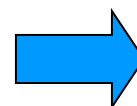
Quit



Quit



Next



$$2\sin x^\circ - 5\cos x^\circ$$

$$k\sin(x - \alpha)^\circ, \quad 0 \leq \alpha \leq 360 \text{ and } k > 0$$

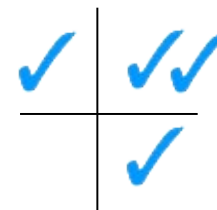
Expand $k\sin(x - a)$: $k\sin(x - a) = k\sin x \cos a - k\cos x \sin a$

Equate coefficients: $k\cos a = 2$ $k\sin a = 5$

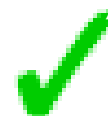
Square and add $k^2 = 2^2 + 5^2$ $k = \sqrt{29}$

Dividing: $\tan a = \frac{5}{2}$ acute $a = 68^\circ$

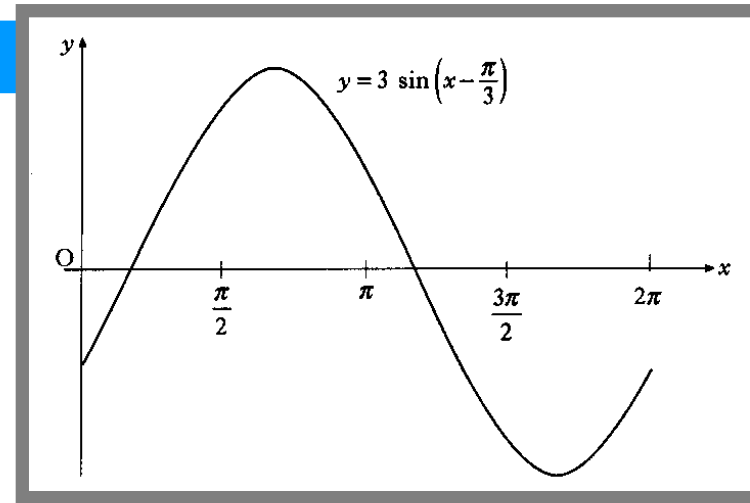
Put together: $2\cos x - 5\sin x = \sqrt{29}\sin(x - 68^\circ)$



$$a = 68^\circ$$



$$y = 3\sin\left(x - \frac{\pi}{3}\right), \text{ for } 0 \leq x \leq 2\pi$$

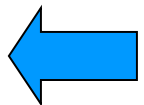


Max for sine occurs $(...) = \frac{\pi}{2}$, $x = \frac{5\pi}{6}$

Max value of sine function:

Max value of function:

Coordinates of max s.p. $\left(\frac{5\pi}{6}, 3\right)$ ✓



Previous

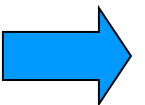
Quit



Quit



Next



$$f(x) = 2\cos x^\circ + 3\sin x^\circ$$

$$k \cos(x - \alpha)^\circ \text{ where } k > 0 \text{ and } 0 \leq \alpha \leq 360$$

$$f(x) = 0.5 \text{ for } 0 \leq x \leq 360$$

Expand $k \cos(x - a)$: $k \cos(x - a) = k \cos x \cos a + k \sin x \sin a$

Equate coefficients: $k \cos a = 2$ $k \sin a = 3$

Square and add $k^2 = 2^2 + 3^2$ $k = \sqrt{13}$

Dividing: $\tan a = \frac{3}{2}$ acute $a = 56^\circ$

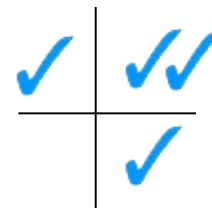
Put together: $2\cos x + 3\sin x = \sqrt{13} \cos(x - 56^\circ)$

Solve equation. $\sqrt{13} \cos(x - 56^\circ) = 0.5$

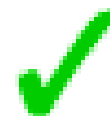
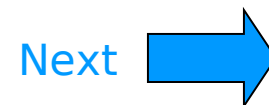
$$\cos(x - 56^\circ) = \frac{0.5}{\sqrt{13}}$$

$$\text{acute}(x - 56^\circ) = 82^\circ$$

$$x = 138^\circ \text{ or } x = 334^\circ$$



$$a = 56^\circ$$



$$k \sin x^\circ = 5$$

$$k \cos x^\circ = 2$$

Use $\tan A = \sin A / \cos A$

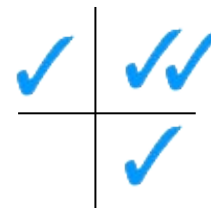
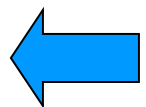
Divide $\tan x = \frac{5}{2}$

Find acute angle acute $x = 68^\circ$

Determine quadrant(s)

State solution

$$x = 68^\circ$$

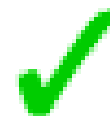


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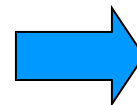
Quit



Quit



Next



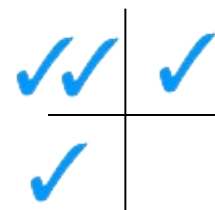
$$2\sin x^\circ - 3\cos x^\circ = 2.5$$

Use $R \cos(x - a)$: $R\cos(x - a) = R\cos x \cos a + R\sin x \sin a$

Equate coefficients: $R\cos a = -3$ $R\sin a = 2$

Square and add $R^2 = 2^2 + (-3)^2$ $R = \sqrt{13}$

Dividing: $\tan a = -\frac{2}{3}$ acute $a = 34^\circ$



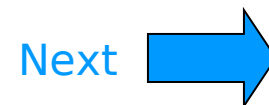
$$a = 146^\circ$$

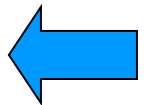
Put together: $2\sin x - 3\cos x = \sqrt{13}\cos(x - 146^\circ)$

Solve equation. $\sqrt{13}\cos(x - 146^\circ) = 2.5$ $\cos(x - 146^\circ) = \frac{2.5}{\sqrt{13}}$

$$\text{acute}(x - 146^\circ) = 46^\circ$$

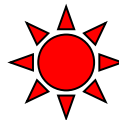
$$x = 192^\circ \text{ or } x = 460^\circ \text{ (out of range, so subtract } 360^\circ) \quad x = 100^\circ \text{ or } x = 192^\circ$$





Previous

Quit



Quit



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