LINEAR EQUATIONS

- Distance Formula
- Gradients
- Collinearity
- The Midpoint Formula
- Gradients of Perpendicular lines
- The Equation of a Straight Line
- Median, Altitude & Perpendicular Bisector
- Concurrency
- Exam Type Questions

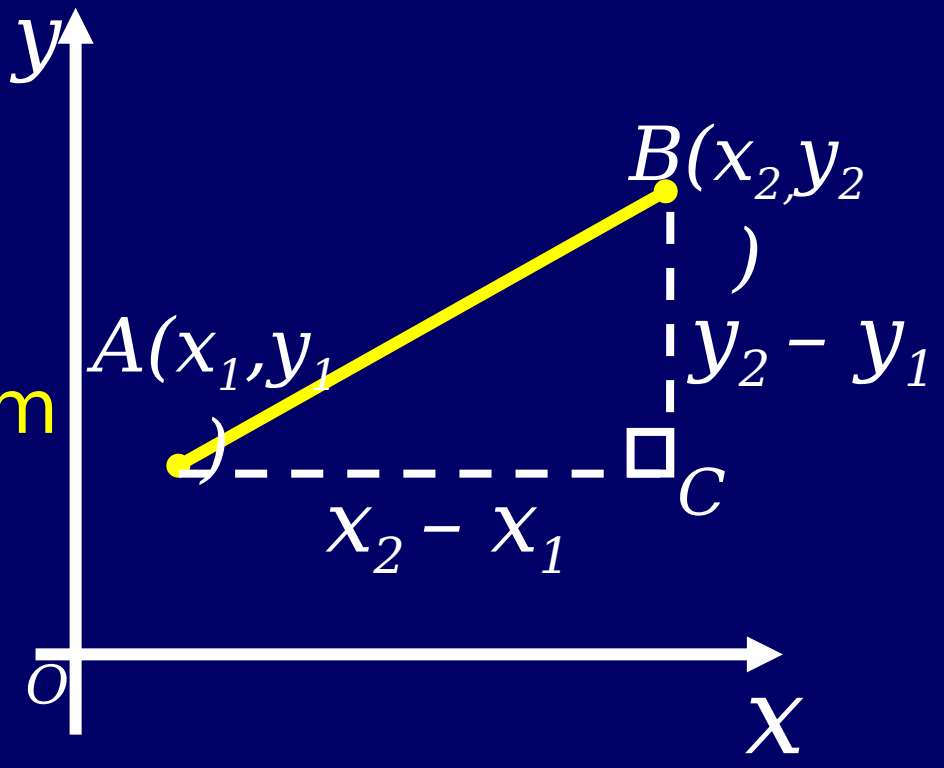
Distance Formula

Length of a straight line

$$(AB)^2 = (AC)^2 + (BC)^2$$

This is just

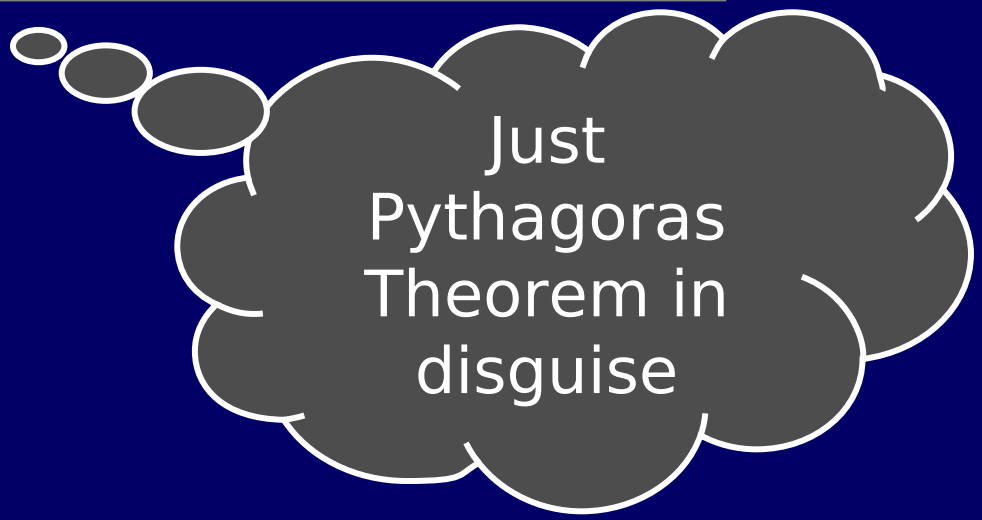
Pythagoras' Theorem



Distance Formula

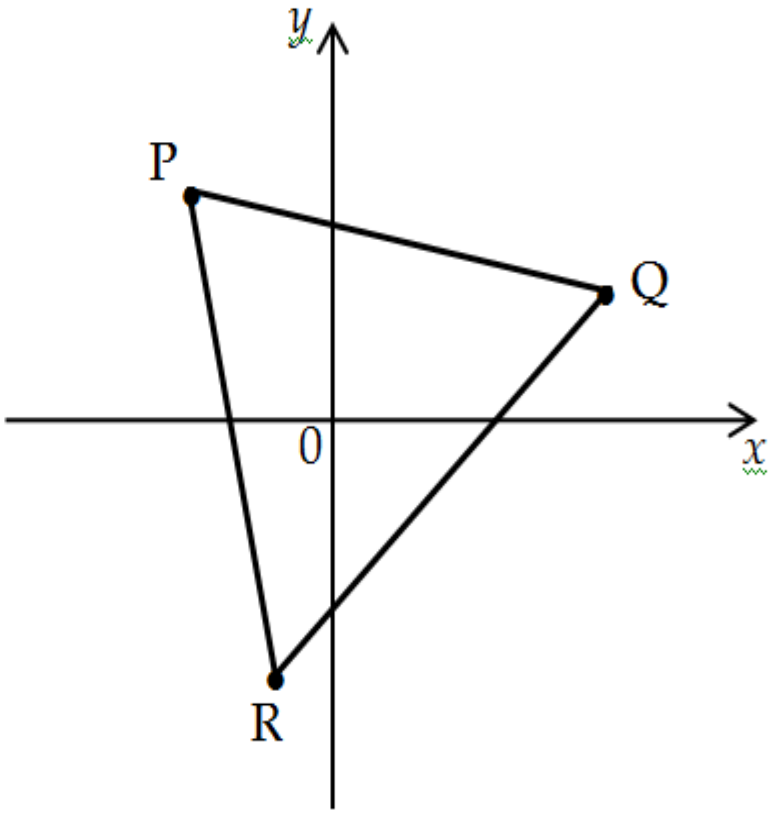
The length (distance) of ANY line can be given by the formula :

$$AB_{distance} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$



$$AB_{distance} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Triangle PQR has vertices P(–3, 5), Q(7, 3) and R(–1, – 5), as shown.



Find the distance between PQ, QR and RP.

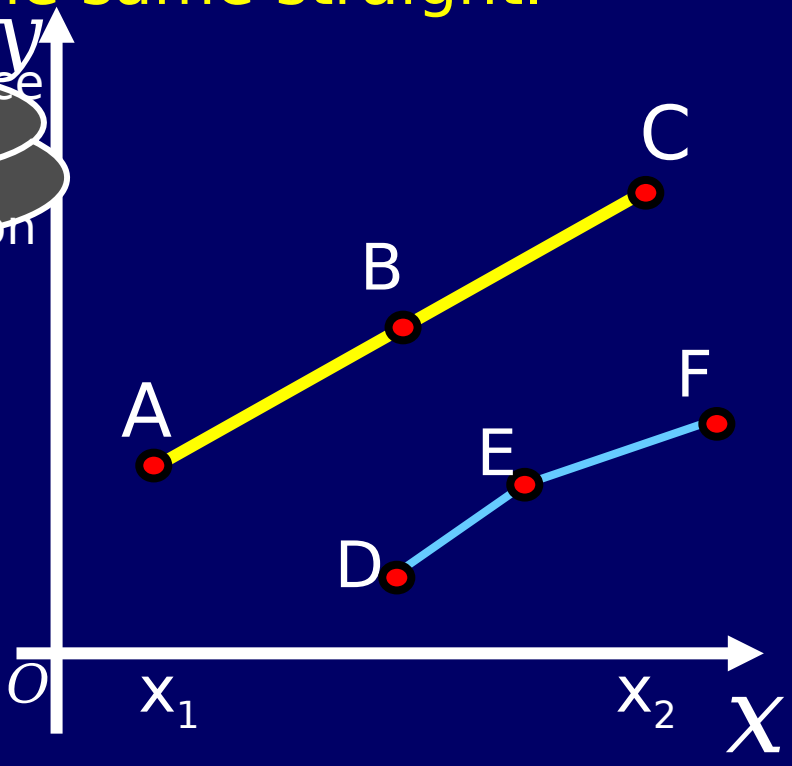
Collinearity

Points are said to be collinear

if they lie on the same straight.

The coordinates A,B C are collinear since they lie on the same straight line.

D,E,F are not collinear they do not lie on the same straight line.



Straight Line Theory

P is the point (4, 1), Q is (5, 2) and R is (7, 4).

(*a*) Show that P, Q and R are collinear.

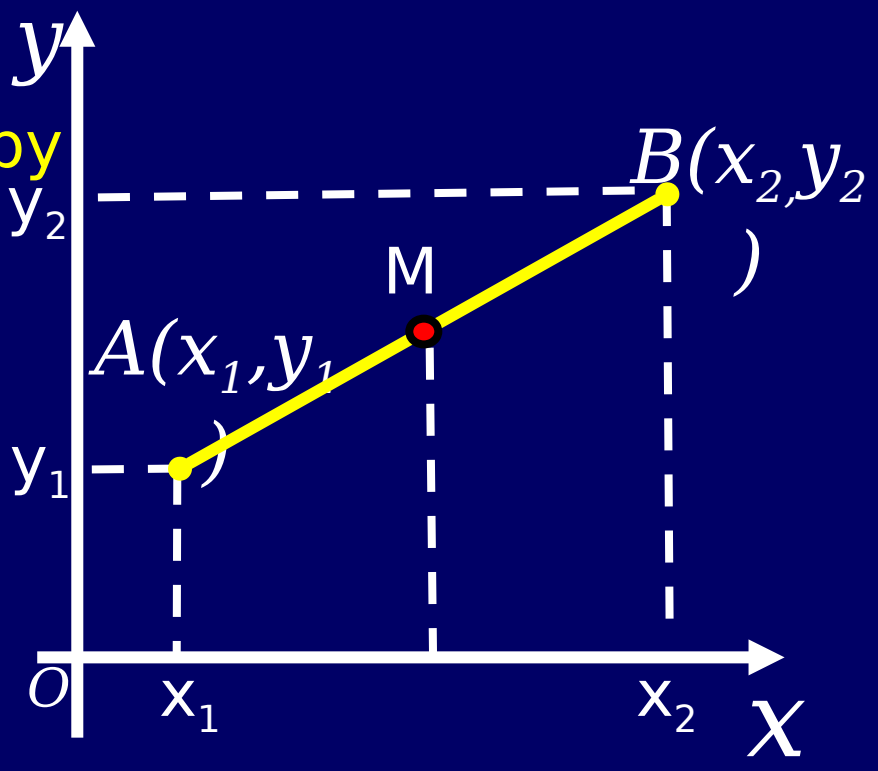
(*b*) Find the ratio in which Q divides PR.

Finding Mid-Point of a line

The mid-point (Median) between 2 points is given by

Simply add both x coordinates together and divide by 2.

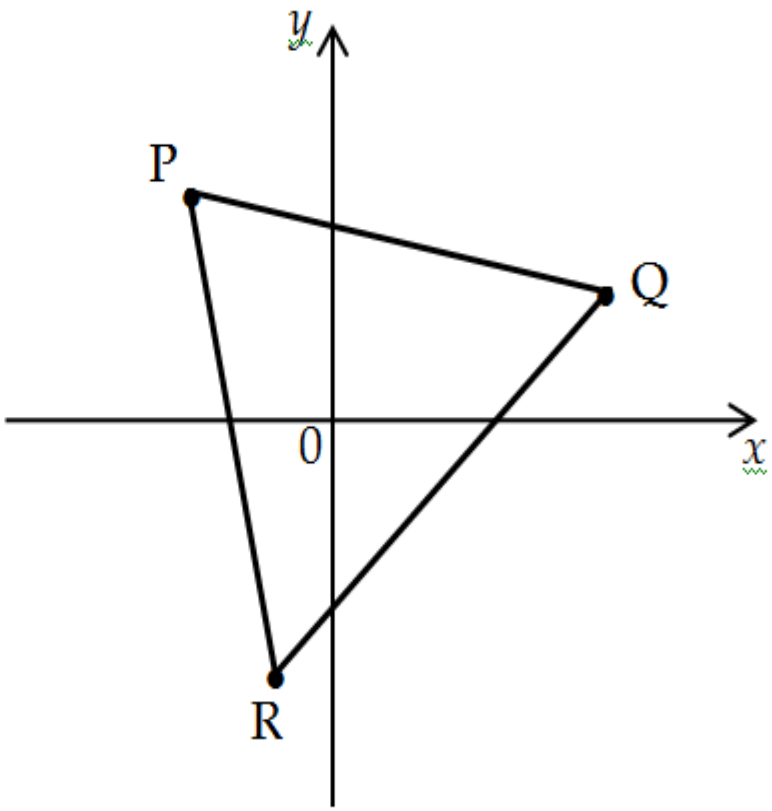
Then do the same with the y coordinates.



$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \right)$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \right)$$

Triangle PQR has vertices P(–3, 5), Q(7, 3) and R(–1, – 5), as shown.



Find the median points for PQ QR and RP.

Straight line Facts

$$y = mx + c$$

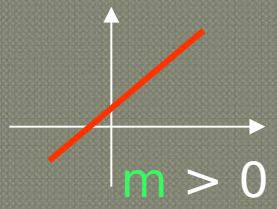
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Y – axis Intercept

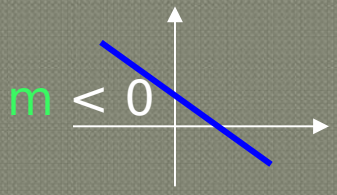
Another version of the straight line general formula is:

$$ax + by + c = 0$$

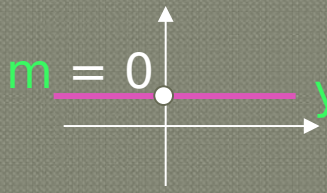
Gradient Facts



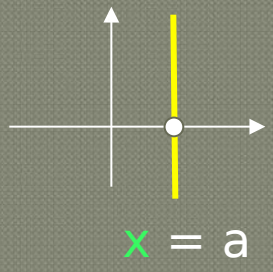
Sloping left to right up has +ve gradient



Sloping left to right down has -ve gradient

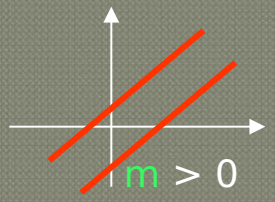


Horizontal line has zero gradient.



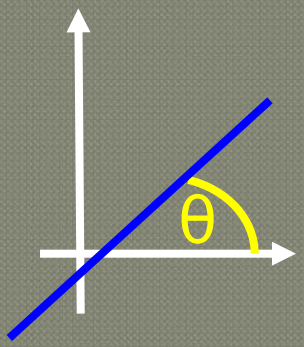
Vertical line has undefined gradient.

Gradient Facts



Lines with the same gradient

means lines are Parallel



The gradient of a line is ALWAYS

equal to the tangent of the angle

made with the line and the positive x-axis

$$m = \tan \theta$$

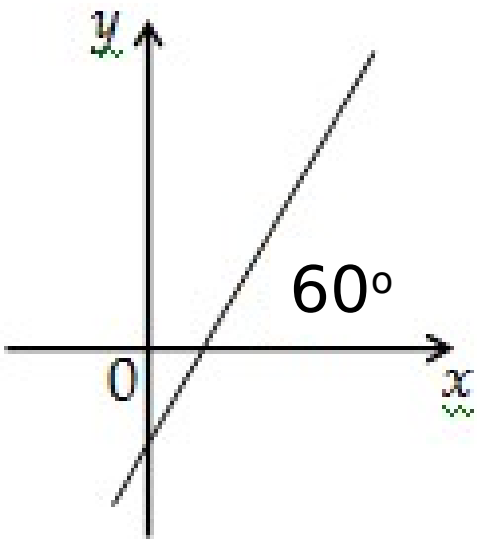
Straight Line Theory

The line with equation $kx - 2y + 9 = 0$ is parallel to the line with gradient 7.

What is the value of k?

The angle between the line shown in the diagram and the x-axis is 60°

What is the gradient of the line?



Straight Line Theory

Here are two statements about the line with equation $3x + 4y - 8 = 0$.

(1) This line is parallel to a line with gradient $-\frac{3}{4}$.

(2) This line cuts the y-axis at the point $(0, 8)$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

Straight Line Theory

A straight line passes through the points $(4, 3)$ and $(0, -1)$.

What is the equation of the line?

A $x+y-1=0$

B $x-y-1=0$

C $x-2y-1=0$

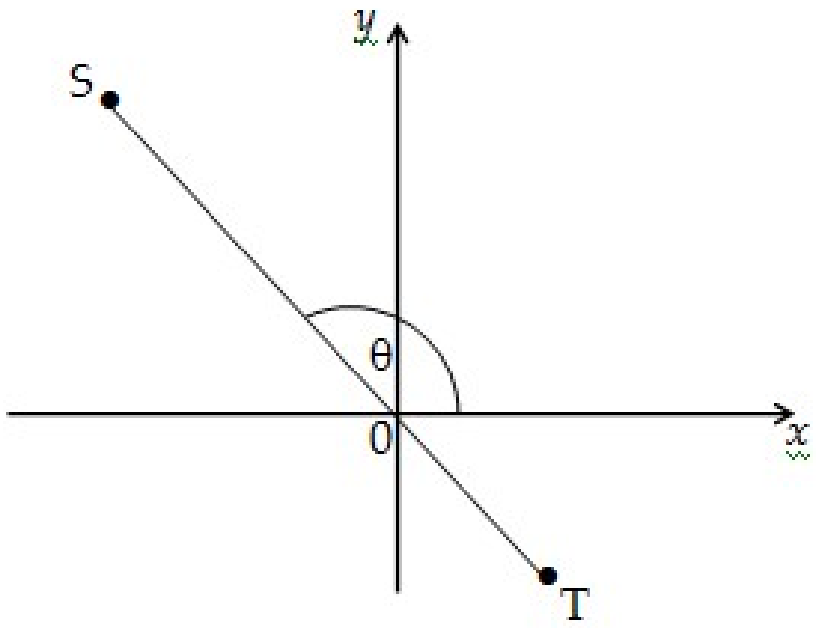
D $3x-4y-1=0$

Straight Line Theory

The diagram shows the line ST with equation $2x + y = 0$.

The angle between ST and the positive direction of the x-axis is θ.

Find an expression for θ.



Straight Line Theory

The line with equation $kx - 4y + 1 = 0$ is parallel to the line with gradient 3.

What is the value of k?

A -3

B $-\frac{1}{3}$

C $\frac{1}{4}$

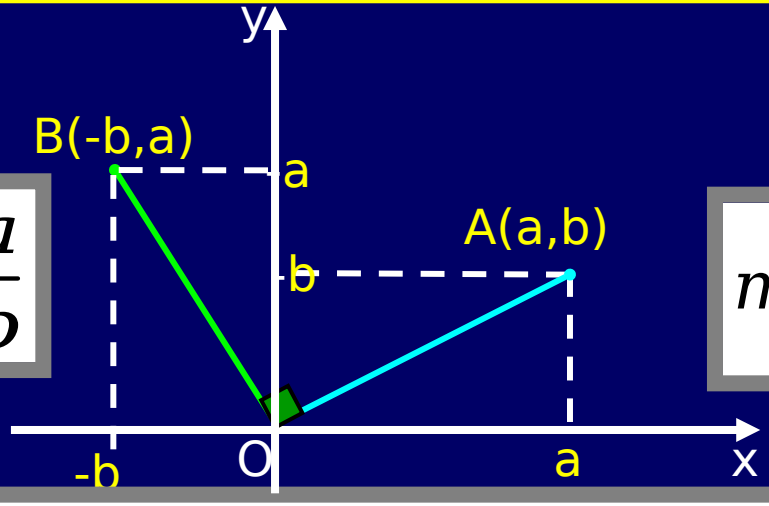
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Gradient of perpendicular lines

Investigation

If 2 lines with gradients m_1 and m_2 are perpendicular then $m_1 \times m_2 = -1$

When rotated through 90º about the origin A (a, b) → B (-b, a)



$$m_{OB} = \frac{a-0}{-b-0} = -\frac{a}{b}$$

$$m_{OA} = \frac{b-0}{a-0} = \frac{b}{a}$$

$$m_{OA} \times m_{OB} = \frac{b}{a} \times \frac{a}{-b} = -\frac{ab}{ab} = -1$$

Demo

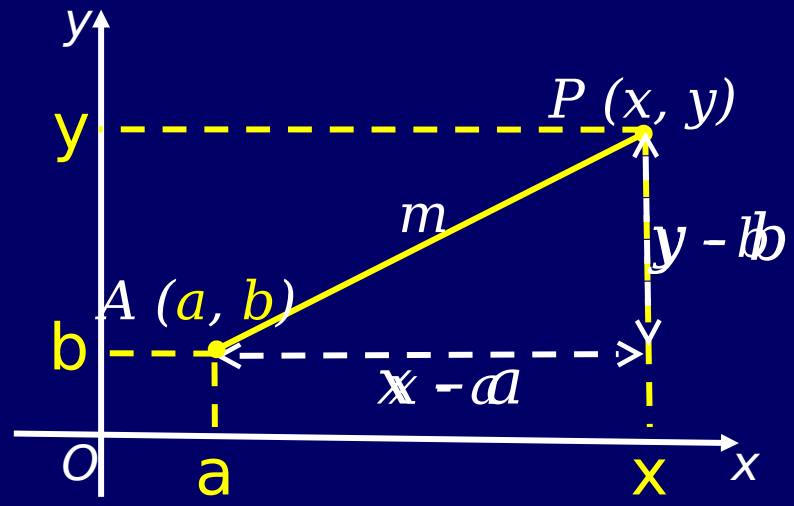
Conversely:

If $m_1 \times m_2 = -1$ then the two lines with gradients m_1 and m_2 are perpendicular

The Equation of the Straight Line

$y - b = m(x - a)$

The equation of any line can be found if we know the gradient and one point on the line.



$$m = \frac{y - b}{(x - a)}$$

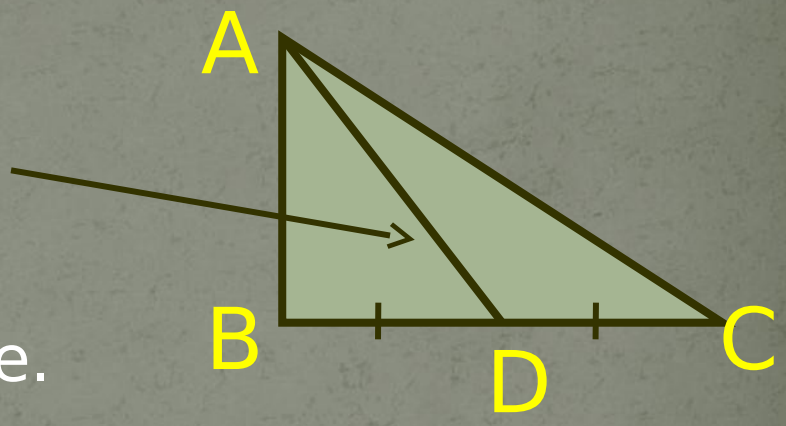
Gradient, m

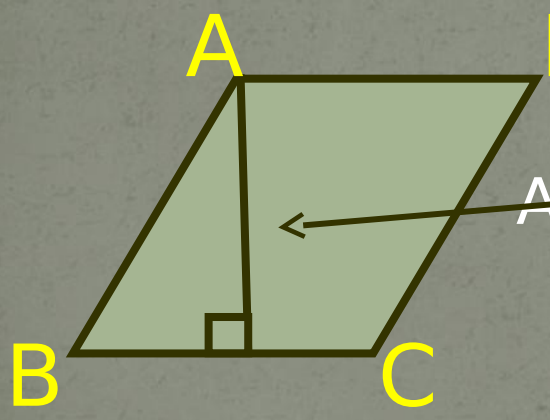
Point (a, b)

$y - b = m(x - a)$ Point on the line (a, b)

Terminology

Median means a line from a vertex to the midpoint of the base.

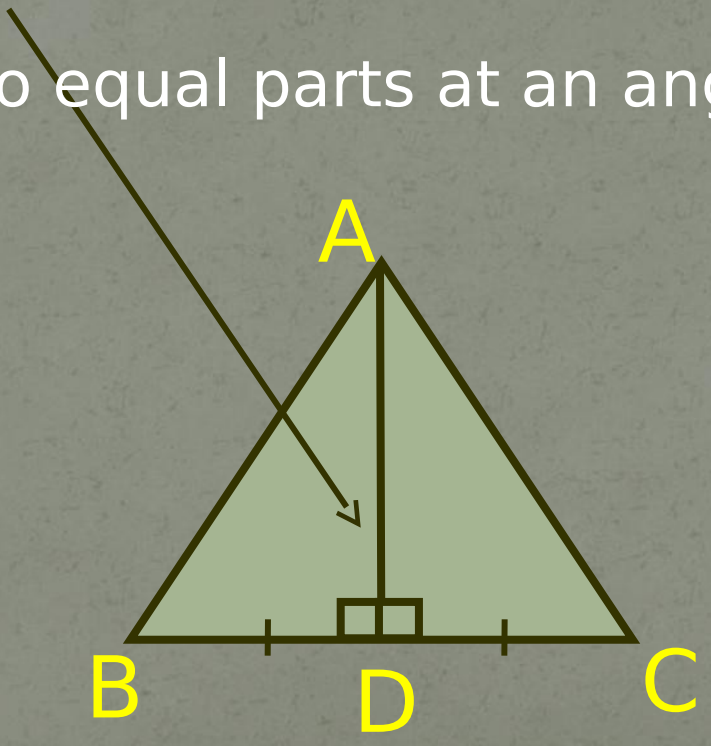




Altitude means a perpendicular line from a vertex to the base.

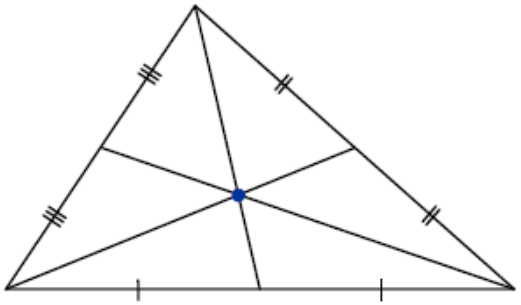
Terminology

Perpendicular bisector - a line that cuts another line into two equal parts at an angle of 90°

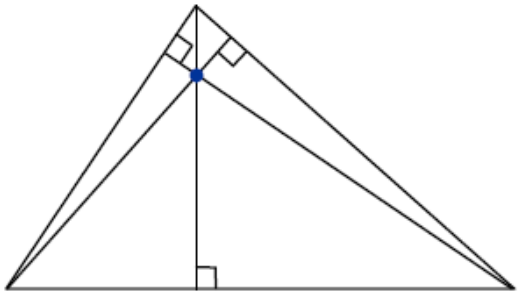


Any number of lines are said to be **concurrent if there is a point through** which they all pass.

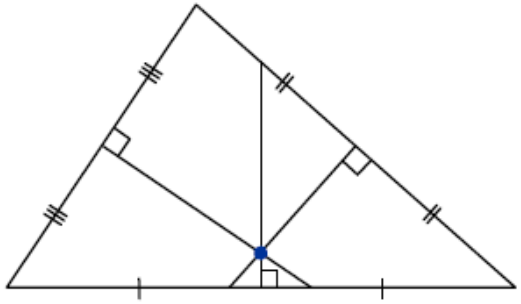
For three lines to be concurrent, they must all pass through a single point.



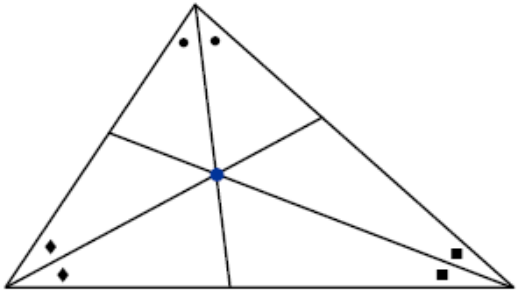
The three medians of a triangle are concurrent.



The three altitudes of a triangle are concurrent.



The three perpendicular bisectors in a triangle are concurrent.



The three angle bisectors of a triangle are concurrent.

Typical Exam Questions

Find the equation of the line which passes through the point (-1, 3) and is perpendicular to the line with $4x + y - 1 = 0$

Find gradient of given line: $4x + y - 1 = 0 \Rightarrow y = -4x + 1 \Rightarrow m = -4$

Find gradient of perpendicular: $m = \frac{1}{4}$ (using formula $m_1 \times m_2 = -1$)

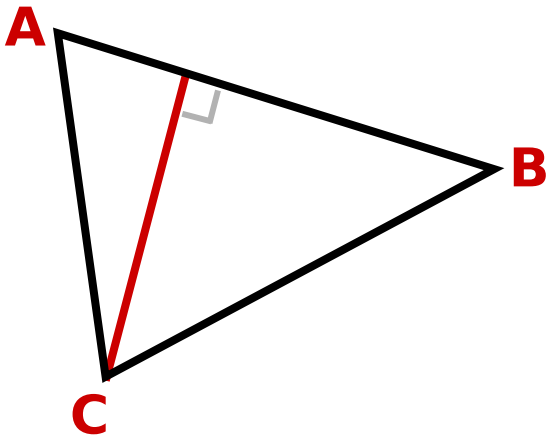
Find equation: $\frac{1}{4} = \frac{y - 3}{x - (-1)} \Rightarrow x + 1 = 4(y - 3) \Rightarrow x + 1 = 4y - 12$

$4y - x - 13 = 0$



Common Straight Strategies for Exam Questions

Finding the Equation of an Altitude



To find the equation of an altitude:

- **Find the gradient of the side it is perpendicular to (m_{AB}).**
- **To find the gradient of the altitude, flip the gradient of AB and change from positive to negative:**

$$m_{altitude} = \frac{-1}{m_{AB}}$$

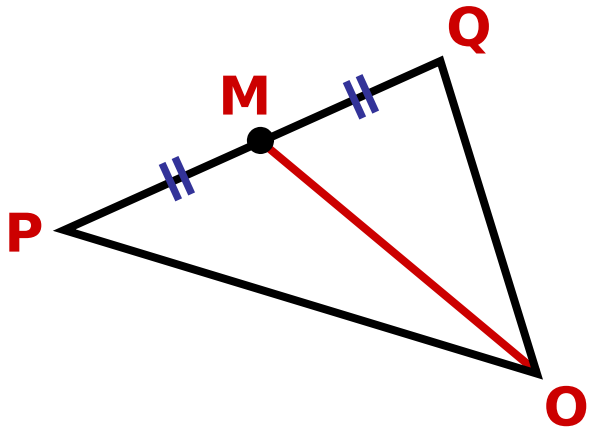
- **Substitute the gradient and the point C into** $y - b = m(x - a)$

Important

Write final equation in the form $Ax + By + C = 0$ **with** Ax **positive.**

Common Straight Strategies for Exam Questions

Finding the Equation of a Median



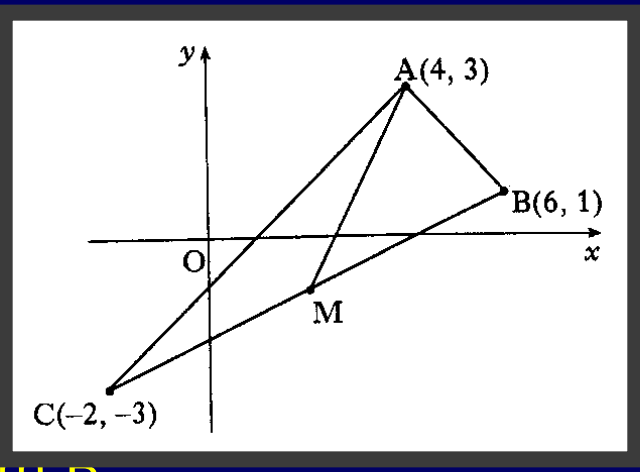
To find the equation of a median:

- **Find the midpoint of the side it bisects, i.e.** $M = \left(\frac{x_2 + }{x_1 2}, \frac{y_2 + }{y_1 2} \right)$
- **Calculate the gradient of the median OM.**
- **Substitute the gradient and either point on the line (O or M) into** $y - b = m(x - a)$

Important

Write answer in the form $Ax + By + C = 0$ **with** Ax **positive.**

Typical Exam Questions



A triangle ABC has vertices A(4, 3), B(6,

and C(–2, –3) as shown in the diagram.

Find the equation of AM, the median from B to C

Find mid-point of BC: $(2,-1)$ Using $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Find gradient of median AM $m=2$ Using $m=\frac{y_2-y_1}{x_2-x_1}$

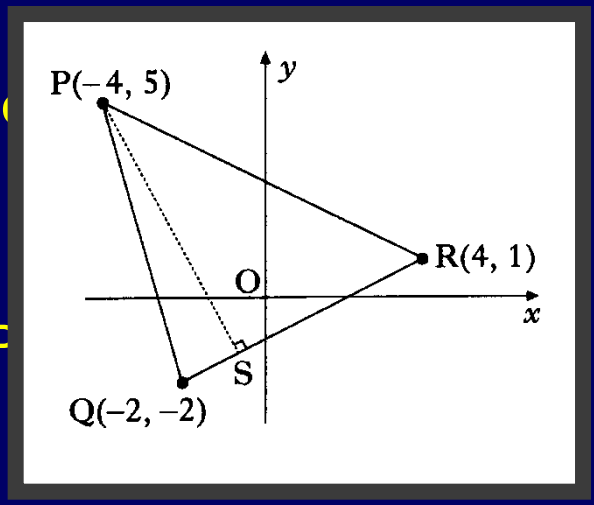
Find equation of median AM $y=2x-5$ Using $y-b=m(x-a)$

Typical Exam Questions

P(–4, 5), Q(–2, –2) and R(4, 1) are the verti

of triangle PQR as shown in the diagram.

Find the equation of PS, the altitude from P



Find gradient of QR: $m = \frac{1}{2}$ Using $m = \frac{y_2 - y_1}{x_2 - x_1}$

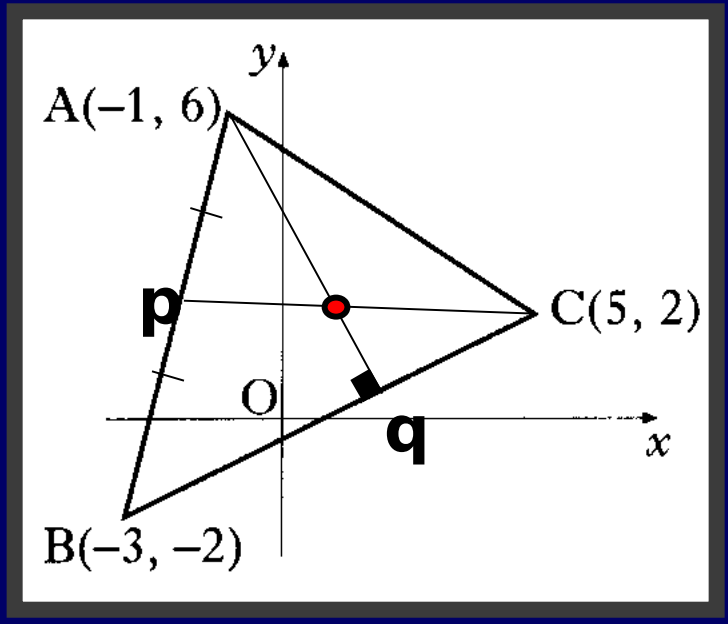
Find gradient of PS (perpendicular to QR)

$m = -2 \quad (m_1 \times m_2 = -1)$

Find equation of altitude PS

$y + 2x + 3 = 0$ Using $y - b = m(x - a)$

Exam Type Questions



Triangle ABC has vertices

A(–1, 6), B(–3, –2) and C(5, 2)

Find:

a) the equation of the line *p*, the median from C of triangle ABC.

b) the equation of the line *q*, the perpendicular bisector of BC.

c) the co-ordinates of the point of intersection of the lines *p* and *q*.

Find mid-point of AB (-2, 2)

Find gradient of p $m=0$

Find equation of *p* $y=2$

Find mid-point of BC (1, 0)

Find gradient of BC $m=\frac{1}{2}$

Find gradient of *q* $m=-2$

Find equation of *q* $y=-2x+2$

Solve p and q simultaneously for intersection (0, 2)

Typical Exam Questions

Find the equation of the straight line which is parallel to the $2x+3y=5$ equation

and which passes through the point (2, –1).

Find gradient of given line:

$$3y=-2x+5 \quad \Rightarrow \quad y=-\frac{2}{3}x+5 \quad \Rightarrow \quad m=-\frac{2}{3}$$

Knowledge: Gradient of parallel lines are the same.

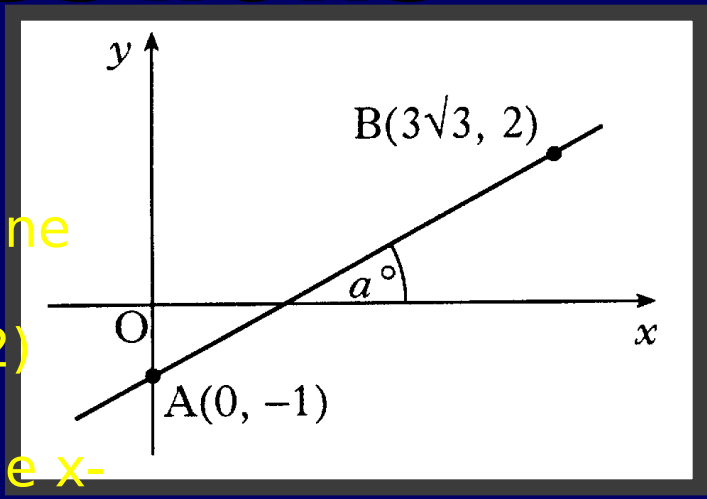
Therefore for line we want to find has gradient $m=-\frac{2}{3}$

Find equation: Using y – b =m(x - a)

$3y+2x-1=0$



Exam Type Questions



Find the size of the angle a° that the line joining the points A(0, -1) and B(3√3, 2) makes with the positive direction of the x-axis.

Find gradient of the line: $m = \frac{2-(-1)}{3\sqrt{3}-0} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$

Use $m = \tan\theta$ $\tan\theta = \frac{1}{\sqrt{3}}$

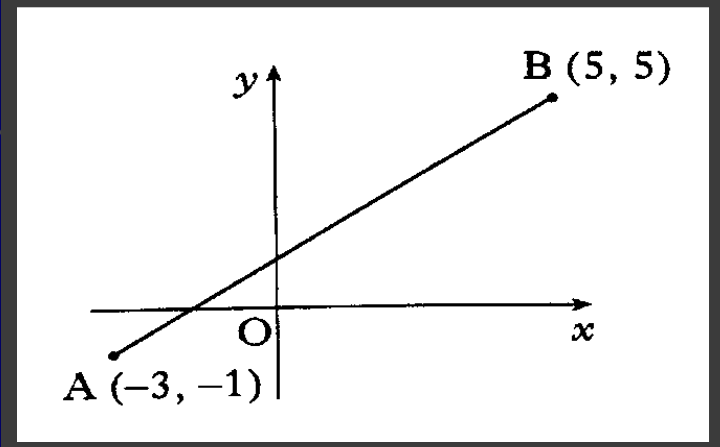
Use table of exact values $\theta = \tan^{-1}\frac{1}{\sqrt{3}}$ $\theta = 30°$

Exam Type Questions

A and B are the points (–3, –1) and (5

Find the equation of

a) the line AB.



b) the perpendicular bis f AB

Find gradient of the AB: $m = \frac{3}{4}$

Find equation of AB $4y = 3x + 5$

Find mid-point of AB $(1, 2)$

Gradient of AB (perp): $m = -\frac{4}{3}$

Use y – b = m(x – a) and point (1, 2) to obtain line of perpendicular bisector of AB we get

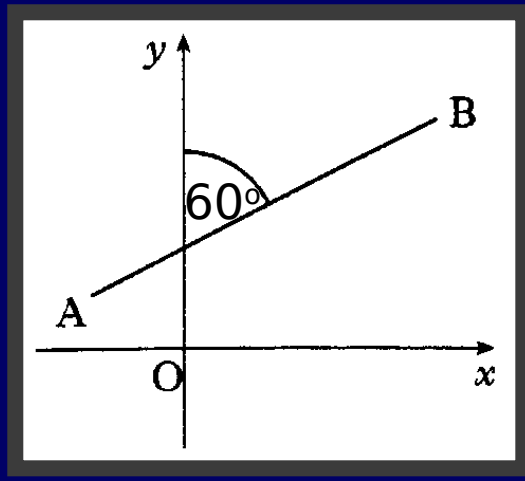
$3y + 4x - 10 = 0$



Typical Exam Questions

The line AB makes an angle of 60° with the y-axis, as shown in the diagram.

Find the exact value of the gradient of AB.



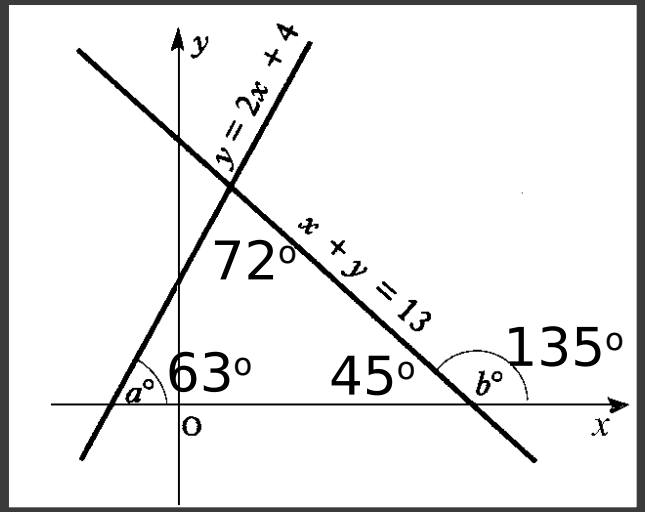
Find angle between AB and x-axis $90^o - 60^o = 30^0$

(x and y axes are perpendicular.)

Use $m = \tan\theta$ $m = \tan 30^o$

Use table of exact values $m = \frac{1}{\sqrt{3}}$

Typical Exam Questions



The lines $y = 2x + 4$ and $x + y = 13$ make angles of a° and b° with the positive direction of the x-axis, as shown in the diagram.

a) Find the values of a and b

b) **Hence** find the acute angle between the two given lines.

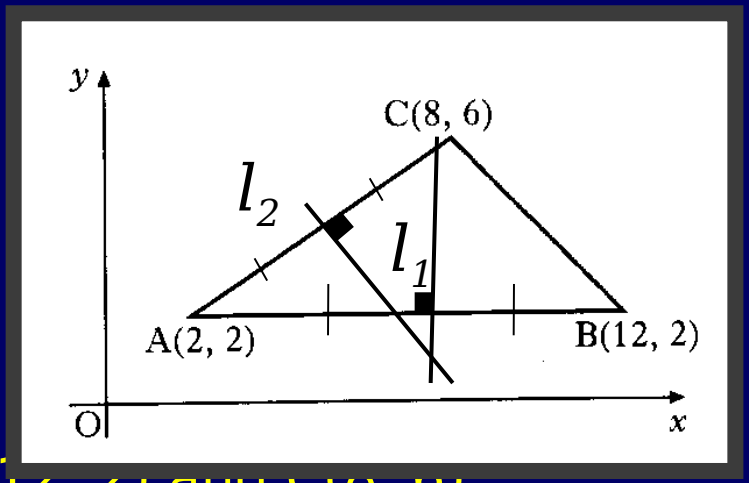
Find gradient of $y = 2x + 4$ $m = 2$ Find a° $\tan a^\circ = 2 \rightarrow a = 63^\circ$

Find gradient of $x + y = 13$ $m = -1$ Find b° $\tan b^\circ = -1 \rightarrow b = 135^\circ$

Find supplement of b $= 180 - 135 = 45^\circ$

Use angle sum triangle = 180° angle between two lines 72°

Exam Type Questions



Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).

a) Write down the equation of l_1, the perpendicular bisector of AB

b) Find the equation of l_2, the perpendicular bisector of AC.

c) Find the point of intersection of lines l_1 and l_2.

Mid-point AB $(7, 2)$

Perpendicular bisector AB $x = 7$

Find mid-point AC $(5, 4)$

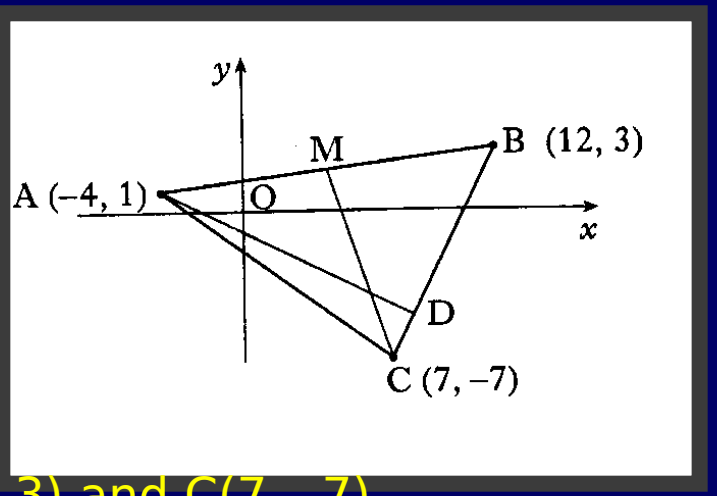
Find gradient of AC $m = \frac{2}{3}$

Gradient AC perp. $m = -\frac{3}{2}$

Equation of perp. bisector AC $2y + 3x = 23$

Point of intersection $(7, 1)$

Exam Type Questions



A triangle ABC has vertices A(–4, 1), B(12,3) and C(7, –7).

a) Find the equation of the median CM.

b) Find the equation of the altitude AD.

c) Find the co-ordinates of the point of intersection of CM and AD

Mid-point AB $(4, 2)$

Gradient CM (median) $m=-3$

Equation of median CM using y – b = m(x – a) $y+3x-14=0$

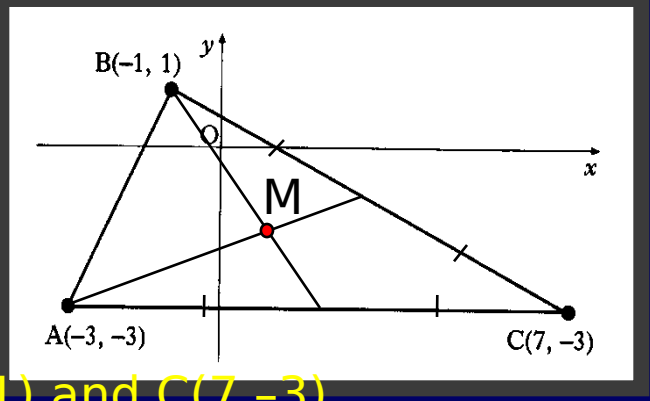
Gradient BC $m=2$

Gradient of perpendicular AD $m=-\frac{1}{2}$

Equation of AD using y – b = m(x – a) $2y+x+2=0$

Solve simultaneously for point of intersection $(6, -4)$

Exam Type Questions



A triangle ABC has vertices A(–3, –3), B(–1, 1) and C(7,–3).

a) Show that the triangle ABC is right angled at B.

b) The medians AD and BE intersect at M.

i) Find the equations of AD and BE. ii) Find find the co-ordinates of M.

Gradient AB $m=2$ Gradient BC $m=-\frac{1}{2}$

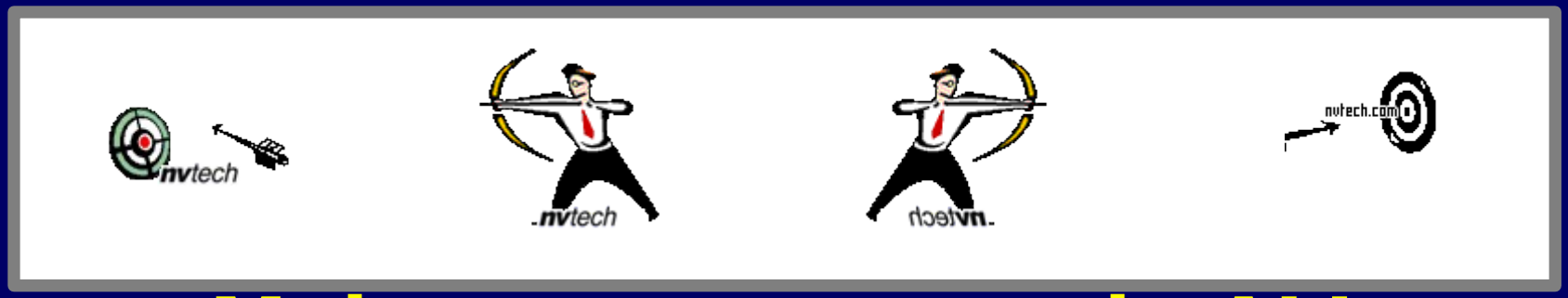
Product of gradients $2 \times -\frac{1}{2} \rightarrow 1$ Hence AB is perpendicular to BC, so B = 90

Mid-point BC $(3,-1)$ Gradient of median AD $m=\frac{1}{3}$ Equation AD $3y-x+6=0$

Mid-point AC $(2,-3)$ Gradient of median BE $m=-\frac{4}{3}$ Equation AD $3y+4x+1=0$

Solve simultaneously for M, point of intersection $\left(1,-\frac{5}{3}\right)$

Are you on Target !



Make sure you can do ALL questions in

HIGHER Straight Line booklet

