

1 Unit 7 Electromagnetic Induction and A. C circuit

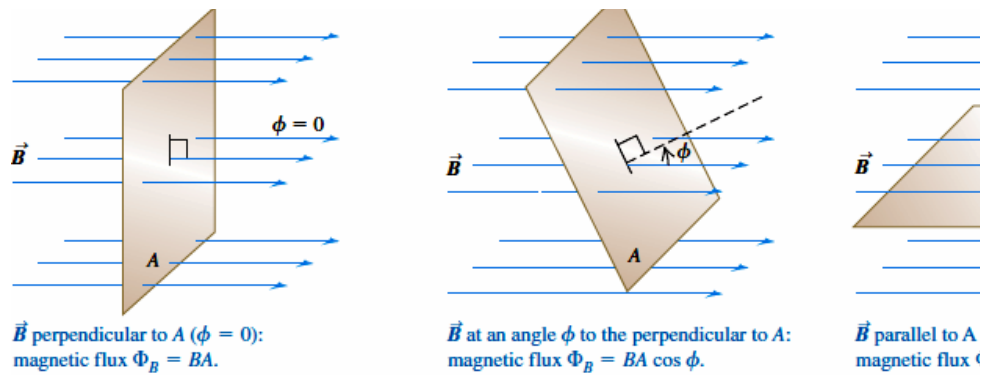
1.1 Phenomena of electromagnetic induction

1.1.1 Magnetic flux

- ⇒ is the magnetic analogous of electric flux.
- ⇒ depends on the number of magnetic field lines passing through an area.
- ⇒ The number of magnetic lines of force passing through an area A is called magnetic flux.
- ⇒ The magnetic flux through the element of area A is mathematically defined as

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$$
- ⇒ If $\vec{B}$ happens to be perpendicular to the surface, then $\cos \theta = 1$ and Equation 7.1 reduces to $\phi_B = BA$
- ⇒ The SI unit of magnetic flux is the unit of magnetic field (1 T) times unit of area (m^2) called Weber (Wb)

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot m^2 = 1 \text{ N} \cdot m / A$$
- $$B = \frac{\phi_B}{A}$$
 - ⇒ is magnetic flux per unit area across an area at right angles to the magnetic field.
 - ⇒ For this reason, magnetic field is sometimes called **magnetic flux density**.



The magnetic flux through a flat surface at various orientations relative to a uniform magnetic field

1.2 Laws of induction

1.2.1 Faraday's law

- ⇒ States that the induced emf in a circuit is *directly proportional* to the time rate of change of the magnetic flux through the circuit.

$$\varepsilon = \left| \frac{\Delta \phi}{\Delta t} \right|$$

- ⇒ If we have a coil with N identical turns, and the magnetic flux varies at the same rate through each turn, the induced emf's in the turns are all equal and is given as

$$\varepsilon = N \left| \frac{\Delta \phi}{\Delta t} \right| \text{-----3}$$

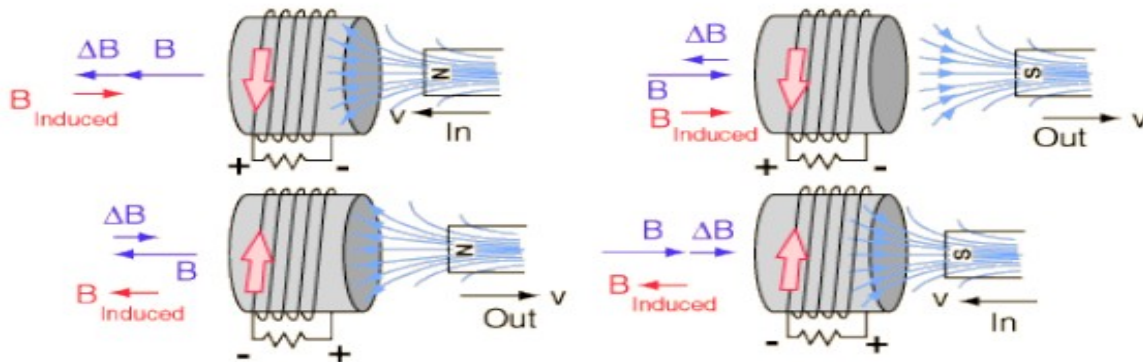
The amount of magnetic flux that interacts with the coil of wire is called the **flux linkage**.

Note that Faraday's law discusses about the magnitude, but not the direction, of an induced emf and the associated current.

The cause of a magnetically induced current or emf may be a **changing flux through a stationary circuit**, or **the motion of a conductor in a magnetic field**, or any combination of the two

1.2.2 Lenz's law

- ⇒ is the consequence of the law of conservation of energy
- ⇒ states that the direction of any magnetically induced current or emf is such as to oppose the direction of the phenomenon causing it



Relation among a changing external magnetic field, the resulting induced current, and the magnetic field induced by the current

- ⇒ Within the loop, the induced magnetic field points *opposite* to the original field if the original field or flux is *increasing*.
- ⇒ Conversely, if the original field is *decreasing*, the additional field caused by the induced current points in the *same* direction as the original field.
- ⇒ In each case, as Lenz's law requires, the induced magnetic field opposes the *change in flux* through the circuit (*not* the flux itself).

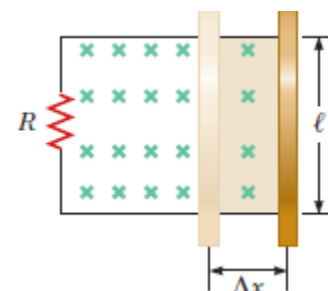
$$\varepsilon = -N \frac{\Delta \phi_B}{\Delta t} \text{-----4}$$

Therefore, generally we write the faraday's law as

- ⇒ The minus sign in Equation 20.2 is included to indicate the polarity of the induced emf. This polarity determines which of two directions current will flow in a loop, a direction given by **Lenz's law**

1.3 Motional emf

- ⇒ It is the emf induced in a conductor moving through a magnetic field.
- ⇒ As the bar moves to the right, the area of the loop increases by the amount, Δx and the magnetic flux through the loop increases by $B\Delta x$.
- ⇒ The increase in flux $\Delta \phi_B$ through the loop in that time is the amount of flux that now passes through the portion of



the circuit that has area , Δx

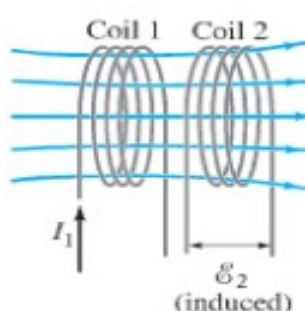
Using Faraday's law and noting that there is one loop (N

$=1$), we find that the magnitude of the induced emf is $\Rightarrow E_{\text{between}} = \frac{\sigma}{\varepsilon_0} \text{-----12.11}$

$$\varepsilon = \frac{\Delta \phi_B}{\Delta t} = Bl \frac{\Delta x}{\Delta t} = Blv \text{-----5}$$

1.4 Mutual inductance

- ☑ The phenomenon of producing an induced emf in a coil due to the change in current in the other coil is known as **mutual induction**.
- ☑ According to Faraday's law, the emf induced in coil 2 is proportional to the rate of change of magnetic flux passing through it. This flux is due to the current in coil 1, and it is often convenient to express the emf in coil 2 in terms of the current in coil 1.



- ☑ Let Φ_1 be the magnetic flux in each loop of coil 2 created by the current in coil 1. If coil 2 contains closely wrapped loops, then $\sum \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ is the total flux passing through coil 2.

- ☑ If the two coils are fixed in space, Φ_E is proportional to the current $\frac{\Phi_E}{I_1}$ in coil 1; the proportionality constant is called the mutual inductance, M_{21} , defined by the Equation

☑

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} \quad \text{-----6}$$

The emf $\mathcal{E}_2$ induced in coil 2 due to a changing current in coil 1 is, by Faraday's law is therefore,

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt} = -M_{21} \frac{dI_1}{dt} \quad \text{-----7}$$

- ☑ **Note:** the mutual inductance of coil 2 with respect to coil 1, M_{21} , is a "constant" in that it does not depend on $\frac{dI_1}{dt}$

- ☑ M_{21} depends on "**geometric**" factors such as the **size, shape, number of turns, and relative positions** of the two coils, and also on whether iron (or some other ferromagnetic material) is **present**.

Hence, for a given arrangement, we do not need the subscripts and we can let

$$\tau = PE \sin \theta = P = \frac{\tau}{E \sin \theta} = \frac{2.5 \times 10^{-2}}{(300) \left(\frac{1}{2} \right)} = P = 1.7 \times 10^{-4} \text{ C.m}$$

And

$$\mathcal{E}_1 = M \frac{dI_1}{dt} \quad \text{-----8}$$

1.5 Self-Inductance

⇒ The property of a coil which enables to produce an opposing induced emf in it when the current in the coil changes is called self induction.

⇒ The magnetic flux Φ_B passing through the N turns of a coil is proportional to the current I in the coil, so we define the self-inductance L (in analogy to mutual inductance, Eq. 7.6) as

$$L = \frac{N\Phi_B}{I} \quad \text{-----9}$$

☑ Then the emf $\mathcal{E}$ induced in a coil of self-inductance L is, from Faraday's law,

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -L \frac{dI}{dt} \quad \text{-----10}$$

☑ Like the SI unit for mutual inductance the SI unit of self inductance is the Henry (H), where $1 \text{ H} = 1 \frac{\text{Vs}}{\text{A}}$

1.5.1 The inductance of a solenoid

To determine the inductance L, it is usually simplest to start with Eq. 15.9; therefore, we need to first determine the flux.

The magnetic field inside a solenoid (ignoring end effects) is constant given by

$\Rightarrow E_x = -\frac{(190-120)}{5-3}$
 $\Rightarrow E_x = -35 \text{ N/C}$ Where $\vec{e}$. The flux is $\Phi_E = \vec{E} \cdot \vec{A}_{\text{right}} + \vec{E} \cdot \vec{A}_{\text{left}}$, so

$$\Phi_E = -E_{\text{right}}\ell^2 + E_{\text{left}}\ell^2 \Rightarrow -E_{\text{right}}\ell^2 + E_{\text{left}}\ell^2 = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

1.5.2 The energy stored in an inductor

When an inductor of inductance L is carrying a current I which is changing at a rate $\frac{dI}{dt}$, energy is being supplied to the inductor at a rate



the work dW done in a time dt is

$$\Phi = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

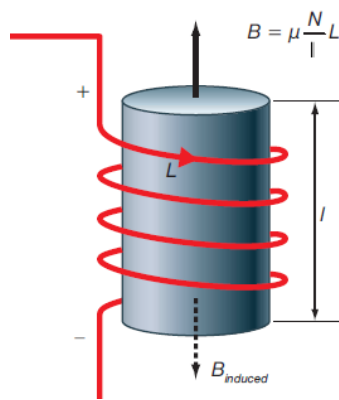
Then the total work done to increase the current from zero to I is

$$\Rightarrow \Phi_1 = \frac{1}{6} \Phi \Rightarrow \Phi_1 = \frac{1}{6} \left(\frac{Q_{\text{enclosed}}}{\epsilon_0} \right) \Rightarrow \Phi_1 = \frac{Q}{6\epsilon_0}$$

This work done is equal to the energy U stored in the inductor when it is carrying a current I (and we take U = 0 when I = 0):

$$U_{21} = V_1 e_2$$

Note: Just as the energy stored in a capacitor can be considered to reside in the electric field between its plates, so the energy in an inductor can be considered to be stored in its magnetic field.



$$\Rightarrow U_{21} = \left(\frac{Ke_1}{d} \right) e_2 \Rightarrow U_{21} = \frac{ke^2}{d}$$

$$\Rightarrow U_{21} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{2 \times 10^{-9}} \Rightarrow U_{21} = \underline{\underline{1.15 \times 10^{-19} \text{ J.}}}$$

We can think of this energy as residing in the volume enclosed by the windings, which is $A l$. Then the energy per unit volume or **energy density** is

$$\Rightarrow \Delta U = (-1.6 \times 10^{-19})(5000 \text{ V})$$

$$\Rightarrow \underline{\underline{\Delta U = -8 \times 10^{-16} \text{ J.}}}$$

1.6 Alternating currents, generators and transformers

1.6.1 Generators

- ☑ Generators and motors are important practical devices that operate on the principle of electromagnetic induction.
- ☑ **Alternating-current (AC) generator** is a device that converts mechanical energy to electrical energy.
- ☑ Ac generator in its simplest form consists of a wire loop rotated in a magnetic field by some external means, inducing an **emf** and a **current** in an external circuit.

The **ends** of the loop are connected to **slip rings** that rotate with the loop. Connections to the external circuit are made by stationary **brushes** in contact with the slip rings

Assuming that the loop is being made to rotate in a uniform magnetic field B with constant angular velocity. From Faraday's law the induced emf is.

$$\Rightarrow \frac{kq_1}{x} - \frac{3kq_1}{d+x} = 0$$

Where A is the area of the loop and θ is the angle between B and A .

$$\Rightarrow x = \frac{d}{2}$$

If the loop contains N loops,

$\Rightarrow x = 12 \text{ cm}$ to the left of q_1

field.

Between the charges
 $\Rightarrow \frac{kdq}{x^2} - \frac{kdq}{(d+x)^2} = 0$. However, $q_2 = -3q_1$
 $\Rightarrow \frac{kdq}{x^2} - \frac{3kdq}{(d+x)^2} = 0$
 $\Rightarrow x = \frac{d}{2}$

$V = 100 \text{ V} - 25 \text{ V/m} \cdot x$ is called the peak voltage.

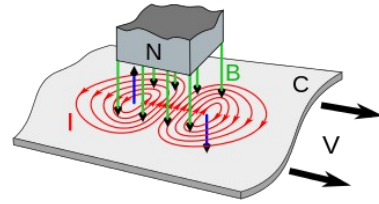
- ☑ Such a rotating coil in a magnetic field is the basic operating principle of an ac generator.

1.6.2 Eddy currents

$\Rightarrow$ A swirling current set up in a conductor in response to a changing magnetic

⇒ By Lenz's law, the current swirls in such a way as to create a magnetic field opposing the change; to do this in a conductor, electrons swirl in a plane perpendicular to the magnetic field.

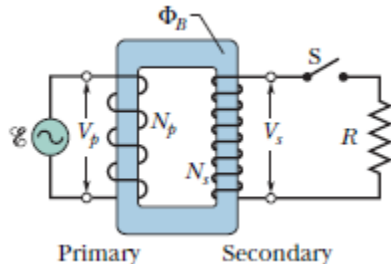
The increasing field at the leading edge of the magnet (left) induces a counterclockwise current, which by Lenz's law creates its own magnetic field, directed up, which opposes the source magnetic field.



1.6.3 Transformers

⇒ **Transformer** is a device with which we can **raise** (for transmission) and **lower** (for use) the ac voltage in a circuit, keeping the product current x voltage essentially constant.

⇒ **The ideal transformer** in Fig. 7.12 consists of two coils, with different numbers of turns, wound around an iron core. (The coils are insulated from the core.)



In use, the primary winding, of N_p turns, is connected to an alternating-current generator

⇒ Since the primary current is out of phase relative to the primary voltage from the generator, no power is delivered from the generator to the transformer.

⇒ However, the small sinusoidally changing primary current produces a sinusoidal changing magnetic flux in the iron core.

⇒ Because $\vec{E}$ varies, it induces an emf $\vec{E} = -\frac{\Delta V}{\Delta x} \Rightarrow E_x = -\frac{dV}{dx}$ in each turn of the secondary and this emf per turn is the same in the primary and the secondary
Therefore,

$$\Rightarrow E_x = -25 \text{ V/m} \Rightarrow \underline{E = (25 \text{ V/m})\hat{i}}$$

$$q = 5 \mu\text{C}$$

Note: If $N_s > N_p$, the device is a *step-up transformer* because it steps the primary's voltage V_p up to a higher voltage V_s . Similarly, if $N_s < N_p$, it is a *step-down transformer*.

If no energy is lost along the way, conservation of energy requires that

$$x = 5 \text{ cm and } y = 5 \text{ cm}$$

1.6.4 Alternating currents

⇒ When a battery is connected to a circuit, the current moves steadily in one direction. This is called a direct current, or **dc**.

⇒ **An alternating current** reverses direction many times per second and is commonly sinusoidal

⇒ The voltage produced by an ac electric generator is sinusoidal. The current it produces is thus sinusoidal

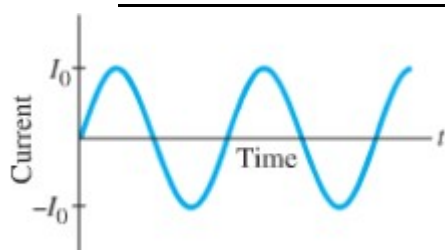
$$\vec{E} = (50 \text{ V/m}) \hat{i}$$

⇒ The frequency f is the number of complete oscillations made per second, and

$$E_x = -\frac{\Delta V}{\Delta x} \Rightarrow \Delta V = -|E_x| \Delta x$$

$$\Rightarrow \Delta V = -(50 \text{ V/m})(5 \text{ cm})$$

$$\Rightarrow \underline{\underline{\Delta V = -2.5 \text{ V}}}$$



An alternating current is as often positive as it is negative. Thus, the **average current is zero**.

⇒ The power transformed in a resistance R at any instant is

$$P = \frac{Q}{t} \text{ --- 12.23} \quad \text{the power is always positive.}$$

⇒ The quantity $\frac{E}{\epsilon_0} \Rightarrow E = \frac{Q}{\epsilon_0 A}$ varies between 0 and 1; and it is not too difficult to show that its average value is $\frac{1}{2}$.

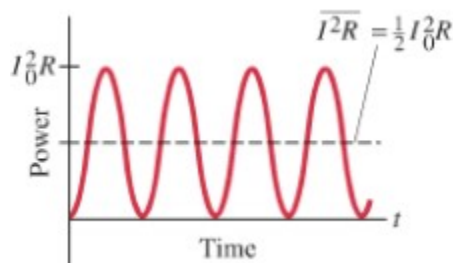
⇒ Thus, the average power transformed, P , is

$$\Rightarrow \frac{V_{ba}}{d} = \frac{Q}{\epsilon_0 A} \Rightarrow V_{ba} = \left(\frac{Q}{\epsilon_0 A} \right) d \quad \text{power can also be written}$$

$$\bar{P} = \frac{V_0^2}{2R} \text{ --- 24}$$

The average or mean value of the square of current or voltage is thus what is important calculating average power:

$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d} \text{ --- 12.24}$$



the
for

The square root of each of these is the rms (root-mean-square) value of the current or voltage.

$$C = KC_0 \Rightarrow C = \frac{K\epsilon_0 A}{d} \text{ --- 12.25}$$

The rms values of V and I are sometimes called the **effective values**. The average power can be written as:

$$P = E_p (E') \quad E = E_0 - E_p$$

- ⇒ Thus, a direct current whose values of I and V equal the rms values of I and V for an alternating current will produce the same power.
- ⇒ Hence it is usually the rms value of current and voltage that is specified or measured.

1.7 Ac circuits

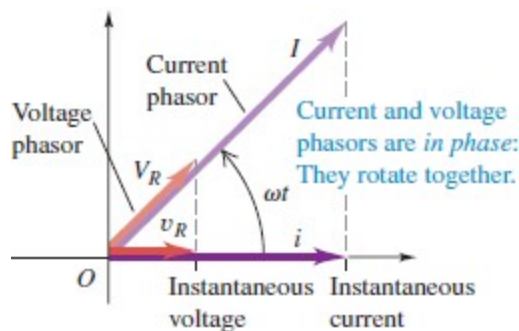
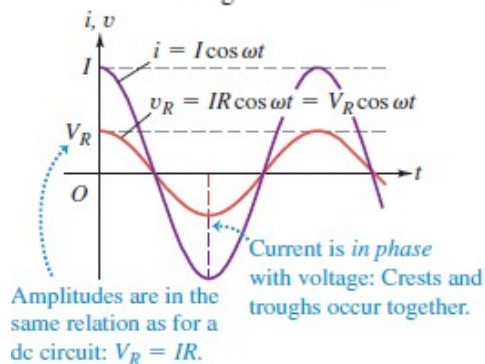
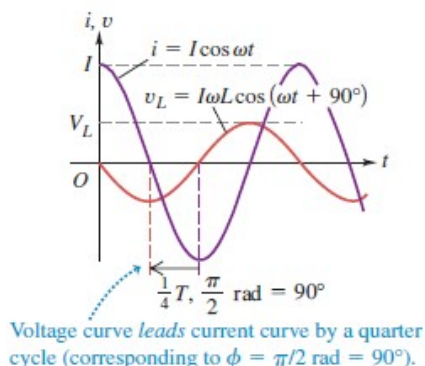
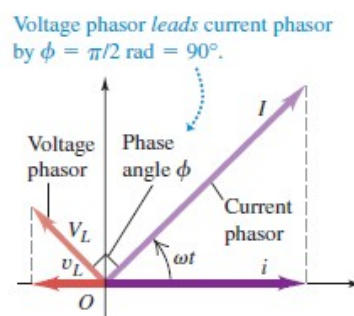
1.7.1 Resistor in an ac Circuit

Consider a resistor with resistance R , with a sinusoidal current given by

$$\frac{E_0}{E} = K$$

- ⇒ The current amplitude (maximum current) is $\varepsilon = K \varepsilon_0$.
- ⇒ We denote the instantaneous potential of point a with respect to point b by

$\mathcal{E}$ from Ohm's law



Note: the instantaneous

voltage v_R and current i are both proportional to $\cos \omega t$ so the voltage is in **step**, or **in phase**, with the current.

The corresponding phasors diagram is shown in the fig.

Because v_R and i are **in phase** and have the same frequency, the current and voltage phasors **rotate together**; they are parallel at each instant. Their

projections on the horizontal axis represent the instantaneous current and voltage, respectively.

1.7.2 Inductor in an ac Circuit

Again, we assume that the current is $\Rightarrow \frac{Q}{C} = \frac{Q_0}{C_0} \Rightarrow \frac{kQ_0}{C} = \frac{Q_0}{C_0}$

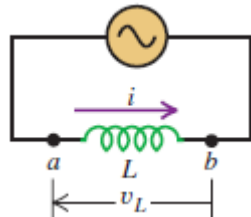
$$\Rightarrow Q_1 = C_1 V$$

The induced emf in the direction of the current is

This

respect to

point b is **the**



corresponds to the potential of point b with

point a ; the potential $Q_2 = C_2 V$ of point a with respect to point b is **negative of** $V = V_1 - V_2$ and

$$\Rightarrow Q_T = Q_1 + Q_2 \Rightarrow Q_T = C_1 V + C_2 V$$

$$\Rightarrow Q_T = (C_1 + C_2) V$$

This result and the phasor diagram show that the voltage can be viewed as a cosine function with a “head start” of 90° , $\pi/2$ radians, or $1/4$ cycle.

If the current is $= C_{eq} = \frac{Q_T}{V} = \frac{(C_1 + C_2)V}{V}$ and the voltage of one point with respect to another is

$$\Rightarrow C_{eq} = C_1 + C_2 \quad \phi \text{ is called the phase angle.}$$

☑ **Note:** For a pure **inductor**, $\phi = \pi/2$ radians, or $1/4$ cycle. For a **pure resistor**, $\phi = 0$ and the voltage and current are **in phase**.

From Eq. 15.22 the voltage amplitude is

$$C_1$$

◇ Definition

Inductive reactance C_2

☑ We define the **inductive reactance** of an inductor as the product of the inductance L and the angular frequency ω

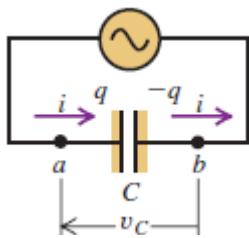
$$C_1$$

And the peak voltage can be written as

$$+ Q$$

The inductive reactance of an inductor is directly proportional both to its L and to the angular frequency. Capacitor in an ac Circuit

- ☑ If a capacitor is connected to an alternating source of voltage, an alternating current will flow continuously.
- ☑ This can happen because when the ac voltage is first turned on, charge begins to flow and one plate acquires a negative charge and the other a positive charge. But when the voltage reverses itself, the charges flow in the opposite direction.



By Kirchhoff's loop rule, the applied source voltage must equal the voltage V across the capacitor at any moment:

$$C_2$$

The current I at any instant is given by

$$— Q$$

$$\Rightarrow Q_1 = Q_2 \quad \text{Then the voltage across the capacitor is}$$

$$\Rightarrow V = V_a - V_b = V_1 + V_2$$

$$\Rightarrow V = \frac{Q}{C_1} + \frac{Q}{C_2} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

◇ Definition

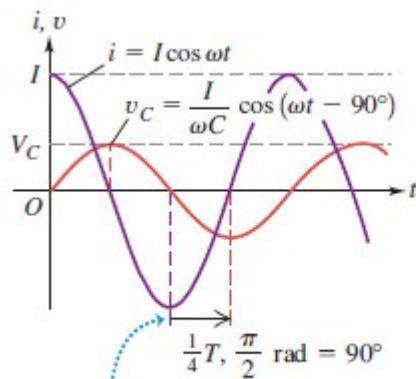
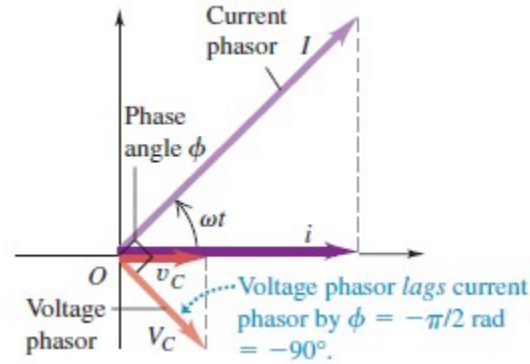
Capacitive reactance $\left(\frac{Q}{C_q} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \right)$

We define the capacitive reactance of a capacitor as the inverse of the product the angular frequency and the capacitance:

$$\Rightarrow \frac{1}{Ceq} =$$

- ◇ **Note:** The **current leads** the voltage across a capacitor by 90° . Alternatively we can say that the **voltage lags** the current by 90° . This is the *opposite of what happens for an inductor*.
- ◇ The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to the angular frequency

The capacitor voltage and current are out of phase by a quarter cycle. The peaks of voltage occur a quarter cycle *after* the corresponding current peaks

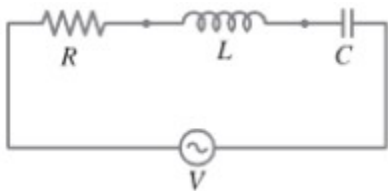


This cosine function has a “late start” of 90° compared with that for the current.

The peaks of voltage occur a quarter cycle *after* the corresponding current peaks, and we say that the voltage *lags* the current by 90° .

1.7.3 R-L-C series circuit

- ⇒ a circuit containing all three elements in series: a resistor R , an inductor L , and a capacitor C .
- ⇒ If a given circuit contains only two of these elements, we can still use the results of this section by setting $R = 0$, $X_L = 0$, or $X_C = 0$, as needed.



At any instant the voltage V supplied by the source will be, by Kirchhoff's loop rule,

$$\frac{1}{C_1} + \frac{1}{C_2}$$

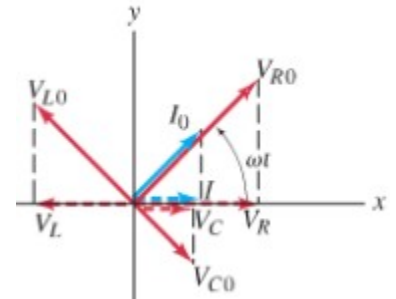
- ⇒ We analyze an R-L-C circuit using a **phasor diagram**. **Phasor diagram** contains arrows (treated like vectors) that are drawn in an xy coordinate system to represent each voltage.
- ⇒ The length of each arrow represents the magnitude of the peak voltage across each element:

$$\Leftrightarrow \frac{1}{C_{eq}} = \sum_{i=1}^n \frac{1}{C_i} \text{ --- 12.27}$$

⇒ $V = q/C$ is **in phase** with the current. Δq **leads** the current by 90° , so it leads by 90° . Δq lags the current by 90° .

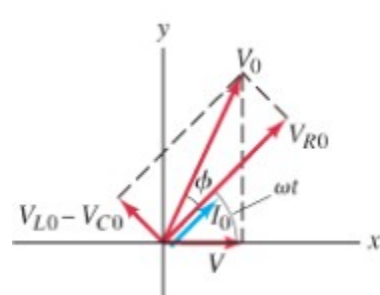
⇒ If we let the vector diagram rotate counterclockwise at frequency f , we get the diagram shown in Fig. 15.20; after a time, t , each arrow has rotated through an angle ωt . Then the projections of each arrow on the x axis represent the voltages across each element at the instant t

⇒ The sum of the projections of the three voltage vectors represents the instantaneous voltage across the whole circuit, V .



⇒ Therefore, the vector sum of these vectors will be the vector that represents the peak source voltage, V_0 as shown in Fig.

15.21 where it is seen that V_0 makes an angle ϕ with V_C and V_R



⇒ so the instantaneous voltage V (projection of V_0 on the x axis) is

$$\Rightarrow V = V_0 \cos \phi$$

⇒ The total impedance Z of the circuit, which **is defined in analogy to resistance and reactance as**

$$\Rightarrow U = \frac{1}{2} C V^2$$

$$C = \frac{\epsilon_0 A}{d}$$

$$V = E d$$

$$\therefore U = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) (E d)^2 \Rightarrow U = \frac{1}{2} \epsilon_0 E^2 (A d)$$

Z is called the impedance of the R-L-C series circuit.

⇒ The phase angle ϕ between voltage and current:

$$\tan \phi = \frac{X_L - X_C}{R}$$

Or u_e

Note: Figure 15.21 was drawn for the case $X_L > X_C$, and the current lags the source voltage by ϕ . When the reverse is true, $X_L < X_C$, then ϕ is less than zero, and the current leads the source voltage.

1.7.4 Power in ac circuits

The average power in an AC circuit is given by: $\Rightarrow \frac{\text{energy}}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2 = 12.30$ However, $U = \frac{1}{2} \left(\epsilon_0 \frac{A}{d} \right) Ed^2 = U = \frac{1}{2} k \epsilon_0 E^2 Ad$, 

$$+ Q_0$$

Therefore,

$\Rightarrow$ The factor $\cos \phi$ is referred to as the **power factor** of the circuit.

- ◇ For a pure resistor, $\cos \phi = 1$ and $-Q_0$
- ◇ For a **capacitor or inductor** alone, $\phi = -90^\circ$ or $+90^\circ$, respectively, so $\cos \phi = 0$ and **no power is dissipated**.

The average power can also be written in terms of the peak current and voltage as

$$R$$

1.7.5 Resonance in AC circuit

- $\Rightarrow$ The peaking of the current amplitude at a certain frequency is called **resonance**.
- $\Rightarrow$ The angular frequency at which the resonance peak occurs is called the **resonance angular frequency**. This is the angular frequency at which the inductive and capacitive reactance are equal, so

$$V_0 = Q_0 / C$$

$$I_0 = \frac{V_0}{R} = \frac{Q_0}{RC} \text{ is called the resonant angular frequency}$$

And the corresponding frequency is given by

$$\Rightarrow I = - \frac{dQ}{dt}$$

Note: At resonance, you need to note the following things

- ☑ IR is maximum
- ☑ $\frac{Q}{C}$ is maximum
- ☑ Z is minimum
- ☑ $Z = R$ which implies that $\Rightarrow \frac{Q}{C} + R \frac{dQ}{dt} = 0$

1.7.6 True, reactive and apparent power

True power

$\Rightarrow$ Power dissipated by load is called true power.

Reactive power (ideal power)

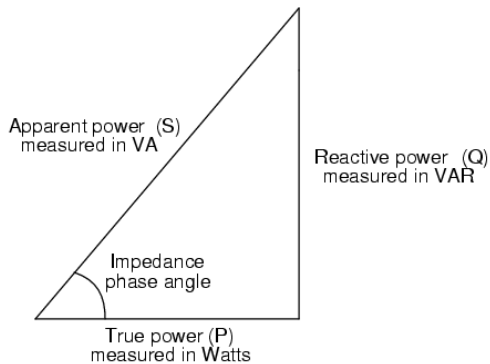
⇒ Power only absorbed and returned in load due to its reactive properties is called reactive power or ideal power.

VA is the unit called Volt-Amps.

VAR is the unit Volt-Amps-Reactive.

VA is the unit Volt-Amps

The "Power Triangle"



called the **power triangle**.

⇒ **Reactive** loads such as inductors and capacitors **dissipate zero power**, yet the fact that they drop voltage and draw current gives the false impression that they actually do dissipate, that is why it is called **ideal**

Apparent power

⇒ The total power in an ac circuit that is both dissipated and absorbed/returned is referred to as apparent power.

The three types of power are related trigonometrically to one another by the so

1.8 Related University entrance exam questions

1. **EHEECE 2000/2008**: an inductive coil has a resistance of 100Ω . When an ac signal of frequency 1000Hz is fed to the coil, the applied voltage leads the current by 45° . What is the inductance of the coil?

A. 10 mH B. 12 mH C. 16 mH D. 20 mH

Soln: For pure inductive circuit $C_1 = 6 \mu F$

$C_2 = 12 \mu F$ Ans : (C)

2. **EHEECE 2001/2009:** The figure below shows a series LCR circuit connected to a variable frequency 230 V source. Which statement is not correct about the circuit?

- A. the source frequency that drives the circuit into resonance is $\frac{25}{\pi}$.
 B. the impedance of the circuit is 40Ω .
 C. the current amplitude at resonance is 8.1A.
 D. the rms potential drop across R at resonance is zero.

Soln: Resonance frequency $f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times \pi \times \sqrt{5H \times 80 \times 10^{-6}F}} = \frac{25}{\pi}$

☑ $X_L = X_C$ and impedance Z at resonance is $Z=R=40\Omega$ and

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} \rightarrow V_{max} = 230\sqrt{2}V \text{ then } I_{max} = \frac{V_{max}}{R} = \frac{230\sqrt{2}V}{40\Omega} = 8.13A$$

The voltage drop across R is $V_R = I_{rms} R = \frac{8.1}{\sqrt{2}} A \times 40\Omega = 229V$ **not equal to**

zero. Ans :(D)

3. **EHEECE 2001/2010:** In a purely inductive AC circuit $L = 35 \text{ mH}$ and the rms voltage is 165 V. What is the inductive reactance in Ω and rms current in ampere in the circuit if the frequency is 40 Hz, respectively?

- A) 8.79, 18.77 B) 7.79, 20.77 C) 7.69, 20.77 D) 9.89, 18.77

Soln: Given $L = 35 \text{ mH}$, $Q_1 = 48 \mu C$ the inductive reactance

$$X_L = 2\pi fL = 8.792\Omega \text{ and}$$

$$V_{rms} = I_{rms} \rightarrow I_{rms} = \frac{V_{rms}}{\square} = 18.77 \text{ Ans :(A)}$$

4. **EHEECE 2002/2010:** Which device function with the principle of mutual induction

- A. electromagnet B. transformer C. electric motor D. transistor

Brief explanation:

Electromagnet: form of solenoid with an iron core

Electric motor: device that converts electrical energy to mechanical energy.

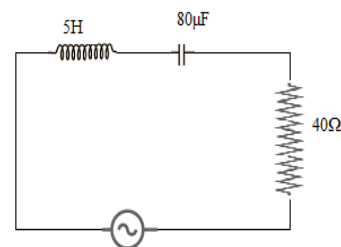
Transistor: electronics elements that is mainly used in amplifying and rectifying.

Transformer is a device mainly used to step up and step down voltages from ac source. It works with the principle of mutual induction. **Ans :(B)**

5. **EHEECE 2002/2010:** A capacitance $C = 2\mu F$ and inductance $L = 10 \text{ H}$ and coil resistance 5Ω are in series in a circuit. When an alternating current of 2 A rms flows, the power in watts in the circuit is

- A. 100 B. 50 C. 20 D. 2

Soln: Since the average power dissipated across L and C is zero, power is the one dissipated across the resistor



$$Q'_1 = ? \text{ and } Q'_2 = ? \text{ . Ans : (C)}$$

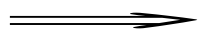
6. **EHEECE 2002/2010:** An induced emf is obtained between the ends of a horizontal steel axle X of a train moving due east. This is because

- A. X points due east
- B. the earth's magnetic field has a horizontal component.
- C. X moves parallel to the earth's field.
- D. the earth has a vertical magnetic field component.

Brief explanation: For an emf to be induced on the steel axle X, we must have a field perpendicular to the direction of motion of the axle. That is, the earth must have a vertical magnetic field component.

Ans : (D)

7. **EHEECE 2002/2010:** The voltage output of a generator is given by



. What is the rms current in the circuit when the generator is connected to a 100Ω resistor?

- A. 1.50 A
- B. 2.12 A
- C. 1.12 A
- D. 1.41 A

Soln: Given $V_1 = V_2 \Rightarrow \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2}$ Required $Q'_1 + Q'_2 = 96\mu C$

$$\Rightarrow Q'_1 = \left[\frac{C_2}{C_1} \right] Q'_2 = 2Q'_2$$

$$\therefore Q'_1 + 2Q'_2 = 96\mu C \Rightarrow 3Q'_2 = 96\mu C$$

$$Q'_2 = 32\mu C \text{ and } Q'_1 = 64\mu C$$

$$V_1 = V_2 = \frac{32\mu C}{6\mu F} = 5.3V \quad \text{Ans : (B)}$$

8. **EHEECE 2002/2010:** The peak voltage of an alternating emf is 141 volt. The effective value of the voltage in volts is

- A. 70.5
- B. 100
- C. 141
- D. 200

Soln: Given $d_0 = d$ and $d_f = 2d$ Required $U_f = ?$

$$U = \frac{Q^2}{2C} \quad \text{Ans : (B)}$$

9. **EHEECE 2002/2010:** In a R-L-C series circuit, suppose $R = 300\Omega$, $L = 60 \text{ mH}$, $C = 0.5\mu F$, $V = 50 \text{ V}$, and $\omega = 10,000 \text{ rad/s}$. what is the average power delivered to the entire circuit?

- A. 1.5 W
- B. 2.3 W
- C. 3.5 W
- D. 1.0 W

Soln:

$$U_o = \frac{Q^2}{2C_o} \text{ and } U = \frac{Q^2}{2C}$$

$$C_o = \frac{\epsilon_0 A}{d_o} \text{ and } C = \frac{\epsilon_0 A}{2d_o}$$

$$\frac{U}{U_o} = \frac{\frac{Q^2}{2C}}{\frac{Q^2}{2C_o}} = \frac{\frac{Q^2}{2\left(\frac{\epsilon_0 A}{2d_o}\right)}}{\frac{Q^2}{2\left(\frac{\epsilon_0 A}{d_o}\right)}} = 2$$

Therefore, $U = 2U_o$

$$V_o$$

Ans : (A)

10. **UEE 2003/2011:** The phase angle in an alternating current circuit is zero degree when the circuit contains:

- A. resistance only B. capacitance only C. inductance only D. series combination of resistance and inductance

Brief explanation: The phase difference becomes zero when the circuit is pure resistive.

11. **UEE 2003/2011:** The impedance of an 8 microfarad capacitor at 1000Hz is:

- A. 600 Ohm B. 22.7 Ohm C. 19.9 Ohm D. 0.44 Ohm

Soln: the capacitive reactance or given by

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 10^3 \times 8 \times 10^{-6}} = 19.9 \Omega. \text{ Ans : (B)}$$

12. **UEE 2003/2011:** What is the inductance of an LC circuit that oscillates at 120Hz when the capacitance is 8.0μF?

- A. 0.22 μH B. 0.22 H C. 2.19 H D) 2.19μH

Soln:

$$\omega = \frac{1}{\sqrt{LC}} \rightarrow L = \frac{1}{4\pi^2 f^2 C} = 0.22 H. \text{ Ans : (A)}$$

13. **UEE 2003/2011:** Which one of the following statement is NOT true?

- A. an electric current can be induced in a circuit with a change in the cross sectional area of the wire.
 B. the induced emf in a circuit is directly proportional to the time rate of change of magnetic flux.
 C. emf can be induced in a circuit by change in the area enclosed by the loop.
 D. an electric current can be induced in a circuit with change of magnetic field.

Brief explanation: Emf cannot be induced in a circuit by varying the area of the loop. Emf is induced if the area crossing the magnetic field varies with time. **Ans : (D)**

14. **UEE 2004/2012:** Two coils, held in fixed positions, have a mutual inductance of 100 μH. What is the peak voltage in one when the sinusoidal current given by $I(t) = (10.0 A) \sin(1000t)$ flows in the other?

- A) 1.0μV B) 100.0 V C) 10.0 V D) 1.0V

Soln:

$$C_1 + C_2 = C_2$$

$$\Rightarrow Q_1 + Q_2 = C_2 V_0$$

Therefore, $V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{C_2}{C_1} \frac{Q_2}{C_2} = \frac{C_2}{C_1} V_0$. **Ans : (D)**

15. **UEE 2004/2012:** What is the main reason why alternating current is used to transmit electricity?

- A) to minimize power gain B) to maximize energy loss
 C) to maximize voltage gain. D) to maximize current gain

Brief explanation: It is quite difficult to reduce a DC high voltage to low voltage without additional loss; it is easy to reduce an AC high voltage to low voltage using a *step-down transformer*. The correct answer is choice C

16. **50. UEE 2004/2012:** Find the impedance for the RLC circuit show in the figure below. Fig 15.25

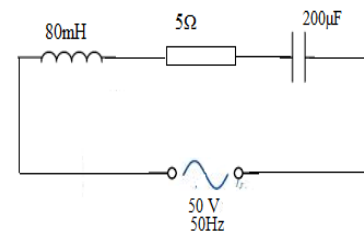
- A) 1.96Ω B) 19.2Ω C) 25.5Ω D) 10.46Ω

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$Z = \sqrt{25 + (25.12 - 15.92)^2} = 10.46 \text{ . Ans : (D)}$$

17. **UEE 2004/2012:** Which statement is correct about a purely inductive circuit?

- A) the current leads the voltage by $\pi/2$
 B) the current is in phase with the voltage.
 C) the current lags behind the voltage by $\pi/2$
 D) the current lags by $\pi/2$ at low frequencies and leads by $\pi/2$ at high frequencies.



Brief explanation: Remembering the **mnemonic**

ELI the ICE man, in a purely inductive load the voltage leads the current by a quarter of a cycle

and in a purely capacitive load the current leads the voltage by a quarter of a cycle. **Ans : (C)**

18. **UEE 2005/2013:** A series RLC circuit has a resistance of 45.0Ω and an impedance of 75.0Ω . What average power is delivered to this circuit when $\Delta V_{rms} = 210V$?

- A) 573 W B) 353 W C) 255 W D) 247 W

Soln:

$$\text{Since } P_{av} = I_{rms} \Delta V_{rms} \cos \Phi = \frac{\Delta V_{rms}^2}{Z} \frac{R}{Z}$$

$$\therefore \frac{210 \times 210 \times 45}{75 \times 75} = 352.8W \text{ . Ans : (B)}$$

19. **UEE 2005/2013:** An alternating current is represented by

$I(t) = (10.0A) \sin(100\pi t)$ what are the amplitude and frequency of the alternating current?

- A) 100 A, 75 Hz B) 10.0 A, 100 Hz
 C) 10.0 A, 50 Hz D) 5.0 A, 25 Hz

Soln: When an ac current is given by the equation of the form

$$I(t) = A \sin(2\pi ft) \quad A = 10.0A \text{ and } \omega = 2\pi f = 100\pi \quad f = 50Hz.$$

Ans : (C)

20. **UEE 2005/2013:** Which device uses the principle of electromagnetic induction for its operation?

- A. Ac generator B. moving coil meter
 C. cathode ray tube D. light emitting diode

Brief explanation: Moving coil meter works on the principle of electromagnetic induction. **Ans : (B)**

21. **UEE 2005/2013:** A step-up transformer is used to

- A. increase the power B. decrease the power
C. increases the voltage D. decrease the voltage.

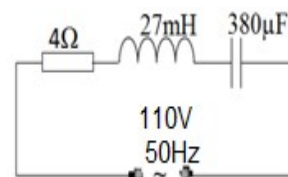
Ans : (C)

22. **UEE 2006/2014:** Determine the total impedance for the circuit shown below

- A) 0Ω B) 4Ω C) 27Ω D) 31Ω

Solution $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z = \sqrt{4^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2} = \sqrt{16 + (8.48 - 8.38)^2} = \sqrt{16.01}$$



$\therefore Q_1 = \left(\frac{C_2}{C_1}\right) Q_1 = C_2 V_0$
 $\rightarrow Q_1 = \left(\frac{C_1 C_2}{C_1 + C_2}\right) V_0$. **Ans : (B)**

23. **UEE 2006/2014:** A 6 V battery is connected across the primary coil of a transformer having 50 turns. If the secondary coil of the transformer has 100 turns, what voltage appears across the secondary?

- A. 24 V B. 12 V C. 6 V D. 3 V

Soln: $Q_2' = C_2 V_0 - Q_1'$ **Ans : (B)**

24. **UEE 2007/2015:** A series **RLC** circuit has a total impedance of **75.0 Ω** . If the resistance is **R = 45.0 Ω** and the average power delivered to the circuit is **P = 352.8W**, what is the root mean square value of the ac voltage source?

- A) 83 V B) 116 V C) 210 V D) 297 V

Soln: Since $P_{av} = I_{rms} V_{rms} \cos \Phi = \frac{V_{rms}^2}{Z} \frac{R}{Z} \rightarrow V_{rms}^2 = \frac{P_{av} Z^2}{R}$

$\rightarrow V_{rms} = Z \sqrt{\frac{P_{av}}{R}} = 75\Omega \times \sqrt{\frac{352.8W}{45\Omega}} = 210V$. **Ans : (C)**

25. **UEE 2007/2015:** The figure shows a rectangular loop of wire of width l and resistance R . One end of the loop is in a uniform magnetic field of strength B directed perpendicular into the plane of the page. The loop is pulled to the right at a constant speed v . What are the magnitude and direction of the induced current in the loop?



- (A) $BlvR$ and Clockwise
 (B) $BlvR$ and Counterclockwise
 (C) Blv/R and Clockwise
 (D) Blv/R and Counterclockwise

Brief explanation:

- ⇒ As the loop moves to the right, the magnetic field through the loop decreases and is directed into the page.
- ⇒ The change in field through the loop is, therefore, directed out of the page.
- ⇒ The induced current must be directed in such a way that the induced field is opposite to the direction of the change in field. That is, the **induced current must be clockwise so that the induced field is into the page.**

The correct answer is C.

26. UEE 2008 Self inductance is a quantity that describes:

- A. the number of turns in a circuit
- B. the induction of an electric field by an AC current in a lone coil.
- C. the induction of a magnetic field by an AC current in a lone coil.
- D. the steady state current in a coil.

Brief explanation: Self induction is the process of producing magnetic field by an AC current in a certain coil itself. **Ans: (C)**

27. UEE 2008 Which conversion process forms the basis for the function of an electric generator?

- A. alternating current to direct
- B. mechanical energy to electrical.
- C. electrical energy to mechanical.
- D. low voltage to high voltage.

Brief explanation: An electric **generator** is a device that converts mechanical **energy to electrical**. A device that converts **electrical energy to mechanical energy** is called an **electric motor**. A device that converts low voltage to high is called **transformer**. Therefore, **choice B is correct**.

28. UEE 2008: What is the change in magnetic flux in a coil of area 5 m^2 as its orientation relative to the perpendicular of a uniform 3.0 T magnetic field changes from 45° to 90°

- A. 0 Wb
- B. -11 Wb
- C. 7.5 Wb
- D. 11 Wb

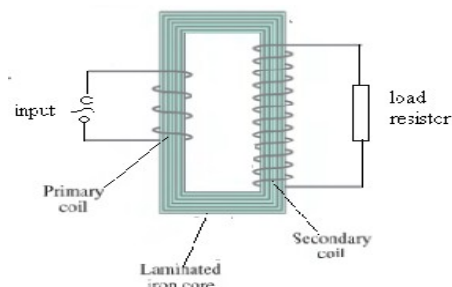
Soln: $\phi_0 = BA \cos 45^\circ \Rightarrow \phi_0 = (3\text{T})(5\text{m}^2)(0.707) = 10.6\text{Wb}$ and

$$\phi_f = BA \cos 90^\circ \Rightarrow \phi_f = 0$$

Therefore, $\Delta\phi = -10.6\text{Wb} \approx -11\text{Wb}$. **Ans: (B)**

29. UEE 2008 A simple transformer is illustrated in the figure below. Why is it that the core is laminated?

- A. to increase the induced current in the primary coil.
- B. to increase the induced current in the secondary coil.
- C. to avoid heat exchange between the primary and the secondary coils
- D. to reduce the loss in power in the core due to eddy currents.



Brief explanation: Eddy currents are main sources of heat in transformers. In actual transformers, the eddy currents are greatly reduced by the use of a

laminated core—that is, a core built up of thin sheets, or laminae. The large electrical surface resistance of each lamina, due to either a natural coating of oxide or an insulating varnish, effectively confines the eddy currents to individual laminae. The possible eddy-current paths are narrower, the induced emf in each path is smaller, and the eddy currents are greatly reduced. **Ans: (D)**

30. **UEE 2010:** An inductor has a self inductance of 12mH. The current in the circuit decreases at a rate of 2.0A/s. What is the magnitude of the induced emf?

- A) $2.4\pi V$ B) $1.2 \pi V$ C) $2.4 \times 10^{-2} V$ D) $1.2 \times 10^{-2} V$

Solution

The magnitude of the induced emf in the inductor is

$$\varepsilon = \left| L \frac{\Delta I}{\Delta t} \right| = (12 \times 10^{-3} H)(2 A/s) \Rightarrow \underline{\varepsilon = 2.4 \times 10^{-2} V}$$

. Choice C is correct.

31. **UEE 2010** Which of the following is correct about the characteristic of current and voltage in ac circuits?

- A) If the current is in phase with the voltage, the circuit is inductive.
- B) If the current leads the voltage, its capacitive reactance is greater than inductive reactance.
- C) If the current leads the voltage, it is inductive circuit.
- D) If the current lags the voltage, the circuit is capacitive.

Brief explanation and solution

Recap the mnemonic “ELI the ICE man.”

- In pure inductive circuit, the voltage leads the current.
- In pure capacitive circuit the current leads the voltage.
- If the current and voltage are in phase, the circuit is resistive.

Choice B is correct.

32. **UEE 2010:** An RLC circuit is designed to have a resistance of 400Ω , capacitive reactance of 350Ω and inductive reactance of 50Ω . The circuit is connected to an alternative voltage source of effective (rms) value 50 V, and frequency 50 Hz. What is the output average power?

- A) 40 W B) 4.0 W C) 400 W D) 62.5 W

Brief explanations and solutions

No power is dissipated in an ideal capacitor or an ideal inductor; the power is dissipated only in the resistance of the circuit (including the resistances of the wires of the circuit and the windings of the inductor):

$$P_{av} = I_{rms} V_{R,rms}. \text{ For RLC series, } V_{Z,rms} = I_{rms} Z \text{ and for resistive, } V_{R,rms} = I_{rms} R$$

$$\Rightarrow \frac{V_{R,rms}}{V_{Z,rms}} = \frac{R}{Z} \Rightarrow V_{R,rms} = V_{Z,rms} \left(\frac{R}{Z} \right)$$

$$V_{R,rms} = V_{Z,rms} \cos \phi \Rightarrow P_{av} = I_{rms} V_{Z,rms} \cos \phi$$

$$\underline{\underline{P_{av} = \left(\frac{V_{Z,rms}^2}{Z^2} \right) R, \quad R=400\,\Omega, \quad X_C=350\,\Omega, \quad X_L=50\,\Omega, \quad V_{Z,rms}=50\,V}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 500\,\Omega$$

$$P_{av} = \left(\frac{V_{Z,rms}^2}{Z^2} \right) R = \left(\frac{2500}{250000} \right) 400 \Rightarrow \underline{\underline{P_{av} = 4.0\,W}}$$

The correct answer is choice B.