

ME 201

Engineering Mechanics: Statics

Chapter 9 – Part A

9.1 Center of Gravity and Center of Mass

9.3 Composite Bodies



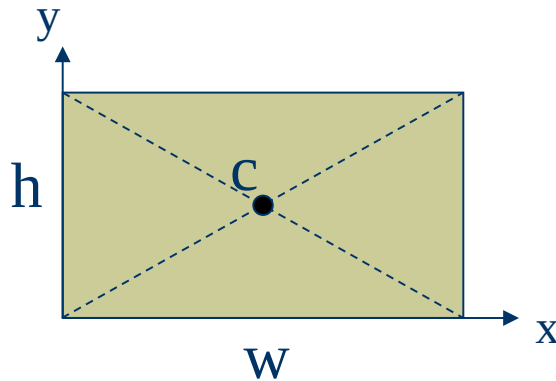
Centroid

- ◆ Point which defines geometrical center of an object
- ◆ Determined by geometry for simple shapes and by computing moments of an element about an axis for more complex shapes
- ◆ Also referred to as *First Moment Area*

Centroid of Simple Shapes

Rectangle

- The centroid of a rectangle or parallelogram lies at the intersection of its diagonals.



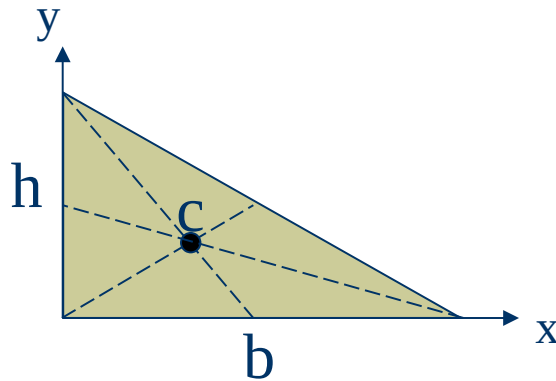
$$\tilde{x} = \frac{w}{2} \quad \tilde{y} = \frac{h}{2}$$

$$A = w \cdot h$$

Centroid of Simple Shapes

Triangle

- The centroid of a triangle lies at the intersection of its median.



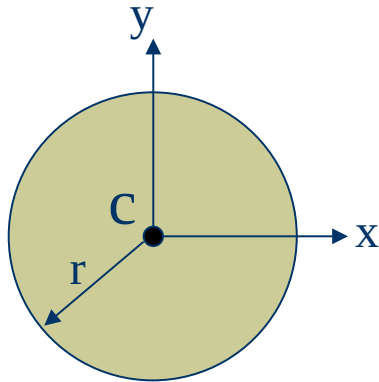
$$\tilde{x} = \frac{b}{3} \quad \tilde{y} = \frac{h}{3}$$

$$A = \frac{1}{2} b \cdot h$$

Centroid of Simple Shapes

Circle

- The centroid of a circle lies at its center point.

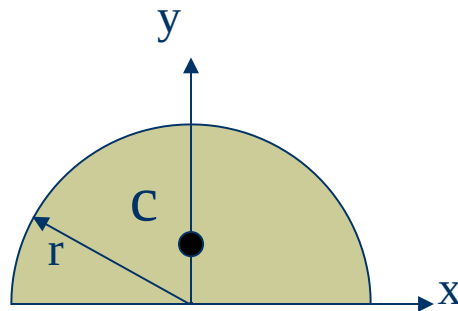


$$\tilde{x} = 0 \quad \tilde{y} = 0$$

$$A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4}$$

Centroid of Simple Shapes

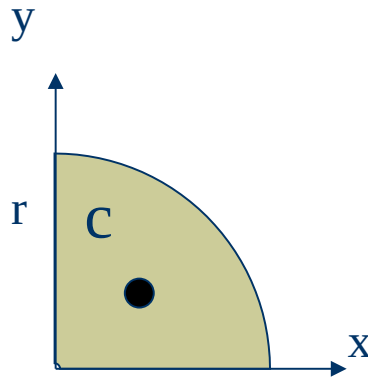
Semi-Circle



$$\tilde{x} = 0 \quad \tilde{y} = \frac{4r}{3\pi}$$

$$A = \frac{1}{2} \pi \cdot r^2$$

Quarter-Circle



$$\tilde{x} = \frac{4r}{3\pi} \quad \tilde{y} = \frac{4r}{3\pi}$$

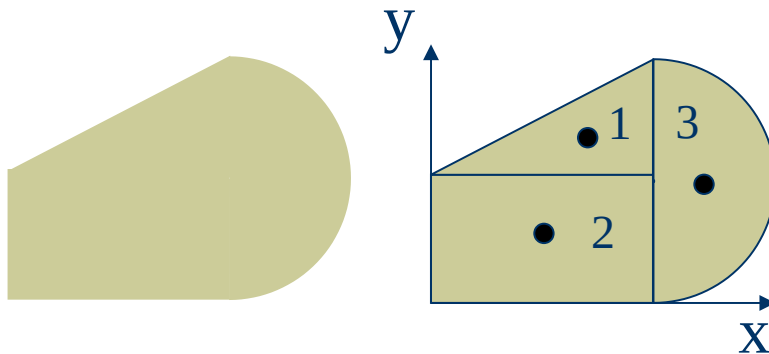
$$A = \frac{1}{4} \pi \cdot r^2$$

Centroids of Composite Areas

- ◆ Centroids of more complex shapes can be found by the following procedure

- Divide into numbered, simple areas
- Find centroid of each area
- Sum moment areas relative to origin

$$\bar{X} = \frac{\sum \tilde{x}A}{\sum A} \quad \bar{Y} = \frac{\sum \tilde{y}A}{\sum A}$$



Shape	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1					
2					
3					
Σ					

Example Problem

Centroid of Composite Area

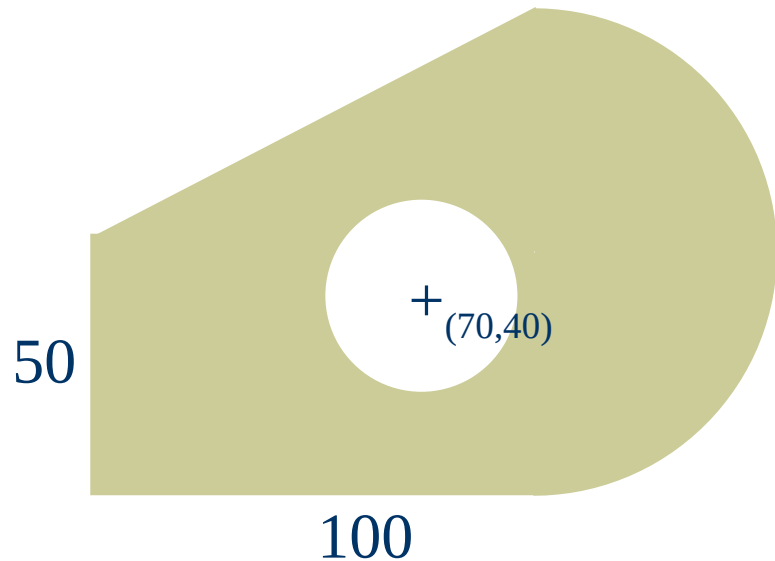
Given:

$$D_{\text{hole}} = 50 \text{ mm}$$

$$R_{\text{arc}} = 50 \text{ mm}$$

Find:

Centroid of
composite



Example Problem Solution

Given:

$$D_{\text{hole}} = 50 \text{ mm}$$

$$R_{\text{arc}} = 50 \text{ mm}$$

Find:

Centroid of composite

Solution:

$$A_1 = 50 * 100 = 5000$$

$$\tilde{x}_1 = 100 / 2 = 50$$

$$\tilde{y}_1 = 50 / 2 = 25$$

$$A_2 = 50 * 100 / 2 = 2500$$

$$\tilde{x}_2 = 100 * 2 / 3 = 66.7$$

$$\tilde{y}_2 = 50 + 50 / 3 = 66.7$$

$$A_3 = \pi * 50^2 / 2 = 3927$$

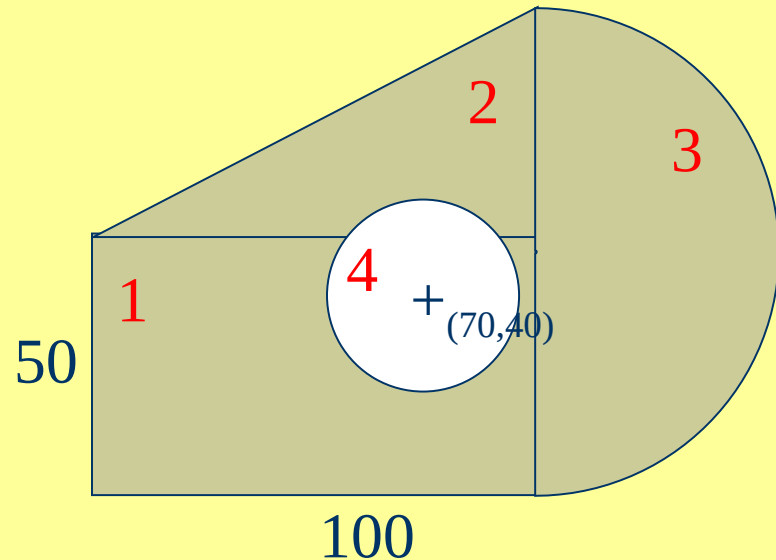
$$\tilde{x}_3 = 100 + 4 * 50 / (3 * \pi) = 121.2$$

$$\tilde{y}_3 = 50$$

$$A_4 = - \pi * 50^2 / 4 = - 1963$$

$$\tilde{x}_4 = 70$$

$$\tilde{y}_4 = 40$$



Example Problem Solution

shape	A	$\tilde{x}$	$\tilde{y}$	$\tilde{x}A$	$\tilde{y}A$
1	5000	50.0	25.0	250000	125000
2	2500	66.7	66.7	166667	166667
3	3927	121.2	50.0	476032	196350
4	-1963	70.0	40.0	-137445	-78540
Σ	9463			755254	409476

$$\bar{X} = \frac{\sum \tilde{x}A}{\sum A} = \frac{755254}{9463} = 79.8 \text{ mm} \quad \bar{Y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{409476}{9463} = 43.3 \text{ mm}$$

Center of Gravity & Mass

- ◆ Centroid (Center of Area)

$$\bar{X} = \frac{\sum \tilde{x}A}{\sum A} \quad \bar{Y} = \frac{\sum \tilde{y}A}{\sum A}$$

- ◆ Center of Gravity

$$\bar{X} = \frac{\sum \tilde{x}W}{\sum W} \quad \bar{Y} = \frac{\sum \tilde{y}W}{\sum W} \quad \bar{Z} = \frac{\sum \tilde{z}W}{\sum W}$$

- ◆ Center of Mass

$$\bar{X} = \frac{\sum \tilde{x}m}{\sum m} \quad \bar{Y} = \frac{\sum \tilde{y}m}{\sum m} \quad \bar{Z} = \frac{\sum \tilde{z}m}{\sum m}$$