ME 201
Engineering Mechanics: Statics

Chapter 9 – Part A

9.1 Center of Gravity and Center of Mass

9.3 Composite Bodies

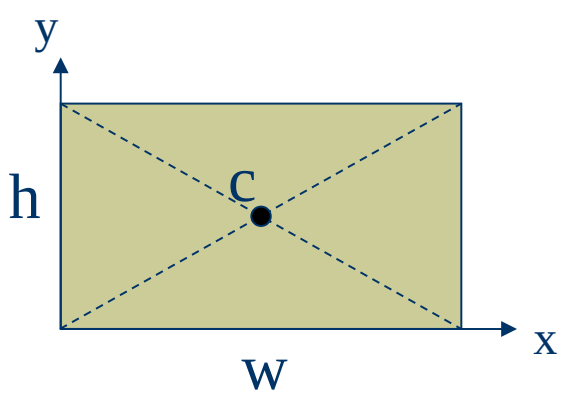
Centroid

- Point which defines geometrical center of an object
- Determined by geometry for simple shapes and by computing moments of an element about an axis for more complex shapes
- Also referred to as *First Moment Area*

Centroid of Simple Shapes

Rectangle

- The centroid of a rectangle or parallelogram lies at the intersection of its diagonals.



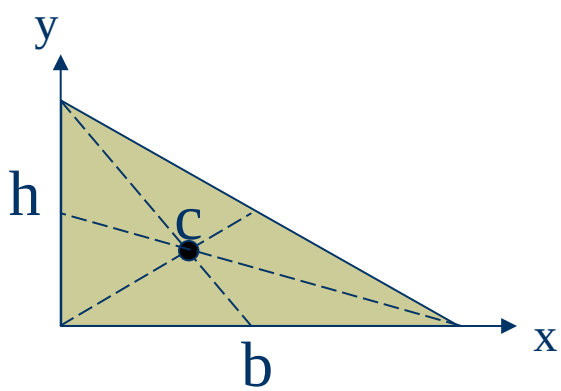
$$\tilde{x} = \frac{w}{2} \qquad \tilde{y} = \frac{h}{2}$$

$$A = w \cdot h$$

Centroid of Simple Shapes

Triangle

- The centroid of a triangle lies at the intersection of its median.



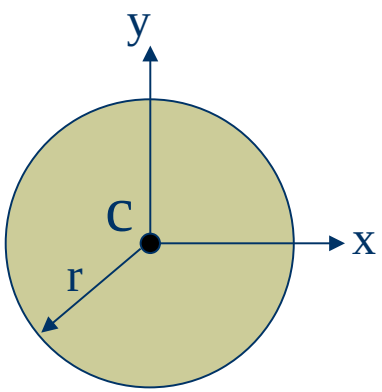
$$\tilde{x} = \frac{b}{3} \qquad \tilde{y} = \frac{h}{3}$$

$$A = \frac{1}{2} b \cdot h$$

Centroid of Simple Shapes

Circle

- The centroid of a circle lies at its center point.

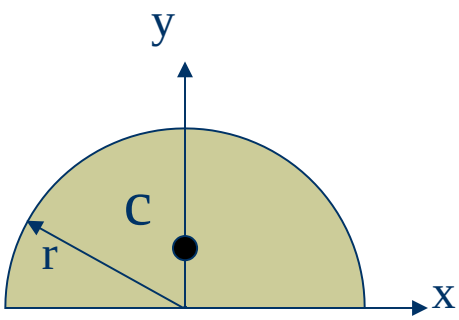


$$\tilde{x} = 0 \qquad \tilde{y} = 0$$

$$A = \pi \cdot r^2 = \frac{\pi \cdot d^2}{4}$$

Centroid of Simple Shapes

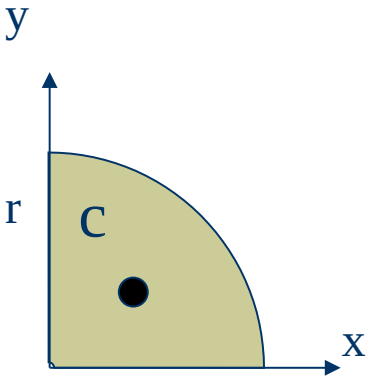
Semi-Circle



$$\tilde{x} = 0 \qquad \tilde{y} = \frac{4r}{3\pi}$$

$$A = \frac{1}{2}\pi \cdot r^2$$

Quarter-Circle



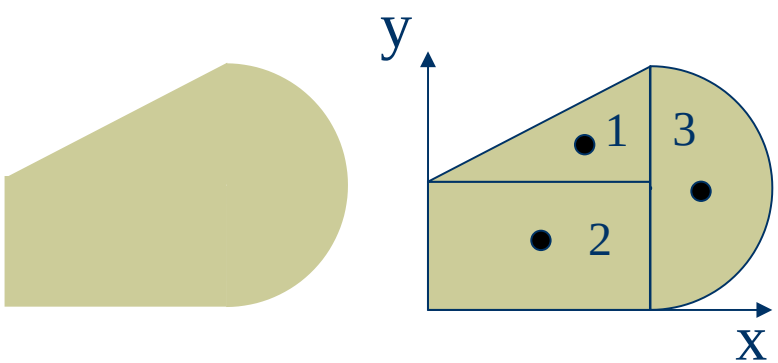
$$\tilde{x} = \frac{4r}{3\pi} \qquad \tilde{y} = \frac{4r}{3\pi}$$

$$A = \frac{1}{4}\pi \cdot r^2$$

Centroids of Composite Areas

- Centroids of more complex shapes can be found by the following procedure
  - Divide into numbered, simple areas
  - Find centroid of each area
  - Sum moment areas relative to origin

$$\overline{X} = \frac{\sum \tilde{x} A}{\sum A} \qquad \overline{Y} = \frac{\sum \tilde{y} A}{\sum A}$$



Shape	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1					
2					
3					
▯					

Example Problem
Centroid of Composite Area

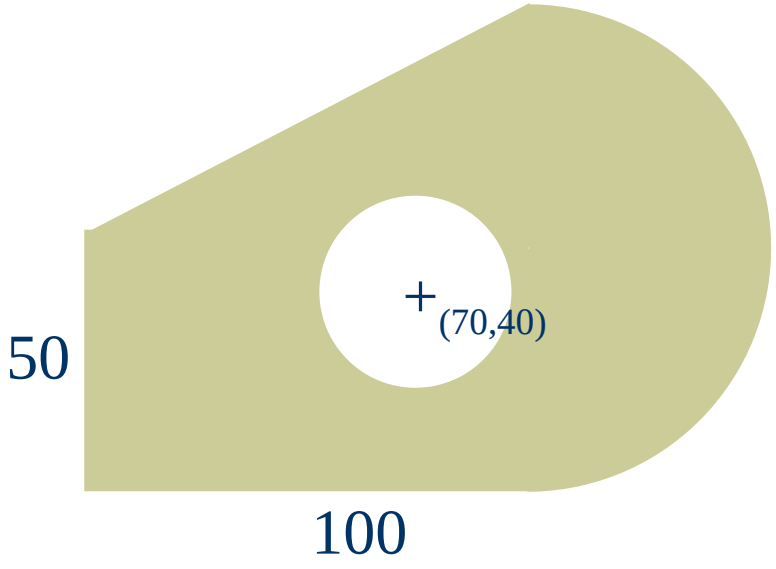
Given:

$D_{hole} = 50$ mm

$R_{arc} = 50$ mm

Find:

Centroid of composite



Example Problem Solution

Given:

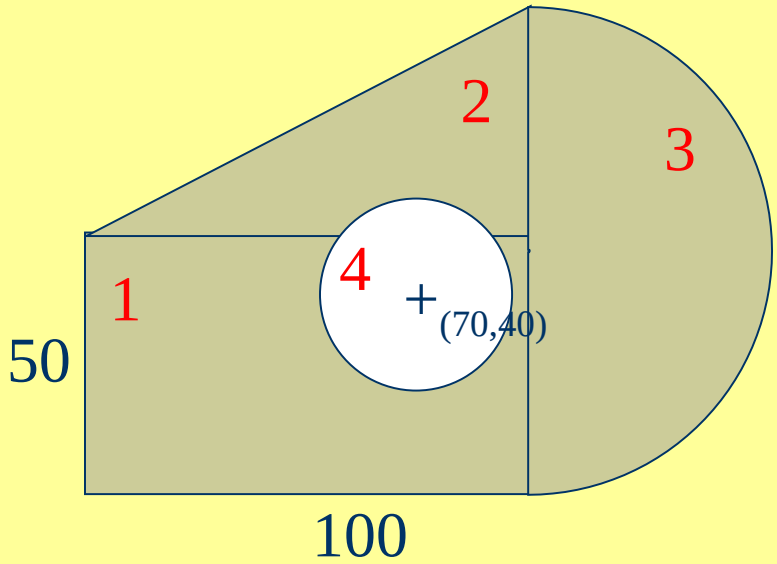
D_{hole} = 50 mm

R_{arc} = 50 mm

Find:

Centroid of composite

Solution:



$$A_1 = 50*100 = 5000$$
$$\tilde{x}_1 = 100/2 = 50$$
$$\tilde{y}_1 = 50/2 = 25$$

$$A_2 = 50*100/2 = 2500$$
$$\tilde{x}_2 = 100*2/3 = 66.7$$
$$\tilde{y}_2 = 50 + 50/3 = 66.7$$

$$A_3 = pi*50^2/2 = 3927$$
$$\tilde{x}_3 = 100 + 4*50/(3*pi) = 121.2$$
$$\tilde{y}_3 = 50$$

$$A_4 = -pi*50^2/4 = -1963$$
$$\tilde{x}_4 = 70$$
$$\tilde{y}_4 = 40$$

Example Problem Solution

shape	A	$\tilde{x}$	$\tilde{y}$	$\tilde{x}A$	$\tilde{y}A$
1	5000	50.0	25.0	250000	125000
2	2500	66.7	66.7	166667	166667
3	3927	121.2	50.0	476032	196350
4	-1963	70.0	40.0	-137445	-78540
$\sum$	**9463**			**755254**	**409476**

$$\overline{X} = \frac{\sum \tilde{x}A}{\sum A} = \frac{755254}{9463} = 79.8\,mm \qquad \overline{Y} = \frac{\sum \tilde{y}A}{\sum A} = \frac{409476}{9463} = 43.3\,mm$$

Center of Gravity & Mass

- Centroid (Center of Area) $\overline{X} = \frac{\sum \tilde{x} A}{\sum A}$ $\overline{Y} = \frac{\sum \tilde{y} A}{\sum A}$
- Center of Gravity $\overline{X} = \frac{\sum \tilde{x} W}{\sum W}$ $\overline{Y} = \frac{\sum \tilde{y} W}{\sum W}$ $\overline{Z} = \frac{\sum \tilde{z} W}{\sum W}$
- Center of Mass $\overline{X} = \frac{\sum \tilde{x} m}{\sum m}$ $\overline{Y} = \frac{\sum \tilde{y} m}{\sum m}$ $\overline{Z} = \frac{\sum \tilde{z} m}{\sum m}$