

LCMs and GCFs



MSJC ~ San Jacinto Campus
Math Center Workshop Series
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Least Common Multiples (LCMs) and Greatest Common Factors (GCFs) play a big role in mathematics involving fractions

- When adding fractions, it is necessary to find a common denominator. We use the LCM as the smallest denominator.
- To reduce fraction, we need to find the GCF.

Least Common Multiples

- The multiples of a number are the products of that number and the Natural numbers (1, 2, 3, 4, . . .)
- The number that is a multiple of two or more numbers is a common multiple of those numbers.
- The Least Common Multiple (LCM) is the smallest common multiple of two or more numbers.

Example:

- The multiples of 4 are
 - 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, . . .
- The multiples of 6 are
 - 6, ~~12~~, ~~18~~, ~~24~~, ~~30~~, ~~36~~, ~~42~~, 48, 54, 60, 66, . . .
- The common multiples of 4 and 6 are
 - 12, 24, 36, 48, . . .
- The LeastCommonMultiples of 4 and 6 is 12
- ✓ Notation: $\text{LCM}(4, 6) = 12$

Finding the LCM

We can find the LCM of two or more numbers by listing out the multiples of each and identifying the smallest common multiple

But, this could be difficult . . .

- Ex: Find LCM(24, 50)

Do you know your multiples of 24 and 50 easily?

We need a more systematic approach to finding LCMs

We will find the LCM of two or more numbers using the *prime factorization* of each number

Review: the *prime factorization* of a number is that number written solely as a product of prime numbers.

Ex: Find the prime factorization of 24

Primes

Quotient
(composites)

2	4
2	12
2	6
2	3

$$24 = \underline{2} * \underline{12}$$

$$24 = \underline{2} * \underline{2} * \underline{6}$$

$$\begin{array}{c} 6 \\ 24 = 2 * 2 * \underline{2} * \underline{3} \end{array}$$

Prime on the right → done
→ clean it up

$$24 = 2^3 * 3$$

Ex: Find the prime factorization of 50

Primes

Quotient
(composites)

5
0
2
5

25

5

$$50 = 2 * 25$$

$$50 = 2 * 5 * 5$$

Prime on the right → done
→ just clean it up

$$50 = 2 * 5^2$$

Ex: Find the LCM(24, 50)

- Find the prime factorization of each number:

$$24 = 2^3 * 3 \quad \text{and} \quad 50 = 2 * 5^2$$

- Arrange the factorizations in a table

# \ prime s	2	3	5
24	2^3	3^1	5^0
50	2^1	3^0	5^2
LCM	8	3	25

- Circle the Largest product in each column.
- The LCM(24, 50) is the product of the circled numbers: $8 * 3 * 25 = 600$

Note:

- The exponent represents the number of times that factor appears in the prime factorization
- In the prime factorization of the LCM of two numbers we can find the prime factorization of each of the numbers:

$$24 = 2*2*2*3 \text{ and } 50 = 2*5*5$$

$$\begin{aligned} \text{LCM}(24, 50) &= 600 = 2*2*2*3*5*5 \\ &= (2*2*2*3)*5*5 \\ &= (2*5*5)*2*2*3 \end{aligned}$$

600 is a multiple of both 24 and 50!

Ex: Find the LCM(44, 60)
Prime Factorizations

$$\begin{array}{r|l} 44 & \\ \hline 2 & 22 \\ 2 & 11 \end{array}$$

$$44 = 2 * 2 * 11$$

$$\begin{array}{r|l} 60 & \\ \hline 2 & 30 \\ 2 & 15 \\ 3 & 5 \end{array}$$

$$60 = 2 * 2 * 3 * 5$$

Ex: Find the LCM(44, 60)

- M: Find the prime factorization of each number:

$$44 = \overline{2*2} * 11 \quad \text{and} \quad 60 = \overline{2*2} * 3 * 5$$

- C: Find the common factors: $\overline{2 * 2}$
- L: Include all the “leftovers”: $3 * 5 * 11$
- The LCM(44, 60) = $2 * 2 * 3 * 5 * 11 = 660$

Ex: Find the LCM(102,
184) Prime Factorizations

$$\begin{array}{r|l} 10 & \\ \hline 2 & 51 \\ 3 & 17 \end{array}$$

$$102 = 2 * 3 * 17$$

$$\begin{array}{r|l} 184 & \\ \hline 2 & 92 \\ 2 & 46 \\ 2 & 23 \end{array}$$

$$184 = 2 * 2 * 2 * 23$$

Ex: Find the LCM(102, 184)

- M: Find the prime factorization of each number:

$$102 = \overline{2} * 3 * 17 \quad \text{and} \quad 184 = \overline{2} * 2 * 2 * 23$$

- C: Find the common factors: $\overline{2}$
- L: Include all the “leftovers”: $2 * 2 * 3 * 17 * 23$
- The $\text{LCM}(44, 60) = 2 * 2 * 2 * 3 * 17 * 23 = 9384$

Ex: Find the LCM(16, 30, 84) Prime Factorizations

16	
2	8
2	4
2	2

30	
2	15
3	5

84	
2	42
2	21
3	7

$$16 = 2 * 2 * 2 * 2 \quad 30 = 2 * 3 * 5 \quad 84 = 2 * 2 * 3 * 7$$

Ex: Find the LCM(16, 30, 84)

- M: Find the prime factorization of each number:

$$16 = \overline{2} * \overline{2} * \overline{2} * \overline{2} \quad 30 = \overline{2} * \overline{3} * \overline{5} \quad \text{and } 84 = \overline{2} * \overline{2} * \overline{3} * \overline{7}$$

- C: Find the common factors: $\overline{2}$
- Continue to find factors that are common to some: $\overline{2} * \overline{3}$

• L: Include all the "leftovers": $\overline{2} * \overline{2} * \overline{2} * \overline{2} * \overline{3} * \overline{5} * \overline{7}$

• The LCM(16, 30, 84) = $\overline{2} * \overline{2} * \overline{2} * \overline{2} * \overline{3} * \overline{5} * \overline{7}$
= 1680

Try a few problems
on the handout

Greatest Common Factors

- The factors of a number are the numbers that divide the original number evenly
- A number that is a factor of two or more numbers is a common factor of those numbers
- The Greatest Common Factor (GCF) is the largest common factor of two or more numbers

Example:

- The factors of 24 are
 - 1, 2, 3, 4, 6, 8, 12, 24
- The factors of 36 are
 - 1, 2, 3, 4, 6, 9, 12, 18, 36
- The common factors of 24 and 36 are
 - 1, 2, 3, 4, 6, 12
- The GreatestCommonFactor of 24 and 36 is 12
- ✓ Notation: $\text{GCF}(24, 36) = 12$

Finding the GCF

We can find the GCF of two or more numbers by listing out the factors of each and identifying the largest common factor

But, this could be difficult when the numbers are very large.

We need a more systematic approach to finding GCFs

We will find the GCF of two or more numbers using the *prime factorization* of each number and using a process nearly identical to the one we used to find LCMs of two or more numbers

Ex: Find the GCF(24, 40)

Prime Factorizations

$$\begin{array}{r|l} 24 & \\ \hline 2 & 12 \\ 2 & 6 \\ 2 & 3 \end{array}$$

$$24 = 2 * 2 * 2 * 3$$

$$\begin{array}{r|l} 40 & \\ \hline 2 & 20 \\ 2 & 10 \\ 2 & 5 \end{array}$$

$$40 = 2 * 2 * 2 * 5$$

Ex: Find the GCF(24, 40)

- Find the prime factorization of each number:
 $24 = 2 * 2 * 2 * 3$ and $40 = 2 * 2 * 2 * 5$
- Arrange the factorizations in a table

# \ prime s	2	3	5
24	2^3	3^1	5^0
40	2^3	3^0	5^1
GCF	8	1	1

- Circle the Smallest product in each column
- The GCF(24, 40) is the product of the circled numbers: $8 * 1 * 1 = 8$

Note:

- The exponent represents the number of times that factor appears in the prime factorization
- In the prime factorization of the numbers, we can find the prime factorization of the GCF:

$$\text{GCF}(24, 40) = 8 = 2*2*2$$

$$24 = 2*2*2*3 = (2*2*2)*3$$

$$40 = 2*2*2*5 = (2*2*2)*5$$

8 is a factor of both 24 and 40!

Ex: Find the GCF(32, 51)

Prime Factorization:

$$\begin{array}{r|l} 32 & \\ \hline 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \end{array}$$

$$32 = 2 * 2 * 2 * 2 * 2$$

$$\begin{array}{r|l} 51 & \\ \hline 3 & 17 \end{array}$$

$$51 = 3 * 17$$

Ex: Find the GCF(32, 51)

- M: Find the prime factorization of each number:

$$32 = 2*2*2*2*2 \quad \text{and} \quad 51 = 3*17$$

- C: Find the common factors: 1
- G: Multiply all the common factors together:
1
 - The GCM(32, 51) = 1

Ex: Find the GCF(102, 84)

Prime Factorization:

$$\begin{array}{r|l} 102 & \\ \hline 2 & 51 \\ 3 & 17 \end{array}$$
$$102 = 2 * 3 * 17$$

$$\begin{array}{r|l} 84 & \\ \hline 2 & 42 \\ 2 & 21 \\ 3 & 7 \end{array}$$
$$84 = 2 * 2 * 3 * 7$$

Ex: Find the GCF(102, 84)

- M: Find the prime factorization of each number:

$$102 = 2 * \overline{3} * 17 \quad \text{and} \quad 84 = 2 * \overline{2} * 3 * 7$$

- C: Find the common factors: $\overline{2 * 3}$
- G: Multiply all the common factors together:
 6
 - The GCM(102, 84) = 6

Ex: Find the GCF(14, 42, 84)

Prime Factorizations

$$\begin{array}{c|c} 14 & \\ \hline 2 & 7 \end{array}$$

$$14 = 2 * 7$$

$$\begin{array}{c|c} 42 & \\ \hline 2 & 21 \\ 3 & 7 \end{array}$$

$$42 = 2 * 3 * 7$$

$$\begin{array}{c|c} 84 & \\ \hline 2 & 42 \\ 2 & 21 \\ 3 & 7 \end{array}$$

$$84 = 2 * 2 * 3 * 7$$

Ex: Find the GCF(14, 42, 84)

- M: Find the prime factorization of each number:

$$14 = \overline{2} * \overline{7} \quad 42 = 2 * \overline{3} * \overline{7} \quad \text{and} \quad 84 = 2 * \overline{2} * 3 * \overline{7}$$

- C: Find the common factors: $\overline{2} * \overline{7}$
- G: Multiply all the common factors together:

14

- The GCM(14, 42, 84) =

14

Try a few problems
on the handout