TRIGONOMETRICAL EQUATIONS


Trigonometry identities of the form sin(A+B)


Double Angle formulae


Trigonometric Equations


Radians & Trig Basics


More Trigonometric Equations


Exam Type Questions

Trig Identities

Supplied on a formula sheet !!

The following relationships are always true for two angles A and B.

1a. $\sin(A + B) = \sin A\cos B + \cos A\sin B$

1b. $\sin(A - B) = \sin A\cos B - \cos A\sin B$

2a. $\cos(A + B) = \cos A\cos B - \sin A\sin B$

2b. $\cos(A - B) = \cos A\cos B + \sin A\sin B$

Quite tricky to prove but some of following examples should show that they do work!!

Trig Identities

Examples 1

(1) Expand cos(U – V). (use formula 2b)

cos(U – V) = cosUcosV + sinUsinV

(2) Simplify sinf°cosg° - cosf°sing° (use formula 1b)

sinf°cosg° - cosf°sing° = sin(f – g)°

(3) Simplify cos8 θ sinθ + sin8 θ cosθ (use formula 1a)

cos8 θ sin θ + sin8 θ cos θ = sin(8 θ + θ) = sin9 θ

Trig Identities

Example 2

By taking A = 60° and B = 30°,

prove the identity for cos(A – B).

NB: cos(A – B) = cosAcosB + sinAsinB

LHS = cos(60 – 30)° = cos30° = $\sqrt{3}/2$

RHS = cos60°cos30° + sin60°sin30°

= (½ x $\sqrt{3}/2$) + ($\sqrt{3}/2$ x ½)

= $\sqrt{3}/4$ + $\sqrt{3}/4$

= $\sqrt{3}/2$

Hence LHS =

Trig Identities

Example 3

Prove that $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

$\sin 15° = \sin(45 - 30)°$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{(\sqrt{3} - 1)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

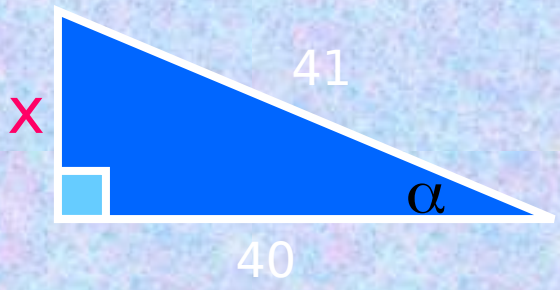
$= \frac{(\sqrt{6} - \sqrt{2})}{4}$

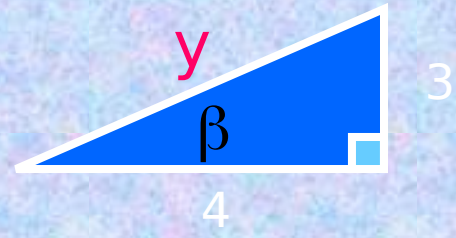
$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

Trig Identities

NAB type Question

Example 4





Show that $\cos(\alpha - \beta) = {}^{187}/_{205}$

Triangle1

If missing side = x

Then $x^2 = 41^2 - 40^2 = 81$

So $x = 9$

$\sin\alpha = {}^{9}/_{41}$ and $\cos\alpha =$ ${}^{40}/$

Triangle2

If missing side = y

Then $y^2 = 4^2 + 3^2 = 25$

So $y = 5$

$\sin\beta = {}^{3}/_{5}$ and $\cos\beta =$ ${}^{4}/$

Trig Identities

$\sin\alpha = {}^{9}/_{41}$ and $\cos\alpha = {}^{40}/_{41}$ $\sin\beta = {}^{3}/_{5}$ and $\cos\beta = {}^{4}/_{5}$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= ({}^{40}/_{41} \times {}^{4}/_{5}) + ({}^{9}/_{41} \times {}^{3}/_{5})$$

$$= {}^{160}/_{205} + {}^{27}/_{205}$$

$$= {}^{187}/_{205}$$

Remember this is a NAB type Question

Trig Identities

NAB type Question

Example 5

Solve $\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ = -0.966$

where $0^\circ < x < 360^\circ$

ALWAYS work out Quad 1 first

1a $\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ \equiv \sin(x + 30)^\circ$

$\sin(x + 30)^\circ = -0.966$

S	A
$180-x^\circ$	x°
$180+x^\circ$	$360-x^\circ$
T	C

Quad 3 and Quad 4

$\sin^{-1} 0.966 = 75^\circ$

Quad 3: angle = $180^\circ + 75^\circ$

$x + 30^\circ = 255^\circ$

$x = 225^\circ$

Quad 4: angle = $360^\circ - 75^\circ$

$x + 30^\circ = 285^\circ$

$x = 255^\circ$

Trig Identities

Example 6

Solve $\sin 5\theta \cos 3\theta - \cos 5\theta \sin 3\theta = \sqrt{3}/2$ where $0 < \theta < \pi$

By rule 1b. $\sin 5\theta \cos 3\theta - \cos 5\theta \sin 3\theta = \sin(5\theta - 3\theta) = \sin 2\theta$

$\sin 2\theta = \sqrt{3}/2$

Repeats every π

Quad 1 and Quad 2

$\sin^{-1} \sqrt{3}/2 = \pi/3$

S	A
$\pi - \theta$	θ
$\pi + \theta$	$2\pi - \theta$
T	C

Quad 1: angle = $\pi/3$

$2\theta = \pi/3$

$\theta = \pi/6$

Quad 2: angle = $\pi - \pi/3$

$2\theta = 2\pi/3$

$\theta = \pi/3$

In this example repeats lie out with limits

Trig Identities

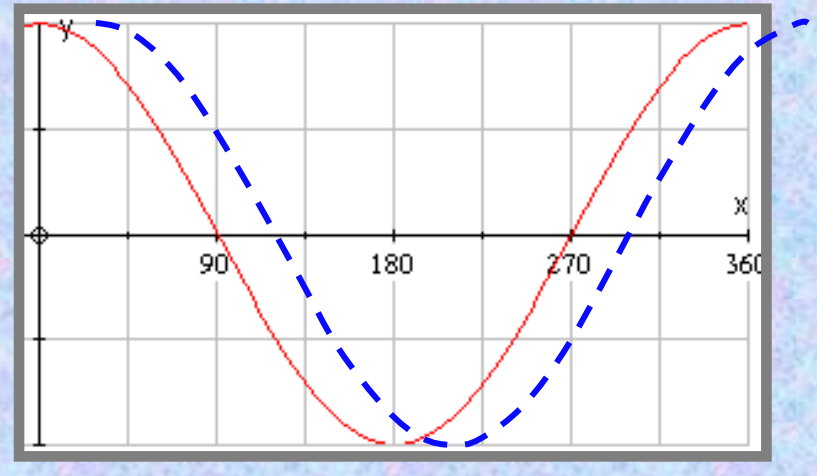
Example 7

Find the value of x that minimises the expression

$$\frac{\cos x^\circ \cos 32^\circ + \sin x^\circ \sin 32^\circ}{}$$

Using rule 2(b) we get $\cos x^\circ \cos 32^\circ + \sin x^\circ \sin 32^\circ = \cos(x - 32)^\circ$

cos graph is roller-coaster



min value is $\underline{-1}$ when angle = 180°

ie $x - 32^\circ = 180^\circ$

ie $x = 212^\circ$

Trig Identities

Paper 1 type questions

Example 8

Simplify $\sin(\theta - \pi/3) + \cos(\theta + \pi/6) + \cos(\pi/2 - \theta)$

$$\sin(\theta - \pi/3) + \cos(\theta + \pi/6) + \cos(\pi/2 - \theta)$$

$$= \sin\theta\cos\pi/3 - \cos\theta\sin\pi/3$$

$$+ \cos\theta\cos\pi/6 - \sin\theta\sin\pi/6$$

$$+ \cos\pi/2\cos\theta + \sin\pi/2\sin\theta$$

$$= 1/2\sin\theta - \sqrt{3}/2\cos\theta + \sqrt{3}/2\cos\theta - 1/2\sin\theta + 0 \times \cos\theta + 1 \times \sin\theta$$

$$= \sin\theta$$

Trig Identities

Paper 1 type questions

Example 9

Prove that

$$(\sin A + \cos B)^2 + (\cos A - \sin B)^2 = 2(1 + \sin(A - B))$$

LHS $= (\sin A + \cos B)^2 + (\cos A - \sin B)^2$

$= \sin^2 A + 2\sin A\cos B + \cos^2 B + \cos^2 A - 2\cos A\sin B + \sin^2 B$

$= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2\sin A\cos B - 2\cos A\sin B$

$= 1 + 1 + 2(\sin A\cos B - \cos A\sin B)$

$= 2 + 2\sin(A - B)$

$= 2(1 + \sin(A - B))$ $=$ RHS

Double Angle Formulae

$$\sin 2A = 2\sin A\cos A$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Two further formulae derived from the $\cos 2A$ formulae.

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

Double Angle formulae

Mixed Examples:

Given that A is an acute angle and $\tan A = \frac{4}{3}$, calculate $\sin 2A$ and $\cos 2A$.

$$\frac{\sin A}{\cos A} = \frac{4}{3}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A + (\tfrac{3}{4}\sin A)^2 = 1$$

$$\sin A = \pm\sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\cos A = \frac{3}{5}$$

Substitute form the tan (sin/cos) equation

+ve because A is acute

3-4-5 triangle !

Similarly:

$$\sin 2A = 2\sin A\cos A = \frac{24}{25}$$

$$\cos 2A = \cos^2 A - \sin^2 A = \frac{9 - 16}{25} = \frac{-7}{25}$$

A is greater than 45 degrees – hence 2A is greater than 90 degrees

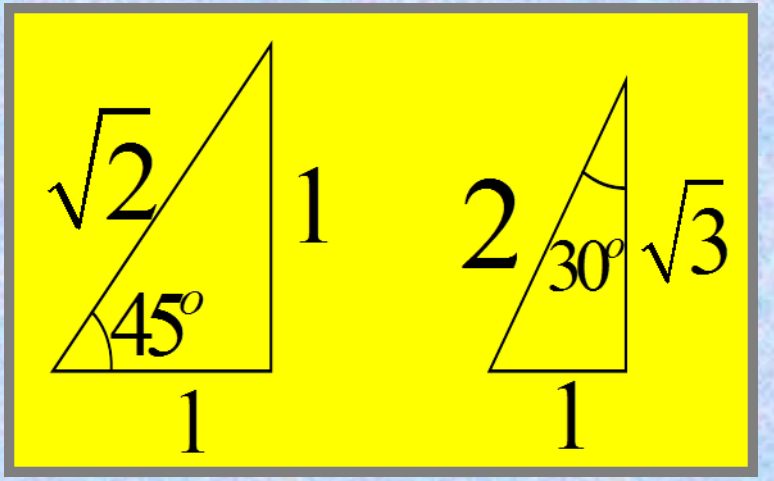
Double Angle formulae

Find the exact value of $\sin 75^o$.

$\sin(75^o) = \sin(45+30)$

$\sin(75^o) = \sin 45\cos 30 + \cos 45\sin 30$

$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$



Prove that $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$

$= \tan\alpha + \tan\beta$

Double Angle formulae

For the diagram opposite show that $\cos LMN = \frac{\sqrt{5}}{5}$.

$\cos LMN = \cos(\alpha + \beta)$

Length of $LM = \sqrt{18} = 3\sqrt{2}$

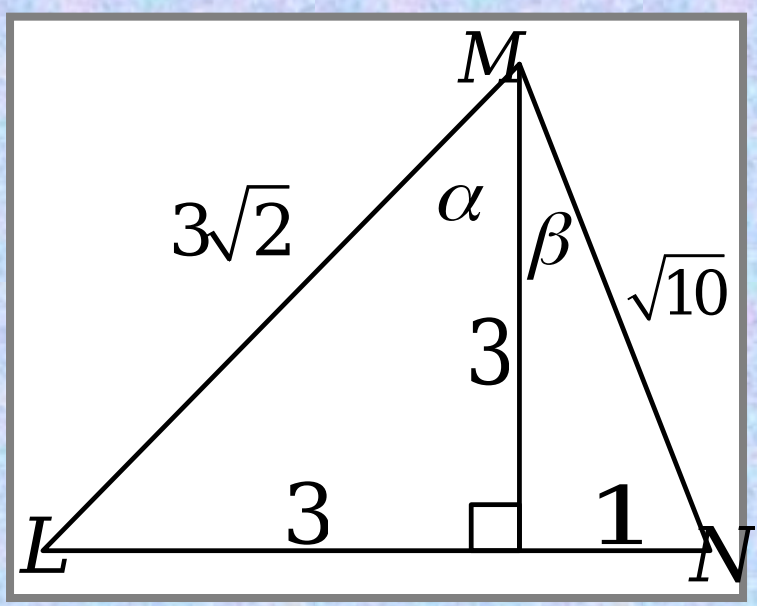
Length of $MN = \sqrt{10}$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \frac{1}{\sqrt{2}}\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{10}}$$

$$= \frac{2}{\sqrt{20}} = \frac{2}{\sqrt{4}\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}$$



Double Angle formulae

Prove that, $\cos^4\alpha - \sin^4\alpha = \cos 2\alpha$.

$\cos^4\alpha - \sin^4\alpha = (\cos^2\alpha)^2 - (\sin^2\alpha)^2$

Using $x^2 - y^2 = (x - y)(x + y)$

$= (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha)$

$\cos^2\alpha + \sin^2\alpha = 1$

$= \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \cos 2\alpha$

Trigonometric Equations

Double angle formulae (like cos2A or sin2A) often occur in trig equations. We can solve these equations by substituting the expressions derived in the previous sections.

Rules for solving equations

$\sin 2A = 2\sin A\cos A$	when replacing sin2A equation
$\cos 2A = 2\cos^2 A - 1$	if cosA is also in the equation
$\cos 2A = 1 - 2\sin^2 A$	if sinA is also in the equation

Trigonometric Equations

Solve: $\cos 2x^o - 4\sin x^o + 5 = 0$ for $0 \le x \le 360^o$.

cos2x and sin x,

so substitute $1-2\sin^2 x$

$(1 - 2\sin^2 x) - 4\sin x + 5 = 0$

$6 - 4\sin x - 2\sin^2 x = 0$

compare with $6 - 4z - 2z^2 = 0$

$(6 + 2\sin x)(1 - \sin x) = 0$

$\sin x = 1$ or ~~$\sin x = -3$~~

$0 \le \sin x \le 1$ for all real angles

$x = 90^o$

Trigonometric Equations

Solve: $5\cos 2x^o = \cos x^o - 2$ for $0 \le x \le 360^o$

cos 2x and cos x,

so substitute $2\cos^2 - 1$

$$5(2\cos^2 x - 1) = \cos x - 2$$

$$10\cos^2 x - \cos x - 3 = 0$$

$$(5\cos x - 3)(2\cos x + 1) = 0$$

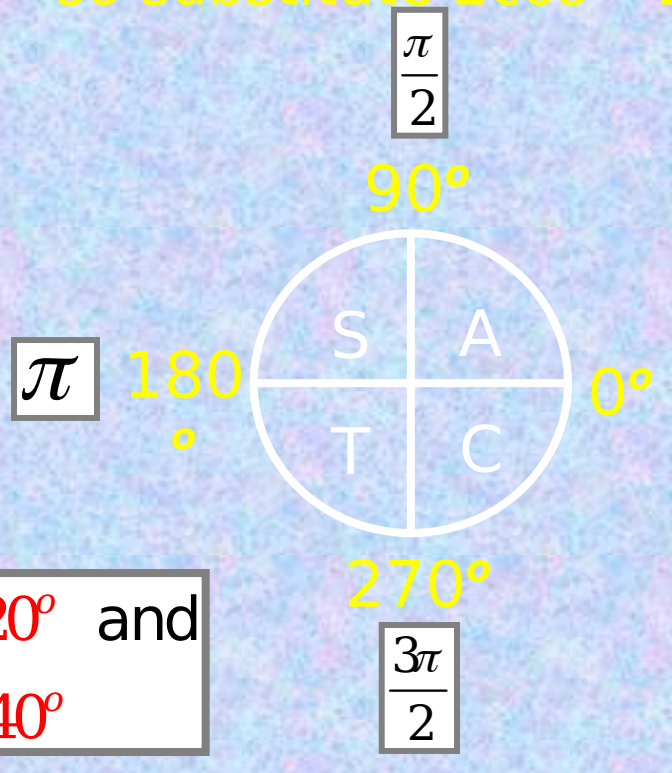
$$\cos x = \frac{3}{5} \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$x = 53.1^o$ and

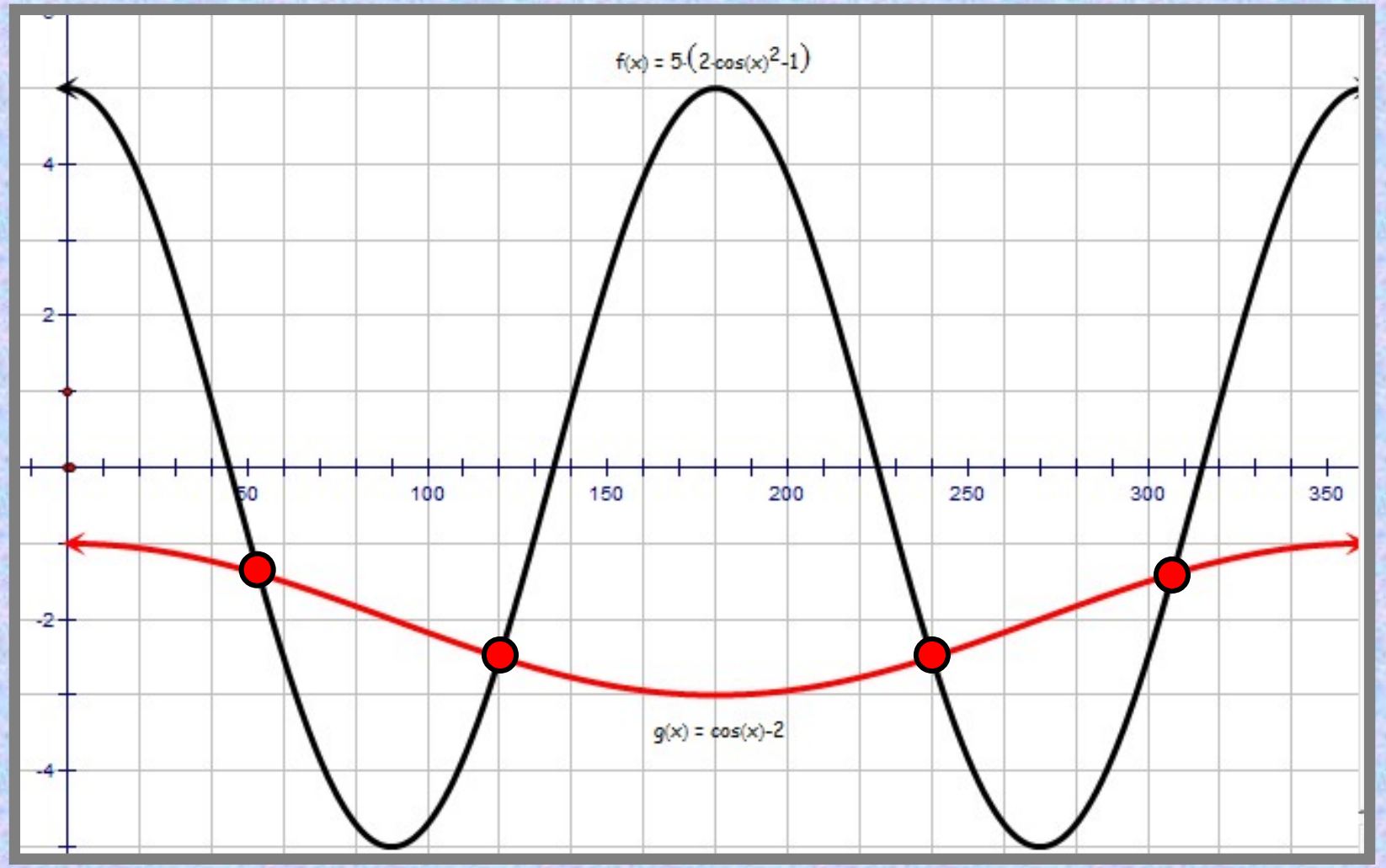
$x = 360 - 53.1 = 306.9^o$

$x = 180 - 60 = 120^o$ and

$x = 180 + 60 = 240^o$

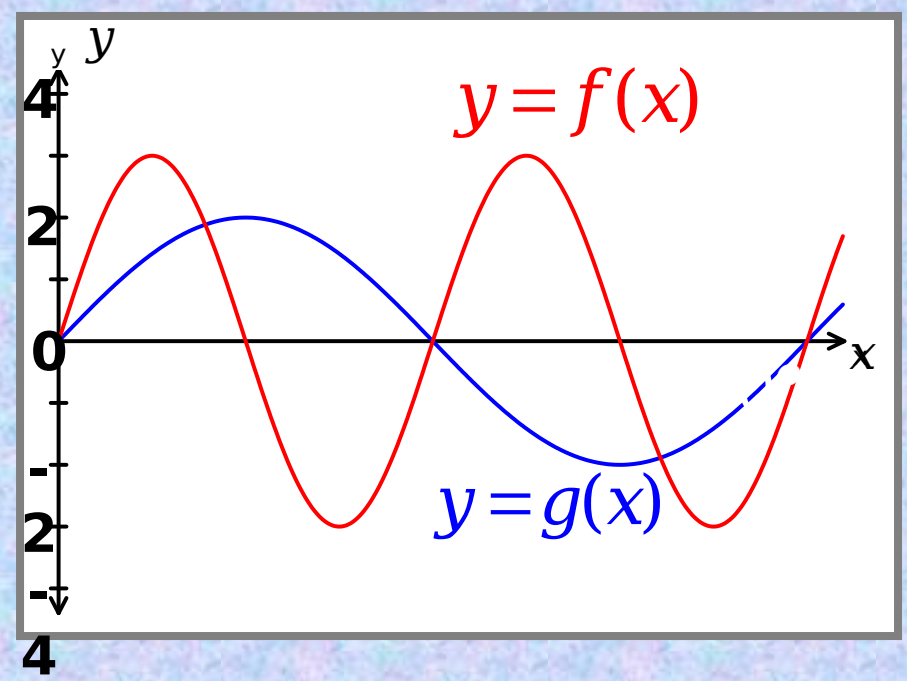


Trigonometric Equations



Trigonometric Equations

The diagram shows the graphs of $f(x) = a\sin bx^o$ and $g(x) = c\sin x^o$ for $0 \le x \le 360^o$.

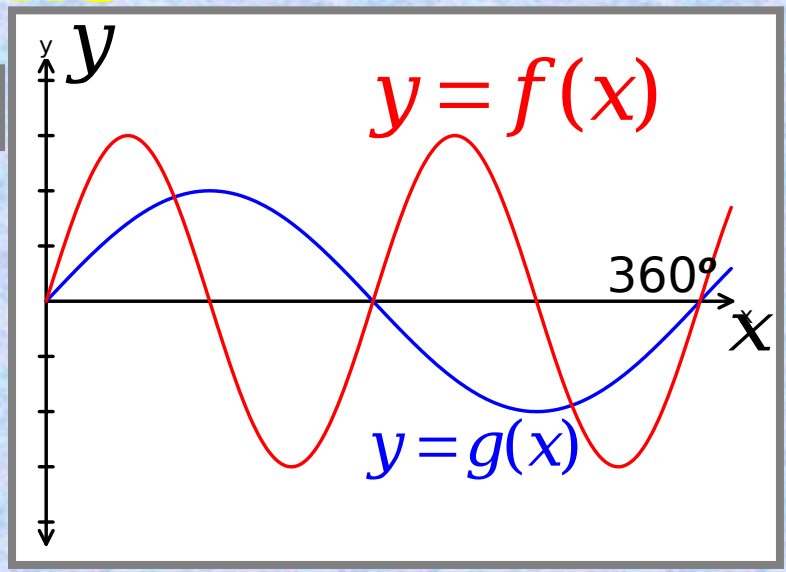


Three problems concerning this graph follow.

Trigonometric Equations

i) State the values of a, b and c.

$f(x) = a\sin bx^o$ $g(x) = c\sin x^o$



The max & min values of sinbx are 1 and -1 resp.

The max & min values of asinbx are 3 and -3 resp $a = 3$

f(x) goes through 2 complete cycles from 0 – 360° $b = 2$

$g(x) = c\sin x^o$

The max & min values of csinx are 2 and -2 $c = 2$

Trigonometric Equations

ii) Solve the equation $f(x) = g(x)$ algebraically.

From the previous problem we now

$f(x) = 3\sin 2x$ and $g(x) = 2\sin x$

Hence, the equation to solve $3\sin 2x = 2\sin x$

$3(2\sin x\cos x) = 2\sin x$ Expand sin 2x

$6\sin x\cos x - 2\sin x = 0$ Divide both sides by 2

$3\sin x\cos x - \sin x = 0$ Spot the common factor in the terms?

$\sin x(3\cos x - 1) = 0$ Is satisfied by all values of x for which:

$\sin x = 0$ or $\cos x = \frac{1}{3}$

Trigonometric Equations

iii) find the coordinates of the points of intersection of the graphs for $0 \le x \le 360^o$.

From the previous problem we

$$\sin x = 0 \quad \text{and} \quad \cos x = \frac{1}{3}$$

$$\sin x = 0$$

$$\cos x = \frac{1}{3}$$

Hence

$x = 0^o$

$x = 180^o$

$x = 360^o$

$x = 70.5^o$

$x = (360 - 70.5)^o = 289.5^o$

Radian Measurements

Reminders

i) Radians

$$\pi \text{ radians} = 180^\circ$$

Converting between degrees and radians:

$$120^\circ = 120 . \frac{\pi}{180}$$

$$= \frac{2\pi}{3} \text{ radians}$$

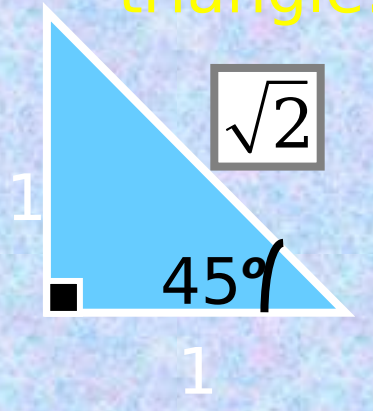
$$\frac{5\pi}{6} = \frac{5\pi}{6} . \frac{180}{\pi}$$

$$= 150^\circ$$

Degree Measurements

ii) Exact Values

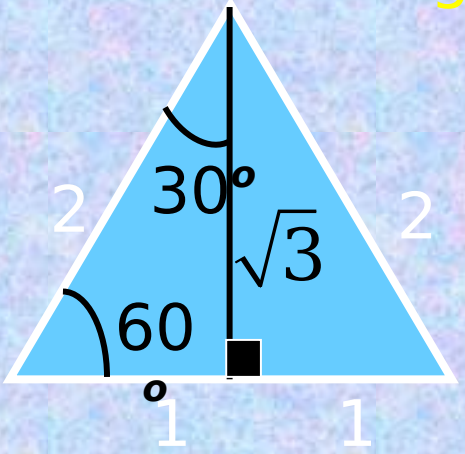
45° right-angled triangle:



$$\cos 45^o = \sin 45^o = \frac{1}{\sqrt{2}}$$

$$\tan 45^o = 1$$

Equilateral triangle:



$$\sin 60^o = \frac{\sqrt{3}}{2}$$

$$\cos 60^o = \frac{1}{2}$$

$$\tan 60^o = \sqrt{3}$$

$$\sin 30^o = \frac{1}{2}$$

$$\cos 30^o = \frac{\sqrt{3}}{2}$$

$$\tan 30^o = \frac{1}{\sqrt{3}}$$

Radians / Degrees

degrees	0°	30°	45°	60°	90°
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

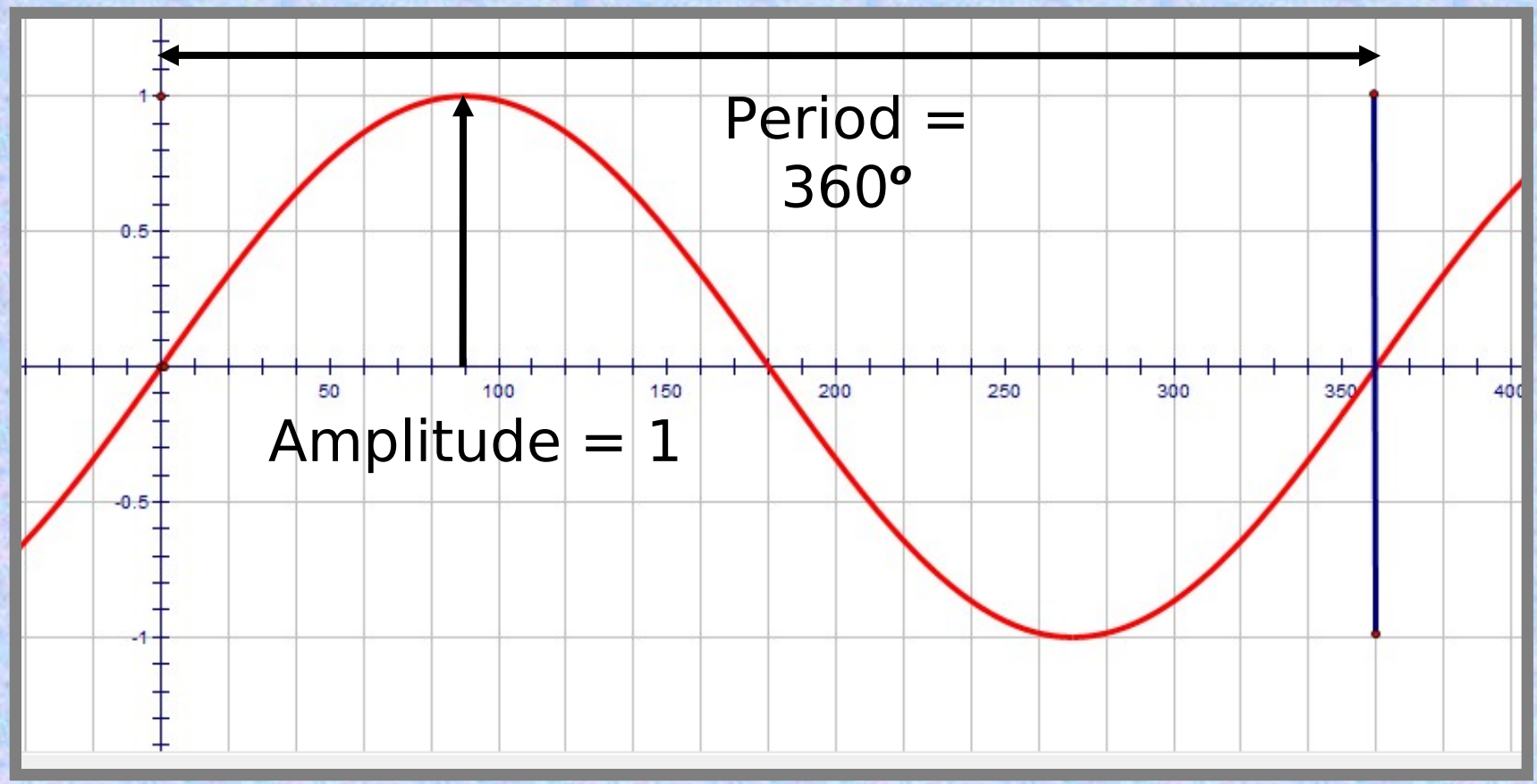
Example: What is the exact value of $\sin 240^\circ$

$240 = 180 + 60$

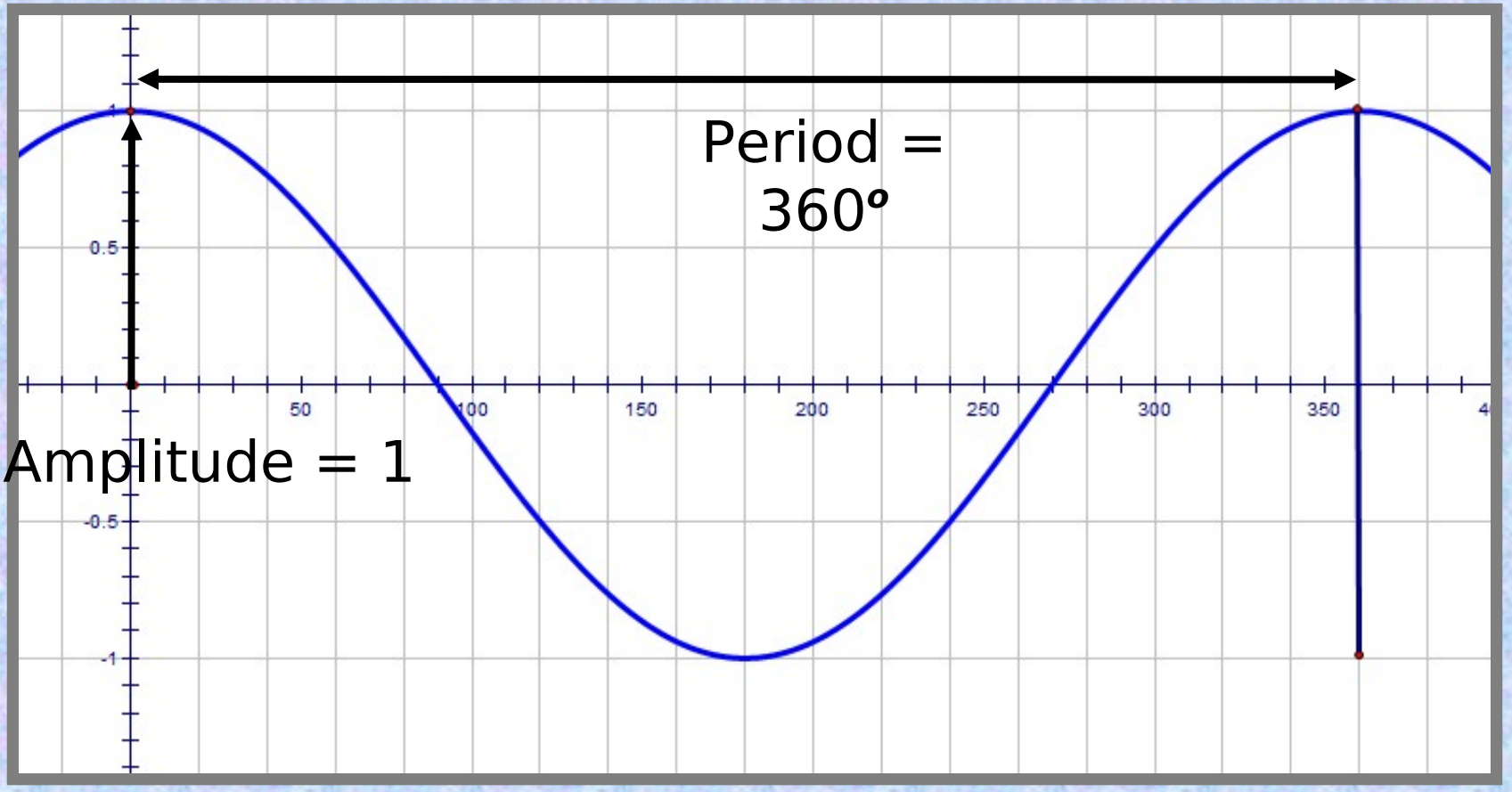
$\sin(180 + \alpha) = -\sin\alpha$

$\sin 240^\circ = -\frac{\sqrt{3}}{2}$

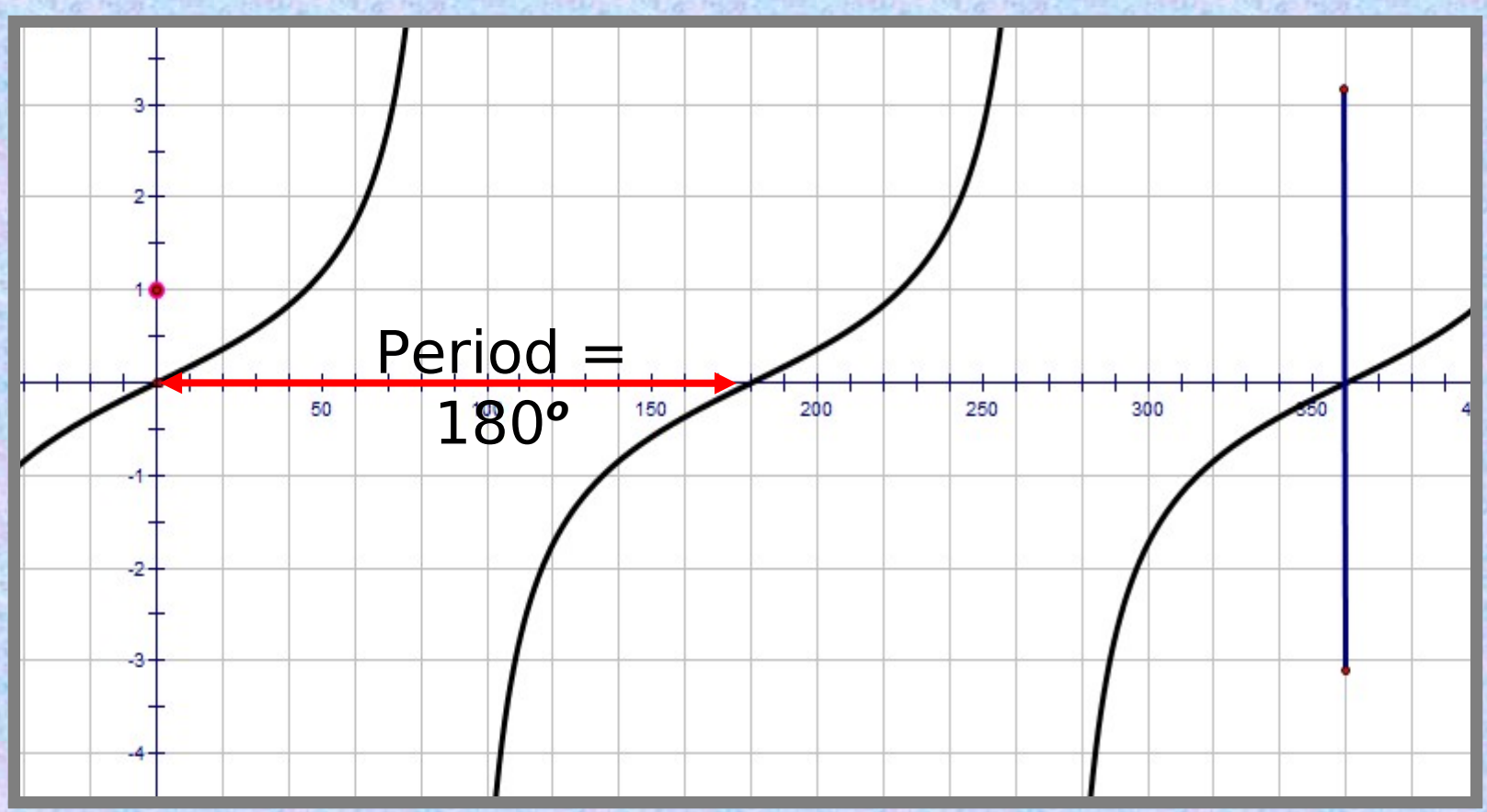
Sine Graph



Cosine Graph



Tan Graph



Amplitude cannot be found for tan function

Solving Trigonometric Equations

Example :

Solve $2\cos 3x - 1 = 0 \qquad (0 \le x \le 360^\circ)$

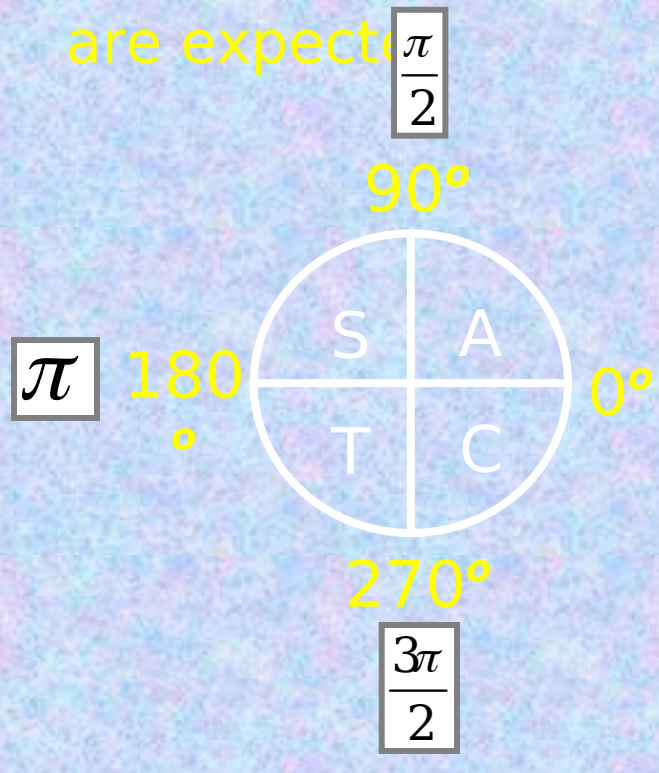
Step 1: Re-Arrange

$$2\cos 3x - 1 = 0$$

$$2\cos 3x = 1$$

$$\cos 3x = \frac{1}{2}$$

Step 2: consider what solutions are expected



Solving Trigonometric Equations

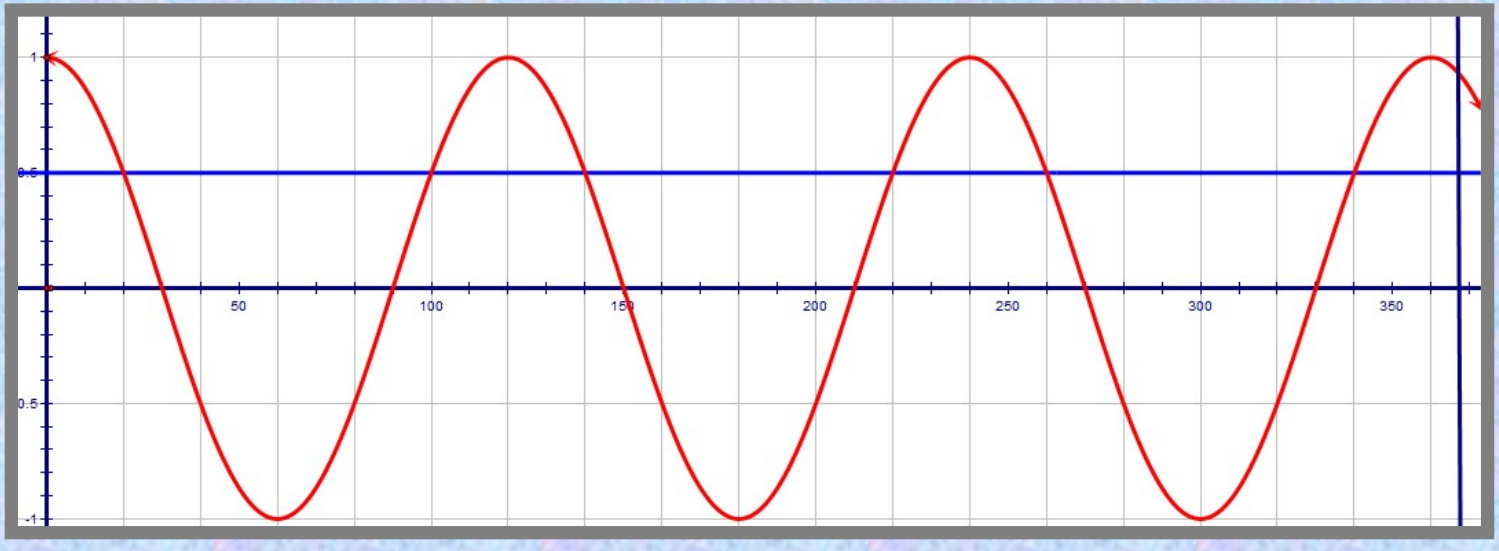
$$\cos 3x = \frac{1}{2}$$

cos 3x is positive so solutions in the first and fourth quadrants

Since $0 \le x \le 360^o$ has 2 solutions

x 3 x 3

Then $0 \le 3x \le 1080^o$ has 6 solutions



Solving Trigonometric Equations

Step 3: Solve the equation

$$\cos 3x = \frac{1}{2}$$

$$3x = \cos^{-1}\left(\frac{1}{2}\right) = 60^{o}$$

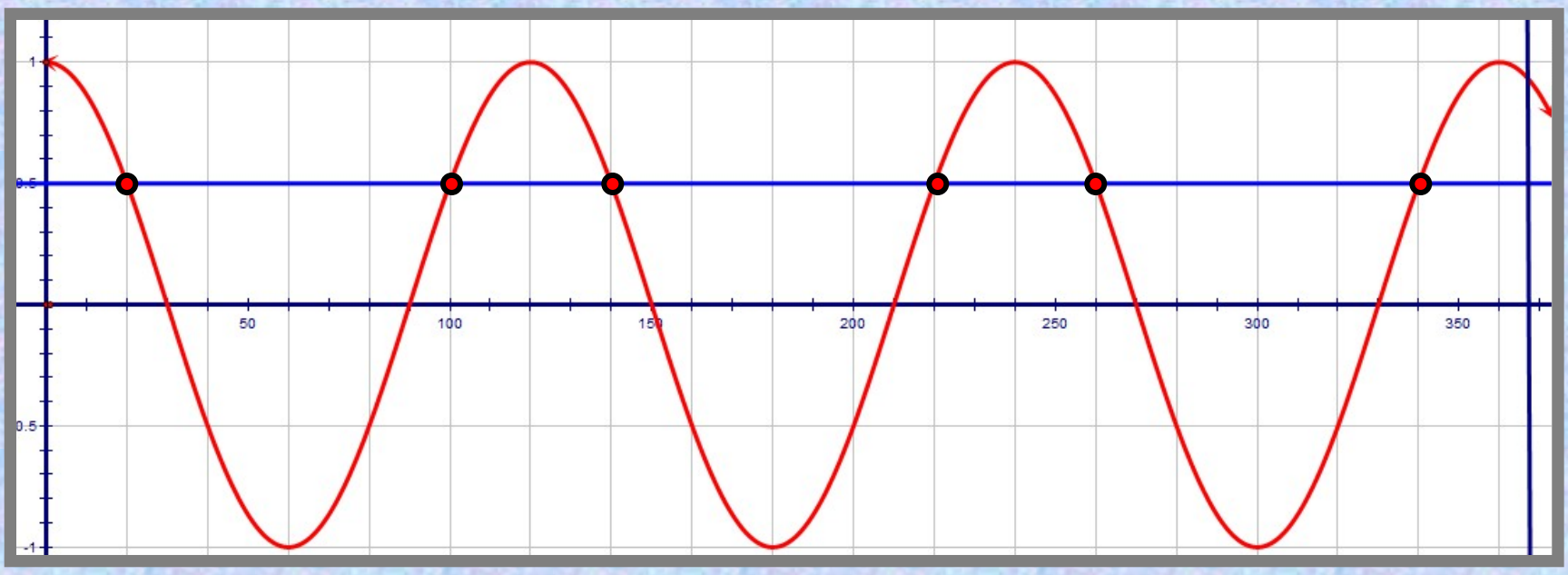
1st quad 4th quad cos wave repeats every 360°

3x =	60°	300°	420°	660°	780°	1020°
x =	20°	100°	140°	220°	260°	340°

Solving Trigonometric Equations

Graphical solution for

$$\cos 3x = \frac{1}{2}$$



Solving Trigonometric Equations

Example :

Solve $1+\sqrt{2}\sin 6t = 0 \qquad (0 \le t \le 180^o)$

Step 1: Re-Arrange

$$\sin 6t = \frac{-1}{\sqrt{2}}$$

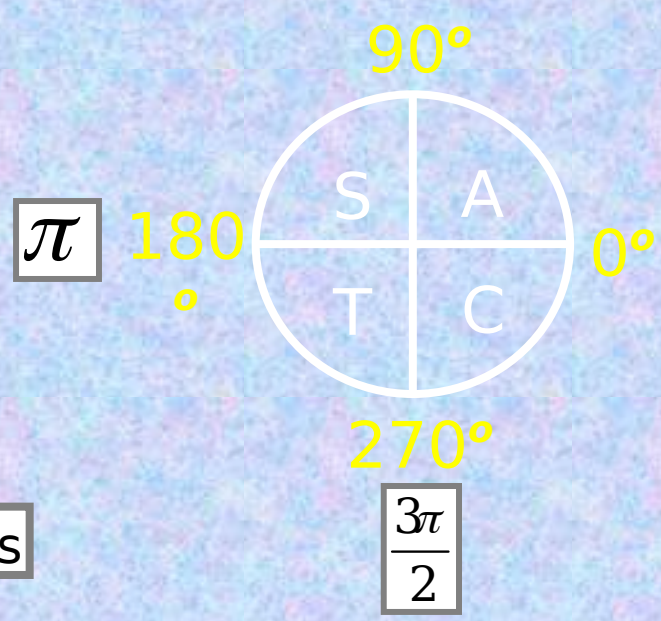
Step 2: consider what solutions are expected

sin *6t* is negative so solutions in the third and fourth

Since $0 \le t \le 180^o$ has 2 solutions

x 6 x 6

Then $0 \le 6t \le 1080^o$ has 12 solutions



π $\frac{\pi}{2}$ $\frac{3\pi}{2}$

Solving Trigonometric Equations

Step 3: Solve the equation

$$\sin 6t = \frac{-1}{\sqrt{2}}$$

$$6t = \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$$

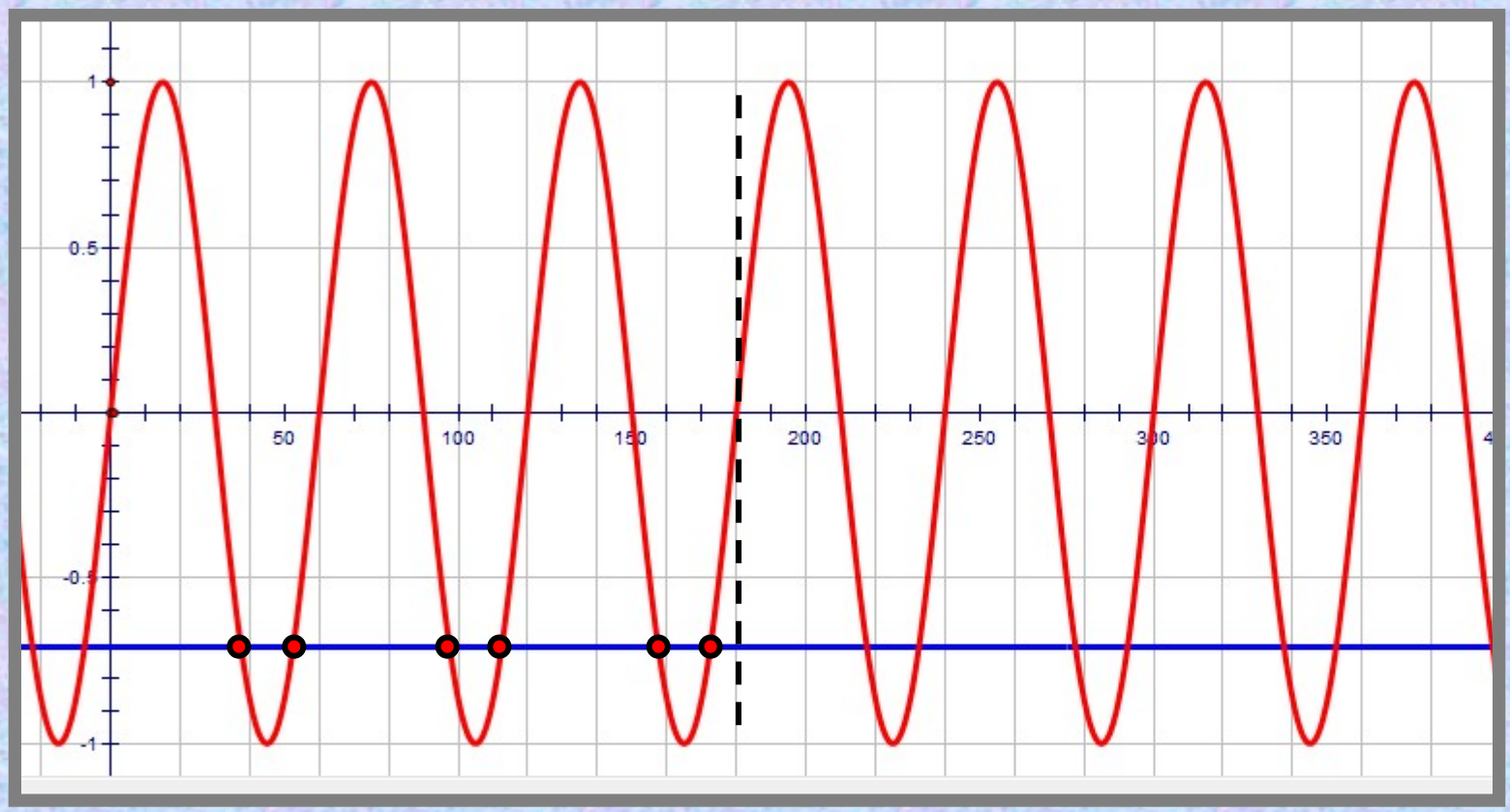
$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^{o} \text{ always 1st Quad first}$$

3rd quad 4th quad sin wave repeats every 360°

6t =	225°	315°	585°	675°	945°	1035°
x =	39.1°	52.5°	97.5°	112.5°	157.5°	172.5°

Solving Trigonometric Equations

Graphical solution fo $\sin 6t = \frac{-1}{\sqrt{2}}$



The solution is to be in radians – but work in degrees and convert at the end.

Example :

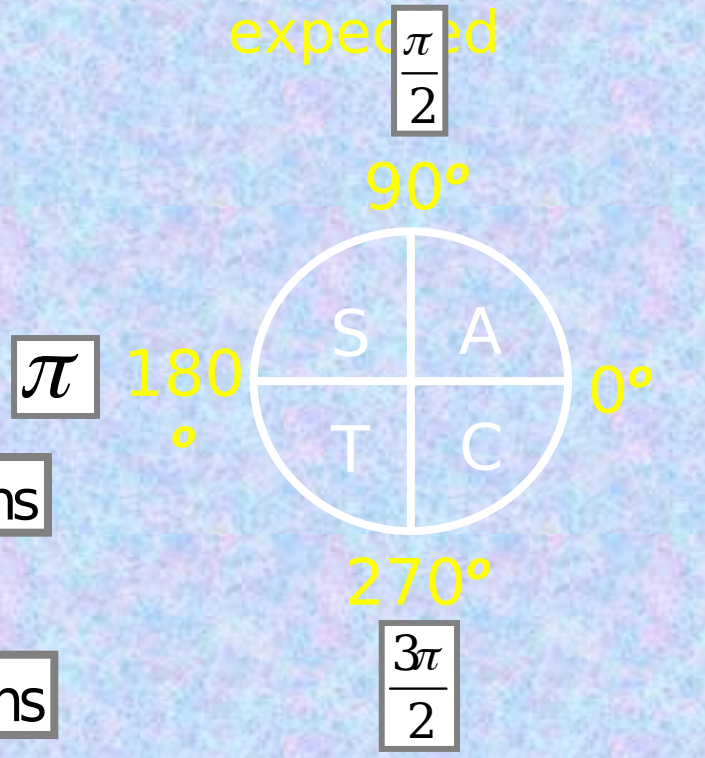
Solve $2\sin(2x - \frac{\pi}{3}) = 1 \qquad (0 \le x \le 2\pi)$

Step 1: Re-Arrange

$$\sin(2x - 60^o) = \frac{1}{2}$$

$(2x - 60^\circ) = \sin^{-1}(1/2)$

Step 2: consider what solutions are expected



Since $0 \le x \le 360^o$ has 2 solutions

x 2

Then $0 \le 2x \le 720^o$ has 4 solutions

x 2

Solving Trigonometric Equations

Step 3: Solve the equation

$\sin(2x - 60^o) = \frac{1}{2}$ $\quad 2x - 60^o = \sin^{-1}\left(\frac{1}{2}\right)$ $\quad \sin^{-1}\left(\frac{1}{2}\right) = 30^o \quad (1^{st}\ \text{quadrant})$

$2x - 60^o = 30^o$ and 150°

1st quad 2nd quad sin wave repeats every

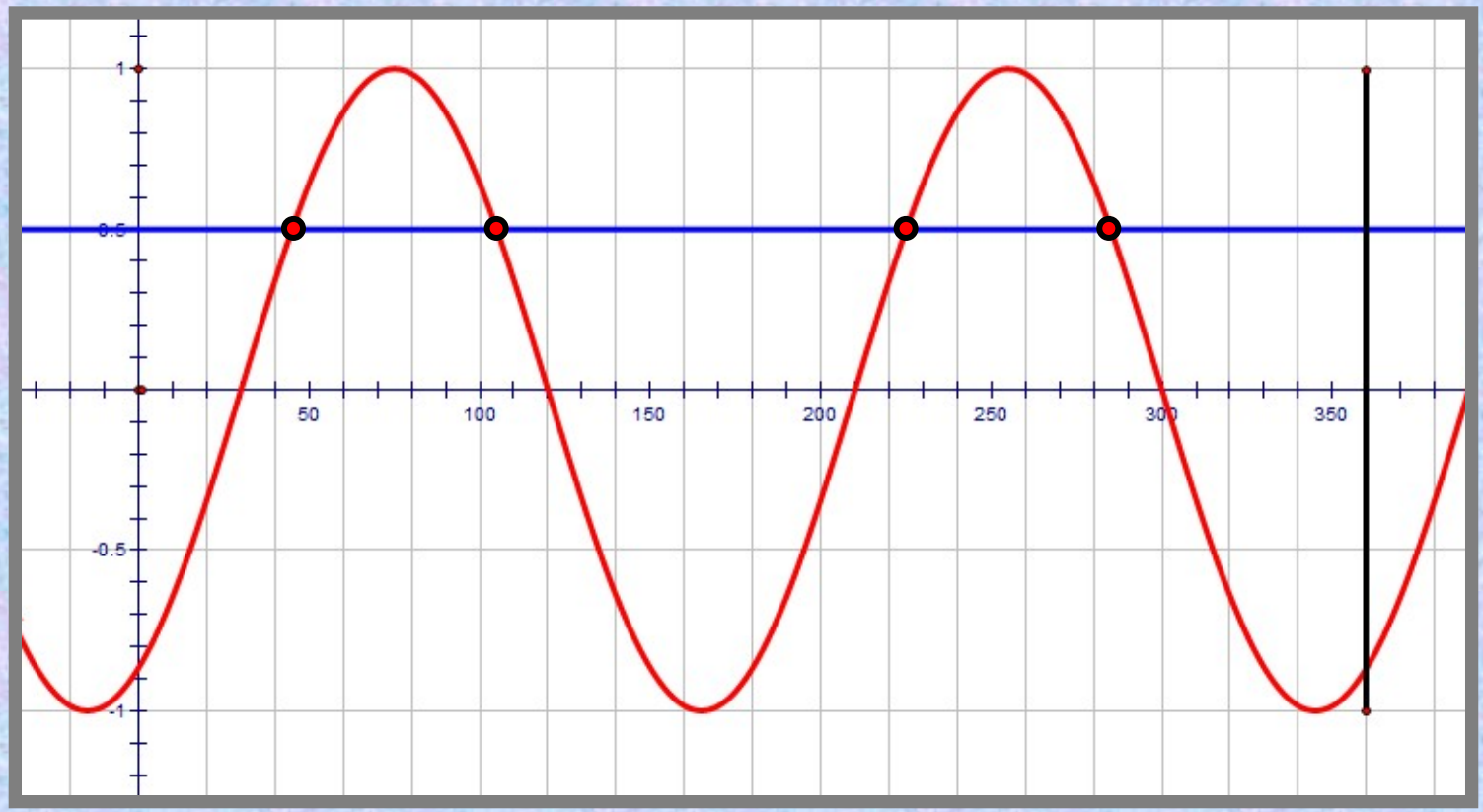
$2x = 90^\circ \quad 210^\circ \quad 450^\circ \quad 570^\circ \quad 360^\circ$

$x = 45^\circ \quad 105^\circ \quad 225^\circ \quad 285^\circ$

$\frac{\pi}{4} \quad \frac{7\pi}{12} \quad \frac{5\pi}{4} \quad \frac{19\pi}{12}$

Solving Trigonometric Equations

Graphical solution for $\sin(2x - 60^\circ) = \frac{1}{2}$



The solution is to be in radians – but work in degrees and convert at the end.

Harder Example:

Solve $\tan^2 x = 3 \quad (0 \le x \le 2\pi)$

Step 1: Re-Arrange

$$\tan x = \pm\sqrt{3}$$

$\tan x = +\sqrt{3}$

2 solutions

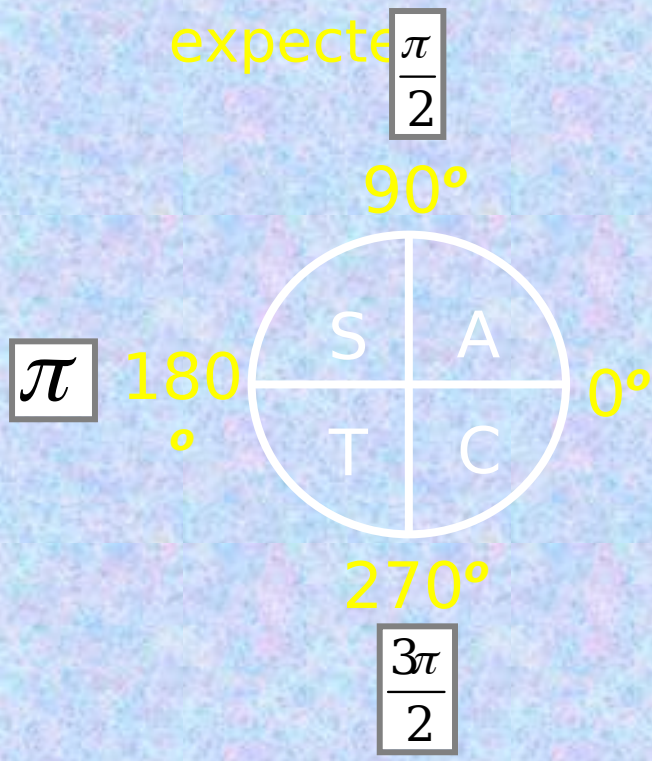
1st and 3rd quads

$\tan x = -\sqrt{3}$

2 solutions

2nd and 4th quads

Step 2: consider what solutions are expected



Solving Trigonometric Equations

Step 3: Solve the equation

$\tan x = +\sqrt{3}$ $\tan x = -\sqrt{3}$ $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$ (60° in the 1st quadrant)

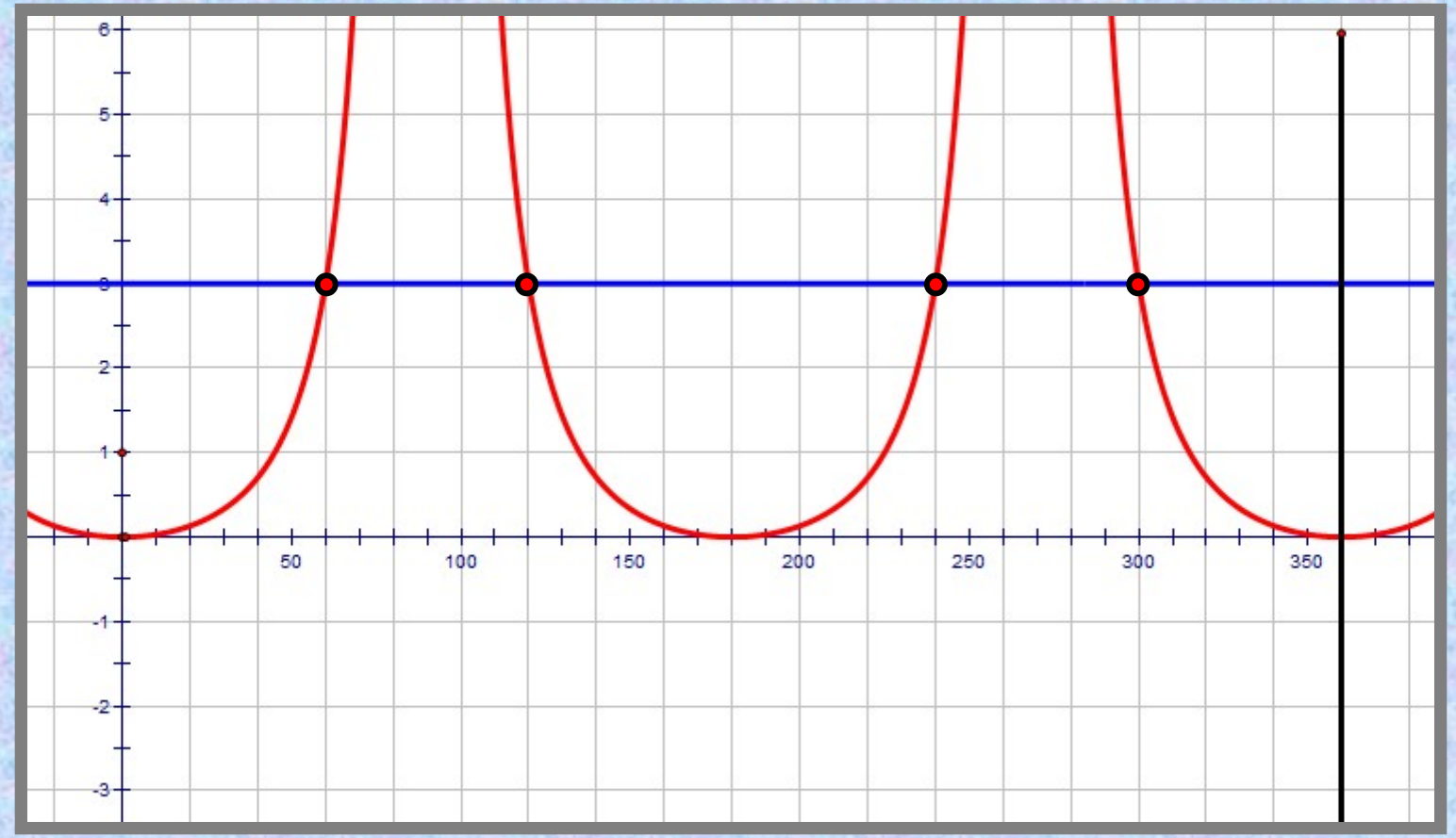
1st quad 2nd quad tan wave repeats every 180°

x = 60° 120° 240° 300°

$\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{4\pi}{3}$ $\frac{5\pi}{3}$

Solving Trigonometric Equations

Graphical solution for $\tan^2 x = 3$



Solving Trigonometric Equations

Harder Example:

Solve $3\sin^2 x - 4\sin x + 1 = 0 \quad (0 \le x \le 360^o)$

Step 1: Re-Arrange

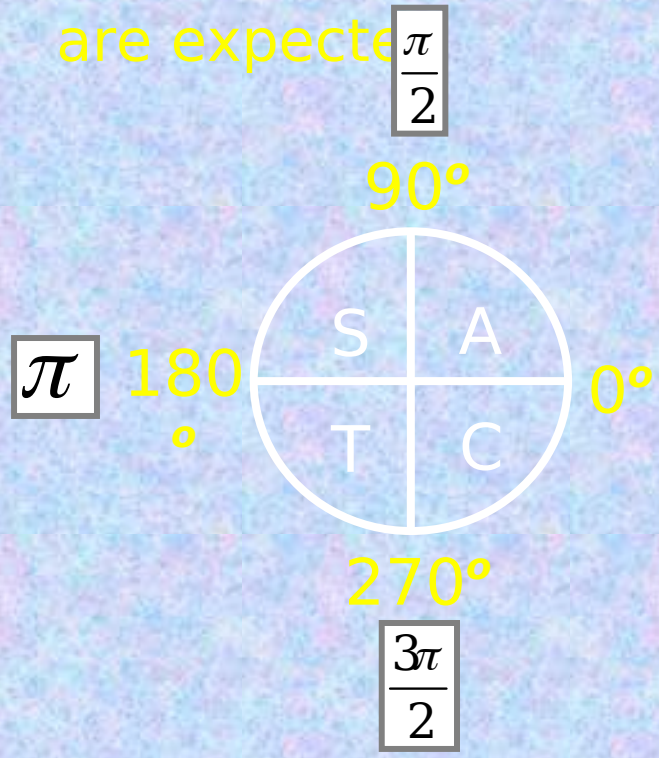
$$(3\sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{3} \qquad \sin x = 1$$

Two solutions

One solution

Step 2: Consider what solutions are expected



Solving Trigonometric Equations

Step 3: Solve the equation

$\sin x = \frac{1}{3}$

Two solutions

1st quad

x = 19.5°

2nd quad

160.5°

$\sin x = 1$

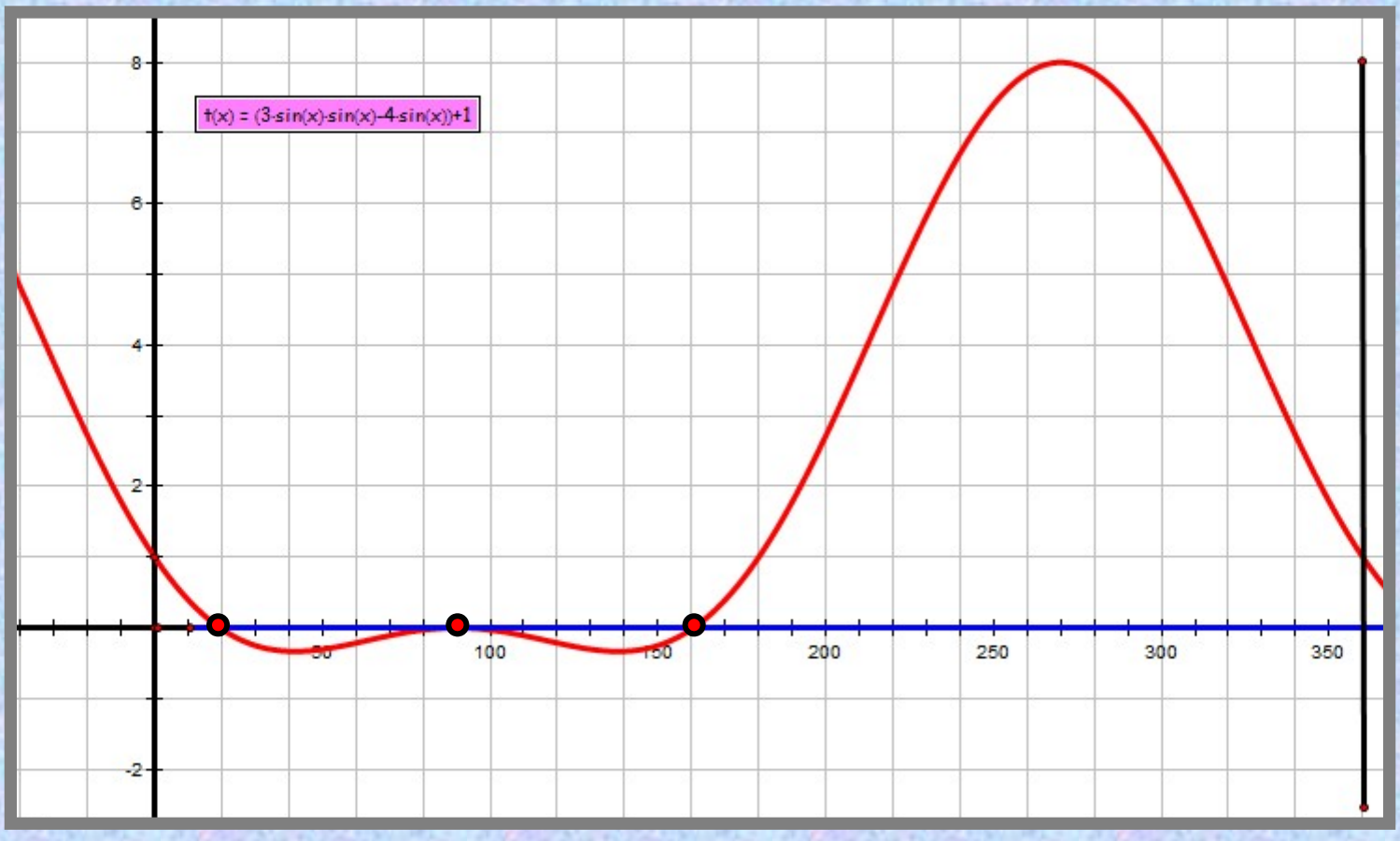
One solution

90°

Overall solution x = 19.5°, 90° and 160.5°

Solving Trigonometric Equations

Graphical solution for $3\sin^2 x - 4\sin x + 1 = 0$



The solution is to be in radians – but work in degrees and convert at the end.

Harder Example:

Solve $5\sin^2 x - 2 = 2\cos x \quad (0 \le x \le 2\pi)$

Step 1: Re-Arrange

$5(1 - \cos^2 x) - 2 = 2\cos x$

$3 - 2\cos x - 5\cos^2 x = 0$

$(3 - 5\cos x)(1 + \cos x) = 0$

$\cos x = \frac{3}{5}$

$\cos x = -1$

Two

One solution

Step 2: Consider what solutions

are expect

Remember this !

$\sin^2 \alpha + \cos^2 \alpha = 1$

$\cos^2 \alpha = 1 - \sin^2 \alpha$

$\sin^2 \alpha = 1 - \cos^2 \alpha$

0°

°

T C

270°

$\frac{3\pi}{2}$

Solving Trigonometric Equations

Step 3: Solve the equation

$$\boxed{\cos x = \frac{3}{5}}$$

$$\boxed{\cos x = -1}$$

Two solutions

One solution

1st quad 3rd quad

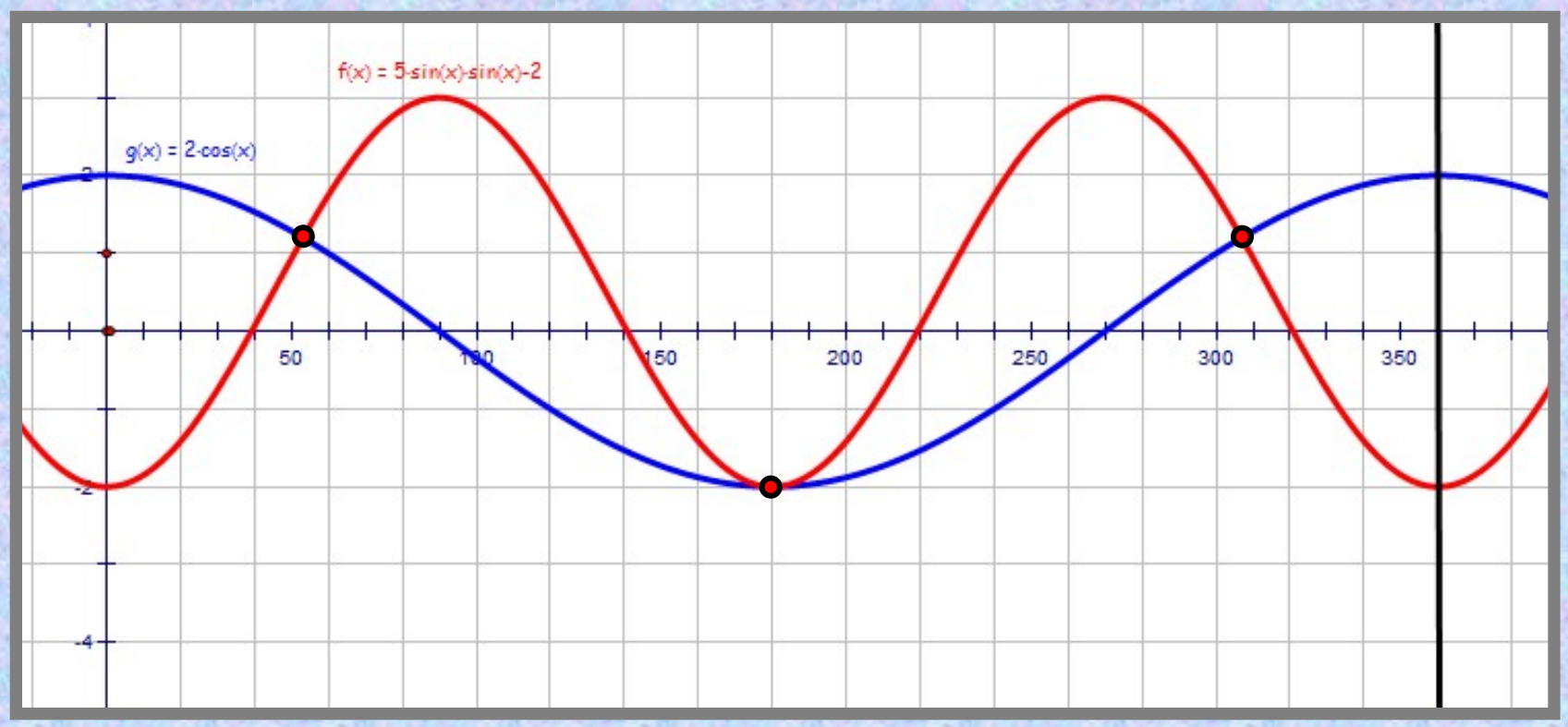
x = 53.1° 306.9°

180°

Overall solution in radians x = 0.93 , π and 5.35

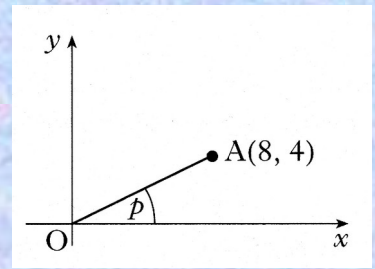
Solving Trigonometric Equations

Graphical solution for $5\sin^2 x - 2 = 2\cos x$

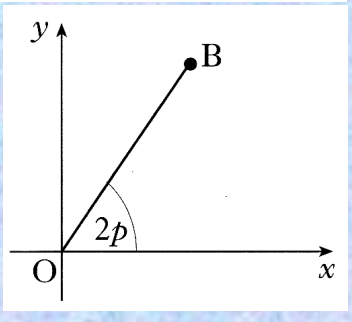


A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis

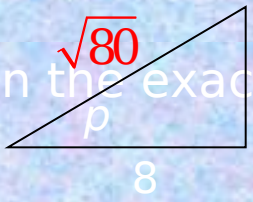
a) Find the exact values of: i) sin ($2p$)
ii) cos ($2p$)



The line OB is inclined at an angle $2p$ radians to the x-axis.



Draw triangle



Pythagoras

Write down values for cos p and sin p

$$\cos p = \frac{8}{\sqrt{80}} \qquad \sin p = \frac{4}{\sqrt{80}}$$

Expand **sin (2p)**

$$\sin 2p = 2\sin p\cos p \Rightarrow 2\times\frac{4}{\sqrt{80}}\times\frac{8}{\sqrt{80}} \Rightarrow \frac{64}{80} \Rightarrow \frac{4}{5}$$

Expand **cos (2p)**

$$\cos 2p = \cos^2 p - \sin^2 p \Rightarrow \left(\frac{8}{\sqrt{80}}\right)^2 - \left(\frac{4}{\sqrt{80}}\right)^2 \Rightarrow \frac{64-16}{80} \Rightarrow \frac{3}{5}$$

Use **m = tan (2p)**

$$\tan 2p = \frac{\sin 2p}{\cos 2p} \Rightarrow \frac{4}{5}\div\frac{3}{5} \Rightarrow \frac{4}{3}$$

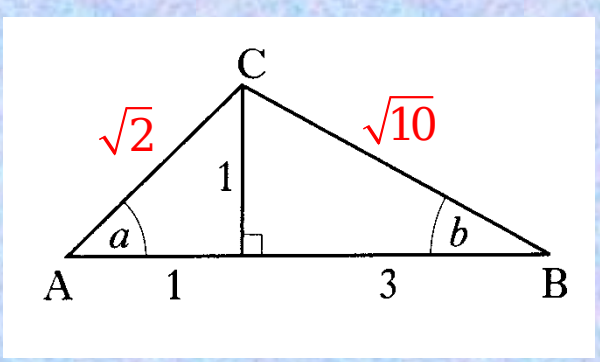


Hint



Previous Quit Quit Next

In triangle ABC show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$



Use Pythagoras

$AC = \sqrt{2}$ $CB = \sqrt{10}$

Write down values for

$\sin a = \frac{1}{\sqrt{2}}$ $\cos a = \frac{1}{\sqrt{2}}$ $\sin b = \frac{1}{\sqrt{10}}$ $\cos b = \frac{3}{\sqrt{10}}$

Expand **sin (a + b)**

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

Substitute values

$\sin(a+b) = \frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}$

Simplify

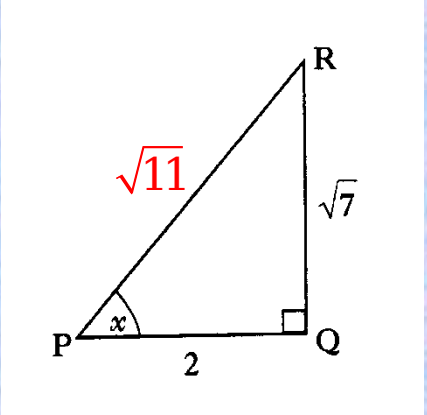
$\sin(a+b) = \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}} \rightarrow \frac{4}{\sqrt{20}} \rightarrow \frac{4}{\sqrt{4\times5}} \rightarrow \frac{4}{2\sqrt{5}} \rightarrow \frac{2}{\sqrt{5}}$

Hint

Previous Quit Quit Next

Using triangle PQR, as shown, find the

exact value of $\cos 2x$



Use Pythagoras

$$PR = \sqrt{11}$$

Write down values for

$\cos x$ and $\sin x$

$$\cos x = \frac{2}{\sqrt{11}} \qquad \sin x = \frac{\sqrt{7}}{\sqrt{11}}$$

Expand $\cos 2x$

$$\cos 2x = \cos^2 x - \sin^2 x$$

Substitute values

$$\cos 2x = \left(\frac{2}{\sqrt{11}}\right)^2 - \left(\frac{\sqrt{7}}{\sqrt{11}}\right)^2$$

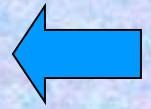
Simplify

$$\cos 2x = \frac{4}{11} - \frac{7}{11} \rightarrow -\frac{3}{11}$$





Hint



Previous

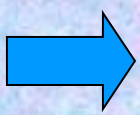
Quit



Quit



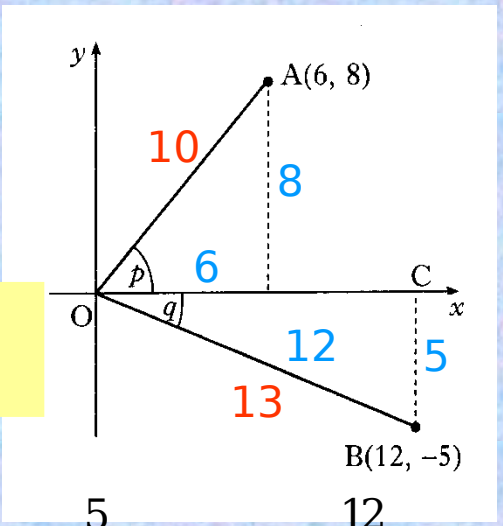
Next



On the co-ordinate diagram shown, A is the point (6, 8) and

B is the point (12, -5). Angle AOC = p and angle COB = q

Find the value of $\sin(p+q)$.



Mark up triangles

Use Pythagoras

$OA = 10 \quad OB = 13$

Write down values for

$\sin p, \cos p, \sin q,$

$\sin p = \frac{8}{10}, \quad \cos p = \frac{6}{10}, \quad \sin q = \frac{5}{13}, \quad \cos q = \frac{12}{13}$

Expand $\sin(p + q)$

$$\sin(p+q) = \sin p \cos q + \cos p \sin q$$

Substitute values

$$\sin(p+q) = \frac{8}{10} \times \frac{12}{13} + \frac{6}{10} \times \frac{5}{13}$$

Simplify

$$\sin(p+q) = \frac{96}{130} + \frac{30}{130} \rightarrow \frac{126}{130} \rightarrow \frac{63}{65}$$



Hint



Previous

Quit



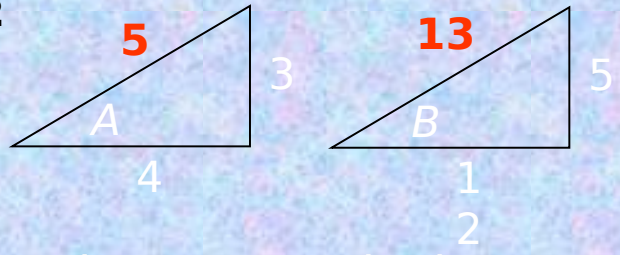
Quit

Next

A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

Find the exact value of

a) $\sin 2A$ b) $\cos 2A$ c) $\sin(2A + B)$



Draw triangles

Use Pythagoras

Hypotenuses are 5 and 13 respectively

Write down sin A, cos A, sin B, cos B

$$\sin A = \frac{3}{5},\ \cos A = \frac{4}{5},\ \sin B = \frac{5}{13},\ \cos B = \frac{12}{13}$$

Expand sin 2A

$$\sin 2A = 2\sin A\cos A \qquad \sin 2A = 2 \times \frac{3}{5} \times \frac{4}{5} \Rightarrow \frac{24}{25}$$

Expand cos 2A

$$\cos 2A = \cos^2 A - \sin^2 A \qquad \cos 2A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \Rightarrow \frac{16}{25} - \frac{9}{25} \Rightarrow \frac{7}{25}$$

Expand sin (2A + B)

$$\sin(2A + B) = \sin 2A\cos B + \cos 2A\sin B$$

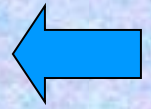
Substitute

$$\sin(2A + B) = \frac{24}{25} \times \frac{12}{13} + \frac{7}{25} \times \frac{5}{13} = \frac{323}{325}$$





Hint



Previous

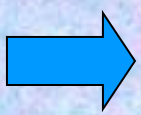
Quit



Quit

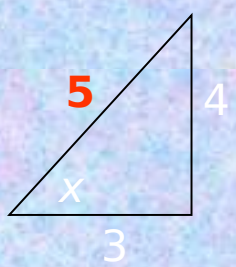


Next



If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$

show that the exact value of $\sin(x+30)°$ is $\frac{4\sqrt{3}+3}{10}$



Draw triangle

Use Pythagoras

Hypotenuse is 5

Write down sin x and cos x

$$\sin x = \frac{4}{5}, \quad \cos x = \frac{3}{5}$$

Expand sin (x + 30)

$$\sin(x+30) = \sin x \cos 30 + \cos x \sin 30$$

Substitute

$$\sin(x+30) = \frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2}$$

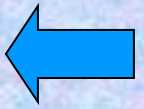
Simplify

$$\sin(x+30) = \frac{4\sqrt{3}}{10} + \frac{3}{10} \Rightarrow \frac{4\sqrt{3}+3}{10}$$





Hint



Previous



Table of exact values

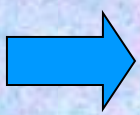
Quit



Quit



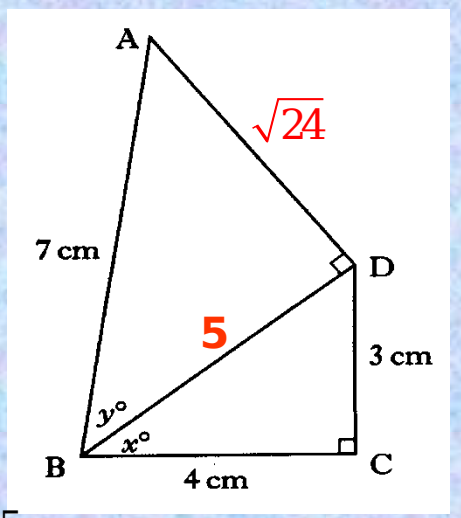
Next



The diagram shows two right angled triangles ABD and BCD with AB = 7 cm, BC = 4 cm and CD = 3 cm.

Angle DBC = $x°$ and angle ABD is $y°$.

Show that the exact value of $\cos(x+y)°$ *is* $\dfrac{20-6\sqrt{6}}{35}$



Use Pythagoras

$BD = 5,\ AD = \sqrt{24}$

Write down sin x, cos x, sin y, cos y.

$\sin x = \frac{3}{5},\ \cos x = \frac{4}{5},\ \sin y = \frac{\sqrt{24}}{7},\ \cos y = \frac{5}{7}$

Expand cos (x + y)

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

Substitute

$\cos(x+y) = \frac{4}{5} \times \frac{5}{7} - \frac{3}{5} \times \frac{\sqrt{24}}{7}$

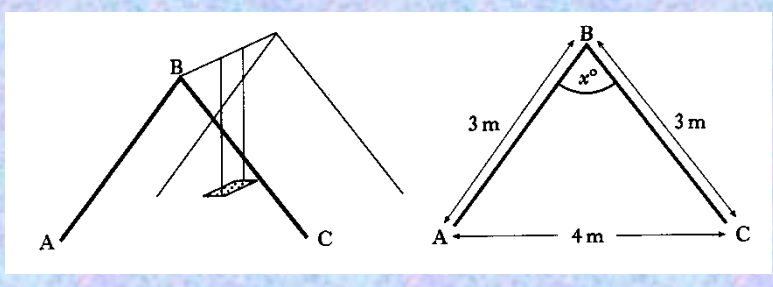
Simplify

$\cos(x+y) = \frac{20}{35} - \frac{3\sqrt{24}}{35} \Rightarrow \frac{20-3\sqrt{4\times 6}}{35} \Rightarrow \frac{20-6\sqrt{6}}{35}$

Hint

Previous Quit Quit Next

The framework of a child's swing has dimensions



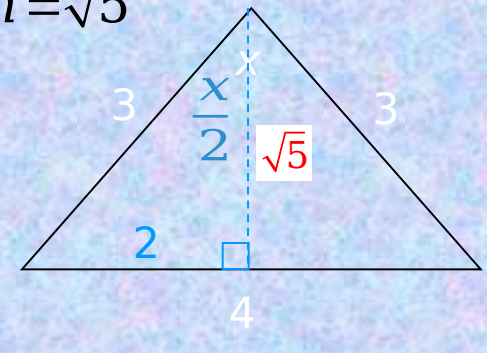
as shown in the diagram. Find the exact value of sin $x°$

Draw triangle

Draw in perpendicular

Use Pythagoras

$h = \sqrt{5}$



Use fact that sin x = sin (½ x + ½ x)

Write down sin ½ x and cos ½ x

$$\sin\left(\frac{x}{2}\right) = \frac{2}{3}, \quad \cos\left(\frac{x}{2}\right) = \frac{\sqrt{5}}{3}$$

Expand sin (½ x + ½ x)

$$\sin\left(\frac{x}{2} + \frac{x}{2}\right) = \sin\frac{x}{2}\cos\frac{x}{2} + \sin\frac{x}{2}\cos\frac{x}{2} = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

Substitute

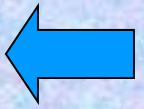
$$\sin\left(\frac{x}{2} + \frac{x}{2}\right) = 2 \times \frac{2}{3} \times \frac{\sqrt{5}}{3}$$

Simplify

$$\sin x = \frac{4\sqrt{5}}{9}$$



Hint



Previous



Table of exact values

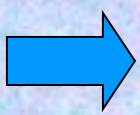
Quit



Quit

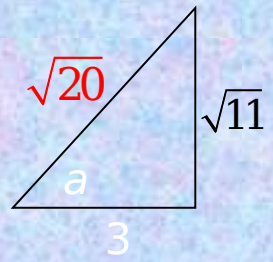


Next



Given that $\tan\alpha = \frac{\sqrt{11}}{3}, \quad 0 < \alpha < \frac{\pi}{2}$

find the exact value of $\sin 2\alpha$



Draw triangle

Use Pythagoras

$\text{hypotenuse} = \sqrt{20}$

Write down values for

cos a and sin a

$\cos a = \frac{3}{\sqrt{20}} \qquad \sin a = \frac{\sqrt{11}}{\sqrt{20}}$

Expand sin 2a

$\sin 2a = 2\sin a \cos a$

Substitute values

$\sin 2a = 2 \times \frac{\sqrt{11}}{\sqrt{20}} \times \frac{3}{\sqrt{20}}$

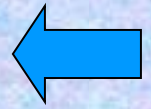
Simplify

$\sin 2a = \frac{6\sqrt{11}}{20} \Rightarrow \frac{3\sqrt{11}}{10}$





Hint



Previous

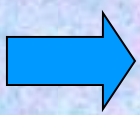
Quit



Quit



Next



Find algebraically the exact value of $\sin\theta^\circ + \sin(\theta+120)^\circ + \cos(\theta+150)^\circ$

Expand sin $(\theta + 120)$

$$\sin(\theta+120) = \sin\theta\ \cos120 + \cos\theta\ \sin120$$

Expand cos $(\theta + 150)$

$$\cos(\theta+150) = \cos\theta\ \cos150 - \sin\theta\ \sin150$$

Use table of exact values

$$\cos 120 = -\cos 60 = -\frac{1}{2} \qquad \cos 150 = -\cos 30 = -\frac{\sqrt{3}}{2}$$

$$\sin 120 = \sin 60 = \frac{\sqrt{3}}{2} \qquad \sin 150 = \sin 30 = \frac{1}{2}$$

Combine and substitute

$$\sin\theta + \sin\theta.\left(-\frac{1}{2}\right) + \cos\theta.\left(\frac{\sqrt{3}}{2}\right) + \cos\theta.\left(-\frac{\sqrt{3}}{2}\right) - \sin\theta.\left(\frac{1}{2}\right)$$

Simplify

$$\sin\theta - \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$$

$$= 0$$



Hint



Previous

Table of exact values

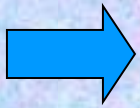


Quit

Quit

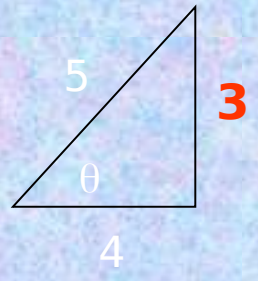


Next



If $\cos\theta = \frac{4}{5}, \quad 0 \le \theta \le \frac{\pi}{2}$ find the exact value of

a) $\sin 2\theta$ b) $\sin 4\theta$



Draw triangle

Use Pythagoras

Opposite side = 3

Write down values for

$\cos\theta = \frac{4}{5}$ $\sin\theta = \frac{3}{5}$

Expand sin 2θ

$\sin 2\theta = 2\sin\theta\cos\theta \Rightarrow 2\times\frac{3}{5}\times\frac{4}{5} \Rightarrow \frac{24}{25}$

Expand sin 4θ (4θ = 2θ + 2θ)

$\sin 4\theta = 2\sin 2\theta\cos 2\theta$

Expand cos 2θ

$\cos 2\theta = \cos^2\theta - \sin^2\theta \Rightarrow \frac{16}{25} - \frac{9}{25} \Rightarrow \frac{7}{25}$

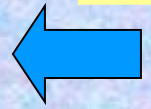
Find sin 4θ

$\sin 4\theta = 2\times\frac{24}{25}\times\frac{7}{25} \Rightarrow \frac{336}{625}$





Hint



Previous

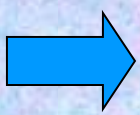
Quit



Quit

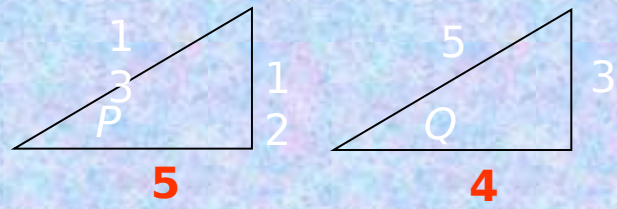


Next



For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$

Show that the exact value of $\sin(P+Q) = \frac{63}{65}$



Draw triangles

Use Pythagoras

Adjacent sides are 5 and 4 respectively

Write down sin P, cos P, sin Q, cos Q

$\sin P = \frac{12}{13},\ \cos P = \frac{5}{13},\ \sin Q = \frac{3}{5},\ \cos Q = \frac{4}{5}$

Expand sin (P + Q)

$\sin(P+Q) = \sin P \cos Q + \cos P \sin Q$

Substitute

$\sin(P+Q) = \frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$

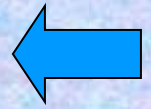
Simplify

$\sin(P+Q) = \frac{48}{65} + \frac{15}{65} \Rightarrow \frac{63}{65}$





Hint



Previous

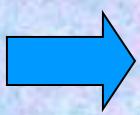
Quit



Quit



Next



Solve the equation $3\cos(2x)+10\cos(x)-1=0$ for $0 \le x \le \pi$

correct to 2 decimal places

Replace cos 2x with Substitute

$$\cos 2x = 2\cos^2 x - 1$$

$$3(2\cos^2 x - 1) + 10\cos x - 1 = 0$$

Simplify

$$6\cos^2 x + 10\cos x - 4 = 0$$

$$3\cos^2 x + 5\cos x - 2 = 0$$

Factorise

$$(3\cos x - 1)(\cos x + 2) = 0$$

Hence

$$\cos x = \frac{1}{3}$$

$$\cos x = -2 \quad \text{Discard}$$

Find acute x

$$\text{acute } x = 1.23 \text{ rad}$$

Determine quadrants



$$x = 1.23 \text{ or } 2\pi - 1.23 \text{ rads}$$

$$x = 1.23 \text{ rads}$$

$$x = 5.05 \text{ rads}$$

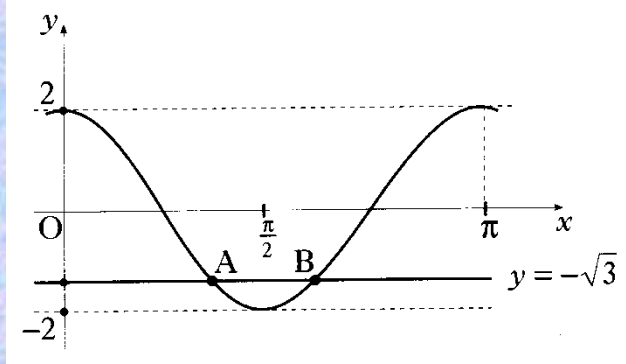
Hint

Previous Quit Quit Next

The diagram shows the graph of a cosine function from 0 to π.

a) State the equation of the graph.

b) The line with equation $y = -\sqrt{3}$ intersects this graph



$$y = 2\cos 2x$$

Equation

points A and B. Find the co-ordinates

Solve simultaneously

$$2\cos 2x = -\sqrt{3}$$

Rearrange

$$\cos 2x = -\frac{\sqrt{3}}{2}$$

Check range

$$0 \le x \le \pi \quad \Rightarrow \quad 0 \le 2x \le 2\pi$$

Find acute 2x

$$acute\ 2x = \frac{\pi}{6}$$

Deduce 2x

$$2x = \frac{6\pi}{6} - \frac{\pi}{6} \ or \ \frac{6\pi}{6} + \frac{\pi}{6} \ rads$$

Determine quadrants

S A

T C

$$x = \frac{5\pi}{12} \ or \ \frac{7\pi}{12}$$

$$B \ is \ B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$$

Hint

Previous

Table of exact values

Quit

Quit

Next

Functions f and g are defined on suitable domains by $f(x) = \sin(x)$ and $g(x) = 2x$

a) Find expressions for:

i) $f(g(x))$ ii) $g(f(x))$

2 $f(g(x)) = g(f(x))$ for $0 \leq x \leq 360°$

1st expression

$$f(g(x)) = f(2x) = \sin 2x$$

expression

$$g(f(x)) = g(\sin x) = 2\sin x$$

Form equation

$$2\sin 2x = 2\sin x \rightarrow \sin 2x = \sin x$$

Replace sin 2x

$$2\sin x\cos x = \sin x$$

Rearrange

$$2\sin x\cos x - \sin x = 0$$

Common factor

$$\sin x(2\cos x - 1) = 0$$

Hence

$$\sin x = 0 \text{ or } 2\cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{2}$$

Determine x

$$\sin x = 0 \Rightarrow x = 0°, 360°$$

$$\cos x = \frac{1}{2} \Rightarrow \text{acute } x = 60°$$

Determine quadrants

S A

T C

$$x = 60°, 300°$$

$$x = 0°, 60°, 300°, 360°$$

Hint

Previous

Table of exact values

Quit

Quit

Next

Functions $f(x)=\sin x,\ \ g(x)=\cos x$ and $h(x)=x+\frac{\pi}{4}$ are defined on a suitable set of real numbers

a) Find expressions for i) $f(h(x))$ ii) $g(h(x))$

$$f(h(x))=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$$

b) i) Show that $f(h(x))-g(h(x))=1$ for $0\le x\le 2\pi$

ii) Find a similar expression for $g(h(x))$

Hence solve the equation

1st expression expression

$$f(h(x))=f\left(x+\frac{\pi}{4}\right)=\sin\left(x+\frac{\pi}{4}\right)$$

$$g(h(x))=g\left(x+\frac{\pi}{4}\right)=\cos\left(x+\frac{\pi}{4}\right)$$

Simplify 1st expr.

$$f(h(x))=\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}$$

Use exact values

$$f(h(x))=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$$

Similarly for 2nd expr.

$$g(h(x))=\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}$$

$$g(h(x))=\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x$$

Form Eqn.

$$f(h(x))-g(h(x))=1$$

Simplifies to Rearrange: acute x

$$\frac{2}{\sqrt{2}}\sin x=1$$

$$\sin x=\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{\sqrt{2}\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$acute\ x=\frac{\pi}{4}$$

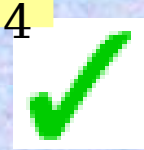
Determine quadrants

$$x=\frac{\pi}{4},\ \frac{3\pi}{4}$$





Hint







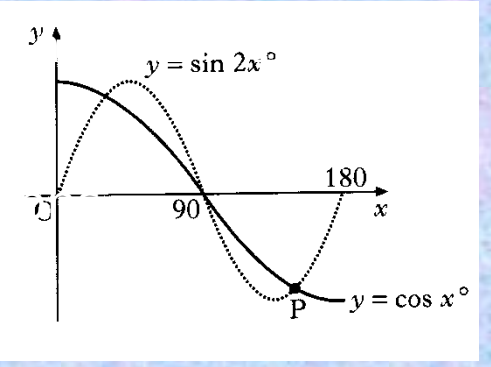
Previous Quit Quit Next

Table of exact values

a) Solve the equation $\sin 2x - \cos x = 0$ in the inter… 180°

b) The diagram shows parts of two trigonometric gra…

$y = \sin 2x$ and $y = \cos x$. Use your solutions in (a… …n the coordinates of the point P.



Replace sin 2x

$2\sin x\cos x - \cos x = 0$

factor

$\cos x(2\sin x - 1) = 0$

Henc… e

Determin… e x

$\cos x = 0$ or $2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2}$

$\cos x = 0 \Rightarrow x = 90°$, ($270°$ *out of range*)

$\sin x = \frac{1}{2} \Rightarrow$ *acute* $x = 30°$

Determine quadrants for sin x

S A

T C

$x = 30°,\ 150°$

Solutions for where graphs cross

$x = 30°, 90°, 150°$

By inspection

$x = 150°$

Find y value

$y = \cos 150°$

$y = -\frac{\sqrt{3}}{2}$

Coords, P

$P\left(150°, -\frac{\sqrt{3}}{2}\right)$

Hint

Previous

Table of exact values

Quit

Quit

Next

Solve the equation $3\cos(2x)+\cos(x)=-1$ for $0 \le x \le 360°$

Replace cos 2x with Substitute

$$\cos 2x = 2\cos^2 x - 1$$

$$3(2\cos^2 x - 1) + \cos x = -1$$

Simplify

$$6\cos^2 x + \cos x - 2 = 0$$

Factorise

$$(3\cos x + 2)(2\cos x - 1) = 0$$

Hence

$$\cos x = -\frac{2}{3} \qquad \cos x = \frac{1}{2}$$

Find acute x

$$\text{acute } x = 48° \qquad \text{acute } x = 60°$$

Determine quadrants

$\cos x = -\frac{2}{3}$ $\quad$ $\cos x = \frac{1}{2}$

$\text{acute } x = 48°$ $\quad$ $\text{acute } x = 60°$





$x = 132°$ $\quad$ $x = 60°$

$x = 228°$ $\quad$ $x = 300°$

Solutions are: x= 60°, 132°, 228° and 300°

Hint

Previous
Table of exact values
Quit
Quit
Next

Solve the equation $2\sin\left(2x-\frac{\pi}{6}\right)=1$ for $0 \le x \le 2\pi$

Rearrange

$$\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$$

Find acute x

$$\text{acute}\left(2x-\frac{\pi}{6}\right)=\frac{\pi}{6}$$

Note range

$$0 \le x \le 2\pi \Rightarrow 0 \le 2x \le 4\pi$$

Determine quadrants



for range $0 \le 2x \le 2\pi$

$$\left(2x-\frac{\pi}{6}\right)=\frac{\pi}{6} \qquad \left(2x-\frac{\pi}{6}\right)=\frac{5\pi}{6}$$

and for range $2\pi \le 2x \le 4\pi$

$$\left(2x-\frac{\pi}{6}\right)=\frac{13\pi}{6} \qquad \left(2x-\frac{\pi}{6}\right)=\frac{17\pi}{6}$$

Solutions are:

$$x=\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$$



Hint







Previous

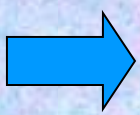
Table of exact values

Quit



Quit

Next



a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that for $\cos\theta$ it has equal roots.

b) Show that there are no real roots for θ

Replace $\cos 2\theta$ with

$$\cos 2\theta = 2\cos^2\theta - 1$$

Rearrange

$$2\cos^2\theta + 8\cos\theta + 8 = 0$$

Divide by 2

$$\cos^2\theta + 4\cos\theta + 4 = 0$$

Factorise

$$(\cos\theta + 2)(\cos\theta + 2) = 0$$

Deduction

Equal roots for $\cos\theta$

Try to solve:

$$(\cos\theta + 2) = 0$$

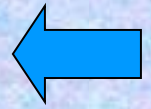
$\cos\theta = -2$ No solution

Hence there are no real solutions for θ





Hint



Previous

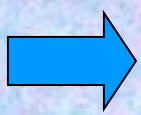
Quit



Quit



Next



Solve algebraically, the equation $\sin 2x + \sin x = 0, \quad 0 \le x \le 360$

Replace sin 2x

$$2\sin x\cos x+\sin x=0$$

Common factor

$$\sin x(2\cos x+1)=0$$

Hence

$$\sin x=0$$

$$\text{or } 2\cos x+1=0 \quad \Rightarrow \quad \cos x=-\frac{1}{2}$$

Determine x

$$\sin x=0 \Rightarrow x=0^\circ, 360^\circ$$

$$\cos x=-\frac{1}{2} \Rightarrow \textit{acute } x=60^\circ$$

Determine quadrants for $\cos x$



$$x=120^\circ, \ 240^\circ$$

X = 0°, 120°, 240°, 360°

Hint

Previous
Table of exact values
Quit
Quit
Next

Find the exact solutions of $4\sin^2 x = 1, \quad 0 \leq x \leq 2\pi$

Rearrange

$$\sin^2 x = \frac{1}{4}$$

Take square roots

$$\sin x = \pm\frac{1}{2}$$

Find acute x

$$\text{acute } x = \frac{\pi}{6}$$

Determine quadrants for sin x – from the square root requires all 4 quadrants



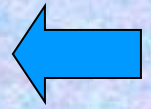
$$x = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}$$





Hint





Previous

Table of exact values

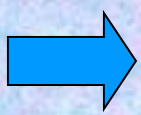


Quit

Quit



Next



Solve the equation $\cos 2x + \cos x = 0$ for $0 \leq x \leq 360°$

Replace cos 2x with Substitute

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - 1 + \cos x = 0$

Simplify

$2\cos^2 x + \cos x - 1 = 0$

Factorise

$(2\cos x - 1)(\cos x + 1) = 0$

Hence

$\cos x = \frac{1}{2}$ $\cos x = -1$

Find acute x

$acute\ x = 60°$ $x = 180°$

Determine quadrants

$\cos x = \frac{1}{2}$

$acute\ x = 60°$



$x = 60°$

$x = 300°$

Solutions are: x= 60°, 180° and 300°

Hint

Previous

Table of exact values

Quit

Quit

Next

Solve algebraically, the equation $\cos 2x + 5\cos x - 2 = 0$ for $0 \le x \le 360°$

Replace cos 2x with Substitute

$$\cos 2x = 2\cos^2 x - 1$$

$$2\cos^2 x - 1 + 5\cos x - 2 = 0$$

Simplify

$$2\cos^2 x + 5\cos x - 3 = 0$$

Factorise

$$(2\cos x - 1)(\cos x + 3) = 0$$

Hence

$$\cos x = \frac{1}{2} \qquad \cos x = -3$$

Find acute x

$acute\ x = 60°$ Discard above

Determine quadrants

$$\cos x = \frac{1}{2}$$

$acute\ x = 60°$



$x = 60°$

$x = 300°$

Solutions are: x= 60° and 300°

Hint

Previous
Table of exact values
Quit
Quit
Next

You have completed all 12 questions in this presentation

Previous

Quit



Quit

Back to start

Table of exact values

	30°	45°	60°
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Return

