

VECTORS



3D Vectors



Properties



Section



formula



Product



Angle between
vectors



Perpendicular



Properties of
Scalar
Product



Exam Type
Questions

Collinearity

Reminder from chapter 1

Points are said to be **collinear** if they lie on the **same straight line**.

For vectors

If $\vec{AB} = k\vec{BC}$, where k is a scalar, then $\vec{AB}$ is parallel to $\vec{BC}$.

If B is also a point common to both $\vec{AB}$ and $\vec{BC}$ then A, B and C are collinear.

Collinearity

Prove that the points A(2,4), B(8,6) and C(11,7) are collinear.

$$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 11 \\ 7 \end{pmatrix} - \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ 1 \end{pmatrix} = 2\vec{BC}$$

Collinearity

Since $AB = 2BC$, AB is parallel to BC .

B is a point common to both AB and BC
so A, B and C are collinear.

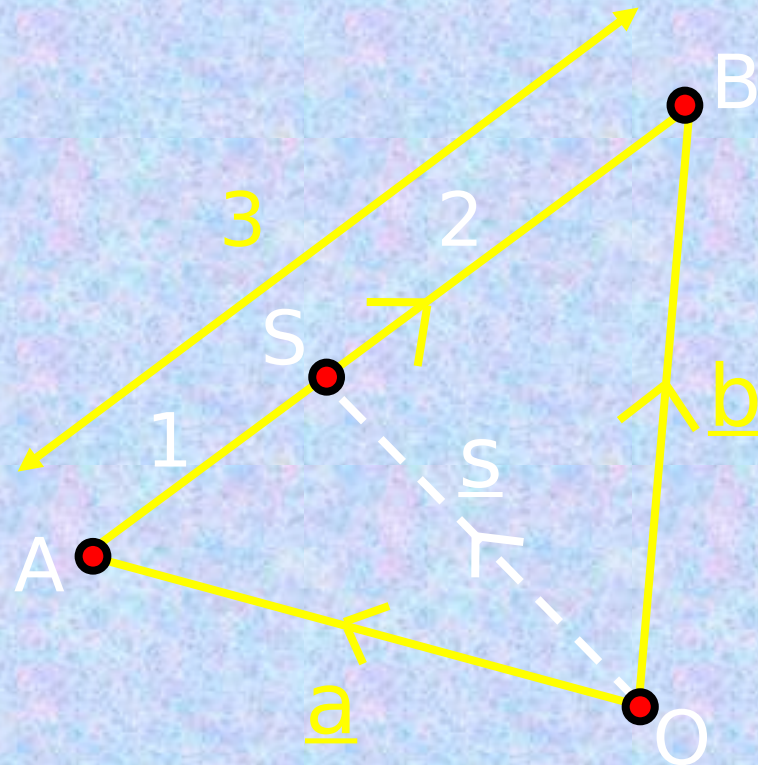
Section Formula

$$\vec{OS} = \vec{OA} + \vec{AS}$$

$$\vec{OS} = \vec{OA} + \frac{1}{3}\vec{AB}$$

$$\underline{s} = \underline{a} + \frac{1}{3}(\underline{b} - \underline{a})$$

$$\underline{s} = \frac{2}{3}\underline{a} + \frac{1}{3}\underline{b}$$



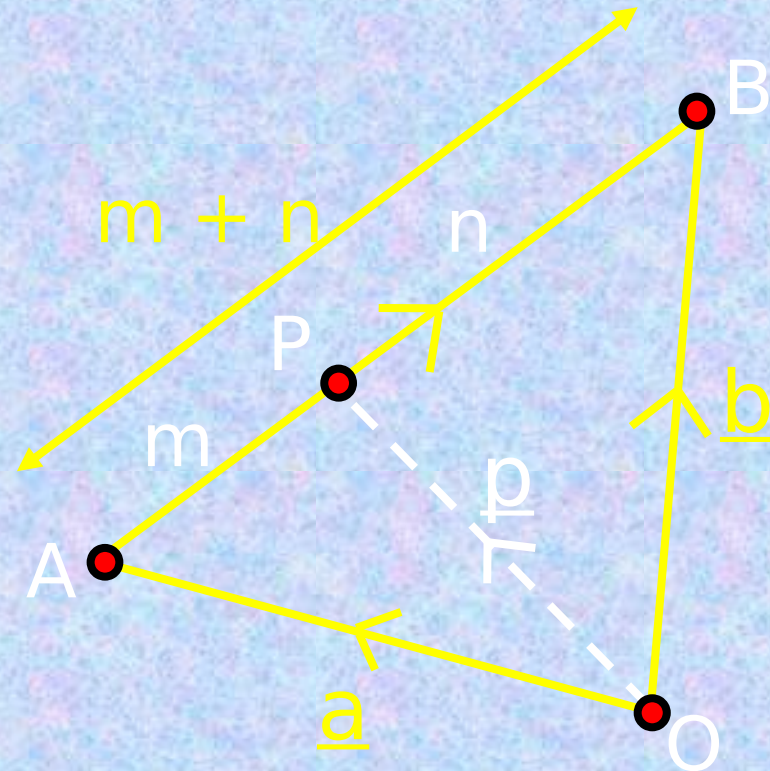
General Section Formula

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = \vec{OA} + \frac{m}{m+n} \vec{AB}$$

$$\underline{\underline{p}} = \underline{\underline{a}} + \frac{m}{m+n} (\underline{\underline{b}} - \underline{\underline{a}})$$

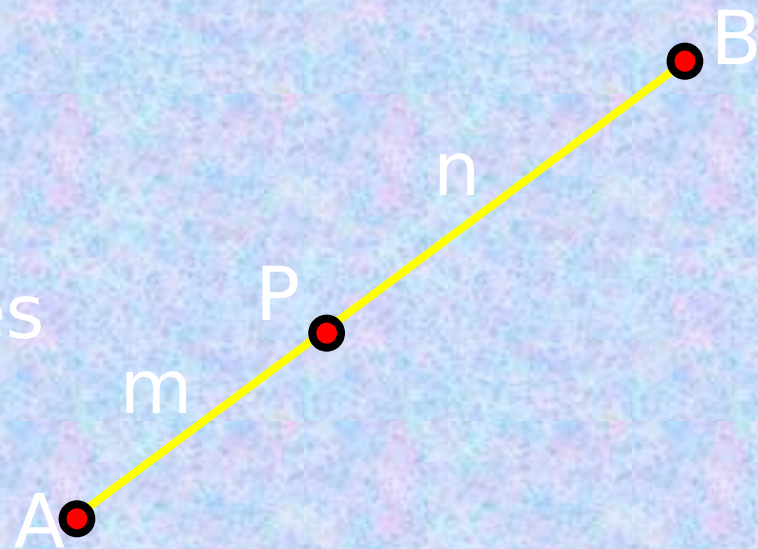
$$\underline{\underline{p}} = \frac{n}{m+n} \underline{\underline{a}} + \frac{m}{m+n} \underline{\underline{b}}$$



General Section Formula

Summarising we have

If $\underline{p}$ is a position vector
of the point P that divides
AB in the ratio $m : n$
then



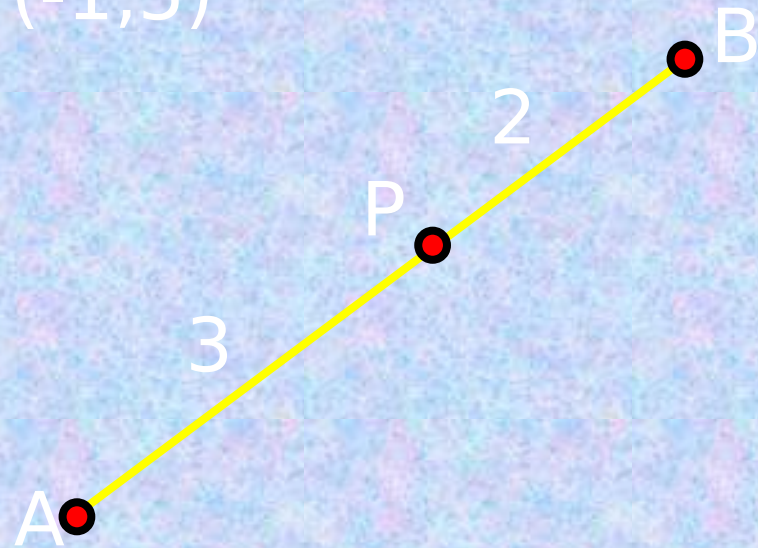
$$\underline{p} = \frac{n}{m+n} \underline{a} + \frac{m}{m+n} \underline{b}$$

General Section Formula

A and B have coordinates $(-1,5)$ and $(4,10)$ respectively.

Find P if AP : PB is 3:2

$$\underline{p} = \frac{2}{5}\underline{a} + \frac{3}{5}\underline{b}$$

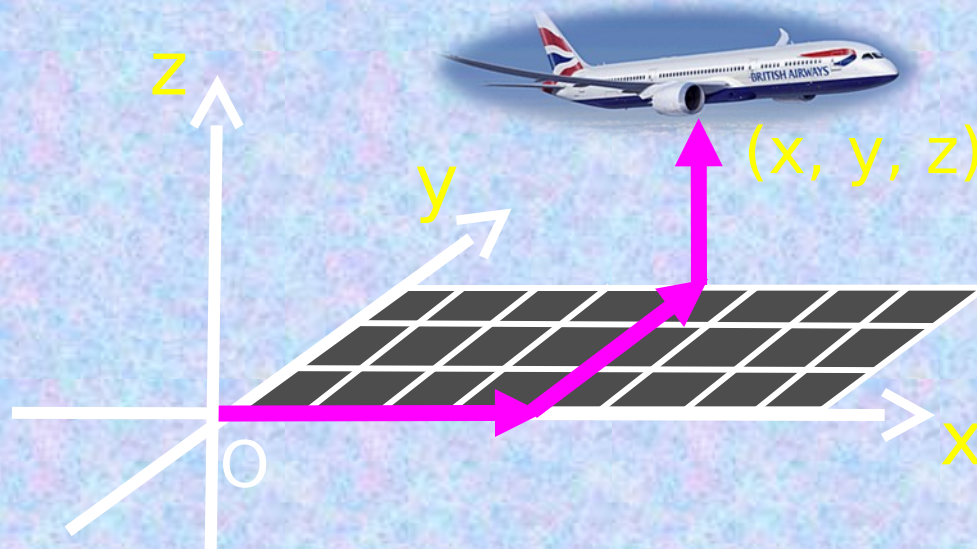


$$\underline{p} = \frac{2}{5} \begin{pmatrix} -1 \\ 5 \end{pmatrix} + \frac{3}{5} \begin{pmatrix} 4 \\ 10 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{12}{5} \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{10}{5} \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

3D Coordinates

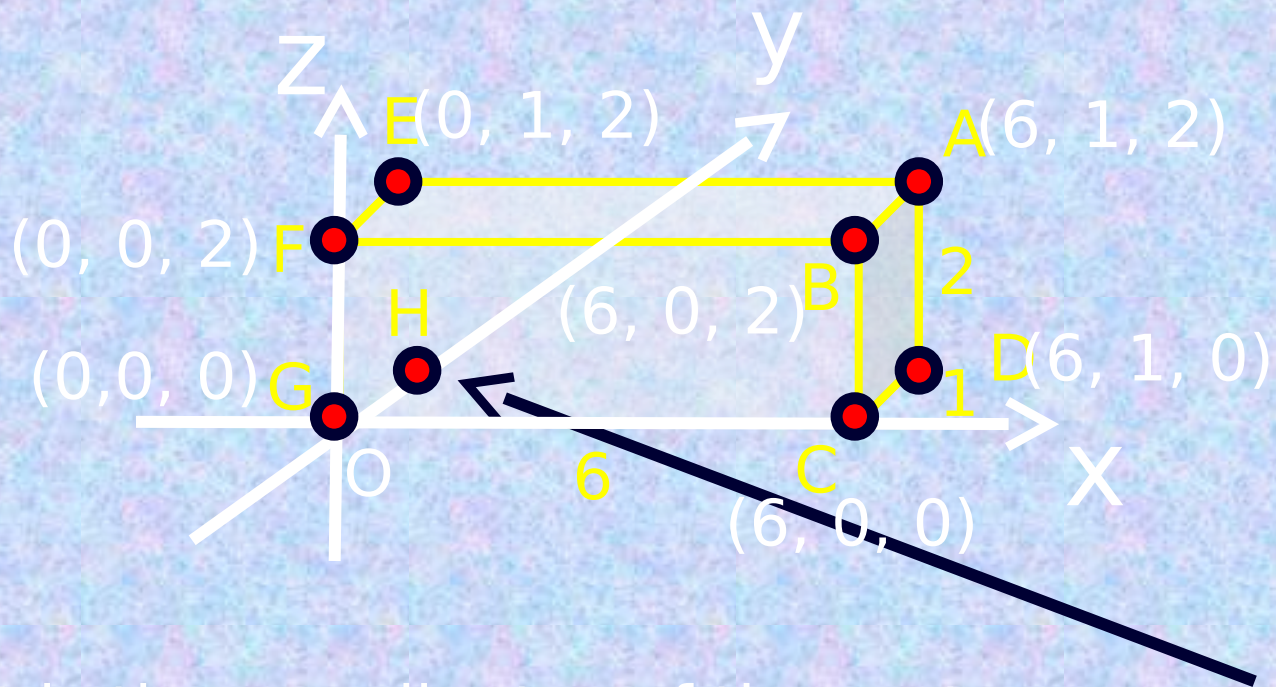
In the real world points in space can be located using a 3D coordinate system.

For example, air traffic controllers find the location a plane by its height and grid reference.



3D Coordinates

Write down the coordinates for the 7 vertices



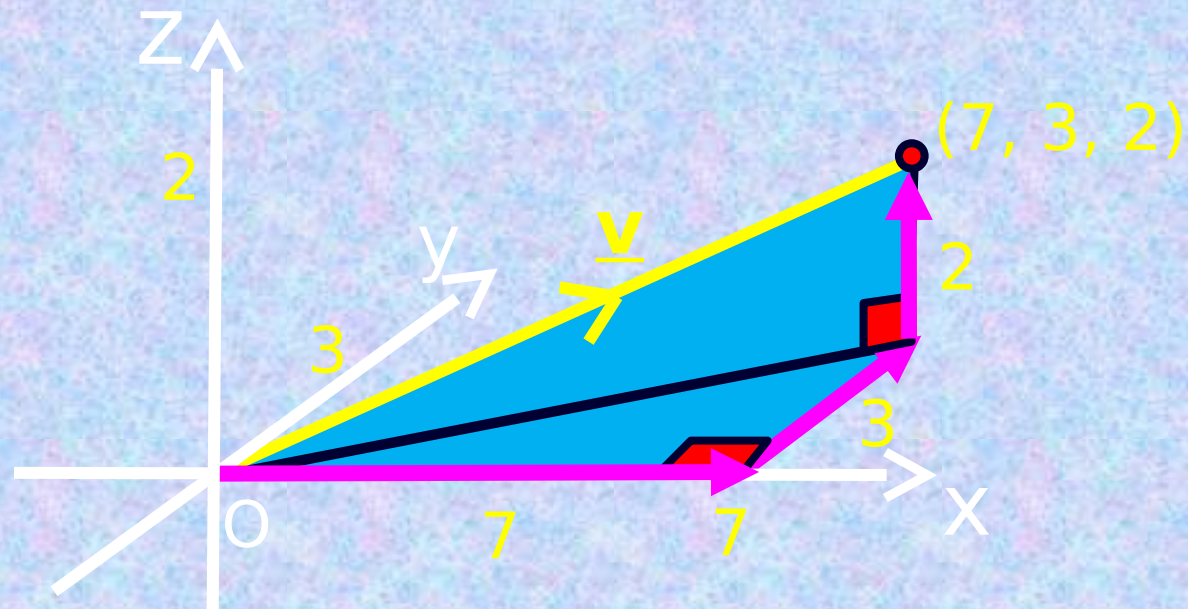
What is the coordinates of the vertex H so that it makes a cuboid shape.

H(0, 1, 0)

3D Vectors

3D vectors are defined by 3 components.

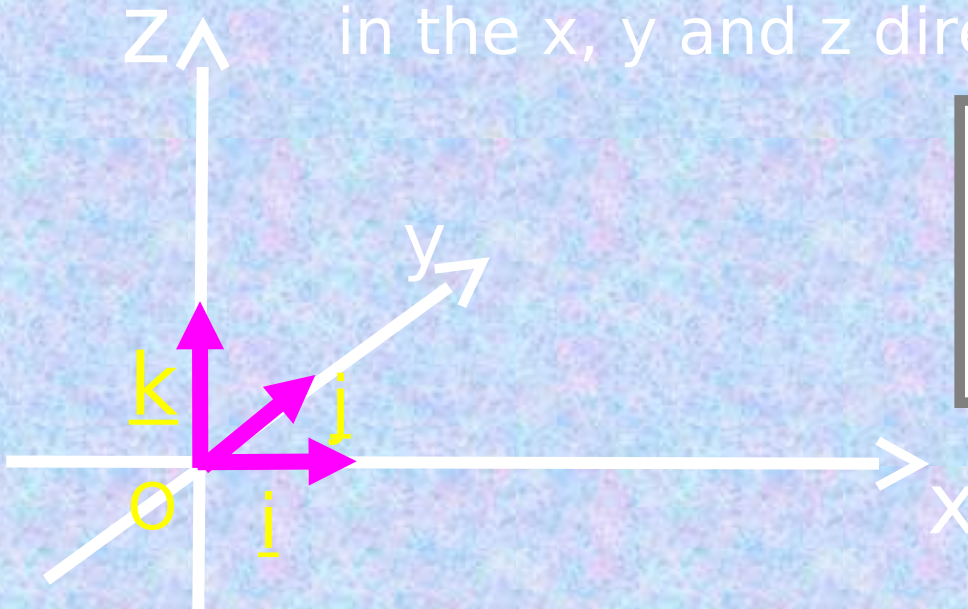
For example, the velocity of an aircraft taking off can be illustrated by the vector **v**.



3D Vectors

Any vector can be represented in terms of the **i**, **j** and **k**

Where **i**, **j** and **k** are unit vectors
in the x, y and z directions.

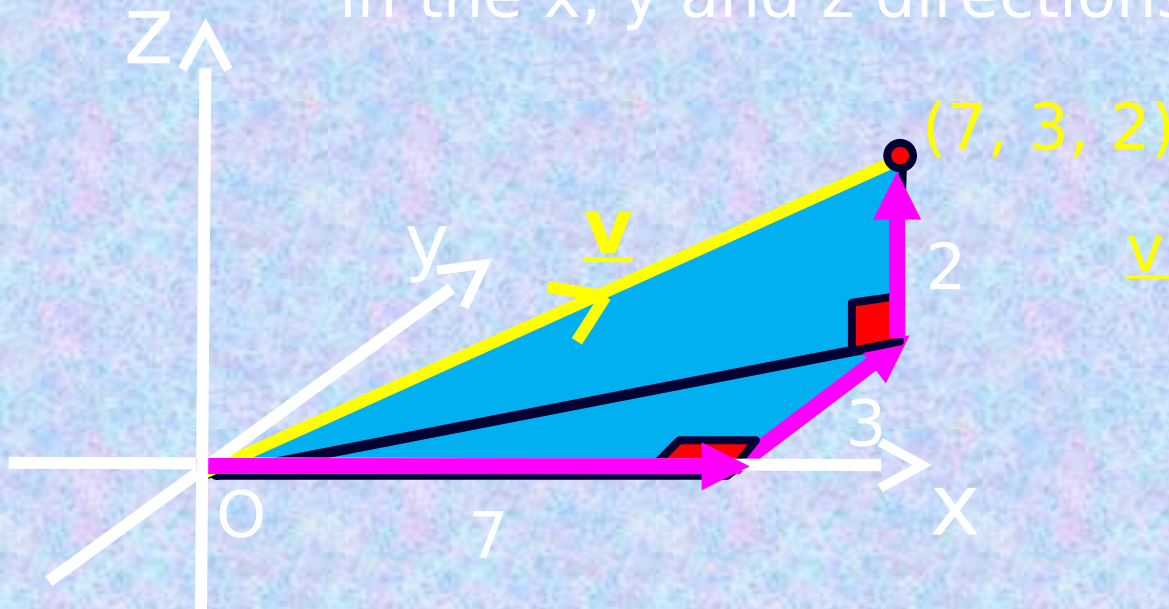


$$\underline{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \underline{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \underline{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

3D Vectors

Any vector can be represented in terms of the **i**, **j** and **k**

Where **i**, **j** and **k** are unit vectors
in the x, y and z directions.



$$\underline{v} = (7\underline{i} + 3\underline{j} + 2\underline{k})$$

$$\underline{v} = \begin{pmatrix} 7 \\ 3 \\ 2 \end{pmatrix}$$

3D Vectors

Good News

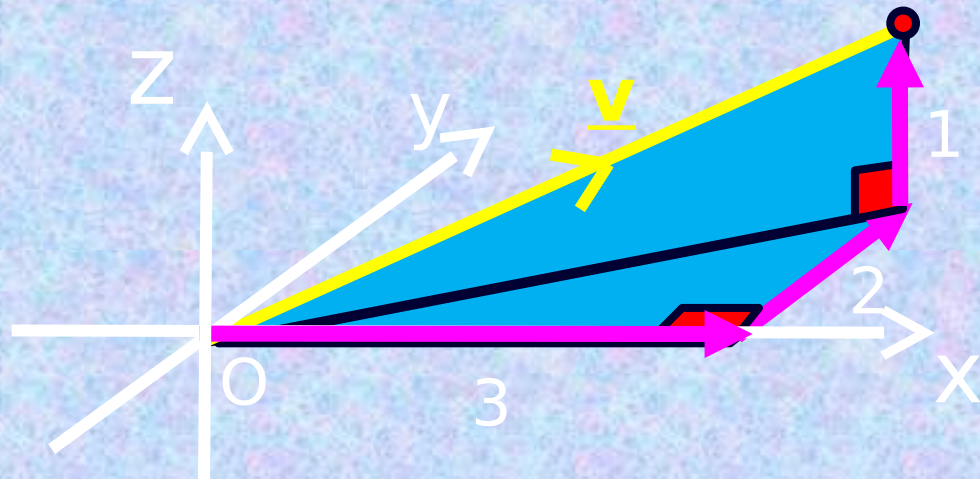
All the rules for 2D vectors
apply in the same way for
3D.

A vector's magnitude (length) is represented by $|\underline{v}|$

$$|\underline{v}| = \sqrt{x^2 + y^2 + z^2}$$

$$|\underline{\mathbf{v}}| = \sqrt{3^2 + 2^2 + 1^2}$$

$$|\underline{v}| = \sqrt{14}$$



Addition of Vectors

Addition of vectors

$$\text{Let } \underline{u} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \text{ and } \underline{v} = \begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix}$$

Then $\underline{u} + \underline{v}$

$$\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix}$$

Addition of Vectors

In general we
have

For vectors u and

v

$$\text{If } \underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ and } \underline{v} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \text{ then}$$

$$\underline{u} + \underline{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$$

Negative Vector

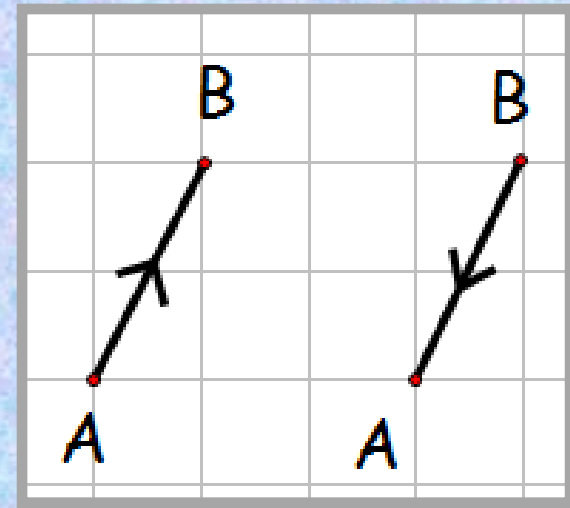
Negative vector

$\underline{BA}$ is the negative of $\underline{AB}$

For any vector $\underline{u}$

$$\underline{u} + (-\underline{u}) = \underline{u} + (-\underline{u}) = \underline{0}$$

$$\text{If } \underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ then } -\underline{u} = \begin{pmatrix} -a \\ -b \\ -c \end{pmatrix}$$



Subtraction of Vectors

Subtraction of vectors

$$\text{Let } \underline{a} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \text{ and } \underline{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

Then $\underline{a} - \underline{b}$

$$= \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

Subtraction of Vectors

For vectors u and v

$$\text{If } \underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ and } \underline{v} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \text{ then}$$

$$\underline{u} - \underline{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} a - d \\ b - e \\ c - f \end{pmatrix}$$

Multiplication by a Scalar

Multiplication by a scalar (a number)

$$\text{If a vector } \underline{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ then } k\underline{v} = \begin{pmatrix} kx \\ ky \\ kz \end{pmatrix}$$

The vector $k\underline{v}$ is parallel to vector $\underline{v}$ (different size)

Hence if $\underline{u} = k\underline{v}$ then $\underline{u}$ is parallel to $\underline{v}$

Conversely if $\underline{u}$ is parallel to $\underline{v}$ then $\underline{u} = k\underline{v}$

Multiplication by a Scalar

Show that the two vectors are parallel.

$$\underline{w} = \begin{pmatrix} 6 \\ 9 \\ 12 \end{pmatrix} \quad \text{then} \quad \underline{z} = \begin{pmatrix} 12 \\ 18 \\ 24 \end{pmatrix}$$

If $\underline{z} = k\underline{w}$ then $\underline{z}$ is parallel to $\underline{w}$

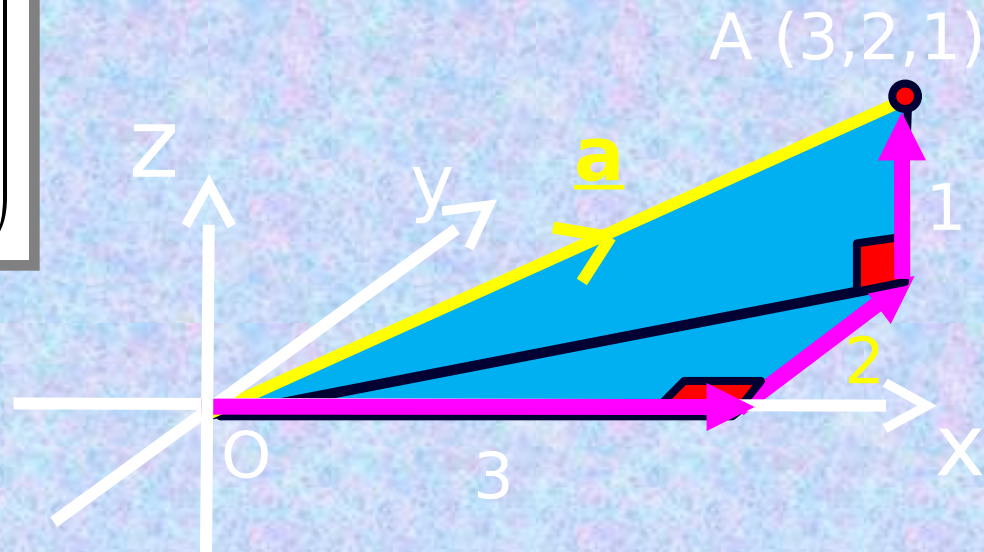
$$\underline{z} = \begin{pmatrix} 12 \\ 18 \\ 24 \end{pmatrix} = 2 \begin{pmatrix} 6 \\ 9 \\ 12 \end{pmatrix}$$

$$\underline{z} = 2\underline{w}$$

Position Vectors

The position vector of a 3D point A is OA , usually written as $\underline{a}$

$$\vec{OA} = \underline{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$



Position Vectors

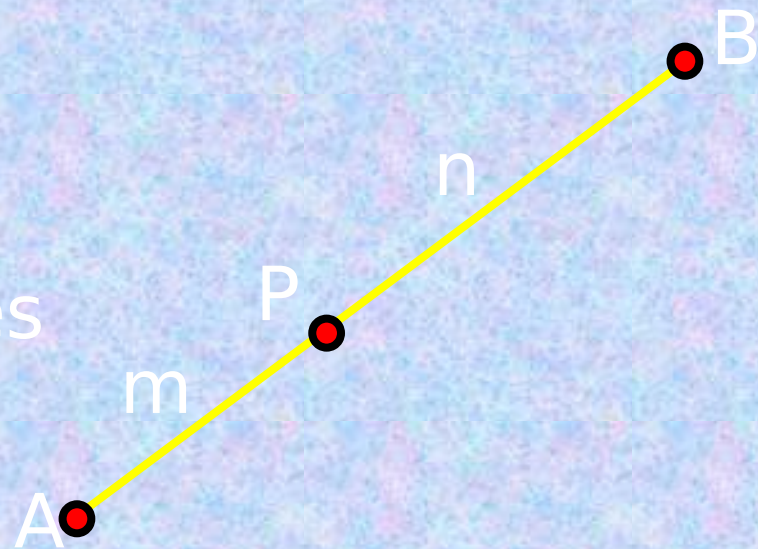
If R is (2,-5,1) and S is (4,1,-3) then $\underline{RS} = \underline{s} - \underline{r}$

$$= \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}$$

General Section Formula

Summarising we have

If $\underline{p}$ is a position vector
of the point P that divides
AB in the ratio $m : n$
then



$$\underline{p} = \frac{n}{m+n} \underline{a} + \frac{m}{m+n} \underline{b}$$

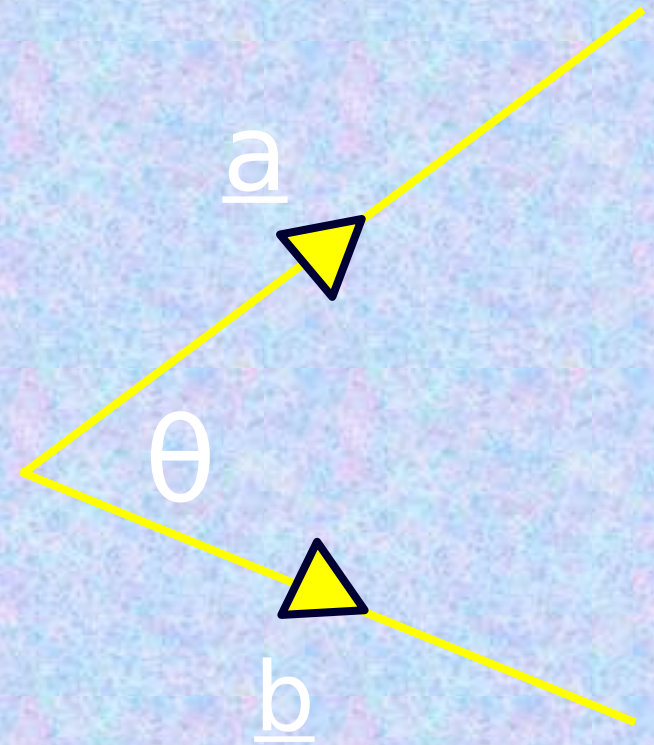
The scalar product

Must be
tail to tail

The scalar product is defined as being:

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$0 \leq \theta \leq 180^\circ$$



The Scalar Product

Find the scalar product for a and b when
a = 4 , b = 5 when (a) $\theta = 45^\circ$ (b) $\theta = 90^\circ$

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$= 4 \times 5 \cos 45^\circ$$

$$\underline{a} \cdot \underline{b} = \frac{20}{\sqrt{2}} = \frac{10 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}} = 10\sqrt{2}$$

The Scalar Product

Find the scalar product for a and b when
a = 4 , b = 5 when (a) $\theta = 45^\circ$ (b) $\theta = 90^\circ$

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos \theta$$

$$= 4 \times 5 \cos 90^\circ$$

$$\underline{a} \cdot \underline{b} = 20 \times 0 = 0$$

Important : If a and b are perpendicular
then

Component Form Scalar Product

If $\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ then

$$\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Angle between Vectors

To find the angle between two vectors we simply use the scalar product formula rearranged

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|}$$

or

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\underline{a}| |\underline{b}|}$$

Angle between Vectors

Find the angle between the two vectors

$$\underline{p} = 3\underline{i} + 2\underline{j} + 5\underline{k} \text{ and } \underline{q} = 4\underline{i} + \underline{j} + 3\underline{k}$$

$$\underline{p} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \text{ and } \underline{q} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$$

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|}$$

$$|\underline{p}| = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{38}$$

$$|\underline{q}| = \sqrt{4^2 + 1^2 + 3^2} = \sqrt{26}$$

Angle between Vectors

Find the angle between the two vectors
below

$$\underline{p} \cdot \underline{q} = 3 \times 4 + 2 \times 1 + 5 \times 3 = 29$$

$$|\underline{p}| = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{38}$$

$$|\underline{q}| = \sqrt{4^2 + 1^2 + 3^2} = \sqrt{26}$$

$$\cos \theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} = \frac{29}{\sqrt{38} \cdot \sqrt{26}} = 0.923$$

$$\theta = \cos^{-1}(0.923) = 22.7^\circ$$

Perpendicular Vectors

Show: $\underline{a} \cdot \underline{b} = 0$

$$\underline{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \quad \text{and} \quad \underline{b} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$$

$\underline{a}$ and $\underline{b}$ are perpendicular

$$\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\underline{a} \cdot \underline{b} = 3 \times 1 + 2 \times 2 + 7 \times (-1)$$

$$\underline{a} \cdot \underline{b} = 3 + 4 + (-7) = 0$$

Perpendicular Vectors

Given $|\underline{a}| \neq 0$ and $|\underline{b}| \neq 0$ and $\underline{a} \cdot \underline{b} = 0$

Then

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} = \frac{0}{|\underline{a}| |\underline{b}|} = 0$$

$$\theta = \cos^{-1}(0) = 90^\circ$$

If $\underline{a} \cdot \underline{b} = 0$ then $\underline{a}$ and $\underline{b}$ are perpendicular

Properties of a Scalar Product

Two properties that you need to be aware of

$$\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$$

$$\underline{a} \cdot (\underline{b} + \underline{c}) = \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$$

Higher Maths

Vectors

Strategies



Click to start

The following questions are on
Vectors

Non-calculator questions will be indicated



You will need a pencil, paper, ruler and rubber.



Click to continue

The questions are in groups



General vector questions
(15)



Points dividing lines in
ratios



Collinear points (8)
Angles between vectors (5)



General Vector Questions



Continue



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Vectors

Higher

Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

Determine whether or not $\mathbf{u}$ and $\mathbf{v}$ are perpendicular to each other.

Is Scalar product = 0

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$$\mathbf{u} \cdot \mathbf{v} = 3 \times 2 + 2 \times -3 + 0 \times 4 \rightarrow \mathbf{u} \cdot \mathbf{v} = 6 + (-6) + 0$$

$$\mathbf{u} \cdot \mathbf{v} = 0$$

Hence vectors are perpendicular



Hint



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Vectors

Higher

For what value of t are the vectors $\mathbf{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?

Put Scalar product = 0

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$$

$$\mathbf{u} \cdot \mathbf{v} = 2t + (-2) \times 10 + 3t \rightarrow \mathbf{u} \cdot \mathbf{v} = 5t - 20$$

$$\text{Perpendicular} \Rightarrow \mathbf{u} \cdot \mathbf{v} = 0 \rightarrow 0 = 5t - 20$$

$$\rightarrow t = 4$$



Hint



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Vectors is a pyramid with rectangular base
ABCD. **Higher**

The vectors AB , AD and AV are given

by $AB = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ $AD = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k}$

$AV = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

CV

Express CV in component form.

Triangle rule Δ
ACV

$AC + CV = AV$

Re-
arrange

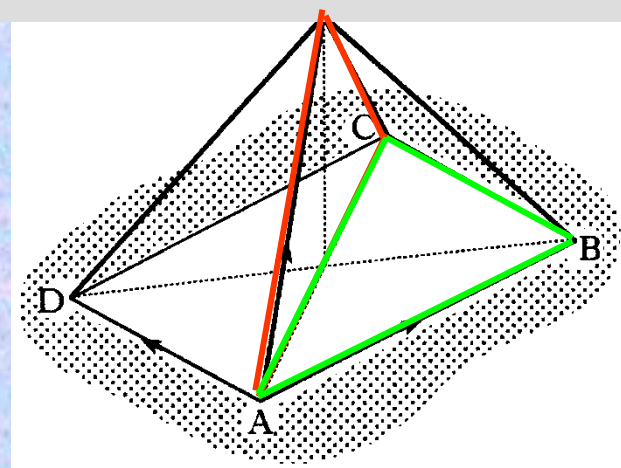
$\rightarrow CV = AV - AC$

Triangle rule Δ
ABC

$AB + BC = AC$ also $BC = AD$

$$\rightarrow CV = AV - (AB + AD) \rightarrow CV = \begin{pmatrix} 1 \\ 7 \\ 7 \end{pmatrix} - \begin{pmatrix} 8 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 10 \\ -2 \end{pmatrix} \rightarrow CV = \begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$$

$\rightarrow CV = -5\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$



Hint



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Vectors

Diagram shows two vectors **a** and **b**, with **Higher** $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2\sqrt{2}$.

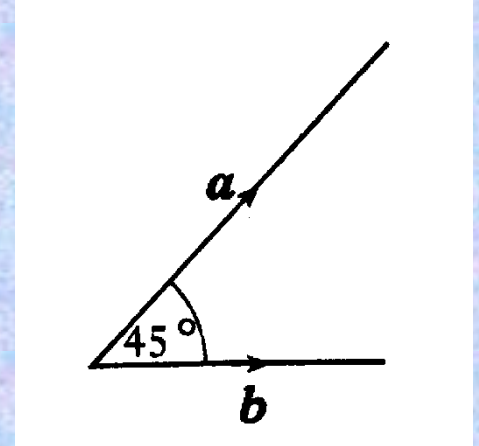
The vectors are inclined at an angle of 45° to each other.

evaluate

i) $\mathbf{a} \cdot \mathbf{a}$ ii) $\mathbf{b} \cdot \mathbf{b}$ iii) $\mathbf{a} \cdot \mathbf{b}$

Another vector **p** is defined by $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$

evaluate $\mathbf{p} \cdot \mathbf{p}$ and hence write down $|\mathbf{p}|$.



$$\text{i) } \mathbf{a} \cdot \mathbf{a} = |\mathbf{a}| |\mathbf{a}| \cos 0^\circ \rightarrow 3 \times 3 \times 1 = 9 \quad \text{ii) } \mathbf{b} \cdot \mathbf{b} = 2\sqrt{2} \times 2\sqrt{2} = 8$$

$$\text{iii) } \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos 45^\circ \rightarrow 3 \times 2\sqrt{2} \times \frac{1}{\sqrt{2}} = 6$$

$$\text{b) } \mathbf{p} \cdot \mathbf{p} = (2\mathbf{a} + 3\mathbf{b}) \cdot (2\mathbf{a} + 3\mathbf{b}) \rightarrow 4\mathbf{a} \cdot \mathbf{a} + 12\mathbf{a} \cdot \mathbf{b} + 9\mathbf{b} \cdot \mathbf{b}$$

$$\rightarrow 36 + 72 + 72 \rightarrow 180 \quad \text{Since } \mathbf{p} \cdot \mathbf{p} = p^2 \quad |\mathbf{p}| = \sqrt{180} \rightarrow 6\sqrt{5}$$



Hint



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Vectors

Higher

Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are defined by $\mathbf{p}=\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{q}=\mathbf{i}+4\mathbf{k}$, and $\mathbf{r}=4\mathbf{i}-3\mathbf{j}$

Express $\mathbf{p}-\mathbf{q}+2\mathbf{r}$ in component form

Calculate $\mathbf{p} \cdot \mathbf{r}$

Find $|\mathbf{r}|$

a) $\mathbf{p}-\mathbf{q}+2\mathbf{r} \rightarrow (\mathbf{i}+\mathbf{j}-\mathbf{k})-(\mathbf{i}+4\mathbf{k})+2(4\mathbf{i}-3\mathbf{j}) \rightarrow 8\mathbf{i}-5\mathbf{j}-5\mathbf{k}$

b) $\mathbf{p} \cdot \mathbf{r} = (\mathbf{i}+\mathbf{j}-\mathbf{k}) \cdot (4\mathbf{i}-3\mathbf{j}) \rightarrow \mathbf{p} \cdot \mathbf{r} = 1 \times 4 + 1 \times (-3) + (-1) \times 0 \rightarrow \mathbf{p} \cdot \mathbf{r} = 1$

c) $|\mathbf{r}| = \sqrt{4^2 + (-3)^2} \rightarrow |\mathbf{r}| = \sqrt{16+9} \rightarrow |\mathbf{r}| = 5$



Hint



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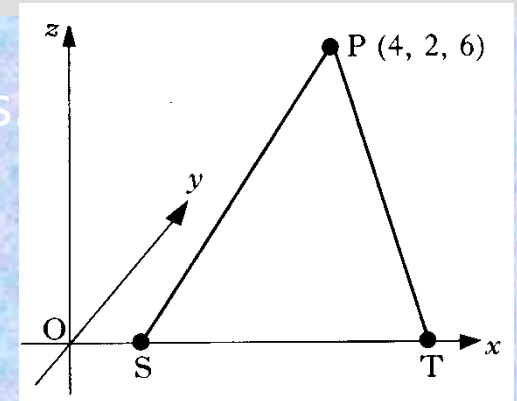
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Vectors

Higher

A diagram shows a point P with co-ordinates (4, 2, 6) and two points S and T which lie on the x-axis. P is 7 units from S and 7 units from T, find the co-ordinates of S and T.



Use distance formula

$$S(a, 0, 0) \quad T(b, 0, 0)$$

$$PS^2 = 49 = (4 - a)^2 + 2^2 + 6^2 \quad \rightarrow \quad 49 = (4 - a)^2 + 40 \quad \rightarrow \quad 9 = (4 - a)^2$$

$$\rightarrow a = 4 \pm 3 \quad \rightarrow \quad a = 7 \text{ or } a = 1$$

Hence there are 2 points on the x axis that are 7 units from P S and T

$$S(1, 0, 0) \quad \text{and} \quad T(7, 0, 0)$$



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Vectors

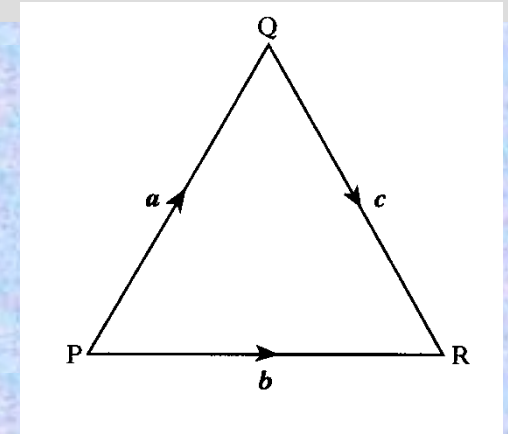
Higher

position vectors of the points P and Q are

$-i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

Express PQ in component form.

Find the length of PQ.



$$a) \quad PQ = q - p \rightarrow PQ = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \rightarrow PQ = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix} = 8i - 4j + k$$

$$b) \quad |PQ| = \sqrt{8^2 + (-4)^2 + 1^2} = \sqrt{64 + 16 + 1} = \sqrt{81} = 9$$



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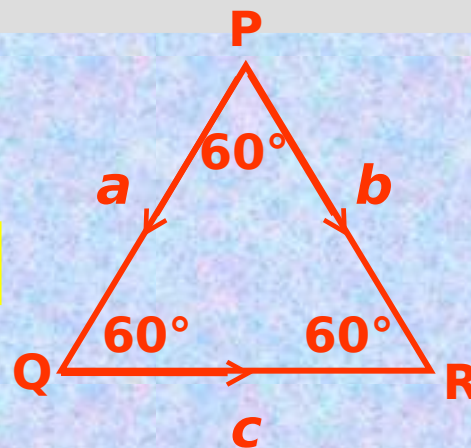
Vectors

Higher
PQR is an equilateral triangle of side 2 units.

$PQ = \mathbf{a}$, $PR = \mathbf{b}$, and $QR = \mathbf{c}$

evaluate $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ and hence identify
two vectors which are perpendicular.

Diagram



$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos 60^\circ \rightarrow \mathbf{a} \cdot \mathbf{b} = 2 \times 2 \times \frac{1}{2} \rightarrow \mathbf{a} \cdot \mathbf{b} = 2$$

NB for $\mathbf{a} \cdot \mathbf{c}$ vectors must point OUT of the vertex (so
angle is 120°)

$$\mathbf{a} \cdot \mathbf{c} = |\mathbf{a}| |\mathbf{c}| \cos 120^\circ \rightarrow \mathbf{a} \cdot \mathbf{c} = 2 \times 2 \times \left(-\frac{1}{2}\right) \rightarrow \mathbf{a} \cdot \mathbf{c} = -2$$

Hence $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 0$ so, $\mathbf{a}$ is perpendicular to $\mathbf{b} + \mathbf{c}$



Hint



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Table of
Exact
Values

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Vectors

Higher

Calculate the length of the vector $2\mathbf{i} - 3\mathbf{j} + \sqrt{3}\mathbf{k}$

$$\text{Length} = \sqrt{2^2 + (-3)^2 + (\sqrt{3})^2} \rightarrow \sqrt{4 + 9 + 3}$$

$$\rightarrow \sqrt{16}$$

$$\rightarrow 4$$



Hint



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Vectors

Higher

Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$ are perpendicular

Put Scalar product = 0

$$0 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$$

$$0 = -4 + 6 - (k - 1) \quad \rightarrow \quad 0 = 2 - k + 1$$

$$\rightarrow k = 3$$



Hint



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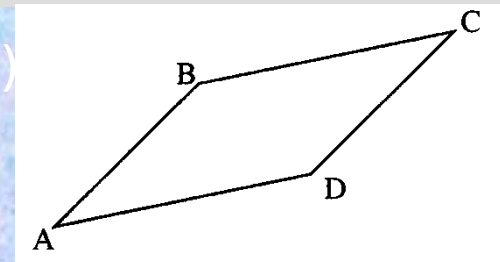
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Vectors

Higher

the point $(2, -1, 4)$, B is $(7, 1, 3)$ and C is $(-6, 4, 2)$.
BCD is a parallelogram, find the co-ordinates of D.



$$AD = BC = \mathbf{c} - \mathbf{b} \quad , \quad BC = \begin{pmatrix} -6 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix} \quad , \quad BC = \begin{pmatrix} -13 \\ 3 \\ -1 \end{pmatrix}$$

D is the displacement $\mathbf{AD}$ from A

$$\text{hence } \mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -13 \\ 3 \\ -1 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} -11 \\ 2 \\ 3 \end{pmatrix} \rightarrow D(-11, 2, 3)$$



Hint



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Vectors

Higher

$$\mathbf{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$$

write down the components of $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$

show that $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular.

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$$

$$\mathbf{u} - \mathbf{v} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$$

look at scalar product

$$(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$$

$$(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$$

$$= (-2) \times (-4) + 8 \times (-2) + 2 \times 4$$

$$= 8 - 16 + 8 = 0$$

Hence vectors are perpendicular



Hint



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Vectors

Vectors **a**, **b** and **c** are defined as follows:

$$\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \mathbf{c} = -\mathbf{j} + \mathbf{k}$$

Evaluate $\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

From your answer to part (a), make a deduction about the vector $\mathbf{b} + \mathbf{c}$

$$\text{a) } \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{a} \cdot \mathbf{b} = 2 + 0 - 1 \quad \mathbf{a} \cdot \mathbf{b} = 1$$

$$\mathbf{a} \cdot \mathbf{c} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad \mathbf{a} \cdot \mathbf{c} = 0 + 0 - 1 \quad \mathbf{a} \cdot \mathbf{c} = -1 \quad \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} = 0$$

b) $\mathbf{b} + \mathbf{c}$ is perpendicular to $\mathbf{a}$



Hint



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Vectors

Higher

the point (-3, 2, 4) and B is (-1, 3, 2)

the components of $\overrightarrow{AB}$

the length of AB

a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$ $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

b) $|\overrightarrow{AB}| = \sqrt{2^2 + 1^2 + (-2)^2}$ $|\overrightarrow{AB}| = \sqrt{4 + 1 + 4}$

$$|\overrightarrow{AB}| = \sqrt{9}$$

$$|\overrightarrow{AB}| = 3$$



Hint



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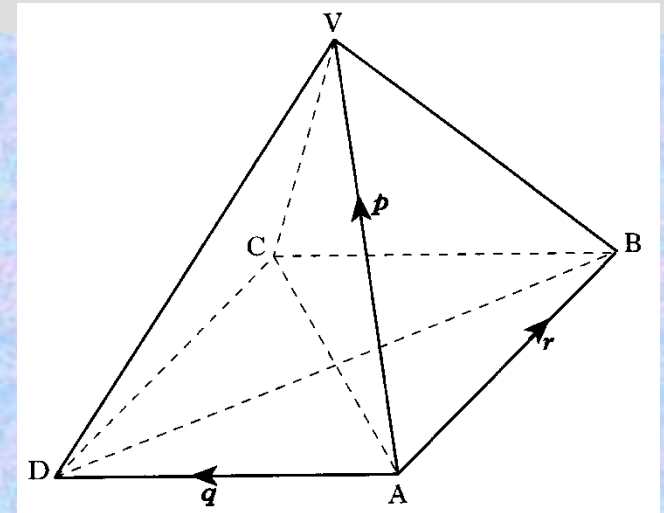
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Vectors

In the square based pyramid, **Higher**
all the eight edges are of length 3
units.

$$AV = \mathbf{p} \quad AD = \mathbf{q} \quad AB = \mathbf{r},$$



Triangular faces are all equilateral

$$\mathbf{p}(\mathbf{q} + \mathbf{r}) = \mathbf{p}\mathbf{q} + \mathbf{p}\mathbf{r}$$

$$\mathbf{p}\mathbf{q} = |\mathbf{p}||\mathbf{q}|\cos 60^\circ \quad \mathbf{p}\mathbf{q} = 3 \times 3 \times \frac{1}{2} \quad \mathbf{p}\mathbf{q} = 4\frac{1}{2}$$

$$\mathbf{p}\mathbf{r} = |\mathbf{p}||\mathbf{r}|\cos 60^\circ \quad \mathbf{p}\mathbf{r} = 3 \times 3 \times \frac{1}{2} \quad \mathbf{p}\mathbf{r} = 4\frac{1}{2}$$

$$\mathbf{p}(\mathbf{q} + \mathbf{r}) = 4\frac{1}{2} + 4\frac{1}{2} \quad \mathbf{p}(\mathbf{q} + \mathbf{r}) = 9$$



Hint



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You have completed all 15 questions in this section



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Points dividing lines in ratios

Collinear Points



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Vectors

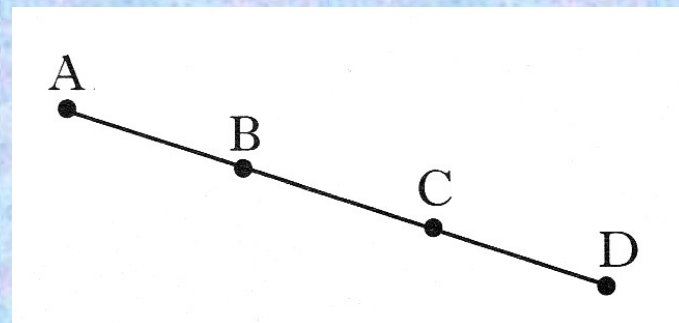
and B are the points $(-1, -3, 2)$

Higher

and $(2, -1, 1)$ respectively.

and C are the points of trisection of AD.

that is, $AB = BC = CD$.



and the coordinates of D

$$\frac{\vec{AB}}{\vec{AD}} = \frac{1}{3} \rightarrow 3\vec{AB} = \vec{AD} \rightarrow 3(\mathbf{b} - \mathbf{a}) = \mathbf{d} - \mathbf{a}$$

$$\rightarrow 3\mathbf{b} - 3\mathbf{a} = \mathbf{d} - \mathbf{a} \rightarrow \mathbf{d} = 3\mathbf{b} - 2\mathbf{a}$$

$$\rightarrow \mathbf{d} = 3 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} \rightarrow \mathbf{d} = \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix} \rightarrow D(8, 3, 1)$$



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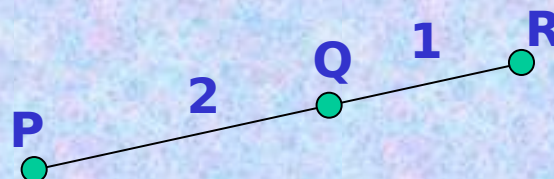


Vectors

Higher

Point Q divides the line joining P(-1, -1, 0) to R(5, 2, -3) in the ratio 2:1.
Find the co-ordinates of Q.

Diagram



$$\frac{\vec{PQ}}{\vec{QR}} = \frac{2}{1} \rightarrow PQ = 2QR \rightarrow \mathbf{q - p = 2r - 2q}$$
$$\rightarrow \mathbf{3q = 2r + p}$$

$$\rightarrow \mathbf{3q = 2 \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}} \rightarrow \mathbf{q = \frac{1}{3} \begin{pmatrix} 9 \\ 3 \\ -6 \end{pmatrix}} \rightarrow \mathbf{Q(3, 1, -2)}$$



Hint



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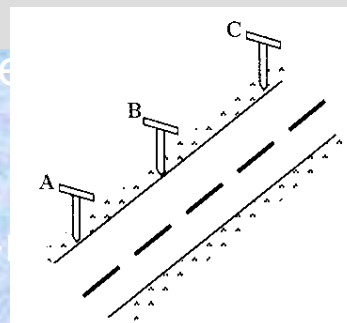
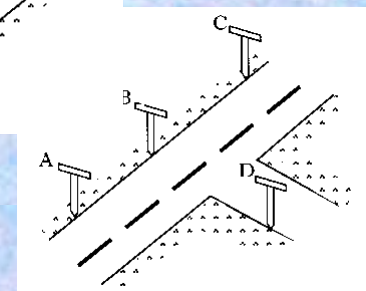
Vectors

Higher

roadmakers look along the tops of a set of T-roads to ensure that straight sections of road are being created.

relative to suitable axes the top left corners of the T-roads are at the points

$A(-8, -10, -2)$, $B(-2, -1, 1)$ and $C(6, 11, 5)$. Determine whether or not the section of road ABC has been built in a straight line.



A further T-rod is placed such that D has co-ordinates $(1, -4, 4)$.

Show that DB is perpendicular to AB. $\vec{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 14 \\ 21 \\ 7 \end{pmatrix} = 7 \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$\vec{AB}$ and $\vec{AC}$ are scalar multiples, so are parallel. A is common. A, B, C are collinear.

b) Use scalar product $\vec{AB} \cdot \vec{BD} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} = 18 - 27 + 9 = 0$

Hence, DB is perpendicular to AB



Hint



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Vectors

ABCD is a pyramid with rectangular base ABCD. Higher.

relative to some appropriate axis,

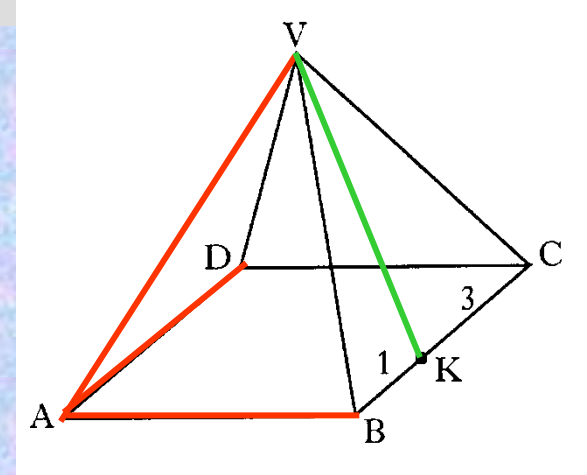
$$\vec{VA} \text{ represents } -7\mathbf{i} - 13\mathbf{j} - 11\mathbf{k}$$

$$\vec{AB} \text{ represents } 6\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$$

$$\vec{AD} \text{ represents } 8\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$$

divides BC in the ratio 1:3

and $\vec{VK}$ component form.



$$\vec{VA} + \vec{AB} = \vec{VB}$$

$$\vec{VK} + \vec{KB} = \vec{VB}$$

$$\vec{KB} = \frac{1}{4}\vec{CB} = \frac{1}{4}\vec{DA} = -\frac{1}{4}\vec{AD}$$

$$\vec{VK} = \vec{VB} - \vec{KB} \rightarrow \vec{VK} = \vec{VA} + \vec{AB} + \frac{1}{4}\vec{AD}$$

$$\rightarrow \vec{VK} = \begin{pmatrix} -7 \\ -13 \\ -11 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 8 \\ -4 \\ -4 \end{pmatrix}$$

$$\rightarrow \vec{VK} = \begin{pmatrix} 1 \\ -8 \\ -18 \end{pmatrix}$$



Hint



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Vectors

Higher

The line AB is divided into 3 equal parts by

points C and D, as shown.

Points A and B have co-ordinates (3, -1, 2) and (9, 2, -4).

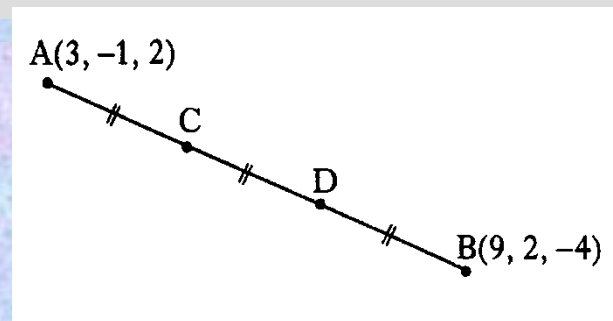
Find the components of $\vec{AB}$ and $\vec{AC}$

Find the co-ordinates of C and D.

$$a) \quad \vec{AB} = \mathbf{b} - \mathbf{a} \quad \vec{AB} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix} \quad \vec{AC} = \frac{1}{3} \vec{AB} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

b) C is a displacement of $\vec{AC}$ from A $\mathbf{c} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \rightarrow C(5, 0, 0)$

similarly $\mathbf{d} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \rightarrow D(7, 1, -2)$



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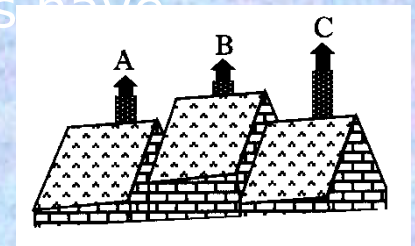
Vectors

Higher

ve to a suitable set of axes, the tops of three chimneys have

inates given by A(1, 3, 2), B(2, -1, 4) and C(4, -9, 8).

that A, B and C are collinear



$$AB = \mathbf{b} - \mathbf{a} \quad , \quad AB = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \quad , \quad AC = \begin{pmatrix} 3 \\ -12 \\ 6 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$$

AC are scalar multiples, so are parallel. A is common. A, B, C are collinear



Hint



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Vectors

Higher

the point (2, -5, 6), B is (6, -3, 4) and C is (12, 0, 1).

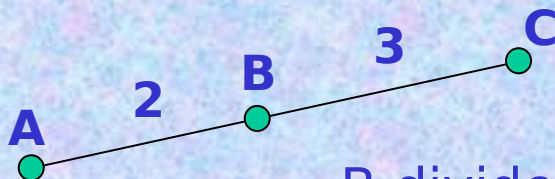
show that A, B and C are collinear

determine the ratio in which B divides AC

$$AB = \mathbf{b} - \mathbf{a} \quad , \quad AB = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad , \quad BC = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

AB and BC are scalar multiples, so are parallel. B is common. A, B, C are collinear

$$\frac{AB}{BC} = \frac{2}{3}$$



B divides AC in ratio 2 : 3



Hint



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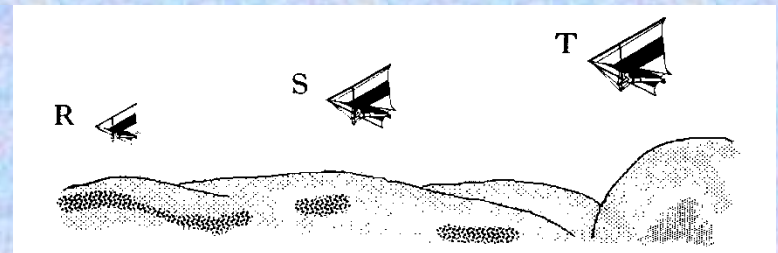
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Vectors

Relative to the top of a hill, three gliders
have positions given by

$R(-1, -8, -2)$, $S(2, -5, 4)$ and $T(3, -4, 6)$.



Prove that R, S and T are collinear

$$\vec{RS} = \vec{s} - \vec{r} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad , \quad \vec{RT} = \begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$\vec{RS}$ are scalar multiples, so are parallel. R is common. R, S, T are collinear



Hint



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You have completed all 8 questions in this section



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Vectors

Angle between two vectors



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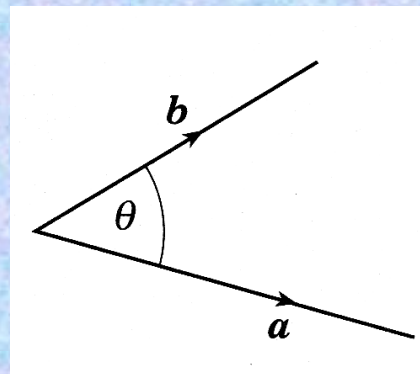
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Vectors

The diagram shows vector **a** and **b**.

If $|\mathbf{a}| = 5$, $|\mathbf{b}| = 4$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 36$

Find the size of the acute angle between **a** and **b**.



$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 36 \rightarrow \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} = 36$$

$$\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}| |\mathbf{a}| = 25 \rightarrow 25 + \mathbf{a} \cdot \mathbf{b} = 36 \rightarrow \mathbf{a} \cdot \mathbf{b} = 11$$

$$\cos \theta = \frac{11}{5 \times 4}$$

$$\theta = \cos^{-1} \left(\frac{11}{20} \right) \quad \theta = 56.6^\circ$$



Hint



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Vectors

Higher

The diagram shows a square based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The co-ordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y-axis.

a) Write down the co-ordinates of B

b) Determine the components $\vec{DA}$ and $\vec{DB}$

c) Calculate the size of angle ADB.

a) $B(6, 6, 0)$

b)

$$\vec{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$$

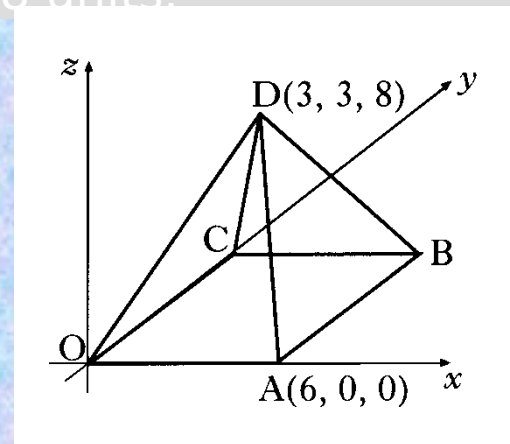
$$\vec{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$$

c) $\cos \theta = \frac{\vec{DA} \cdot \vec{DB}}{|\vec{DA}| |\vec{DB}|}$

$$\vec{DA} \cdot \vec{DB} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix} = 64$$

$$\cos \theta = \frac{64}{\sqrt{82} \sqrt{82}}$$

$$\theta = 38.7^\circ$$



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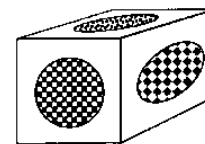
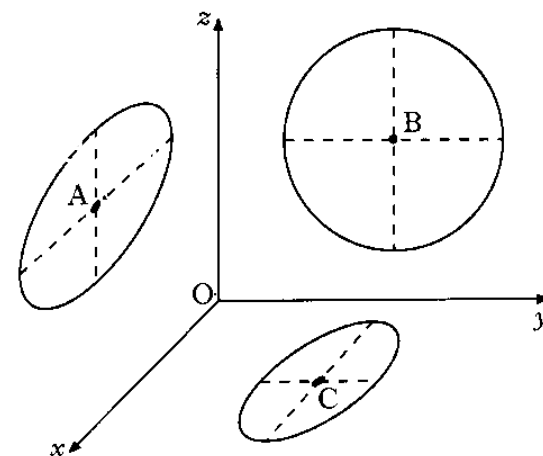
Vectors

in the shape of a cuboid designed with Higher of different
on each face.

Diagram shows three of the circles, where the origin represents
of the corners of the cuboid.

centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5

the size of angle ABC



Vectors to
point

$$\vec{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}, \quad \vec{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$$

$$\vec{BA} \cdot \vec{BC} = 24 + 0 - 6 = 18$$

away from

$$|\vec{BA}| = \sqrt{36 + 25 + 1} = \sqrt{62}$$

$$|\vec{BC}| = \sqrt{16 + 36} = \sqrt{52}$$

$$\cos \theta = \frac{18}{\sqrt{62}\sqrt{52}}$$

$$\theta = 71.5^\circ$$



Hint



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Vectors

A cuboid measuring 11cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Higher

Co-ordinate axes are taken as shown.

- a) The point A has co-ordinates (0, 9, 8) and C has co-ordinates (17, 0, 8).

Write down the co-ordinates of B

- b) Calculate the size of angle ABC

$$B(3, 2, 15)$$

$$BC = \begin{pmatrix} 15 \\ -2 \\ -7 \end{pmatrix}$$

$$BA = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$$

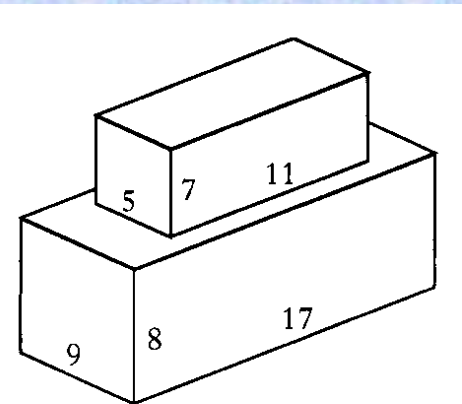
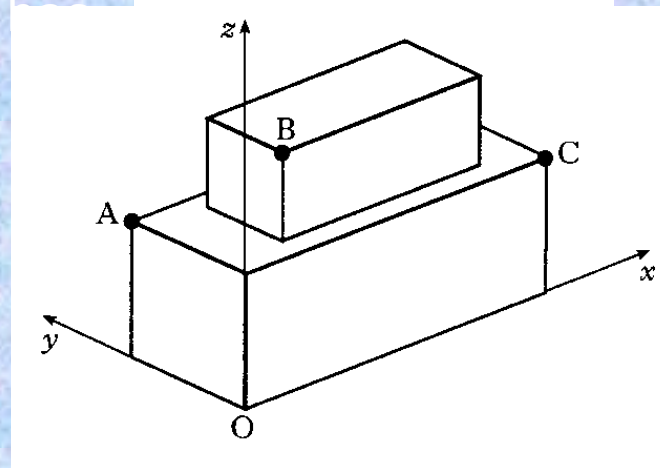
$$BA \cdot BC = -45 - 14 + 49 = -10$$

$$|BC| = \sqrt{225 + 4 + 49} = \sqrt{278}$$

$$|BA| = \sqrt{9 + 49 + 49} = \sqrt{107}$$

$$\cos \theta = \frac{-10}{\sqrt{278}\sqrt{107}}$$

$$\theta = 93.3^\circ$$



Hint



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Vectors

Higher

triangle ABC has vertices A(2, -1, 3), B(3, 6, 5) and C(6, 6, -2)

Find $\vec{AB}$ and $\vec{AC}$

Calculate the size of angle BAC.

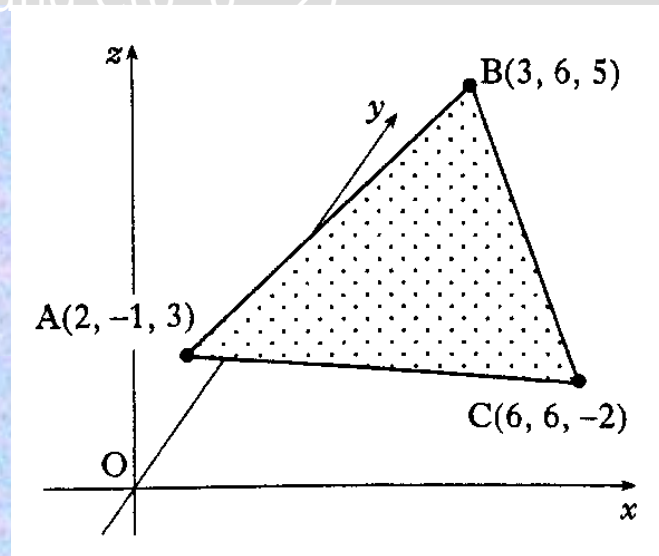
Hence find the area of the triangle

$$a) \quad \vec{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}, \quad \vec{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$$

$$b) \quad |\vec{AB}| = \sqrt{1^2 + 7^2 + 2^2} = \sqrt{54} \quad |\vec{AC}| = \sqrt{90} \quad \vec{AB} \cdot \vec{AC} = 4 + 49 - 10 = 43$$

$$\cos \theta = \frac{43}{\sqrt{54}\sqrt{90}} = 0.6168 \quad \theta = \cos^{-1} 0.6168 = 51.9^\circ \quad \angle BAC = 51.9^\circ$$

$$c) \quad \text{Area of } \triangle ABC = \frac{1}{2} ab \sin C = \frac{1}{2} \sqrt{90} \sqrt{54} \sin 51.9^\circ = 27.43 \text{ unit}^2$$



Hint



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