Number



Counting Numbers

- Also known as Natural numbers = 1, 2, 3, 4, 5...

Multiples

- Achieved by multiplying the counting numbers by a certain number

e.g. List the first 5 multiples of 6

6 × 1 = 6 6 × 2 = 12 6 × 3 = 18 2 3

Common Multiples

- Are multiples shared by numbers

e.g. List the common multiples of 3 and 5

Multiples of 3: 3, 6, 9, 12, 15, ... Multiples of 5: 5, 10, 15, 20, 25, ...

Common Multiples of 3 and 5: 15, ...

- The lowest common multiple (LCM) is the lowest number in the list

e.g. The LCM of 3 and 5 is: 15

Factors

- Are all of the counting numbers that divide evenly into a number
- Easiest to find numbers in pairs

e.g. List the factors of 20 1, 2, 4, 5, 10, 20

Common Factors

- Are factors shared by numbers

e.g. List the common factors of 12 and 28

Factors of 12: 1,2,3,4, 6,12 Factors of 28: 1,2,4,7,14 28

Common Multiples of 12 and 28: 1,2,4

- The highest common factor (HCF) is the highest number in the list

e.g. The HCF of 3 and 5 is: 4

Prime Numbers

Note: 1 is NOT a prime number and 2 is the only EVEN prime number.

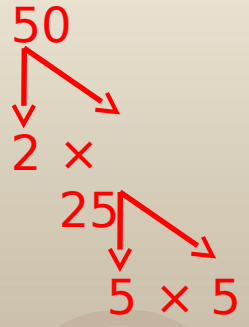
- Have only 1 and itself as factors

e.g. List the first 5 prime numbers 2, 3, 5, 7, 11

Prime Factors

- All numbers can be made by multiplying only prime numbers
- Can be written as a Prime Factor tree.

e.g. Write 50 as a product of prime numbers (factors)

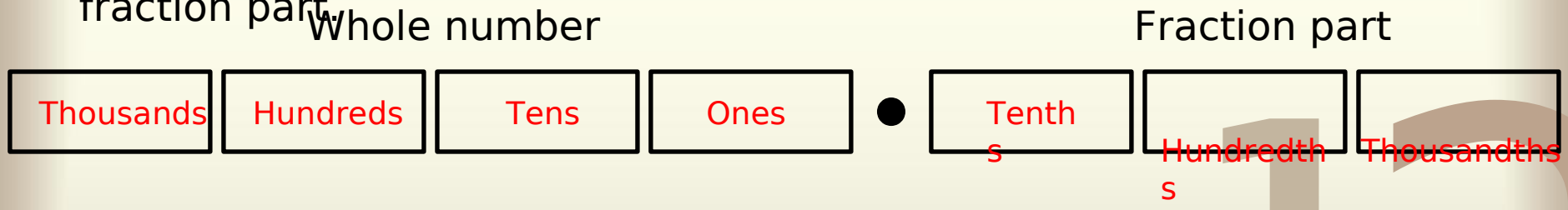


When listing prime factors, list all repeats too.

50 as a product of primes is: $2 \times 5 \times 5$

Decimals

- Also known as decimal fractions
- Place values of decimals are very important to know.
- There are two parts to numbers, the whole number part and fraction part.



1. ADDING DECIMALS

- Use whatever strategy you find most useful

e.g.

a) 2.7 + 4.8 = 7.5

b) 3.9 + 5.2 = 9.1

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c) 23.74 + 5.7 = 29.44

d) 12.8 + 16.65 = 29.45

2. SUBTRACTING DECIMALS

- Again use whatever strategy you find most useful

e.g.

a) 4.8 – 2.7 = 2.1

b) 5.2 – 3.9 = 1.3

c) 23.4 - 5.73 = 17.67

d) 16.65 – 12.8 = 3.85

3. MULTIPLYING DECIMALS

- Again use whatever strategy you find most useful

a) 0.5 × 9.24 = 4.62

b) 2.54 × 3.62 = 9.1948

One method is to firstly ignore the decimal point and then when you finish multiplying count the number of digits behind the decimal point in the question to find where to place the decimal point in the answer

4. DIVIDING DECIMALS BY WHOLE NUMBERS

- Whole numbers = 0, 1, 2, 3, 4, ...
- Again use whatever strategy you find most useful

a) 8.12 ÷ 4 = 2.03

b) 74.16 ÷ 6 = 12.36

c) 0.048 ÷ 2 = 0.024

d) 0.0056 ÷ 8 = 0.0007

e) 2.3 ÷ 5 = 0.46

f) 5.7 ÷ 5 = 1.14

5. DIVIDING BY DECIMALS

- It is often easier to move the digits left in both numbers so that you are dealing with whole numbers

a) 18.296 ÷ 0.04

1829.6 ÷ 4 457.4

=

b) 2.65 ÷ 0.5

26.5 ÷ 5 = 5.3

6. ROUNDING DECIMALS

i) Count the number of places needed AFTER the decimal point

ii) Look at the next digit

- If it's a 5 or more, add 1 to the previous digit
- If it's less than 5, leave previous digit unchanged

iii) Drop off any extra digits

e.g. Round 6.12538 to:

a) 1 decimal place (1 d.p.)

Next digit 2

= = leave unchanged

= 6.1

b) 4 d.p.

Next digit 8

= = add 1

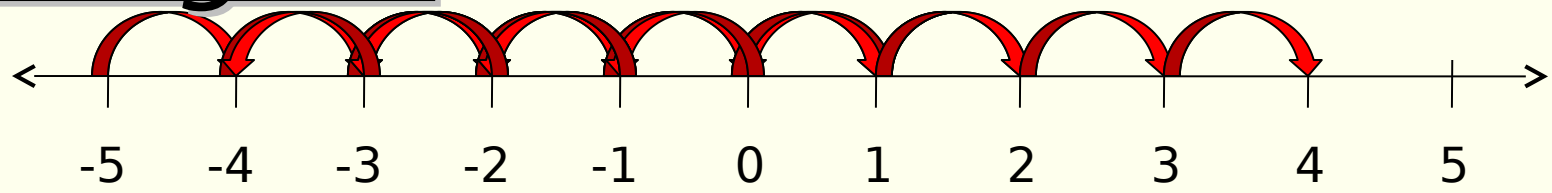
= 6.1254

The number of places you have to round to should tell you how many digits are left after the decimal point in your answer. i.e. 3 d.p. = 3 digits after the decimal point.

When rounding decimals, you DO NOT move digits

- ALWAYS round sensibly i.e. Money is rounded to 2 d.p.

Integers



1. ADDING INTEGERS

- One strategy is to use a number line but use whatever strategy suits you

i) Move to the right if adding positive integers

ii) Move to the left if adding negative integers

e.g.

a) -3 + 5 = 2

b) -5 + 9 = 4

c) 1 + -4 = -3

d) -1 + -3 = -4

2. SUBTRACTING INTEGERS

- One strategy is to add the opposite of the second integer to the first

e.g.

a) 5 - 2 = 3

b) 4 - - 2 = 4 + 2 = 6

c) 1 - -6 = 1 + 6 = 7

- For several additions/subtractions work from the left to the right

a) 2 - -8 + -3 = 10 + -3 = 7

b) -4 + 6 - -3 + -2 = 2 - -3 + -2 = 5 + -2 = 3

3. MULTIPLYING/DIVIDING INTEGERS

- If both numbers being multiplied have the same signs, the answer is positive
- If both numbers being multiplied have different signs, the answer is negative

e.g.

a) 5 × 3 = 15

b) -5 × -3 = 15

c) -5 × 3 = -15

d) 15 ÷ 3 = 5

e) -15 ÷ -3 = 5

f) 15 ÷ -3 = -5

BEDMAS

- Describes order of operations

B rackets

E xponents (Also known as powers/indices)

D ivision

M ultiplication

Work left to right if only these two

A ddition

S ubtraction

Work left to right if only these two

e.g. 4 × (5 + -2 × 6)

= 4 × (5 + -12)

= 4 × (-7)

= -28

POWERS

- Show repeated multiplication

e.g.

a) $3 \times 3 \times 3 \times 3 = 3^4$

b) $2^2 = 2 \times 2$

- Squaring = raising to a power of: 2

e.g. 6 squared $= 6^2$
$= 6 \times 6$
$= 36$

- Cubing = raising to a power of: 3

e.g. 4 cubed $= 4^3$
$= 4 \times 4 \times 4$
$= 64$

1. WORKING OUT POWERS

e.g.

a) $3^3 = 3 \times 3 \times 3$
$= 27$

b) $5^4 = 5 \times 5 \times 5 \times 5$
$= 625$

On a calculator you can use the x^y or ^ button.

2. POWERS OF NEGATIVE NUMBERS

a) $-5^3 = -5 \times -5 \times -5$
$= -125$

b) $-6^4 = -6 \times -6 \times -6 \times -6$
$= 1296$

If using a calculator you must put the negative number in brackets!

With an ODD power, the answer will be negative

With an EVEN power, the answer will be positive

SQUARE ROOTS

- The opposite of squaring

e.g. The square root of 36 is 6 because: $6 \times 6 = 6^2 = 36$

e.g.

a) $\sqrt{64} = 8$

b) $\sqrt{169} = 13$

- On the calculator use the $\sqrt{}$ button or $\sqrt{x}$ button

e.g.

a) $\sqrt{10} = 3.16$ (2 d.p.)

- Other roots can be calculated using the $x\sqrt{}$ button or $^{x}\sqrt{y}$ button

e.g. $^{4}\sqrt{1296}$ 4 = shift $x\sqrt{}$ 1296

= 6

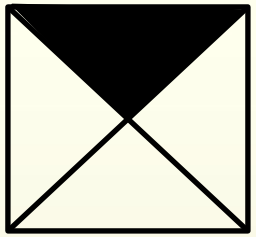
This is because $6 \times 6 \times 6 \times 6 = 6^4$

And $6^4 = 1296$

FRACTIONS

- Show how parts of an object compare to its whole

e.g.



Fraction shaded = $\frac{1}{4}$

1. SIMPLIFYING FRACTIONS

- Fractions must ALWAYS be simplified where possible
- Done by finding numbers (preferably the highest) that divide exactly into the numerator and denominators of a fraction

e.g. Simplify

a) $\frac{5}{10} \div 5 \div 5 = \frac{1}{2}$

b) $\frac{6}{9} \div 3 \div 3 = \frac{2}{3}$

c) $\frac{45}{60} \div 5 \div 5 \div 3 \div 3 = \frac{3}{4}$

2. MULTIPLYING FRACTIONS

- Multiply numerators and bottom denominators separately then simplify.

e.g. Calculate:

a) $\frac{3}{5} \times \frac{1}{6} = \frac{3 \times 1}{5 \times 6}$

$= \frac{3}{30} \div \frac{3}{3}$

$= \frac{1}{10}$

b) $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5}$

$= \frac{6}{20} \div \frac{2}{2}$

$= \frac{3}{10}$

- If multiplying by a whole number, place whole number over 1.

e.g. Calculate:

a) $\frac{3}{20} \times 5 = \frac{3}{20} \times \frac{5}{1}$

$= \frac{3 \times 5}{20 \times 1}$

$= \frac{15}{20} \div \frac{5}{5}$

$= \frac{3}{4}$

b) $\frac{2}{3} \times 15 = \frac{2}{3} \times \frac{15}{1}$

$= \frac{2 \times 15}{3 \times 1}$

$= \frac{30}{3} \div \frac{3}{3}$

$= \frac{10}{1}$ $(= 10)$

3. RECIPROCALS

- Simply turn the fraction upside down.

e.g. State the reciprocals of the following:

a) $\frac{3}{5} = \frac{5}{3}$

b) $4 = \frac{4}{1}$

$= \frac{1}{4}$

4. DIVIDING BY FRACTIONS

- Multiply the first fraction by the reciprocal of the second, then simplify.

e.g. Simplify:

a) $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3}$

$= \frac{2 \times 4}{3 \times 3}$

$= \frac{8}{9}$

b) $\frac{4}{5} \div 3 = \frac{4}{5} \div \frac{3}{1}$

$= \frac{4}{5} \times \frac{1}{3}$

$= \frac{4 \times 1}{5 \times 3}$

$= \frac{4}{15}$

5. ADDING/SUBTRACTING FRACTIONS

a) With the same denominator:

- Add/subtract the numerators and leave the denominator unchanged. Simplify if possible.

e.g. Simplify:

a) $\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$

b) $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$

b) With different denominators:

- Multiply denominators to find a common denominator.
- Cross multiply to find equivalent numerators.
- Add/subtract fractions then simplify.

e.g. Simplify:

a) $\frac{1}{4} + \frac{2}{5} = \frac{5\times1 + 4\times2}{4\times5} = \frac{5+8}{20} = \frac{13}{20}$

b) $\frac{9}{10} - \frac{3}{4} = \frac{4\times9 - 10\times3}{10\times4} = \frac{36-30}{40} = \frac{6}{40} = \frac{3}{20}$

6. MIXED NUMBERS

- Are combinations of whole numbers and fractions.

a) Changing fractions into mixed numbers:

- Divide denominator into numerator to find whole number and remainder gives fraction .

e.g. Change into mixed numbers:

a) $\frac{13}{6} = 2\frac{1}{6}$

b) $\frac{22}{5} = 4\frac{2}{5}$

b) Changing mixed numbers into improper fractions:

- Multiply whole number by denominator and add denominator.

e.g. Change into improper fractions:

a) $4\frac{3}{4} = \frac{4 \times 4 + 3}{4} = \frac{19}{4}$

b) $6\frac{1}{3} = \frac{6 \times 3 + 1}{3} = \frac{19}{3}$

- To solve problems change mixed numbers into improper fractions first.

e.g. $1\frac{1}{2} \times 2\frac{2}{3} = \frac{1 \times 2 + 1}{2} \times \frac{2 \times 3 + 2}{3}$

$= \frac{3}{2} \times \frac{8}{3}$

$= \frac{24}{6}$

$= \frac{4}{1} \quad (= 4)$

Note: All of the fraction work can be done on a calculator using the a b/c button

7. RECURRING DECIMALS

- Decimals that go on forever in a pattern
- Dots show where pattern begins (and ends) and which numbers are included.

e.g. Write as a recurring decimals:

a) $\frac{2}{3} = 0.66666\ldots = 0.\dot{6}$

b) $\frac{2}{11} = 0.181818\ldots = 0.\dot{1}\dot{8}$

c) $\frac{1}{7} = 0.142857142\ldots = 0.\dot{1}4285\dot{7}$

8. FRACTIONS AND DECIMALS

a) Changing fractions into decimals:

- One strategy is to divide numerator by denominator

e.g. Change the following into decimals:

a) $\frac{2}{5} = 0.4$

b) $\frac{5}{6} = 0.8\dot{3}$

b) Changing decimals into fractions:

- Number of digits after decimal point tells us how many zero's go on the bottom

e.g. Change the following into fractions:

a) $0.75 = \frac{75}{100}$ $(\div 4)$ $= \frac{3}{4}$

Don't forget to simplify!

b) $0.56 = \frac{56}{100}$ $(\div 4)$ $= \frac{14}{25}$

Again *a b/c* button can be used

9. COMPARING FRACTIONS

- One method is to change fractions to decimals

e.g. Order from SMALLEST to LARGEST:

$\frac{1}{2}$ $\frac{2}{5}$ $\frac{2}{3}$ $\frac{4}{9}$ $\frac{2}{5}$ $\frac{4}{9}$ $\frac{1}{2}$ $\frac{2}{3}$

0.5 0.4 0.$\dot{6}$ 0.4

ESTIMATION

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- Involves guessing what the real answer may be close to by working with whole numbers
- Generally we round numbers to 1 significant figure first

e.g. Estimate

a) 4.986×7.003 = 5×7 = 35

b) 413×2.964 = 400×3 = 1200

PERCENTAGES

- Percent means out of 100

1. PERCENTAGES, FRACTIONS AND DECIMALS

a) Percentages into decimals and fractions:

- Divide by (decimals) or place over (fractions) 100 and simplify if possible

e.g. Change the following into decimals and fractions:

a) $65\% \div 100 = 0.65$

$= \frac{65}{100} \quad (\div 5)$

$= \frac{13}{20}$

b) $6\% \div 100 = 0.06$

$= \frac{6}{100} \quad (\div 2)$

$= \frac{3}{50}$

c) $216\% \div 100 = 2.16$

$= \frac{216}{100} \quad (\div 4)$

$= \frac{54}{25} \quad (= 2\frac{4}{25})$

b) Fractions into percentages:

- Multiply by 100

e.g. Change the following fractions into percentages:

a) $\frac{2}{5} = \frac{2}{1} \times \frac{100}{5}$

$= \frac{200}{5}$

$= 40\%$

b) $\frac{5}{4} = \frac{5}{1} \times \frac{100}{4}$

$= \frac{500}{4}$

$= 125\%$

c) $\frac{3}{7} = \frac{3}{1} \times \frac{100}{7}$

$= \frac{300}{7}$

$= 42.86\%$

c) Decimals into percentages:

- Multiply by 100

e.g. Change the following decimals into percentages:

a) 0.26 × 100 = 26 %

b) 0.78 × 100 = 78 %

c) 1.28 × 100 = 128%

2. PERCENTAGES OF QUANTITIES

- Use a strategy you find easy, such as finding simpler percentages and adding, or by changing the percentage to a decimal and multiplying

e.g. Calculate:

a) 47.5% of $160

10% 16

5% = 8

2.5% = 4

Therefore 45% = 16 × 4 + 8 + 4

= $76

b) 75% of 200 kg = × 200

= 150

0.7

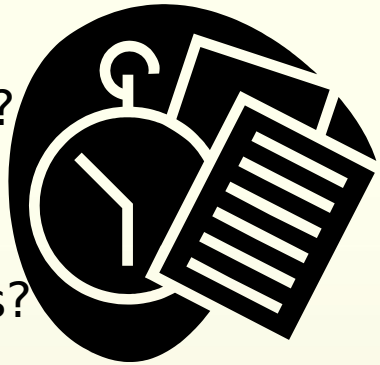
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kg



4. ONE AMOUNT AS A PERCENTAGE OF ANOTHER

- A number of similar strategies such as setting up a fraction and multiplying by 100 exist.



e.g. Paul got 28 out of 50. What percentage is this?

100 ÷ 50 = 2 (each mark is worth 2%)

28 × 2 = 56%

e.g. Mark got 39 out of 50. What percentage is this?

$\frac{3}{9} \times 100 = 78\%$

5. WORKING OUT ORIGINAL QUANITIES

- Convert the final amount's percentage into a decimal.
- Divide the final amount by the decimal.

e.g. 16 is 20% of an amount. What is this amount?

20% as a decimal = 0.2

Amount = 16 ÷ 0.2 = 80

To spot these types of questions, look for words such as 'pre', 'before' or 'original'

e.g. A price of $85 includes a tax mark-up of 15%. Calculate the pre-tax price.

Final amount as a percentage = 100 + 15 = 115

Final amount as a decimal = 1.15

Pre-tax price = 85 ÷ 1.15 = $73.91

4. INCREASES AND DECREASES BY A PERCENTAGE

a) Either find percentage and add to or subtract from original amount

e.g. Carol finds a $60 top with a 15% discount. How much does she pay?

10% = 6 15% $9

5% = 3 Therefore she pays = 60 - 9

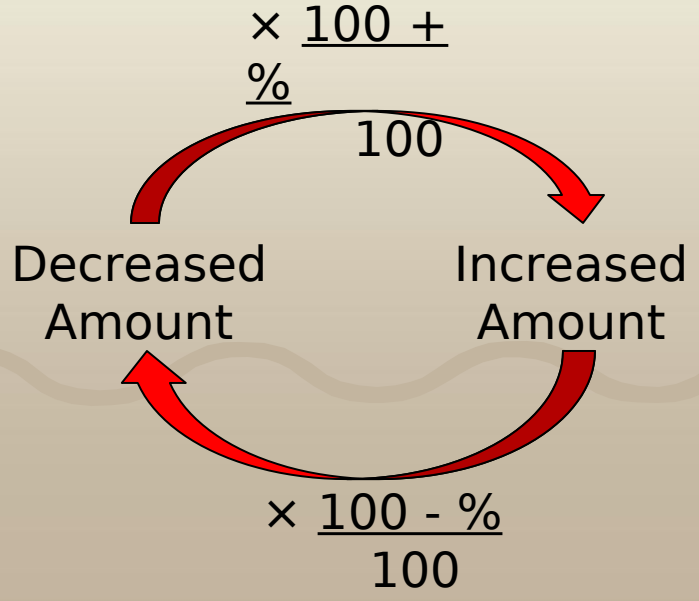
= $51

e.g. A shop puts a mark up of 20% on items. What will be the selling price for an item the shop buys for $40?

$0.2 \times 40 = \$8$ Therefore the selling price = 40 + 8

= $48

b) Or use the following method:



a) Increase $40 by 20%

$$40 \times \frac{100 + 20}{100} = 40 \times \frac{120}{100}$$

$= 40 \times 1.2$

$= \$48$

b) Decrease $60 by 15%

$$60 \times \frac{100 - 15}{100} = 60 \times \frac{85}{100}$$

$= 60 \times 0.85$

$= \$51$

5. PERCENTAGE INCREASE/DECREASE

- To calculate percentage increase/decrease we can use:

$$\text{Percentage increase/decrease} = \frac{\textbf{decrease/increase}}{\textbf{original amount}} \times 100$$

e.g. Mikes wages increased from $11 to $13.50 an hour.

a) How much was the increase?

$13.50 - 11 = \$2.50$

b) Calculate the percentage increase

$\frac{2.50}{11} \times 100 = 22.7\%$ (1 d.p.)

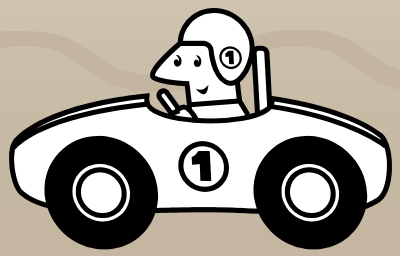
e.g. A car originally brought for $4500 is resold for $2800. What was the percentage decrease in price?

Decrease $= 4500 - 2800$

$= \$1700$

Percentage Decrease $= \frac{1700}{4500} \times 100$

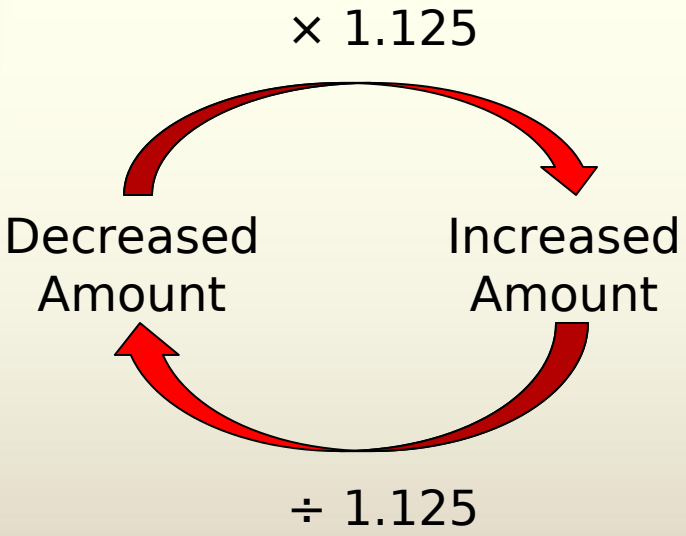
$= 37.8\%$ (1 d.p.)



To spot these types of questions, look for the word 'percentage'

GST

- Is a tax of 12.5%
- To calculate GST increase/decreases use:



a) Calculate the GST inclusive price if $112 excludes GST

112 × 1.125 = $126

bi) An item sold for $136 includes GST

136 ÷ 1.125 = $120.89

ii) How much is the GST worth?

136 - 120.89 = $15.11

INTEREST FROM BANKS

- Two types

1) Only paid interest once at the end.

2) Simple Compound: Interest is added to the deposit on which further interest is earned.

Formula for Simple Interest: $I = \frac{P \times R \times T}{100}$

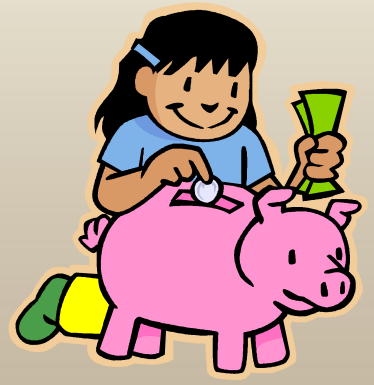
Where **I** = Interest earned, **P** = deposit, **R** = interest rate, **T** = time

e.g. Calculate the interest on $200 deposited for 3 years at an interest rate of 8% p.a.

p.a. = Per annum (year)

$$I = \frac{200 \times 8 \times 3}{100}$$

$I = \$48$



Compound interest is covered in more depth in Year 11

RATIOS

- Compare amounts of two quantities of similar units
- Written with a colon
- Can be simplified just like fractions and should always contain whole numbers

e.g. Simplify 200 mL : 800 mL

÷200 ÷200

1 mL : 4 mL

Must have the same units!

e.g. Simplify 600 m : 2 km

600 m : 2000 m

÷200 ÷200

3 m : 10 m

1. RATIOS, FRACTIONS AND PERCENTAGES

e.g. Fuel mix has 4 parts oil to 21 parts petrol.

a) What fraction of the mix is petrol?

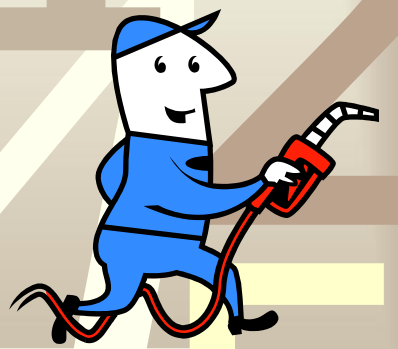
Total parts: 4 + 21 = 25

Fraction of petrol: $\frac{21}{25}$

b) What percentage of the mix is oil?

Total parts = 35

Percentage of oil: $\frac{4}{25} \times \frac{100}{1} = 16\%$



2. SPLITTING IN GIVEN RATIOS

- Steps: i) Add parts
  ii) Divide total into amount being split
  iii) Multiply answer by parts in given ratio

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e.g. Split $1400 between two people in the ratio 2:5

Order of a ratio is very important

Total parts: 2 + 5 = 7

Divide into amount: 1400 ÷ 7 = 200

Multiply by parts: 200 × 2 = $400 200 × 5 = $1000

Answer: $400 : $1000

e.g. What is the smallest ratio when $2500 is split in the ratio 5:3:2

Total parts: 5 + 3 + 2 = 10

Divide into amount: 2000 ÷ 10 = 250

Multiply by parts: 250 × 2 = $500

Answer: $500



RATES

- Compare quantities in different units

e.g. A cyclist covers a distance of 80 km in 4 hours. Calculate the cyclists speed

Rate = km per hour

80 ÷ 4 = 20 km/hr

e.g. A person can shell oysters at a rate of 12 per minute

a) How many oysters can they shell in 4 minutes?

Rate = oysters per minute

12 × 4 = 48 oysters

b) How long will it take them to shell at least 200 oysters?

200 ÷ 12 = 16.6

Therefore it will take 17 minutes



SIGNIFICANT FIGURES

- A way of representing numbers
- Count from the first non-zero number

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e.g. State the number of significant figures (s.f.) in the following:

a) 7553 4 s.f.

b) 4.06 3 s.f.

c) 0.012 2 s.f.

Zero's at the front are known as place holders and are not counted

DECIMAL PLACES

- Another way of representing numbers
- Count from the first number after the decimal point

e.g. State the number of decimal places (d.p.) in the following:

a) 70.6523 d.p.

b) 0.021 3 d.p.

c) 46 0 d.p.

ROUNDING

1. DECIMAL PLACES (d.p.)

i) Count the number of places needed AFTER the decimal point

ii) Look at the next digit

- If it's a 5 or more, add 1 to the previous digit
- If it's less than 5, leave previous digit unchanged

iii) Drop off any extra digits

e.g. Round 6.12538 to:

a) 1 decimal place (1 d.p.)

Next digit 2

= = leave unchanged

= 6.1

b) 4 d.p.

Next digit 8

= = add 1

= 6.1254

The number of places you have to round to should tell you how many digits are left after the decimal point in your answer. i.e. 3 d.p. = 3 digits after the decimal point.

When rounding decimals, you DO NOT move digits

2. SIGNIFCANT FIGURES (s.f.)

i) Count the number of places needed from the first NON-ZERO digit

ii) Look at the next digit

- If it's a 5 or more, add 1 to the previous digit
- If it's less than 5, leave previous digit unchanged

iii) If needed, add zeros as placeholders to keep the number the same size

e.g. Round 0.00564 to:

a) 1 significant figure (1 s.f.)

Next digit 2

= = leave unchanged

= 6.1

e.g. Round 18730 to:

a) 2 s.f.

Next digit 7

= = add 1

= 19000

Don't forget to include zeros if your are rounding digits BEFORE the decimal point. Your answer should still be around the same place value

- ALWAYS round sensibly i.e. Money is rounded to 2 d.p.

STANDARD FORM

1. MULTIPLYING BY POWERS OF 10

- Digits move to the left by the amount of zero's



a) $2.56 \times 10 = 25.6$



b) $0.83 \times 1000 = 830$

As a power of 10: $10 = 10^1$

As a power of 10: $1000 = 10^3$

Therefore, when multiplying by a power of 10, the power tells us how many places to move the digits to the left

2. STANDARD FORM

- Is a way to show very large or very small numbers
- Is written in two parts:

A number between 1 - 10 × A power of 10

e.g.

2.8×10^{14} Positive power = large number

5.58×10^{-4} Negative power = small number

3. WRITING NUMBERS INTO STANDARD FORM

- Move decimal point so that it is just after the first significant figure
- Number of places moved give the power
- If point moves left the power is positive, if it moves right, the power is negative

e.g. Convert the following into standard form

If there is no decimal point, place it after the last digit

a) 7 3 1 0 0 0 = 7.31×10^{3}

b) 3 . 6 6 = 3.66×10^{0}

c) 0 . 0 0 0 8 2 = 8.2×10^{-4}

4. STANDARD FORM INTO ORDINARY NUMBER

- Power of 10 tells us how many places to move the decimal point
- If power is positive, move point right. If power is negative move point left
- Extra zeros may need to be added in

a) 6.5×10^{4} = 65000

b) 7.312×10^{0} = 7.312

c) 6.9×10^{-2} = 0.069