# Simplified procedure of designing beams by allowing moment redistribution

## According to EBCS 2 – Part 2

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# Conventional Procedure

**Control over depth of neutral axis**

**To ensure that all beams have the desirable characteristics of visible warning, if failure is sudden as well as reasonable ductility at failure, it is recommended that depth of neutral axis be limited.**

$$\frac{x}{d} \le 0.8(\delta - 0.44)$$

**In EBCS 2, article 3.7.9, ,**

**where $\delta$ = % moment redistribution, = Moment after redistribution**

**Original moment**

**When no moment is redistributed, $\delta$ = 1.**

$$\frac{x}{d} = 0.45$$

**In such a case, , or, $x = 0.45d$**

***$M_u = 0.8x\ b\ f_{cd}\ (d - 0.4x) = 0.8(0.45d)\ b\ f_{cd}\ [d - 0.4(0.45d)]$***

***$M_u = 0.2952\ f_{cd}\ b\ d^2$.***

# Procedure given in EBCS 2 - Part 2

- Here, the value 0.2952 is given a notation $\mu^*_{us}$
- Therefore, $$\mu^*_{us} = \frac{M_{us}}{f_{cd} b d^2}$$
- If the percentage redistribution is allowed, up to 30%, there can be different values of $\mu^*_{us}$ for different percentage of redistribution permitted.
- In the above derivation, $x = 0.45d$ is a determination of neutral axis depth for zero percent redistribution of moments.
- Here the value 0.45 is given a notation $k_x$ *(Neutral axis depth constant)*
- Similar values of $k_x$ can be possible for different values of moment redistribution.

- From the preliminary principles,

$$M = T \cdot Z = A_s f_{yd} z = \frac{M_{us}}{z f_{yd}}$$

here $z$ is the lever arm between tension and compression forces which can be related to effective depth $d$ by lever arm constant $k_z$ as

$$z = k_z * d$$

- $k_m$ is a value given in the formulations as follows

$$k_m = \sqrt{\frac{M_{us}}{bd^2}}$$

- $k_s$ is another value which is included in the equation for area of steel determination i.e.

$$A_s = \frac{k_s M_{us}}{d}$$

Or $$A_s = \frac{M_{us}}{z\, f_{yd}} = \frac{M_{us}}{z\, f_{yd}} * \frac{d}{d} = \frac{M_{us}}{k_z d\, f_{yd}} * \frac{d}{d} = \frac{M_{us}}{d} * \frac{1}{k_z f_{yd}}$$

Where, $k_s = 1/k_z.f_{yd}$

Tabulations are made for the values of $k_m$, $k_s$, $k_x$ and $k_z$ for different values of percentage moment redistribution, using

# General Design Table No. 1a
## (no moment redistribution)

| $k_m$ | | | | | $k_s$ | | | | | | | Redist-ribution |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C15 | C20 | C25 | C30 | C40 | S300 | S400 | S460 | $\varepsilon_c$ | $\varepsilon_{s1}$ | $k_x$ | $k_z$ | |
| 15 | 17 | 19 | 21 | 24 | 3.95 | 2.96 | 2.58 | 0.94 | 10 | 0.09 | 0.97 | |
| 19 | 22 | 25 | 27 | 31 | 3.99 | 2.99 | 2.60 | 1.27 | 10 | 0.11 | 0.96 | |
| 23 | 26 | 29 | 32 | 37 | 4.04 | 3.03 | 2.63 | 1.59 | 10 | 0.14 | 0.95 | |
| 26 | 30 | 33 | 37 | 42 | 4.08 | 3.06 | 2.66 | 1.92 | 10 | 0.16 | 0.94 | |
| 29 | 33 | 37 | 40 | 47 | 4.12 | 3.09 | 2.69 | 2.24 | 10 | 0.18 | 0.93 | |
| 31 | 35 | 40 | 43 | 50 | 4.17 | 3.13 | 2.72 | 2.56 | 10 | 0.20 | 0.92 | |
| **31.12** | **35.94** | **40.18** | **44.02** | **50.83** | **4.18** | **3.13** | **2.72** | **2.63** | **10** | **0.208** | **0.918** | **30%** |
| 33 | 38 | 42 | 46 | 53 | 4.21 | 3.16 | 2.75 | 2.89 | 10 | 0.22 | 0.91 | |
| 34 | 40 | 44 | 49 | 56 | 4.26 | 3.19 | 2.78 | 3.22 | 10 | 0.24 | 0.90 | |
| 36 | 42 | 46 | 51 | 59 | 4.31 | 3.23 | 2.81 | 3.50 | 9.7 | 0.26 | 0.89 | |
| **37.36** | **43.13** | **48.23** | **52.83** | **61** | **4.36** | **3.27** | **2.84** | **3.50** | **8.7** | **0.288** | **0.880** | **20%** |
| 37 | 43 | 48 | 53 | 61 | 4.36 | 3.27 | 2.84 | 3.50 | 8.6 | 0.29 | 0.88 | |
| 39 | 45 | 50 | 55 | 63 | 4.41 | 3.30 | 2.87 | 3.50 | 7.7 | 0.31 | 0.87 | |
| 40 | 46 | 52 | 56 | 65 | 4.46 | 3.34 | 2.91 | 3.50 | 6.9 | 0.34 | 0.86 | |
| 41 | 47 | 53 | 58 | 67 | 4.51 | 3.38 | 2.94 | 3.50 | 6.2 | 0.36 | 0.85 | |
| **41.42** | **47.83** | **53.47** | **58.58** | **67.64** | **4.53** | **3.39** | **2.95** | **3.50** | **6.0** | **0.368** | **0.847** | **10%** |
| 42 | 49 | 54 | 60 | 69 | 4.56 | 3.42 | 2.98 | 3.50 | 5.6 | 0.38 | 0.84 | |
| 43 | 50 | 56 | 61 | 71 | 4.62 | 3.46 | 3.01 | 3.50 | 5.1 | 0.41 | 0.83 | |
| 44 | 51 | 57 | 63 | 72 | 4.67 | 3.51 | 3.05 | 3.50 | 4.6 | 0.43 | 0.82 | |
| **44.79** | **51.72** | **57.83** | **63.35** | **73.15** | **4.71** | **3.53** | **3.07** | **3.50** | **4.3** | **0.448** | **0.814** | |

No redistribution ($k_x = 0.448$)

| $k_m/k^*_m$ | $k_m$ | | | | | $k_s$ | | |
|---|---|---|---|---|---|---|---|---|
| | C15 | C20 | C25 | C30 | C40 | S300 | S400 | S460 |
| | 44.79 | 51.72 | 57.83 | 63.35 | 73.15 | 4.71 | 3.53 | 3.07 |
| 1.03 | 46 | 53 | 59 | 65 | 75 | 4.68 | 3.51 | 3.05 |
| 1.05 | 47 | 54 | 61 | 67 | 77 | 4.65 | 3.49 | 3.04 |
| 1.08 | 48 | 56 | 63 | 69 | 79 | 4.63 | 3.47 | 3.02 |
| 1.11 | 50 | 58 | 64 | 71 | 81 | 4.60 | 3.45 | 3.00 |
| 1.15 | 51 | 59 | 66 | 73 | 84 | 4.57 | 3.43 | 2.98 |
| 1.19 | 53 | 61 | 69 | 75 | 87 | 4.54 | 3.40 | 2.96 |
| 1.23 | 55 | 64 | 71 | 78 | 90 | 4.51 | 3.38 | 2.94 |
| 1.28 | 57 | 66 | 74 | 81 | 94 | 4.48 | 3.36 | 2.92 |
| 1.33 | 60 | 69 | 77 | 84 | 97 | 4.45 | 3.34 | 2.90 |
| 1.39 | 62 | 72 | 81 | 88 | 102 | 4.43 | 3.32 | 2.89 |
| 1.46 | 66 | 76 | 85 | 93 | 107 | 4.40 | 3.30 | 2.87 |

Notes:

1 See Table 1b for Design Equations

2 Initial values for 0%,10%,20%&30% Moment Redistribution are shown shaded in Tables No.1a & 1b

Factors $\rho$ and $k_s'$ are applicable to all values of moment redistribution

Correction factor $\rho$

| $k_m/k^*_m$ | $d_2/d$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.07 | 0.08 | 0.1 | 0.12 | 0.14 | 0.16 | 0.18 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1.03 | 1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 |
| 1.05 | 1 | 1.00 | 1.00 | 1.00 | 1.01 | 1.01 | 1.01 |
| 1.08 | 1 | 1.00 | 1.00 | 1.01 | 1.01 | 1.01 | 1.02 |
| 1.11 | 1 | 1.00 | 1.01 | 1.01 | 1.01 | 1.02 | 1.02 |
| 1.15 | 1 | 1.00 | 1.01 | 1.01 | 1.02 | 1.02 | 1.03 |
| 1.19 | 1 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 | 1.04 |
| 1.23 | 1 | 1.00 | 1.01 | 1.02 | 1.03 | 1.03 | 1.04 |
| 1.28 | 1 | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 |
| 1.33 | 1 | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 |
| 1.39 | 1 | 1.00 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 |
| 1.46 | 1 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.07 |

| $k_m/k^*_m$ | $k_s'$ | | |
|---|---|---|---|
| | S300 | S400 | S460 |
| 1 | 0 | 0 | 0 |
| 1.03 | 0.20 | 0.15 | 0.13 |
| 1.05 | 0.40 | 0.30 | 0.26 |
| 1.08 | 0.60 | 0.45 | 0.39 |
| 1.11 | 0.80 | 0.60 | 0.52 |
| 1.15 | 1.00 | 0.75 | 0.65 |
| 1.19 | 1.20 | 0.90 | 0.78 |
| 1.23 | 1.40 | 1.05 | 0.91 |
| 1.28 | 1.60 | 1.20 | 1.04 |
| 1.33 | 1.80 | 1.35 | 1.17 |
| 1.39 | 2.00 | 1.50 | 1.30 |
| 1.46 | 2.20 | 1.65 | 1.43 |

Correction factor $\rho'$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| All $K_m/K^*_m$ | 1 | 1.01 | 1.03 | 1.06 | 1.08 | 1.11 | 1.13 |

# General Design Table No. 1b
## (with moment redistribution)

| 10% redistribution ($k_x = 0.368$) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $k_m$ | | | | | $k_s$ | | |
| $k_m/k^*_m$ | C15 | C20 | C25 | C30 | C40 | S300 | S400 | S460 |
| 1 | 41.42 | 47.83 | 53.47 | 58.58 | 67.64 | 4.53 | 3.39 | 2.95 |
| 1.03 | 42 | 49 | 55 | 60 | 69 | 4.52 | 3.39 | 2.95 |
| 1.05 | 44 | 50 | 56 | 62 | 71 | 4.52 | 3.39 | 2.95 |
| 1.08 | 45 | 52 | 58 | 63 | 73 | 4.52 | 3.39 | 2.95 |
| 1.11 | 46 | 53 | 60 | 65 | 75 | 4.51 | 3.38 | 2.94 |
| 1.15 | 48 | 55 | 61 | 67 | 78 | 4.51 | 3.38 | 2.94 |
| 1.19 | 49 | 57 | 63 | 70 | 80 | 4.51 | 3.38 | 2.94 |
| 1.23 | 51 | 59 | 66 | 72 | 83 | 4.50 | 3.38 | 2.94 |
| 1.28 | 53 | 61 | 68 | 75 | 86 | 4.50 | 3.37 | 2.93 |
| 1.33 | 55 | 64 | 71 | 78 | 90 | 4.50 | 3.37 | 2.93 |
| 1.39 | 58 | 67 | 74 | 82 | 94 | 4.49 | 3.37 | 2.93 |
| 1.46 | 61 | 70 | 78 | 86 | 99 | 4.49 | 3.37 | 2.93 |

Notes:

1 $k^*_m$ is $k_m$ value corresponding to $\mu^*_{u,s}$ (shown shaded in Tables No.1a &1b)

2 Units:

M (kNm)
b,d (m)
$A_s$ ($mm^2$)

Correction factor $\rho'$

| All | $d_2/d$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.07 | 0.08 | 0.1 | 0.12 | 0.14 | 0.16 | 0.18 |
| $K_m/K^*_m$ | 1 | 1.01 | 1.03 | 1.06 | 1.08 | 1.11 | 1.13 |

20% redistribution ($k_x = 0.288$)

| $k_m/k^*_m$ | $k_m$ | | | | | $k_s$ | | |
|---|---|---|---|---|---|---|---|---|
| | C15 | C20 | C25 | C30 | C40 | S300 | S400 | S460 |
| 1 | 37.36 | 43.13 | 48.23 | 52.83 | 61 | 4.36 | 3.27 | 2.84 |
| 1.03 | 38 | 44 | 49 | 54 | 63 | 4.34 | 3.26 | 2.83 |
| 1.05 | 39 | 45 | 51 | 56 | 64 | 4.33 | 3.25 | 2.83 |
| 1.08 | 40 | 47 | 52 | 57 | 66 | 4.32 | 3.24 | 2.82 |
| 1.11 | 42 | 48 | 54 | 59 | 68 | 4.31 | 3.23 | 2.81 |
| 1.15 | 43 | 50 | 55 | 61 | 70 | 4.30 | 3.22 | 2.80 |
| 1.19 | 44 | 51 | 57 | 63 | 72 | 4.29 | 3.22 | 2.80 |
| 1.23 | 46 | 53 | 59 | 65 | 75 | 4.28 | 3.21 | 2.79 |
| 1.28 | 48 | 55 | 62 | 68 | 78 | 4.26 | 3.20 | 2.78 |
| 1.33 | 50 | 57 | 64 | 70 | 81 | 4.25 | 3.19 | 2.77 |
| 1.39 | 52 | 60 | 67 | 74 | 85 | 4.24 | 3.18 | 2.77 |
| 1.46 | 55 | 63 | 71 | 77 | 89 | 4.23 | 3.17 | 2.76 |

Correction factor $\rho'$

| | $d_2/d$ | | | |
|---|---|---|---|---|
| All | 0.07 | 0.08 | 0.1 | 0.12 |
| $K_m/K^*_m$ | 1 | 1.01 | 1.03 | 1.06 |

30% redistribution ($k_x = 0.208$)

| | $k_m$ | | | | | $k_s$ | | |
|---|---|---|---|---|---|---|---|---|
| $k_m/k^*_m$ | C15 | C20 | C25 | C30 | C40 | S300 | S400 | S460 |
| 1 | 31.07 | 35.88 | 40.12 | 43.95 | 50.74 | 4.18 | 3.13 | 2.72 |
| 1.03 | 32 | 37 | 41 | 45 | 52 | 4.17 | 3.13 | 2.72 |
| 1.05 | 33 | 38 | 42 | 46 | 53 | 4.17 | 3.13 | 2.72 |
| 1.08 | 34 | 39 | 43 | 48 | 55 | 4.17 | 3.13 | 2.72 |
| 1.11 | 35 | 40 | 45 | 49 | 57 | 4.17 | 3.12 | 2.72 |
| 1.15 | 36 | 41 | 46 | 50 | 58 | 4.16 | 3.12 | 2.71 |
| 1.19 | 37 | 43 | 48 | 52 | 60 | 4.16 | 3.12 | 2.71 |
| 1.23 | 38 | 44 | 49 | 54 | 62 | 4.16 | 3.12 | 2.71 |
| 1.28 | 40 | 46 | 51 | 56 | 65 | 4.16 | 3.12 | 2.71 |
| 1.33 | 41 | 48 | 53 | 59 | 68 | 4.15 | 3.11 | 2.71 |
| 1.39 | 43 | 50 | 56 | 61 | 71 | 4.15 | 3.11 | 2.71 |
| 1.46 | 46 | 53 | 59 | 64 | 74 | 4.15 | 3.11 | 2.70 |

Correction factor $\rho' = 1$ for $d_2/d \leq 0.07$
(higher values of $d_2/d$ not recommended)

$$k_m = \frac{\sqrt{M_{Sd,s} / b}}{d}$$

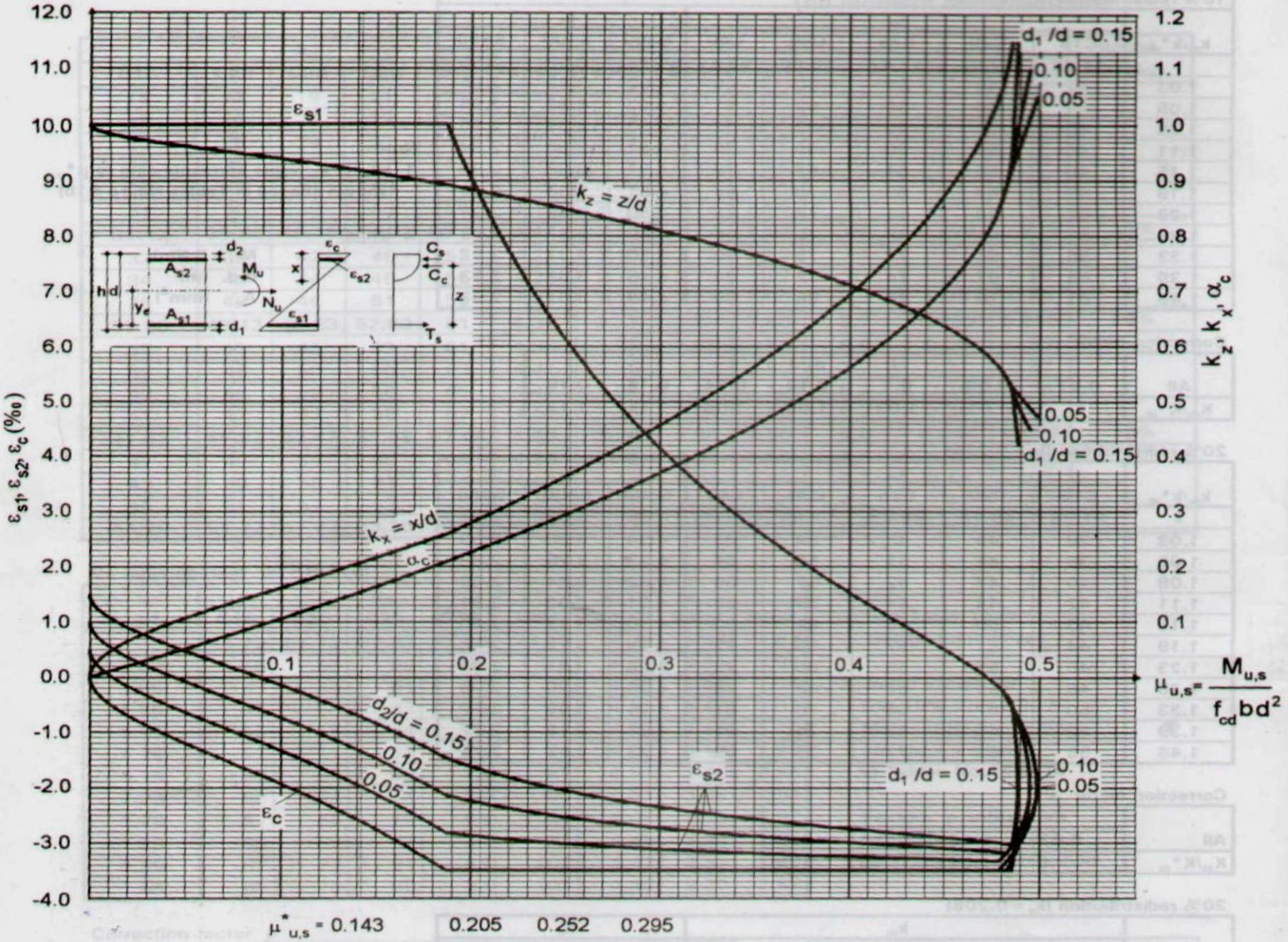
$$M_{Sd,s} = M_{Sd} - N_{Sd}\, y_e$$

$$A_{s1} = \frac{M_{Sd,s}}{d} k_s \rho + \frac{N_{Sd}}{f_{yd}}$$

$$A_{s2} = \frac{M_{Sd,s}}{d} k'_s \rho'$$

**Rectangular Section ... Moment with or without Axial load, with or without Compression Reinforcement**

# General Design Chart No. 1



(1) If $\mu_{Sd,s} \leq \mu^*_{u,s}$ compression reinforcement is not required:

$$A_{s1} = \frac{M_{Sd,s}}{z f_{yd}} + \frac{N_{Sd}}{f_{yd}}$$

(2) If $\mu_{Sd,s} > \mu^*_{u,s}$ compression reinforcement is required:

$$A_{s2} = \frac{M_{Sd,s} - M^*_{u,s}}{(d - d_2)\sigma_{s2}}$$

$$A_{s1} = \frac{M^*_{u,s}}{z f_{yd}} + \frac{M_{Sd,s} - M^*_{u,s}}{(d - d_2) f_{yd}} + \frac{N_{Sd}}{f_{yd}}$$

**Bending with or without Normal Load, with or without Compression Reinforcement**

# 3. Design Examples

## 3.1 Singly Reinforced Section without Normal Load

**Given:**

| | |
|---|---|
| Cross-section: | b/h/d = 300/600/550 mm |
| Action Effects: | Design moment = 180 kNm |
| Material Data: | Concrete C20 |
| | Steel S 460 |

**Required:** Amount of tension reinforcement for both Class I and Class II Works

## Solution

Using General Design Table 1a:

| | | Class I Works | Class II Works |
|---|---|---|---|
| $k_m$ | = | $\frac{\sqrt{M_{Sd,s}/b}}{d}$ | |
| | = | $\frac{\sqrt{180/0.3}}{0.55}$ | |
| | = | 44.5 | $1.05 \times 44.5 = 46.8$ |
| $k_s$ | = | 2.87 | $1.05 \times 2.94 = 3.09$ |
| $A_{s1}$ | = | $\frac{k_s M_{Sd,s}}{d}$ | |
| | = | $\underline{939\ mm^2}$ | $\underline{1011\ mm^2}$ |

# Using General Design Chart 1

(Note $f_{ck}$ = 16 for C20, see Table 2.3 of EBCS - 2)

$$\mu_{Sd,s} = \frac{M_{Sd,s}}{f_{cd} b d^2}$$

$$= \frac{180 \times 10^6 \times 1.5}{0.85 \times 16 \times 300 \times 550^2} \qquad \frac{180 \times 10^6 \times 1.65}{0.85 \times 16 \times 300 \times 550^2}$$

$$= 0.219 \qquad 0.240$$

$k_z$ = 0.87 0.84

$z$ = 0.87 × 550 = 479 mm 0.84 × 550 = 462 mm

$$A_{s1} = \frac{M_{Sd,s}}{z \times f_{yd}}$$

$$= \frac{180 \times 10^6 \times 1.15}{479 \times 460} \qquad \frac{180 \times 10^6 \times 1.20}{462 \times 460}$$

= 939 mm² 1016 mm²

## 3.2 Singly Reinforced Section with Normal Load

**Given:**

| | |
|---|---|
| Cross-section: | b/h/d = 300/600/550 mm |
| Action Effects: | moment due to permanent load = 96 kNm<br>moment due to variable load = 72 kNm<br>Permanent normal load = -180kN |
| Material Data: | Concrete C25<br>Steel S 460 |

**Required:** Amount of tension reinforcement

$= \quad 57.4 < k_{m}' = 57.83$ compression reinforcement is not required

$k_s = 3.06$

$$A_{s1} = \frac{k_s \times M_{Sd,s}}{d} + \frac{N_{Sd}}{f_{yd}}$$

$$A_{s1} = \frac{3.06 \times 298.5}{0.55} - \frac{1.3 \times 180 \times 10^3 \times 1.15}{460}$$

$= 1661 - 585$

$= \underline{1076\ mm^2}$

If effect of axial load is neglected as permitted by EBCS-2 1995 section 4.3.1(2) for small axial loads ($N_{sd} = 1.3 \times 180 = 234$ kN $< 0.1\ f_{ck}A_c = 360$ kN), the following is obtained:

$$M_{sd,s} = 240 \text{ kNm}$$

$$k_m = \frac{\sqrt{240 / 0.3}}{0.55}$$

$$= 51.4$$

$$k_s = 2.89$$

$$A_{s1} = \frac{2.89 \times 240}{0.55}$$

$$= \underline{1261.0 \text{ mm}^2}$$

(Note ≈15% reduction in reinforcement due to compressive force)

**Using General Design Chart No. 1:**

$$\mu_{Sd,s} = \frac{M_{Sd,s}}{f_{cd} b d^2}$$

$$= \frac{298.5 \times 10^6 \times 1.5}{0.85 \times 20 \times 300 \times 550^2}$$

$$= 0.290$$

$k_x$ = 0.44 < 0.448 compression reinforcement is not required

$k_z$ = 0.82

$z$ = $0.82 \times 550 = 451$ mm

$$A_{s1} = \frac{M_{Sd,s}}{z \times f_{yd}} - \frac{N_{Sd}}{f_{yd}}$$

$$= \frac{298.5 \times 10^6 \times 1.15}{451 \times 460} - \frac{1.3 \times 180 \times 10^3 \times 1.15}{460}$$

$$= 1655 - 585$$

$$= \underline{1070 \text{ mm}^2}$$

## 3.3 Doubly Reinforced Section with Moment Redistribution

**Given:**

| | |
|---|---|
| Cross-section: | b/h/d = 300/600/550 mm |
| Action Effects: | Design moment = 240 kNm (after moment redistribution) |
| Material Data: | Concrete C25 |
| | Steel S 460 |

Percent Plastic Moment Redistribution: 20%

**Required:** Amount of tension and compression reinforcement

Table 1a
Table 1b
Table 1a2

**Solution:**

**Using General Design Table 1a and 1b:**

$$k_m = \frac{\sqrt{M_{Sd,s} / b}}{d}$$

$$= \frac{\sqrt{240 / 0.3}}{0.55}$$

$= 51.4 > k^*_m = 48.23$ compression reinforcement is required. Assume $d_2/d = 0.07$

$k_m/k^*_m = 1.07$

$k_s = 2.83$ from Table No. 1b

$k'_s = 0.33$ from Table No. 1a

$$A_{S1} = \frac{k_s \times \rho \times M_{Sd,s}}{d} + \frac{N_{Sd}}{f_{yd}}$$

**Table 1a2**
**Table 1b rp**

$$= \frac{2.83 \times 1 \times 240}{0.55}$$

$$= \underline{1235\ mm^2}$$

$$A_{S2} = \frac{k'_s \times \rho' \times M_{Sd,s}}{d} + \frac{N_{Sd}}{f_{yd}}$$

$$= \frac{0.33 \times 1 \times 240}{0.55}$$

$$= \underline{144\ mm^2}$$

## Using General Design Chart No. 1:

$$\mu_{Sd,s} = \frac{M_{Sd,s}}{f_{cd} b d^2}$$

$$= \frac{240 \times 10^6 \times 1.5}{0.85 \times 20 \times 300 \times 550^2}$$

$$= 0.233$$

$k^*_z = 0.88$

$z^* = 0.88 \times 550 = 484$ mm

$\mu^*_{u,s} = 0.205$

$$M^*_{u,s} = \frac{0.205 \times 0.85 \times 20 \times 300 \times 550^2}{1.5}$$

$$= 210.84 \times 10^6 \text{ Nmm}$$