

Chapter Four Risk and Return

4.1. Understanding and Measuring risk and Return

Return: if you buy an asset of any sort, your gain (or loss) from that investment is called the return on your investment. This return will usually have two components: *income component and capital gain or capital loss on the investment.*

The rate of return can be calculated with the following formula:

$$K_t = \frac{P_t - P_{t-1} + D}{P_{t-1}}$$

Where, K_t , required rate of return
 P_t , Price of asset at time t

Example,

<u>End of year</u>	<u>Price of asset</u>	<u>Dividend</u>
1	birr 21	0
2	19	1
3	18	1
4	21	1.05
5	24	1.05
6	26	1.10

Required: calculate rate of return?

<u>Year</u>	<u>P_t</u>	<u>P_{t-1}</u>	<u>D</u>	<u>K_t</u>
1	21	-	0	-
2	19	21	1	$19-21+1/21 = -0.0476$
3	18	19	1	$18-19+1/19 = 0$
4	21	18	1.05	$21-18+1.05/18 = 0.225$
5	24	21	1.05	$24-21+1.05/21 = 0.1929$
6	26	24	1.10	$26-24+1.10/24 = 0.1292$

Probability distribution

Probabilities: Probabilities can be used to more precisely assess an asset's risk. The probability of a given outcome is its chance of occurring.

The expected value of a return: is the most likely return on a given asset

$$K = \sum_{i=1}^n k_i \times P_{ri}$$

Where, K_i = return for the i th outcome

P_{ri} = probability of occurrence of the i th out come

n = number of out comes considered

- measure of central tendency:- These are used to identify an average or center point among a data set. Example- mean, median, or mode:
- A measure of dispersion or variability:- These reflect the spread of the data points. Example variance, standard deviation, skewness, or range:

Mean:

The mean of rate of return is calculated as follows:

$$\text{Mean} = \sum r$$

Example,

The mean for the above example is $\frac{0.0476+0+0.225+0.1929+0.1292}{5}$
 $=0.0999$
 $=\underline{\underline{10\%}}$

Standard Deviation: The most common statistical indicator of an asset's risk is the standard deviation. It measures the dispersion around the expected value. The expression for the standard deviation of returns, σ_k , is given as:

$$\sigma_k = \sqrt{\sum (K_i - K)^2 \times \text{Pri}}$$

Expected values of returns for assets A and B

Example

Asset A		
<u>Possible outcomes</u>	<u>Probability (1)</u>	<u>Return (%) (2)</u>
<u>Weighted value (3) = (1)X(2)</u>		
Pessimistic	0.25	13
3.25		
Most likely	0.50	15
7.50		

Optimistic	0.25	17
<u>4.25</u>		

Expected return

= 15

Asset B

<u>Possible outcomes</u>	<u>Probability (1)</u>	<u>Return (%) (2)</u>
<u>Weighted value (3) =(1)X(2)</u>		
Pessimistic	0.25	7
1.75		
Most likely	0.50	15
7.50		
Optimistic	0.25	23
<u>5.75</u>		

Expected return

15

Standard deviation of the returns for Assets A and B

Example,

Asset A

<u>i</u>	<u>ki</u>	<u>k</u>	<u>ki-k</u>	<u>(ki-k)²</u>	<u>Pri</u>	<u>(Ki-k)² X Pri</u>
1	13	15	-2	4	0.25	1
2	15	15	0	0	0.50	0
3	17	15	2	4	0.25	<u>1</u>
						$\sum (ki - k)^2 \times Pri = 2$

Therefore, $\sigma_k A = \sqrt{2} = \underline{\underline{1.41\%}}$

Asset B

<u>i</u>	<u>ki</u>	<u>k</u>	<u>ki-k</u>	<u>(ki-k)²</u>	<u>Pri</u>	<u>(Ki-k)² X Pri</u>
1	7	15	-8	64	0.25	16
2	15	15	0	0	0.50	0
3	23	15	8	64	0.25	<u>16</u>
						$\sum (ki - k)^2 \times Pri = 32$

Therefore, $\sigma_A = 32 = 5.66\%$

In general, the higher the standard deviation, the greater the risk. The higher risk of asset B is clearly reflected in its higher standard deviations.

Variance: shows how far the actual return deviates from the average in a typical year. It is the average squared difference between the actual return and the average return.

Coefficient of variation

It is a measure of relative dispersion used in comparing the risk of assets

where

$$C_v = \frac{\sigma}{K}$$

Where, σ is standard deviation

Example, Coefficient of variation for asset A is $1.41/15 = 0.094$

Coefficient of variation for asset B is $5.66/15 = 0.377$

Asset B has the higher coefficient of variation and is therefore more risky than asset A. The higher the coefficient of variation is, the greater the risk.

Note: since both assets have the same expected return, the coefficient of variation has not provided any more information than the standard deviations. The real utility of the coefficient of variation is in comparing assets that have different expected returns.

Example,

	<u>Asset X</u>	<u>Asset Y</u>
Expected return	12%	20%
Standard deviation	9%	10%
Coefficient of variation	$9/12 = 0.75$	$10/20 = 0.5$

If the firm were to compare the assets only with standard deviations, it would prefer asset X, since asset X has a lower standard deviation than asset Y. However, using coefficient of variations, risk is lower in asset Y than asset X. Therefore, the use of the coefficient of variation to compare

asset risk is **effective** because it also considers the relative size, or expected return of the assets.

Covariance: statistical representation of the degree to which the returns on two assets move together over time; calculated as the sum of the product of each asset's deviation from its expected value over time divided by the number of time periods. The covariance of asset A and B can be computed as follows:

$$\tau_{AB} = \frac{\sum (r_A - r_A)(r_B - r_B)}{n}$$

Correlations

Correlation is a statistical measure of the relationship, if any, between series of numbers representing data of any kind. If two series move in the same direction, they are positively correlated. If the series move in opposite directions, they are negatively correlated. Correlation is a statistical representation of the degree to which returns on two securities vary together over time, with a maximum value of +1 (perfect positive correlation) and a minimum value of -1 (perfect negative correlation)

Correlation coefficient: a measure of the degree of correlation between two series.

- *Perfectly positively correlated:* describes two positively correlated series that have a correlation coefficient of +1
- *Perfectly negatively correlated:* describes two negatively correlated series that have a correlation coefficient of -1
- *Uncorrelated:* describes two series that lack any relationship or interaction and therefore have a correlation coefficient close to zero.

Example, For positively correlated

- *cyclical business, i.e having high sales when the economy is expanding and low sales during recession*

For negatively correlated

- Counter cyclical business: having low sales during economic expansion and high sales during recession

$$\text{Coefficient correlation} = \frac{\text{Covariance}}{\sigma_A \sigma_B}$$

Risk Preference

The three basic preference behaviors are

- Risk averse: the attitude toward risk in which an increased return would be required for an increase in risk.
- Risk indifferent: the attitude toward risk in which no change in return would be required for an increase in risk.
- Risk seeking: the attitude toward risk in which a decreased return would be accepted for an increase in risk

Risk and time

The variability of the returns and the risk increases with the passage of time.

Risk of a portfolio

The risk of any single proposed asset investment should not be views independent of other assets. New investments must be considered in light of their impact on the risk and return of the portfolio of assets. The financial manager's goal for the firm is to create **efficient portfolio**. Efficient portfolio is a portfolio that maximizes return for a given level of risk or minimizes risk for a given level of return. The statistical concept of correlation is useful in the process of diversification that is used to develop an efficient portfolio.

A Portfolio is a collection of securities held by a single investor, whether an individual or institution. The main incentive for forming portfolios is **diversification**, which is the allocation of investable funds to a variety of sources.

A **perfect market** is a market without any impediments to trading, such as transaction costs or costly information. The assumptions are:

1. Securities markets operate with no transaction costs
2. All investors have free access to the complete body of information about everything relevant to the pricing of securities.
3. All investors appraise this information in a similar way; that is, they have homogeneous expectations
4. Investors are interested only in the risk and expected return characteristics of securities. They seek securities with higher expected returns and try to avoid risk.
5. All investors in the marketplace have the same one period time horizon.

The concept of dominance

When an investor has investment opportunities in which this risk/return trade off is not confronted, one investment opportunity “dominates” the other. In establishing a definition of dominance, one security dominates another if it meets at least one of the following three conditions:

- A. One security offers greater expected return, but the same risk, than another security
- B. One security offers the same expected return, but lower risk, than another security
- C. One security offers greater expected return, but lower risk, than another security

Portfolio return and standard deviation

The return on a portfolio is calculated as a weighted average of the returns on the individual assets.

$$K_p = (w_1 \times k_1) + (w_2 \times k_2) + \dots + (w_n \times k_n)$$

Where, K_p is portfolio return
 w_n = the proportion of the portfolio total value

in three financial

assets. The birr amounts committed to each asset, the return value are as follows:

<u>Asset</u>	<u>Birr investment</u>	<u>Return</u>
A	20,000	9%
B	30,000	11%
C	<u>50,000</u>	14%
Total	100,000	

Required: Calculate the expected return for portfolio?

The proportion (W) of each asset can be obtained as follows: Asset A (w_1) = $20000/100000 = 0.2$ and similarly you can calculate for asset B and C. The proportion for asset B and C is 0.3 and 0.5, respectively.

$$\begin{aligned}
 K_p &= (w_1 \times k_1) + (w_2 \times k_2) + \dots + (w_n \times k_n) \\
 &= 0.2(9\%) + 0.3(11\%) + 0.5(14\%) \\
 &= \underline{\underline{12.1\%}}
 \end{aligned}$$

Exercise:

If security A has an expected return of 10% and security B has an expected return of 15%, how should you weight your holdings to get an expected return of 12%?

Solution

The sum of investment in security A and B must be 100%, therefore, we can write as

$$A + B = 1 \dots \dots \dots \text{Equation 1}$$

$$\begin{aligned}
 &\text{With the formula of expected return on portfolio } 0.1A + 0.15B \\
 &= 0.12 \dots \dots \dots \text{Equation 2}
 \end{aligned}$$

Solve the two equations simultaneously; the proportion security A and B is 60% and 40 %

Example,

Year	Return	
	Asset A	Asset B
1	14%	7%
2	9%	12%
3	12%	6%
4	4%	10%
5	11%	5%

Required: Compute

1. Variance and standard of asset A
2. Variance of asset B
3. Covariance of asset A and B

4. Coefficient of correlation between asset A and B

Solution:

Year	<u>rA</u>	<u>rB</u>	<u>(rA-rA)</u>	<u>(rB-rB)</u>	<u>(rA-rA) (rB-rB)</u>	<u>(rA-rA)²</u>	<u>(rB-rB)²</u>
1	0.14	0.07	(0.14-0.1)=0.04	(0.07-0.08)=-0.01	-0.0004	0.0016	0.0001
2	0.09	0.12	(0.09-0.1)=-0.01	(0.12-0.08)=0.04	-0.0004	0.0001	0.0016
3	0.12	0.06	(0.12-0.1)=0.02	(0.06-0.08)=-0.02	-0.0004	0.0004	0.0004
4	0.04	0.10	(0.04-0.1)=-0.06	(0.10-0.08)=0.02	-0.0012	0.0036	0.0004
5	0.11	0.05	(0.11-0.1)=0.01	(0.05-0.08)=-0.03	<u>-0.0003</u>	<u>0.0001</u>	<u>0.0009</u>

$$\Sigma = -0.0027$$

$$0.0058 \quad 0.0034$$

1) The variance of asset A is

2) The variance of asset B is

$$\begin{aligned} \sigma_A^2 &= \frac{(rA-rA)^2}{n} \\ &= \frac{0.0058}{5} = 0.0012 \\ \sigma_B^2 &= \frac{(rB-rB)^2}{n} \\ &= \frac{0.0034}{5} = 0.0007 \end{aligned}$$

$$\begin{aligned} \text{3) The covariance of asset A and B is } \sigma_{AB} &= \frac{\Sigma (rA-rA) (rB-rB)}{n} \\ &= -0.0027/5 = \mathbf{-0.0005} \end{aligned}$$

A positive covariance indicates that the values of two variables tend to increase and decrease together. A negative covariance indicates that the two variables tend to move in opposite directions.

4) The coefficient correlation of asset A and B are computed as follows:

$$\begin{aligned} \text{Coefficient correlation} &= \frac{\text{Covariance}}{(\text{Sd}) (\text{Sd})} \end{aligned}$$

Exercise

ABC corporation owns a portfolio which consists of two common stocks: stock M and stock E. The amount invested in each stock is birr 120,000 and birr 280,000, respectively. The rates of returns on each stock in three economic conditions are given below.

<u>Economic condition</u> <u>on stock E</u>	<u>Probability</u>	<u>Return on stock M</u>	<u>Return</u>
Recession	0.2	7%	6%
Stagnant	0.5	9%	10%
Expanding	0.3	12%	15%

Required: 1) calculate expected rate of return on stock M and E?

2) The portfolio returns of stocks M and E?

Solution:

1) The expected return for stock M = $0.2(7\%) + 0.5(9\%) + 0.3(12\%)$
= 9.5%

The expected return for stock E = $0.2(6\%) + 0.5(10\%) + 0.3(15\%)$
= 10.7%

2) The portfolio returns of stock M and E

Stock	Amount invested	Proportion	Return
M	120,000	0.3	9.5
E	280,000	0.7	10.7

Therefore, the portfolio return is $0.3(9.5\%)+0.7(10.7\%)$

$$\underline{=10.34\%}$$

The formula for the variance of a two asset portfolio using the correlation coefficient is:

$$\sigma_P^2 =$$

Where w_1 and w_2 are the proportion of the components of asset 1 and asset 2
 σ_1 and σ_2 are standard deviations of the components of asset 1 and 2
 $r_{1,2}$ is the correlation coefficient between the returns of component assets 1 and 2

Example, you are trading in a market that has only two securities available.

Security A has an expected return of 8 % and a standard deviation of 40 %.

Security B has an expected return of 20% and a standard deviation of 120%.

1. If you place 40 % of your money in A and the remaining 60% in B, what is your expected return?

2. If the correlation between the returns of securities A and B is 0.8, what is the variance and the standard deviation of the portfolio?

Solution

1. The expected return on any portfolio depends on the percentage invested in each stock and the expected return of the stock:

$$\text{Expected return} = 0.4(8\%)+0.6(20\%) = \underline{15.2\%}$$

1. The problem provides almost all of the information necessary to solve the formula for the variance of a portfolio of imperfectly correlated assets:

You must remember that the variance of a security's return is the square of the standard deviation so:

$$\sigma_A^2 = (0.4)^2 = 0.16 \text{ and } \sigma_B^2 = (1.2)^2 = 1.44$$

and the proportion $w_1 = 40\%$ and $w_2 = 60\%$

Therefore, Variance of portfolio = $(0.4)^2(0.16) + (0.6)^2(1.44) + 2(0.4)(0.6)(0.4)(1.2)(0.8)$

$$= 0.0256 + 0.5184 + 0.1843 = \underline{0.7283}$$

$$\begin{aligned}\text{Standard deviation} &= \text{square root of variance} \\ &= (0.7283)^{1/2} = 0.853 = \underline{\underline{85.3\%}}\end{aligned}$$

Diversification and risk reduction

Increasing the number of financial assets in a portfolio is referred to as *diversification*. The portion of a portfolio's risk that can be reduced or eliminated by diversification is called diversifiable risk. (unsystematic risk). It is unique to a particular firm and/or the industry in which it operates. The goods and services provided by the industry, action of competitors, the quality of the firm management, operating leverage, capital structure, financial leverage, and marketing strategies are some of the factors that combine to produce unsystematic risk.

The portion of a portfolio's risk that can not be eliminated by diversification is called non- diversifiable risk (Systematic risk). It represents portion of total portion risk caused by factors that affect the prices of all securities. Example, national economic and political development, business cycle, inflation, unemployment, fiscal and monetary policy.

A portfolio that contains a large number of securities exhibits only systematic risk. An investor's expected rate of return holding a portfolio of risky financial assets is thus based on the expected rate of return and the systematic risk contained in the portfolio and not on the risk-return characteristics of individual assets.

The beta coefficient

It is the slope of the security characteristic line, which shows the volatility of a security's returns relative to that of the market portfolio. It is index of systematic risk. This index measures the amount of systematic risk contained in individual securities and portfolio relative to *financial markets*. This index also is used to determine the rate of return that an investor expects from individual securities and portfolios. Systematic risk is often referred to as market risk. Beta coefficient can be obtained for actively traded stocks from published sources such as value line investment survey. The beta coefficient for the market is considered to be equal to 1.0, all other

betas are viewed in relation to this value. Asset betas make take on values that are either positive or negative, but positive beta is the norm. One important point for beta is beta tend to change in particular ways over time.

Beta and their interpretation

<u>Asset's Beta</u>	<u>Interpretation</u>
Positive	Move in same direction as market
Zero	Unaffected by market movement
Negative	direction to market

Beta= Covariance of security with market
Market variance
 $B_A = \frac{\sigma_{AM}}{\sigma_M^2}$

Example, the following presents estimates of market returns and hypothetical security A.

<u>Year</u>	<u>Market risk (rm)</u>	<u>Return of asset A (rA)</u>
1	0.0830	0.115
2	0.2216	0.24
3	0.0089	0.0975
4	0.0306	-0.0652
5	0.1275	0.1123
6	0.2057	0.1879
7	0.13	0.1443
8	-0.007	0.1234

Required: compute

- A. Variance of the market and asset A
- B. Covariance of asset A and market
- C. Beta coefficient

Solution:

<u>Year</u>	<u>(rm)</u>	<u>(rA)</u>	<u>(rA-rA)</u>	<u>(rm-rm)</u>	<u>(rm-rm)²</u>	<u>(rA-rA) (rm-rm)</u>
1	0.0830	0.115	-0.0044	-0.0170	0.0003	0.0001
2	0.2216	0.24	0.1206	0.1216	0.0148	0.0147
3	0.0089	0.0975	-0.0219	-0.0911	0.0083	0.0020

4	0.0306	-0.0652	-0.1846	-0.0694	0.0048	0.0128
5	0.1275	0.1123	-0.0071	0.0275	0.0008	-0.0002
6	0.2057	0.1879	0.0685	0.1057	0.0112	0.0072
7	0.13	0.1443	0.0249	0.0300	0.0009	0.0007
8	<u>-0.007</u>	<u>0.1234</u>	0.0040	-0.1070	<u>0.0114</u>	<u>-0.0004</u>
Σ	0.8003	0.9552			0.0525	

0.0369

$$\text{Mean (rA)} = \frac{\Sigma rA}{n} = \frac{0.9552}{8} = \underline{0.1194}$$

$$\text{Mean (rm)} = \frac{\Sigma rm}{n} = \frac{0.8003}{8} = \underline{0.10}$$

$$\text{Variance of market (} \sigma_m^2 \text{)} = \frac{\Sigma (rm - \bar{rm})^2}{n} = \frac{0.0525}{8} = \underline{\underline{0.0066}}$$

$$\text{Covariance (} \sigma_{Am} \text{)} = \frac{0.0369}{8} = \underline{\underline{0.0046}}$$

$$\text{Beta (B}_A\text{)} = \frac{\sigma_{Am}}{\sigma_m^2} = \frac{0.0046}{0.0066} = \underline{\underline{0.6970}}$$

Note: a beta of 1.0 means that a security contains the same degree of systematic risk as found in the market. In actual practice, all corporate beta coefficients have a value that is greater than zero and less than 3.0

Portfolio Betas

The beta of a portfolio can be easily estimated by using the beta of the individual assets it includes. Let w_i , the proportion of the portfolio's total birr value represented by assets and b_i , the beta of asset i , the portfolio beta, B_p using B_i and w_i as defined as:

$$B_p = \sum_i w_i \times B_i$$

Where, w_i is the proportion of security

It states that the weighted average of the financial asset beta contained in portfolio.

Example, four financial assets are purchased with birr 50,000. The percentage composition of the portfolio and its corresponding betas are:

Financial assets	Birr investment	X_i	B_i
A	20,000	40%	1.3
B	15,000	30%	1.1
C	10,000	20%	1.0
D	5,000	10%	0.8

Required: 1) compute the portfolio beta (B_p)?

2) Suppose that asset A is sold for birr 30,000 and the proceeds are invested as asset E with a beta of 1.4, compute the new portfolio beta?

$$\begin{aligned}
 1. \text{ Portfolio beta}(B_p) &= \sum_i w_i B_i = w_a \times b_1 + w_b \times b_2 + w_c \times b_3 \\
 &= 0.4(1.3) + 0.3(1.1) + 0.2(1.0) + 0.1(0.8) \\
 &= \underline{\underline{1.13}}
 \end{aligned}$$

2. <u>Financial assets</u>	<u>Birr investment</u>	<u>X_i</u>	<u>B_i</u>
B	15,000	25%	1.1
C	10,000	17%	1.0
D	5,000	8%	0.8
E	<u>30,000</u>	<u>50%</u>	1.4
Total	60,000	100%	

$$\begin{aligned}
 B_p &= 0.25(1.1) + 0.17(1.0) + 0.08(0.8) + 0.5(1.4) \\
 &= \underline{\underline{1.21}}
 \end{aligned}$$

Risk and Return: The Capital Asset Pricing Model (CAPM)

The most important aspect of risk is the overall risk of the firm as viewed by investors in the market place. Overall risk significantly affects investment opportunities and the owner's wealth. The basic theory that links together

risk and return for all assets is called the William Sharp's Capital Asset Pricing Model (CAPM)

Types of risk

1. Diversifiable risk (unsystematic risk): is the portion of an asset's risk that is attributable to firm specific, random causes, can be minimized through diversification. It is unique risk or asset specific risk.

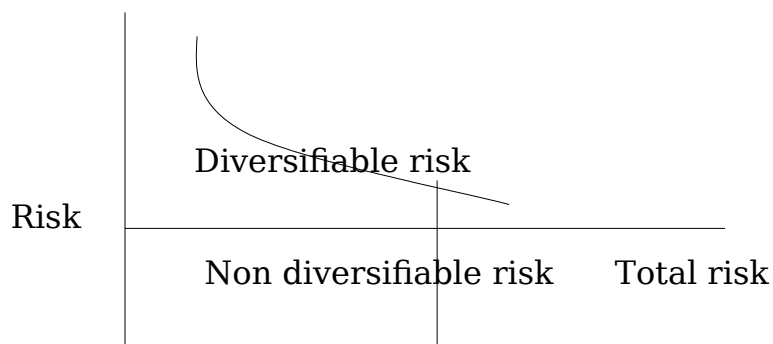
Example, A firm specific event such as strikes, lawsuits, regulatory action and loss of a key account.

2. Non diversifiable risk (systematic risk): is the relevant portion of an asset's risk attributable to market factors that affect all firms, can not be eliminated through diversification. It is also said to be market risk.

Example, war, inflation, international incidents and political events.

3. Total risk: is the combination of a security's non diversifiable and diversifiable risk.

Total security risk = Non diversifiable + Diversifiable risk



No of securities (Assets) in portfolio.

Any investor can create a portfolio of assets that will eliminate all diversifiable risk, the only relevant risk is non diversifiable risk. Therefore, any investor must concern only with non diversifiable risk.

William Sharp CAPM Model

$$K_i = R_f + [b_i (K_m - R_f)]$$

Where, K_i is required return of asset i

R_f is risk free rate of return

B_i is beta coefficient

K_m is market return; the return on the market

The required return on an asset, K_i , is an increasing function of beta, b_i , which measures non diversifiable risk. In other words, the higher the risk, the higher the required return, and the lower the risk, the lower the required return. The model can divide in to two parts:

- 1) the risk free rate, R_f and
- 2) the risk premium, $b_i(k_m - R_f)$

The $(k_m - R_f)$ portion of the risk premium is called the market risk premium, since it represents the premium the investor must receive for taking the average amount of risk associated with holding the market portfolio of assets.

Example, ABC corporation wishes to determine the required return of an asset Z that has a beta b_Z of 1.5. The risk free rate of return is 7 %, the return on the market portfolio of assets is 11%.

Required: Determine

1. Required return of asset Z
2. The market risk premium
3. The risk premium

Solution

$$\begin{aligned}
 K_Z &= R_f + [b_i (k_m - R_f)] \\
 &= 7 + [1.5 (11 - 7)] \\
 &= 7 + 6 \\
 &= 13
 \end{aligned}$$

$$\begin{aligned}
 \text{Market risk premium} &= k_m - R_f \\
 &= 11 - 7 \\
 &= 4\%
 \end{aligned}$$

$$\begin{aligned}
 \text{Risk premium} &= b_i (k_m - R_f) \\
 &= 1.5(11 - 7) \\
 &= 6\%
 \end{aligned}$$

The security Market line (SML)

When the capital asset pricing model (CAPM) is depicted graphically, it is called the security market line (SML). The SML reflects for each level of non diversifiable risk (beta) the required return in the market place.

